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Confinement of Laser-Produced Plasma in Resonant Cavities by RF Electromagnetic Fields

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ABSTRACT

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This is a final report on one phase of a continuing problem.

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CONFINEMENT OF LASER-PRODUCED PLASMA IN RESONANT CAVITIES BY RF ELECTROMAGNETIC FIELDS

INTRODUCTION

In the late 1950's, several papers were published regarding the use of rf electromagnetic fields to confine thermonuclear plasma. These papers have been summarized by Glasstone (1). As a result of these publications it has been generally agreed that such an approach to fusion power is "unpromising." The reasons for rejecting this approach were:

1. ohmic energy losses in the conducting cavity walls, surrounding the plasma, were huge compared to all other energies involved, including possible thermonuclear yield, and

2. the electric fields required were impractically large (exceeding 10^6 V/cm).

It is important to note that the lack of promise ascribed to rf confinement schemes had nothing to do with plasma stability considerations. Indeed, even today, neither theory nor experiment casts appreciable light on the plasma confinement properties of such systems (2).

The above two reasons for rejecting rf confinement schemes as contenders for thermonuclear reactors are no longer valid. The electric fields required are almost within the reach of current rf technology (Ref. 2, p. 236). Furthermore, there seems to be no fundamental reason which would prevent the achievement of such fields. The calculations which follow will show that the advent of high-Q superconducting cavities (3,4) has reduced ohmic wall losses to the point where net fusion energy release is, in principle, achievable.

Given the situation just stated, one must say that while rf concepts are definitely still in the thermonuclear race, the stability of plasmas in such systems is not guaranteed, nor is it assured that the formidable technological problems associated with such systems can be overcome. Magnetic confinement may still be preferable. However, the possible advantages of rf systems must now be taken seriously and, most importantly, *they must be tested experimentally*.

The possible advantages of rf confinement, vis-a-vis steady-state or pulsed magnetic confinement, are two:

1. by a suitable combination of rf fields (5), it may be possible to create a spherically-symmetric pressure well in the "minimum-B" sense, and

2. the rf system possesses inherently a "dynamic stabilization" mechanism (6) which may reduce or prevent microinstabilities.

These potential advantages have not been adequately tested either by experiment or by theory (see Ref. 2, p. 241). This may be because the theory has been motivated by experiments, but the experiments themselves have been unable to sufficiently separate plasma production and heating processes from the confinement process and have not used dense, high-temperature plasma.

The advent of laser methods (7) for producing, in situ, dense high-temperature plasma may provide the technique whereby plasma production and heating can be cleanly separated from confinement aspects of the study. This possibility is strengthened by observations (8) in which the dynamically expanding laser-produced plasma was arrested in its high-β state by magnetic pressure.

In the subsequent sections of this report, representative calculations will be presented which quantify the preceding remarks.

FREQUENCY AND PRESSURE

The use of rf electromagnetic fields to confine plasma rests on two basic principles:

1. radiation impinging on a plasma is largely reflected if the rf radiation frequency (ω_0) is less than the electron plasma frequency (ω_{pe}), and

2. reflected radiation exerts a pressure $\left\langle \frac{E^2 + B^2}{8\pi} \right\rangle$ on the reflecting surface.

Frequency Condition

The electron plasma frequency is given by

$$\omega_p = \left(\frac{4\pi n e^2}{m_e}\right)^{1/2} = 5.6 \times 10^4 \; n^{1/2}$$

or

$$f_p = \frac{\omega_p}{2\pi} = 9 \times 10^3 \; n^{1/2} (\mathrm{sec}^{-1})$$

where n is the electron density in cm^{-3}. Representative values for plasmas of thermonuclear interest are

n (cm^{-3})	f_p (sec^{-1})
10^{13}	2.7×10^{10}
10^{14}	9.0×10^{10}
10^{15}	2.7×10^{11}

The fact that the plasma frequency f_p is so high means that there is a wide choice of confining frequencies for which the condition $\omega_0 < \omega_p$ is satisfied.

The actual choice of ω_0 will be suggested by considerations such as (a) the availability of high-power sources, (b) the dimensions of the cavity, (c) the dimensions of the plasma, (d) the choice of resonance modes, (e) any relevant theory, and (f) experimental data on plasma stability.

If the rf is also to be used for dynamic stabilization, it might be useful if $\omega_0 \gtrsim \omega_{pi}$ where ω_{pi} is the ion plasma frequency. For the choice of plasma parameters to be used in the next section, $\omega_{pi} \approx 2\times10^{10}$. We will therefore pick $\lambda_0 = 10$ cm, i.e., $\omega_0 \approx 2\times10^{10}$ for our representative calculation.

Pressure Balance

A confined plasma exerts an outward pressure of nkT. This must be balanced by the radiation pressure

$$\left\langle \frac{E^2 + B^2}{8\pi} \right\rangle$$

The ratio of these two pressures, defined as β, is

$$\beta = \frac{nkT}{\left\langle \frac{E^2 + B^2}{8\pi} \right\rangle} .$$

In what follows, we assume that $\beta = 1$. This is a reasonable assumption for dense plasmas because the rf fields are confined to a skin depth near the surface if $\omega_0 < \omega_{pe}$.

An important technological question concerns the maximum electric field which can be sustained in a cavity of reasonable dimensions. An estimate of what is required is obtained if we set

$$\left\langle \frac{E^2 + B^2}{8\pi} \right\rangle = E^2_{max}/8\pi .$$

Then

$$E^2_{max} = 8\pi\, nkT .$$

In the next section, we will select a thermonuclear plasma with $kT = 15$ keV and $n = n_D + n_T + n_e = 5.2\times10^{14}$ cm^{-3}. This results in $nkT = 1.2$ joules/cm^3 and $E_{max} = 5.3\times10^6$ V/cm.

While such fields are about an order of magnitude beyond the state of the art, they have been characterized as being "almost within the reach of current rf technology" (Ref. 2, p. 236). Furthermore, there seems to be no fundamental reason which would prevent the achievement of such fields, provided that the fields are tangential to the plasma and cavity surfaces, i.e., in TE modes. A separate experimental program could be undertaken to demonstrate the achievement of electric fields in excess of 10^6 V/cm in cavities without plasma.

ENERGY BALANCE

Considerations related to obtaining net energy from rf-confined thermonuclear plasma can be broken into two parts: (a) those general features which are held in common with magnetic confinement schemes, and (b) those features which are unique.

General Features

A plasma may be said to be thermonuclear if its temperature exceeds about 5 keV. In addition, its density must be such that a useful reaction rate is obtained, e.g., above about 10^{13} cm^{-3}.

The fusion energy released by such a plasma depends linearly on the length of time an average ion is confined. This energy must be sufficient to overcome certain losses. These losses are, to a large extent, independent of the method of confinement, and they have been lumped together in an energy balance criterion called the "Lawson criterion" (9).

The Lawson criterion states that $n\tau > 10^{14}$ cm^{-3}-sec^{-1} for a 1:1 deuterium-tritium (D-T) plasma, where n is the plasma density and τ is the confinement time. The energy losses which are included in the Lawson criterion are (a) the kinetic energy content of the plasma, (b) bremsstrahlung, and (c) the efficiency of converting to electricity, assumed to be 30%.

The problem of meeting this criterion is one of achieving an adequate confinement time τ. Whether rf-confined plasmas will exhibit better or worse τ's than their magnetic counterparts is not obvious. It is plausible, on qualitative theoretical grounds, that the plasma may become unstable due to overheating in the sheath. It is this author's opinion, however, that such a conclusion should rest on experimental evidence. Such evidence is not presently available.

To be of practical interest, a thermonuclear plasma should generate fusion energy at rates between 10–1000 W/cm^3. For spherical plasmas the following representative reactors would result:

Power Density (W/cm^3)	Plasma Volume (cm^3)	Plasma Radius (cm)	Total Power (MW)
20	5×10^6	100	100
100	10^6	65	100
100	10^7	130	1000
1000	10^6	65	1000

The fusion power density in a thermonuclear D-T plasma can be written as

$$P_T = R_{DT} U_{DT} = 3.6 \times 10^{-12} R_{DT}$$

$$= 3.6 \times 10^{-12} n_D n_T (\overline{\sigma v}) , \quad \text{watts/cm}^3 ,$$

where R_{DT} is the reaction rate, U_{DT} is the energy per fusion reaction, and $(\overline{\sigma v})$ is the reaction rate parameter in cm^3-sec^{-1}.

We now arbitrarily select for our thermonuclear plasma a power density of 20 W/cm^3 and a temperature of 15 keV. From Ref. 1, p. 19, we find that $\overline{\sigma v} = 3\times10^{-16}$, and hence we compute the required density as $n_D = 1.3\times10^{14}$ cm^{-3}.

Clearly there are many possible variations in the choice of the above parameters. Our goal has been to select one interesting combination for the energy balance computation which follows.

Unique Features

Since any confined plasma must at least meet the Lawson criterion, we must look elsewhere for the critical criteria for assessing the energy balance problem for rf

systems. The new feature which enters the energy balance picture is the ohmic rf energy loss in the conducting cavity wall. This loss is so large that it completely dominates for ordinary conductors (6). It was primarily because of this that rf schemes were virtually abandoned (1). The results of an energy balance calculated in 1958 show how serious the situation was (6):

Frequency (MHz)	80	Plasma Energy Content (MJ)	13
Cavity Radius (m)	3	Power Loss to Walls (MW)	164,000
Plasma Radius (m)	1	Thermonuclear Power (MW)	324

The rf power dissipated in the cavity walls can be written as (10)

$$\frac{\partial W}{\partial t} = \frac{\omega_0 W}{Q}$$

where ω_0 is the angular frequency of the radiation (= 2×10^{10} sec^{-1} for our case), W is the rf energy stored in the cavity, in joules, and Q is the ratio of energy stored in the cavity to energy dissipated in the walls, per cycle.

The rf energy W required in the cavity can be evaluated by setting

$$\frac{W}{V_c} = nkT$$

where V_c is the volume of the cavity. This is equivalent to the assumption that $\beta = 1$. From the table in the previous subsection we see that for the 100-MW total power case at a power density of 20 W/cm^3, the plasma volume is $V_p = 5\times10^6$ cm^3, giving a plasma radius of 100 cm. We now, arbitrarily, choose $R_c = 130$ cm, i.e., $V_c = 10^7$ cm^3. Noting that $nkT = 1.2$ J/cm^3, we find that $W = 12$ MJ.

Our expression for the rate of energy dissipation in the cavity walls then reduces to

$$\frac{\partial W}{\partial t} = \frac{2.3\times10^{17}}{Q} \text{ watts} .$$

This power loss must be offset by the thermonuclear power $P_T V_p > \partial W/\partial t$. For our chosen case $P_T V_p = 10^8$ W. Thus the criterion for net energy release is

$$\frac{P_T V_p}{\frac{\partial W}{\partial t}} = \frac{Q}{2.3\times10^9}$$

which implies that $Q > 2.3\times10^9$ for net energy release.

Ordinary room temperature cavities have Q values typically in the range 10^4–10^5. A simple superconducting cavity (10) has a Q value of around 10^6. Since about 1962, however, dramatic improvements have been achieved in the Q's of superconducting cavities (3,4). Most recently, Q's exceeding 10^{10} have been achieved in both lead and niobium cavities by using superfluid helium and operating at temperatures below 2°K.

Two different sets of experiments were performed. In the first, $Q \approx 5\times10^9$ was achieved at $T \approx 1.8^\circ$K in cylindrical lead cavities (7.5 cm diam, 7.5 cm length) using frequencies $\omega_0 \approx 1.8\times10^{10}$ sec^{-1}. In the second set, $Q > 10^{10}$ was achieved at a slightly lower temperature in cylindrical niobium cavities (5 cm diam, 5 cm length) using frequencies $\omega_0 \approx 7\times10^{10}$. These achievements were made with electric fields up to 2.5×10^5 V/cm.

It should perhaps be noted that according to the theory of conventional superconductors (10) the wall losses should increase as ω^3. Thus, if other factors had been equal, one might have expected a *decrease* in Q by a factor of about 60 in the second set of experiments. In fact an *increase* in Q by a factor of about 10 was found. Insufficient data exists to draw any firm conclusion from this observation. Very likely the decrease due to increased ω was offset by the slightly lower temperature and different material. It seems likely that the dependence of Q on ω is much smaller than ω^3 for this new class of superfluid-helium-cooled superconductors.

It is pertinent to inquire as to the scaling of Q with cavity size since reactor-sized cavities are about 40 or more times larger than the cavities used in these experiments. Now

$$Q = \frac{\omega_0 W}{\partial W/\partial t}.$$

From strictly geometrical considerations, $\omega_0 \sim 1/\lambda_0 \sim 1/R$, for a given mode. $W \sim V_c \sim R^3$, and $\partial W/\partial t \sim 4\pi R^2 \sim R^2$. For a given mode, therefore, Q is independent of size, from geometrical considerations. For a fixed frequency, however, Q increases directly with cavity radius.

From physics considerations, $\partial W/\partial t$ depends on ω. If $\partial W/\partial t \sim \omega^3 \sim 1/R^3$, then $Q \sim R^3$. As noted, it seems unlikely that $\partial W/\partial t \sim \omega^3$ for this class of superconductors. It does seem likely, however, that $\partial W/\partial t$ is an *increasing* function of ω, and consequently that Q is an *increasing* function of R. Therefore the achievement of $Q > 10^{10}$ for 2.5-cm (radius) cavities corresponds to the achievement of considerably higher Q for reactor-sized cavities. For example, for a fixed frequency Q (130 cm) $\approx 6\times10^{11}$.

Example Calculation

The ratio $P_T V_p/\partial W/\partial t$ is plotted in Fig. 1 as a function of Q. The energy balance level is shown by the dotted line. We note that the advent of high Q superconducting cavities permits the achievement, in principle, of a net-energy-producing thermonuclear plasma based on rf confinement.

The actual numerical values in Fig. 1 are based on the following selection of parameters:

Plasma Temperature kT	15 keV
Plasma Density $n = n_D + n_T + n_e$	5.2×10^{14} cm^{-3}
Plasma Energy Density nkT	1.2 J/cm^3
Plasma Volume V_p	5×10^6 cm^3
Electric Field $\mathcal{E}$	5.3×10^6 V/cm
Plasma Confinement Time τ	$n\tau > 10^{14}$ sec-cm^{-3}

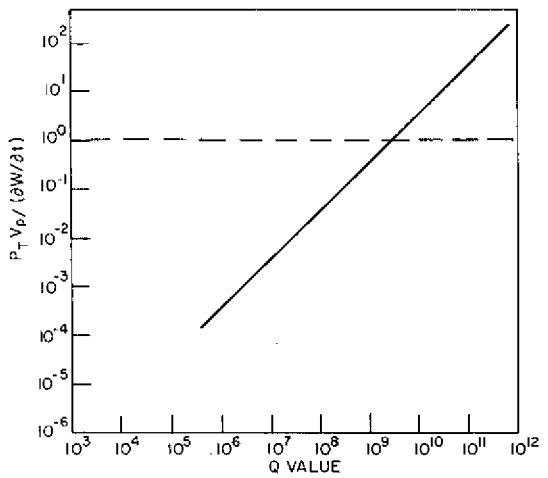


Fig. 1 - Plot of superconducting-cavity Q value vs the criterion for a net energy release, $P_T V_p / (\partial W/\partial t)$, in rf-confined plasmas

Rf Frequency $\omega_0 = 2\pi f_0$	2×10^{10} sec^{-1}
Electron Plasma Frequency ω_{pe}	9×10^{11}
Ion Plasma Frequency ω_{pi}	1.8×10^{10}
Cavity Volume V_c	10^7 cm^3
Cavity Radius R_c	130 cm
Cavity Stored Energy $W = nkTV_c$	12 MJ
Plasma Stored Energy $nkTV_p$	6 MJ
Fusion Power Density P_T	20 W/cm^3
Fusion Power $P_T V_p$	100 MW.

Clearly there are many other possible combinations of parameters which would give a less favorable result. But no serious attempt has been made to optimize the problem.

DISCUSSION

The foregoing analysis indicates that a net-energy rf-confined plasma is possible *in principle*. This conclusion does not take into account a number of important practical aspects of the situation.

Electric Field and Plasma Beta

The electric field requirement is given by

$$E_{max} = \sqrt{\frac{3.6\times10^{-6}\, nkT}{\beta}}\ , \quad V/\text{cm}\ .$$

The E_{max} computed for $n = 5.2\times10^{14}$, $kT = 15$ keV, and $\beta = 1$ was $E_{max} = 5.3\times10^6$ V/cm. Going to higher density, higher temperature, or lower β increases this requirement on E_{max}. While such an E_{max} might be attainable soon under very carefully controlled

conditions, it will be quite another matter to achieve it in the presence of plasma, in large systems, with large stored energy. The problem is eased considerably if mode selection is limited to cases in which the electric field is tangential to the wall surface.

Rf Power Requirements

The buildup of rf fields in a cavity follows (10) $\exp(\omega t/Q)$. Consequently, the stored rf energy W is built up in the cavity in a time of the order $t = Q/\omega$. For our case (W = 12 MJ and $Q \approx 6\times10^{11}$) the rf power required is thus of the order of 400 kW. In the wavelength range $\lambda_0 \approx 10$ cm, continuous-wave rf power is readily available at such powers. Thus, provision of the required rf power to an *empty* reactor-sized cavity does not strain present-day rf technology.

On the other hand, the rf fields will also dissipate energy in the plasma sheath, and this dissipation represents an additional demand on the rf supply. It is doubtful whether ohmic heating of the plasma sheath can be computed reliably, even roughly. It is customary (2,6) to compute the rf dissipation in the plasma by using a model which is only valid for weak fields which do not appreciably alter the presumed Maxwellian temperature distribution of the sheath. Motz and Watson (Ref. 2, p. 237) note the absence of an applicable high-field theory and conclude that the weak-field computation "probably enormously exaggerates the heating effect, since (as the theory of electron runaway shows) the viscous drag of a plasma on an electron moving in a strong electric field, such that

$$\omega_{pe}^2/\omega^2 >> 1 ,$$

is very much less than the drag on a thermal electron. Thus it is probable that [the power dissipated in the plasma] has a negligible effect on the power balance, though it remains a serious burden on the rf generator."

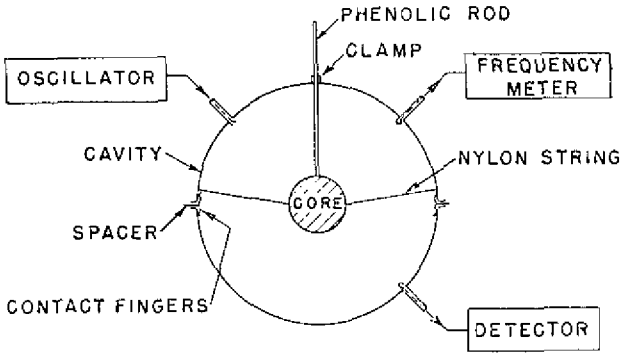
It can also be argued, however, that excessive electron heating in the sheath may lead to turbulence, resulting in an anomalously high collision frequency and, consequently, increased dissipation in the sheath.

It should be noted at this point that the actual effect of the electric field at the plasma surface depends on the selection of cavity modes. In the magnetic quadrupole (TE_{210}) or magnetic sextupole (TE_{310}) modes, for example, field nodes of the vacuum field appear on axis. Furthermore, the magnetic field pressure increases more rapidly than the electric field pressure off axis. The plasma boundary in such a case should be determined by the magnetic field and be in a region of low electric field. Such configurations have been proposed by Hatch (11) and investigated (in an analog experiment using a copper ball) by Hatch and Butler (12) (see Fig. 2).

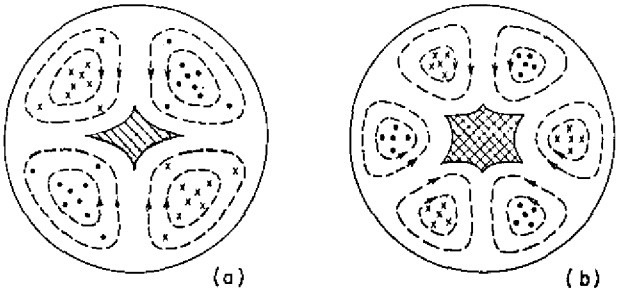
The resolution of this question of plasma sheath heating and its effect on the energy balance and rf power requirements represents the most important area for additional research since its outcome may well be decisive in determining the feasibility of rf reactors in practice.

Maintenance of a Superconductor Near a Thermonuclear Plasma

The maintenance of a superconducting surface under conditions of heat loading requires a refrigeration effort which was not completely taken into account in the foregoing analysis. The removal of $\partial W/\partial t$ from the superconducting wall requires a refrigeration power P_R given by



Cross-sectional diagram of resonant cavity. The spacer ring makes the cavity slightly nonspherical and orients the resonant field configurations.



(TE_{210}) (TE_{310})

Approximate stable core configurations for magnetic multipole modes, sketched in accordance with the principle of minimum perturbation.

Fig. 2 - Schematics of magnetic multipole (cavity) modes in an analog experiment conducted by Hatch and Butler. This figure is taken directly from their work (Ref. 12).

$$P_R = \frac{1}{\eta_R} \frac{\partial W}{\partial t} .$$

At best,

$$\eta_R = \frac{T_2}{T_1 - T_2}$$

where

$$T_2 \approx 2^\circ\text{K} \quad \text{and} \quad T_1 \approx 300^\circ\text{K} .$$

So,

$$\eta_R \approx \frac{1}{150} .$$

If we define energy balance by $P_T V_p > P_R$, the requirement on Q is $Q \approx 3\times10^{11}$ for break-even. If we desire $P_T V_p \approx 10\ P_R$ in a practical situation, then $Q \approx 3\times10^{12}$. Such Q's do

not seem unachievable, since the technology is in a very primitive state now and since additional improvements can be expected with larger size. Further, if high-Q cavities can be developed operating at higher temperatures, the situation is dramatically improved.

In addition to heat loading from the rf fields, the superconducting cavity surface is also subjected to *bremsstrahlung* from the plasma. The bremsstrahlung flux is given approximately by

$$P_B = 2.14 \times 10^{-30} n_D^2 T_e^{1/2} \quad \mathrm{W/cm^3}$$

with n_D in cm^{-3} and T_e in keV. For our representative calculation, $n_D = 1.3 \times 10^{14}$, $T_e = 15$ keV, $V_p = 5 \times 10^6\ \mathrm{cm}^3$, and $P_B = 700$ kW.

If all this energy is deposited at 2°K and removed with an efficiency $\eta_R \approx 1/150$, the resulting refrigeration power required equals the thermonuclear power generated. To achieve net energy release, only a fraction f of the bremsstrahlung radiation can be allowed to be dissipated at 2°K. If, as a practical requirement, we specify that the thermonuclear power exceed the refrigeration power by a factor of 5, then

$$P_T V_p \approx 5\ (f P_B / \eta_R) \ .$$

Then for our case where $P_T V_p = 10^8$ and $P_B = 7 \times 10^5$, this reduces to the requirement $f \approx 0.20$.

If the temperature is increased from 15 to 20 keV, the ratio $P_T V_p / P_B$ increases by a factor 1.6. The requirement that the thermonuclear power be five times the refrigeration power then relaxes to

$$f \approx 0.32 \ .$$

This requirement that only about 32% of the incident bremsstrahlung power be deposited at 2°K is a difficult one. It is not, a priori, impossible to achieve however. Considerably more thought and effort should be devoted to possible engineering solutions to this problem.

An alternative to seeking an engineering solution to achieving $f = 32\%$ would be to seek the development of low-loss superconductors operating above 2°K. Thus, if we set $f = 1$ and $P_T V_p = 5\ (P_B / \eta_R)$, then

$$\eta_R = \frac{5 P_B}{P_T V_p} = 3.5 \times 10^{-2} \ .$$

So

$$\frac{T_2}{T_1 - T_2} \approx \frac{T_2}{T_1} = \frac{T_2}{300} = 3.5 \times 10^{-2}$$

from which

$$T_2 = 10°\mathrm{K} \ .$$

Clearly, some combination of developments could be sought which would lower f and increase T_2.

Heat will also be deposited by neutrons. However, this should present a less difficult engineering problem than that involved for bremsstrahlung. This is because of the longer mean free path for energy deposition in the case of neutrons.

AN EXPERIMENT USING LASER-PRODUCED PLASMAS

Main Requirements

Recognizing that there are serious technological questions facing any rf-confined thermonuclear reactor scheme, we can still ask whether or not a critical scale-model experiment might be performed to test the sheath physics and the confinement properties of plasmas in rf fields. The main requirements for such an experiment are the following:

1. The vacuum field configuration should possess a potential well shape in the "minimum-B" sense; i.e., from a pressure point of view, a hypothetical plasma should experience a restoring force whose magnitude increases in all directions from its location.

2. A $\beta = 1$ condition should obtain somewhere within the cavity.

3. The plasma should be reasonably dense (at least 10^{12}) and reasonably hot (at least 100 eV), and its velocity distribution should be reasonably thermalized.

4. The actual plasma should be produced in such a way that initially it occupies a well-defined location in space, preferably at the minimum of the potential well.

Since energy balance is not an important criterion in the experiment, it is clear that superconducting cavities need not be employed. We will now consider, briefly, the above four criteria in reverse order.

Plasma Production (Requirements 3 and 4)

In many plasma physics experiments, the plasma, in its production and heating stage, interacts so strongly with the confining field that the desired conclusions about the confinement properties of the field itself cannot be obtained. This problem is somewhat alleviated, though not eliminated, by using high-power lasers to produce the plasma. This method has the advantages that the plasma is produced "instantaneously" in a high-temperature condition, at high density, and is approximately spherical in shape and occupies a well-defined and controllable position in space. It has a possible disadvantage that the energy of the plasma is very rapidly converted into kinetic energy of radial expansion. This means that the confining field must be relied upon to stop the radial expansion so that the velocity distribution may become thermalized. Only then do we have the desired experimental situation.

Fortunately there is evidence, obtained with magnetic fields, both for being able to stop (8) the radial expansion at approximately the $\beta = 1$ radius, and also for subsequent thermalization of the particles (13). Figure 3, taken from Ref. 8, shows the plasma radius as a function of time.

Plasma Beta (Requirement 2)

It is essential that a $\beta = 1$ pressure balance be obtained within the experimental cavity. To see whether this is presently feasible, the following considerations are relevant. For $\beta = 1$,

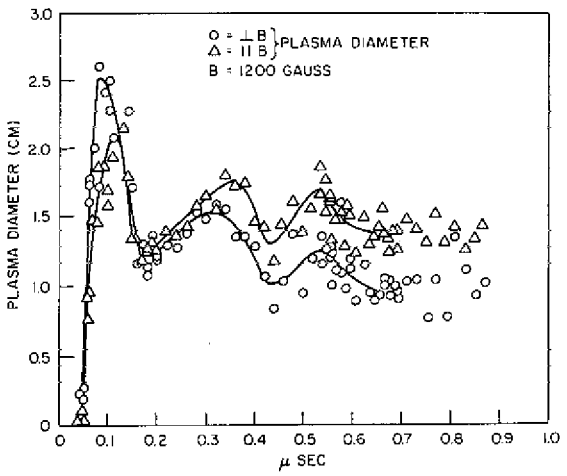


Fig. 3 - Radial and axial plasma diameter, with magnetic mirror field, as a function of time

$$E^2_{max} = 3.6 \times 10^{-6}\, nkT$$

where E is in V/cm, n is in cm^{-3}, and kT is in eV. It is well within the state of the art to make a laser-produced plasma with 10^{15} total ions and $kT = 100$ eV. It is also within state of the art to make $E_{max} = 10^5$ V/cm. Using these values, we can compute the density for the $\beta = 1$ case as

$$n = 2.7 \times 10^{13}\ cm^{-3}$$

and

$$n_i = \frac{n}{2} = 1.4 \times 10^{13}\ cm^{-3}$$

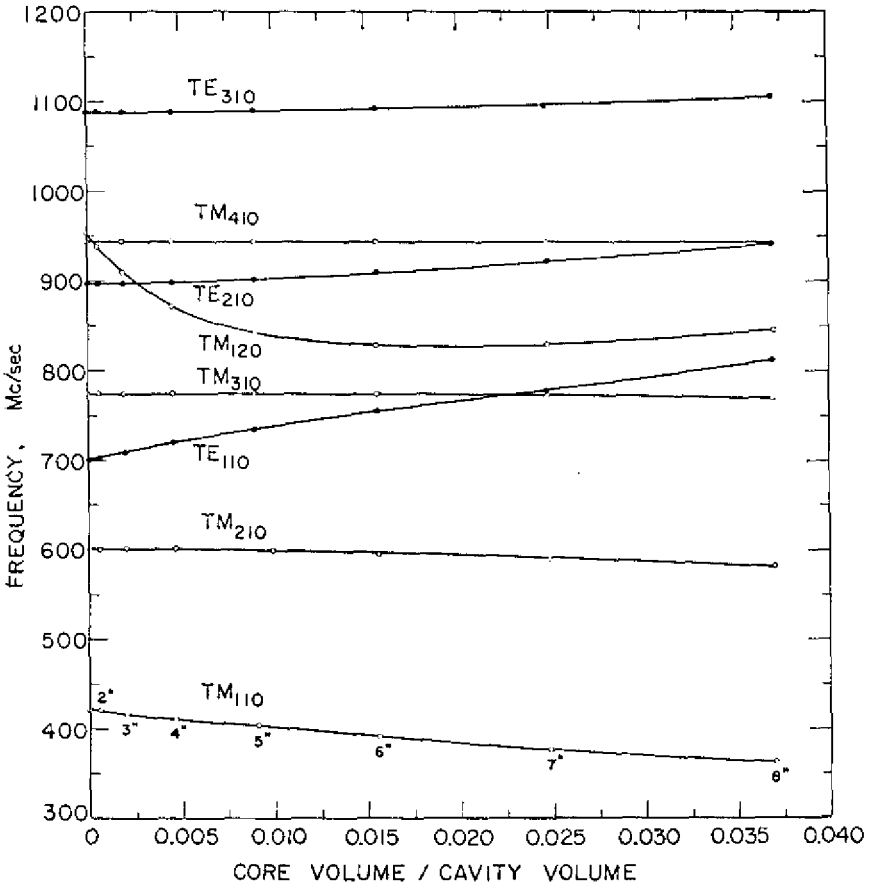
The radius of this plasma can be estimated simply by assuming a spherical shape and uniform density distribution. Thus,

$$n_i = \frac{N_i}{V_p} = \frac{N_i}{\frac{4}{3}\pi R_p^3}$$

where $V_p = 37\ cm^3$ and $R_p = 3$ cm. This is a conveniently small radius. If we then choose a cavity radius of 15 cm, the volume of the cavity is 14,000 cm^3. Setting $W/V_c = 1.6 \times 10^{-19}\, nkT$, we obtain $W = 6$ J.

The cw power required is $\partial W/\partial t = \omega_0 W/Q$. The ion plasma frequency is $f_{pi} = 7 \times 10^8$; the electron plasma frequency is $f_{pe} = 3.3 \times 10^{10}$. From Hatch and Butler (12) we note (Fig. 4) that the cavity has a magnetic quadrupole (TE_{210}) resonance frequency at 1.8×10^9 and a magnetic sextupole resonance frequency at 2.2×10^9. Choosing a normal-conducting cavity with $Q = 10^5$ results in

$$\frac{\partial W}{\partial t} \approx 750\ \text{kW}\ .$$



Size equilibrium curves. Open circles are for TM modes, closed circles are for TE modes. Core diameters indicated on TM_{110} curve apply to corresponding data points for other modes.

Fig. 4 - This plot is taken from the work of Hatch and Butler (Ref. 12). Frequencies given should be doubled for our calculation since our cavity size is half that used by those authors.

In practice, one would require several times this. This is available from commercial sources, in either pulsed or continuous-wave form.

Field Configuration (Requirement 1)

There are a variety of field configurations which deserve study. Hatch (11) has pointed out the advantages of magnetic multipole modes (see Fig. 2). These modes possess potential well shapes, although they do not result in spherically symmetric pressure distributions. However, it may well be possible ultimately to create a desired pressure distribution by a superposition of modes. Knox (5), for example, has computed a spherically symmetric pressure distribution by the superposition of three magnetic dipole modes.

It is also of interest to explore the behavior of plasma confined by higher order modes since, ultimately, in large reactor-sized cavities, one might be forced to consider higher order mode operation. Stability in such a mode might be provided by field compression, which is also discussed by Knox.

A comprehensive experimental program would be required to explore these questions.

CONCLUSIONS

The foregoing analysis and discussion lead to the following conclusions:

1. The old bases for rejecting the possibility of the rf confinement of thermonuclear plasma are no longer valid. The very high electric fields required are today within reach. The achievement of superconducting cavities with Q values exceeding 10^{10} permits the achievement, in principle, of a net-energy-producing rf-confined plasma.

2. Klystrons are currently available which meet the requirements for reactor-sized cavities.

3. Insufficient information currently exists to realistically assess the effects due to interaction of the rf field with the plasma sheath. These effects include energy dissipation and plasma stability. This is the most important area for further study.

4. The thermonuclear power exceeds the refrigeration power required to maintain the wall at 2°K by a factor of 5 if only 32% of the incident bremsstrahlung flux is permitted to be deposited at 2°K.

5. Clarification of the feasibility of a thermonuclear reactor based on rf confinement will require an experimental program. Many of the physics questions may be studied in a laser-produced plasma experiment, such as the one described.

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