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Comparison Study of Unequal Error Protection Methods for One-Dimensional Signal Constellations

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COMPARISON STUDY OF UNEQUAL ERROR PROTECTION METHODS FOR ONE-DIMENSIONAL SIGNAL CONSTELLATIONS

1. INTRODUCTION

There are various methods to achieve unequal error protection for different classes of data. This report describes two approaches compares their performances. The first approach is based on a clever design of signal constellations with nonuniformly spaced signal points scheme and is described by Ramchandran (K. Ramchandran, et al., 1993) for High Definition Television (HDTV) transmission application. The second approach is based on time-sharing concept. In a straightforward time-sharing scheme, a fixed multiplexing rule is chosen, and signals are transmitted according to this predetermined rule. Calderbank and Seshadri (R. Calderbank, 1993), (R. Calderbank, 1990) have described a method where a variable multiplexing rule is used instead yielding a nonzero rate. Additional data can then be transmitted by the particular choice of multiplexing rule. In this paper, a performance comparison of the two methods is completed.

As a further development, the concept of unequal error protection is applied to MPEG (Motion Pictures Experts Group) video transmission. Compressed video data consists of important data and less important data. The importance has two connotations—one, the effect of errors on the visual quality of a single frame and the other, the effect of propagatable errors in a sequence of frames. In the proposed scheme for video transmission described in this report, both types of distortions are minimized.

2. BROADCAST CHANNELS

Consider the broadcast scenario of a single source and two receivers, as in Fig. 1, with Receiver 1 a near or strong receiver and Receiver 2 a far or weak receiver. Suppose each receiver has associated channel capacity of C_1 and C_2 , $C_1 > C_2$. Strategies on how to design for this canonical case have been well studied. Straightforward schemes include choosing to broadcast at the minimum C_2 . Another method is that of time-sharing, broadcasting at C_1 for a time period of length T_1 , and then broadcasting at time C_2 for time T_2 . Each of these techniques considers the transmission to a particular receiver independently.

Cover shows that for a slight decrease in the rate for the weaker channel, a dramatic increase in the rate of transmission can be made for the stronger channel (T. Cover, 1972). Cover demonstrates that high joint rates of transmission are best achieved by superimposing high-rate and low-rate information rather than by using time-sharing.

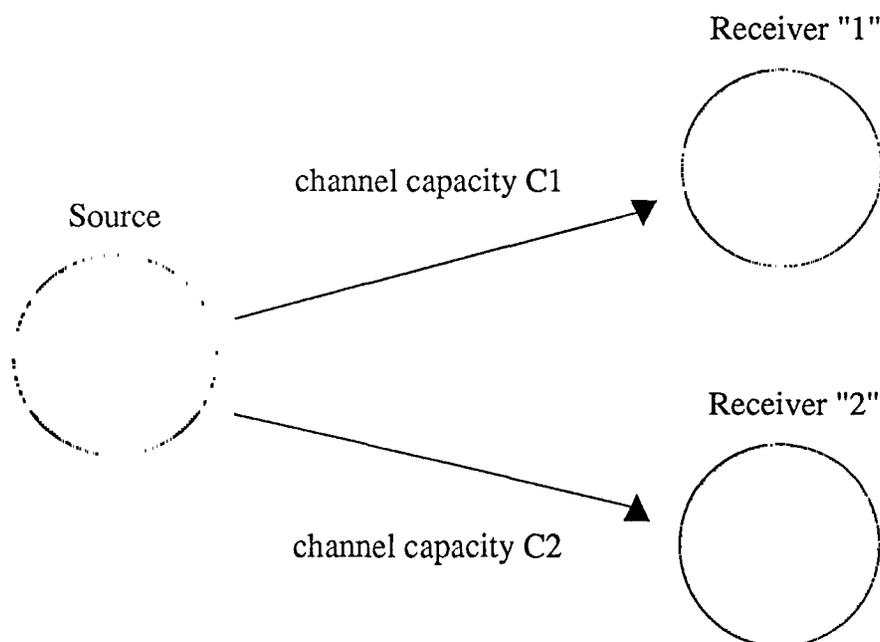


Fig. 1 — Broadcast scenario; one source, two receivers

3. SIGNALING WITH UNEQUAL ERROR PROTECTION

Ramchandran's (K. Ramchandran, et al. 1993) scheme for unequal error protection defines a signal constellation consisting of clouds, which describe the important data; within each cloud is a mini constellation, which defines the less important data. To decode important information, the receiver determines to which cloud a transmitted signal point belongs. To decode the less important data, the receiver determines the specific point within the cloud. A strong receiver is able to decode a particular signal by decoding the cloud and then the point within a cloud. A weak receiver may only be able to decode the cloud. Transmission of high-resolution data is achieved by embedding that information within the less important points. Henceforth this approach for unequal error protection signaling will be referred to as the embedded method.

When implementing Calderbank and Seshadri's (R. Calderbank, 1993), (R. Calderbank, 1990) method for unequal error protection, we first define two concentric disjoint signal constellations—region I and region II. Signals from region I represent less important data, and signals from region II represent important data. Suppose we have a set of N multiplexing rules, $\log_2 N$ bits of information can be conveyed by the particular choice of multiplexing rule. Henceforth this method of unequal error protection signaling will be referred to as the multiplexing method.

We compare the performance of these two methods in terms of the ratio of the minimum squared distance to the average signal power. For this analysis, we restrict the signal constellations to be of one dimension. Also no additional coding of important data or less important data is implemented, such as trellis coded modulation (TCM). Although such techniques as TCM can provide powerful methods of improving transmission reliability, these schemes are not used to provide a fair and simple comparison between embedded and multiplexing methods.

First we consider the embedded scheme and define a one-dimensional signal constellation, as in Fig. 2. This constellation consists of two clouds separated by a distance of $2d_2$, with two signal points per cloud separated by a distance of $2d_1$. This corresponds to one bit of important data per signal, and one bit of less important data per signal. Each signal in this constellation is transmitted with equal probability, and thus the average signal power of this constellation is determined as:

$$P_e = d_2^2 + d_1^2. \tag{1}$$

The minimum squared distance for the important data is:

$$d^2(\text{imp}) = 4(d_2 - d_1)^2. \tag{2}$$

The minimum squared distance for the less important data is:

$$d^2(\text{less}) = 4d_1^2. \tag{3}$$

The figure of merit for the important data is:

$$\xi_{e_{imp}} = \frac{d^2}{P} = \frac{4(\alpha - 1)^2}{\alpha^2 + 1}. \tag{4}$$

1 important bit/signal
1 less important bit/signal

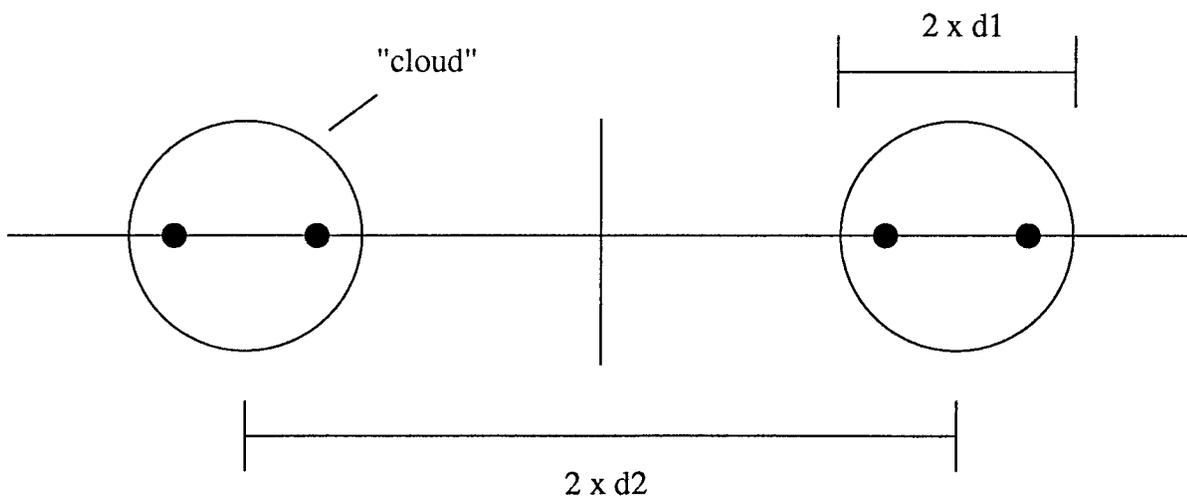


Fig. 2 — Embedded signal constellations restricted to one-dimension

The figure of merit for less important data is:

$$\xi_{less} = \frac{4}{\alpha^2 + 1}, \quad (5)$$

where α is:

$$\alpha \doteq \frac{d_2}{d_1}. \quad (6)$$

Note that in the case of $\alpha = 2$, we simply have a 4-PAM (Pulse Amplitude Modulation) signal constellation.

Next we consider the multiplexing method for unequal error protection signaling. We define a one-dimensional signal constellation as before. Region I, or the inner region, of four signal points defines the less important data. The outer region, or region II, of two signal points defines the important data. Figure 3 describes the encoding scheme.

Two important data bits determine one of four different multiplexing rules. Each multiplexing rule is a block length of four. Two positions out of the block length of four will be from region I, and the other two positions will be signal points from region II. Consider the bits conveyed per signal. Two important data bits were used to determine the multiplexing gain. For each block transmitted, two points were from region II, each of these signal points conveys one bit. Therefore we have a total of four important data bits per four signals transmitted.

Now consider the less important data. For each block, there are two points from region I. Each of these signal points conveys two bits, so there are a total of four bits per block, or four bits per four signals. Thus, for this multiplexing scheme, we have one bit/signal for both important data and less important data. Thus we have the equivalent bits per signal ratio as the embedded scheme described previously. See Fig. 4.

Although the bits-per-signal ratio are equivalent for both schemes, note that this is the average bits per-signal-ratio. In the examples described, the embedded scheme transmits exactly one bit of important data and exactly one bit of less important data for each signal. We describe the embedded method as achieving an instantaneous bit-per-signal rate. On the other hand, for the multiplexing scheme, the bit-per-signal ratio of one is achieved for every block of four signals transmitted. The multiplexing scheme does not attain an instantaneous bits per signal rate. This is a significant difference between the two schemes and will be discussed later.

We determine the average signal power for the multiplexing scheme. The signals within a region are transmitted with equal probability, so the average signal power from region I is:

$$P_I = 5d_1^2. \quad (7)$$

The average signal power from region II is:

$$P_{II} = d_2^2. \quad (8)$$

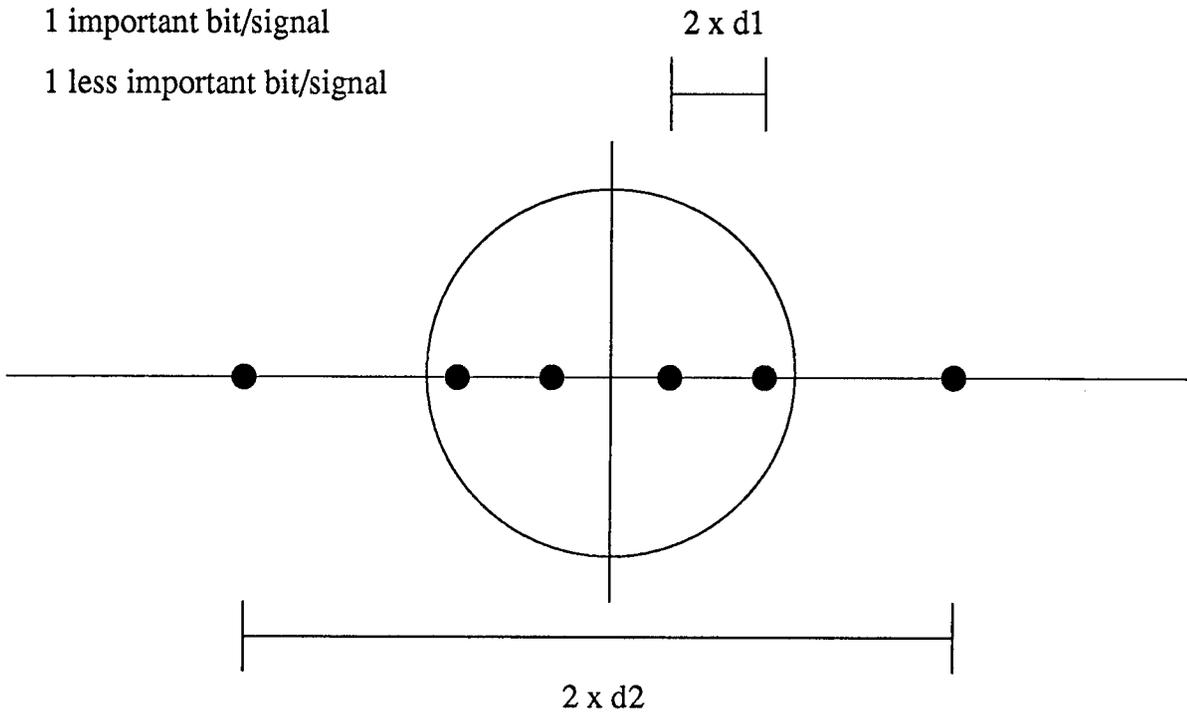


Fig. 3 — Multiplexing signal constellations restricted to one-dimension

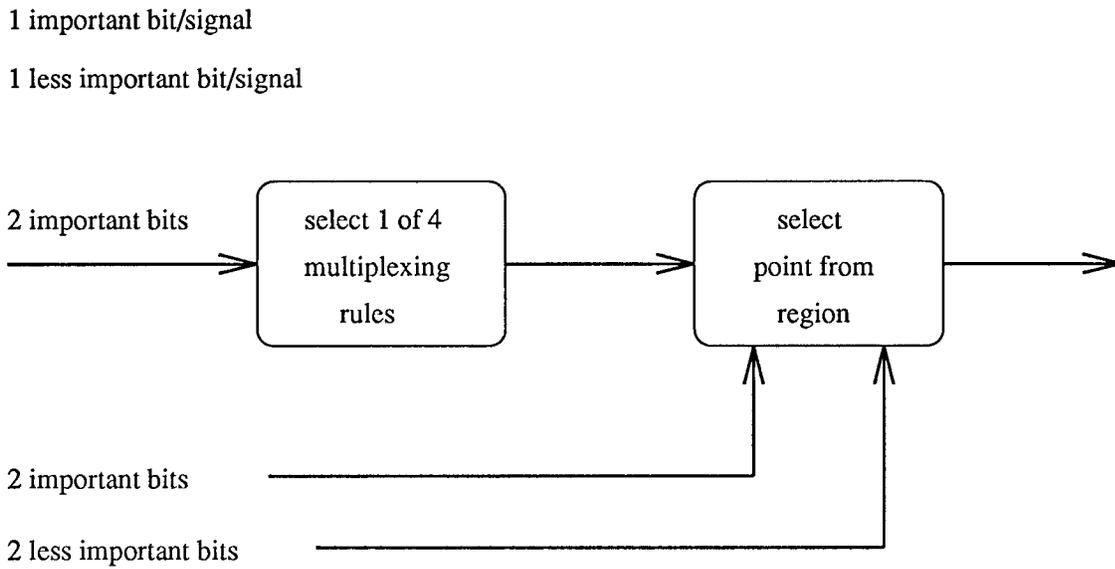


Fig. 4 — Encoded of multiplexing signaling method

In this case, the average frequency of transmission from region I is $1/2$, so the overall average signal power is:

$$P_m = \frac{5}{2}d_1^2 + \frac{1}{2}d_2^2. \quad (9)$$

The minimum squared distance in region I is:

$$d^2(\text{region I}) = 4d_1^2. \quad (10)$$

The minimum squared distance in region II is:

$$d^2(\text{region II}) = 4d_2^2. \quad (11)$$

In the multiplexing scheme for unequal error protection signaling, the ability to distinguish signal points from regions I and II have to be considered. We determine the minimum squared distance between region I and region II as:

$$d^2(\text{region I, region II}) = (d_2 - 3d_1)^2. \quad (12)$$

The minimum squared distance for the important data is the minimum of the distance of the signal points from region II and the distance of the multiplexing rule,

$$d^2(\text{imp}) \geq \min(H(d_2 - 3d_1)^2, 4d_2^2). \quad (13)$$

The minimum squared distance for less important data is:

$$d^2(\text{less}) = 4d_1^2. \quad (14)$$

The figure of merit for important data is:

$$\xi_{imp} = \frac{\min(2H(\alpha - 3)^2, 8\alpha^2)}{5 + \alpha^2}. \quad (15)$$

The figure of merit for less important data is:

$$\xi_{less} = \frac{8}{5 + \alpha^2}. \quad (16)$$

The Hamming distance of the multiplexing rule plays a major role in the performance of this scheme. By increasing the distance of the multiplexing rule, we can improve the distance for the important data. Increased Hamming distance can be accomplished by extending the block length. For example, for a block length of four, we can have set of four codewords with Hamming distance of two. By increasing the block length to eight, we can implement an $[8,4,4]$ Hamming code to achieve a Hamming distance of four. The bits-per-signal ratio is preserved, and there is no power penalty incurred.

For higher Hamming distances, increased block lengths are required. Constructing a set of codewords to satisfy a particular Hamming distance within a given block length can be a difficult process. Fortunately, there exists a table of constant weight codes for block length of $n < 28$ that is detailed by (A. Brouwer, et al. 1990). Now we compare the ratio of the figure of merits of the two schemes—multiplexing and embedded. For each case we have one bit-per-signal, and thus we have a fair comparison. For less important data, we have:

$$\gamma_{less} = 2 \frac{\alpha_e^2 + 1}{\alpha_m^2 + 5}. \quad (17)$$

For important data we have:

$$\gamma_{imp} = \frac{\min(H(\alpha_m - 3)^2, 4\alpha_m^2) \alpha_e^2 + 1}{2(\alpha_e - 1)^2 \alpha_m^2 + 5}. \quad (18)$$

Equations (17) and (18) are plotted in Figs. 5 through 8. Block length is increased to achieve a distance of $H = 8$ for multiplexing rule.

For reasonable values of α_e and α_m , the multiplexing scheme outperforms the embedded scheme for both the important data and less important data.

Figures 5 and 6 show three-dimensional plots. The multiplexing scheme (α_m) and the embedded scheme (α_e), are both plotted against dB gain. This is done for both important data and less important data. Note that Figs. 7 and 8 are diagonal cross-sections of the 3-D plot. For reasonable values of α_m and α_e , the multiplexing scheme outperforms the embedded scheme. The gain in performance is a result of increasing the distance of the multiplexing rule. There is no analogous mechanism for the embedded scheme. The penalty for the performance gain of the multiplexing scheme is that the block length is increased. Instantaneous bits per signal rate can no longer be achieved. Small increases in the block length typically present no problem in terms of system performance.

4. VIDEO TRANSMISSION AND COMPRESSION

With the development of high-definition television and the activity of video compression standards, the issue of reliable video transmission is a current topic of interest. Continued improvements in video compression techniques have established the ability to achieve terrestrial broadcasting of video images. A growing popular standard for video compression is MPEG II. In fact, the HDTV proposal of the Advanced Television Research Consortium (Thomson Consumer Electronics, David Sarnoff Research Center, NBC, and Compression Labs) implements a slight modification of the MPEG compression standard (D. LeGall, 1991), which has been termed MPEG++ (R. Siracusa, 1993). There is currently extensive discussion within the standards groups on the compatibility of HDTV and MPEG II.

Redundancy in video data can be categorized as two types: spatial redundancy and temporal redundancy. The MPEG compression algorithm employs the technique of block-based motion compression to reduce temporal redundancy. For spatial redundancy, MPEG uses the DCT (Discrete-Cosine Transform) and a combination of weighted scalar quantization and run-length coding to achieve data compression (D. LeGall, 1991), (N. Lodge, 1992).

A digitized frame from a video sequence is first divided into 8×8 blocks of pixels and then translated to the frequency domain by DCT. The distribution of frequency coefficients in the transformed block is typically far from uniform. The energy of typical images is highly concentrated in the lower frequencies. Commonly there are strings of 0's as values for the higher frequencies. MPEG takes advantage of this characteristic by using variable length coding, and significant compression is achieved.

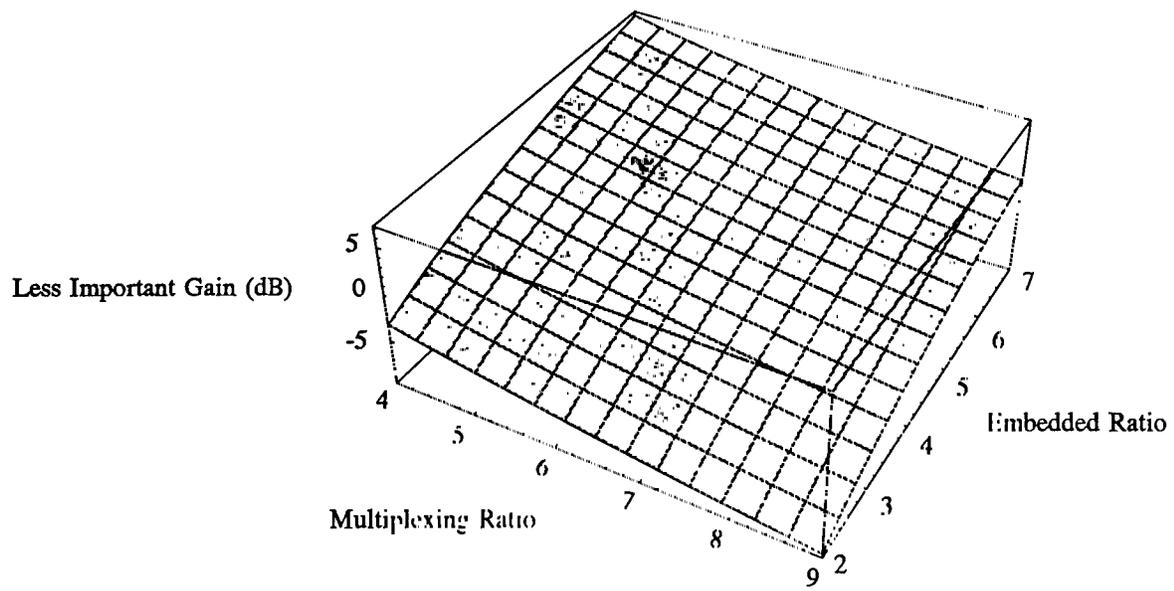


Fig. 5 — Less important gain of multiplexing over embedding graphed over the multiplexing and embedding ratios

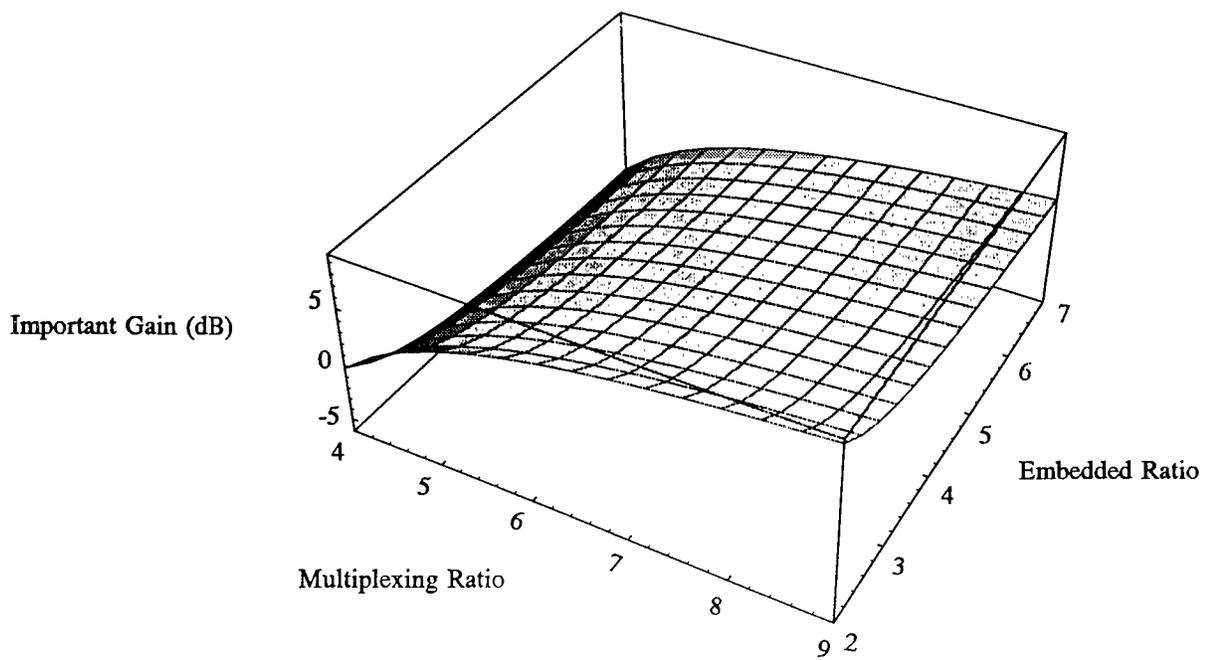


Fig. 6 — Important gain of multiplexing over embedding graphed over the multiplexing and embedding ratios

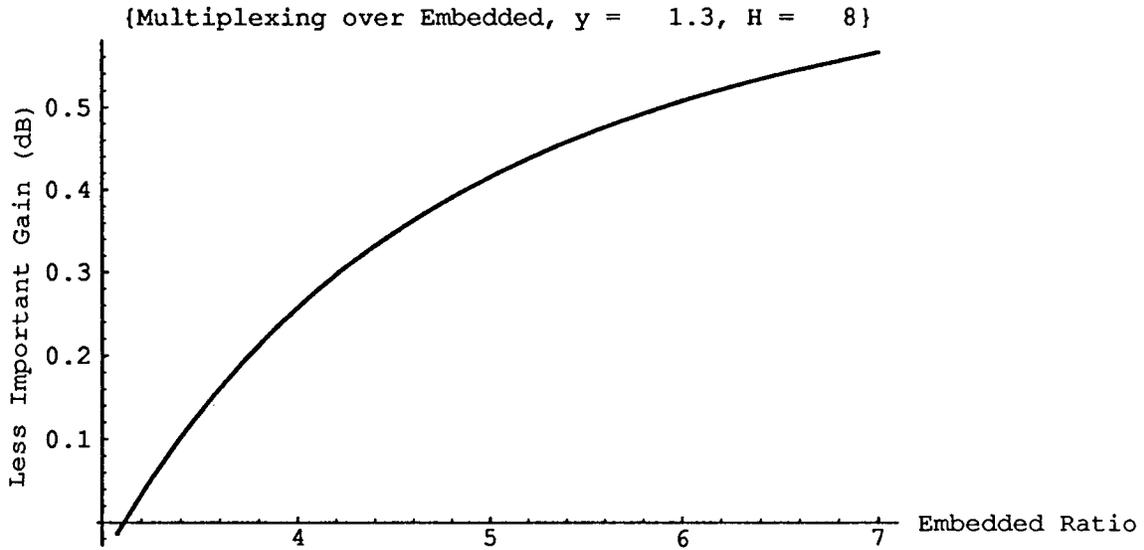


Fig. 7 — Less important gain of multiplexing over embedding graphed over the embedding ratio

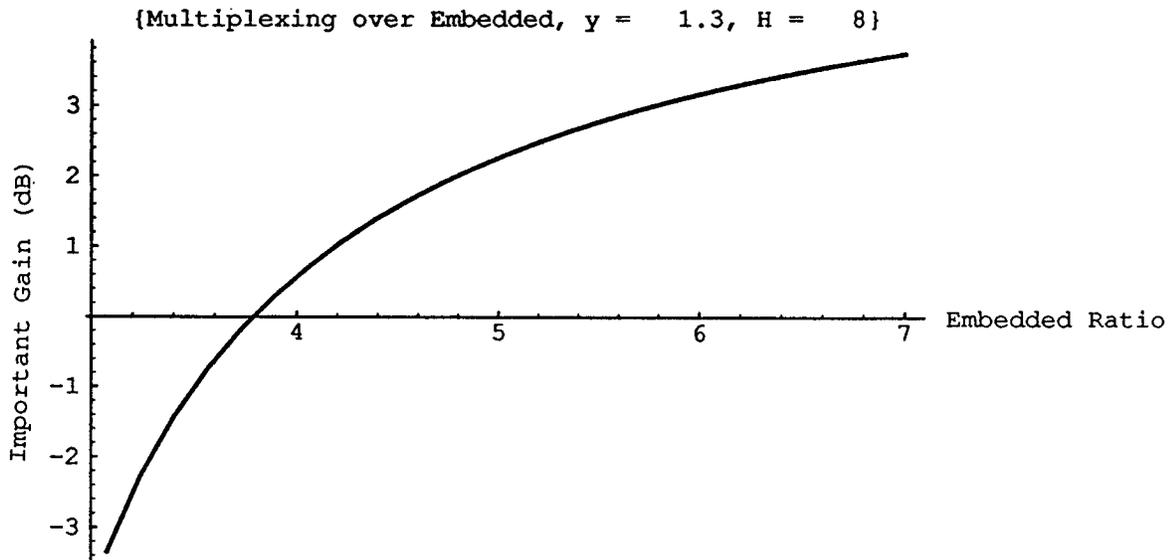


Fig. 8 — Important gain of multiplexing over embedding graphed over the embedding ratio

The human visual perception of quantization errors varies with the frequency. Errors in the higher frequency coefficients are less noticeable to the viewer than errors in lower frequency coefficients. As an example, an error in the DC coefficient results in a high visual image distortion. The MPEG standard takes advantage of this trait of the human visual system by using coarser quantizers for higher frequencies and thus achieves further compression.

The other major characteristic of video data that allows for significant compression is temporal redundancy. Rather than encoding each frame individually, only the residual difference of a frame needs to be transmitted. The MPEG standard uses a motion-compensation technique to determine the residual difference. A motion vector is obtained by minimizing a cost function that measures the mismatch between a block and its prediction.

A frame in MPEG can be coded as one of three types: intrapictures (I), predicted pictures (P), and interpolated pictures (B-for bidirectional prediction). I-frames are coded with no motion compensation, and serve as reference points for P-frames and B-frames. P-frames are coded with motion compensation based on the previous I-frame or P-frame. B-frames are coded with motion compensation, with reference in both forward and backward directions.

Figure 9 shows a typical MPEG frame sequence. In this example, the P-frame is a prediction-frame with the I-frame as the reference point. P-frames may serve as reference-points as well, if there are multiple P-frames between successive I-frames (as is the case for the example shown). The important characteristic of the different types of frames is that bidirectional frames provide the highest amount of compression, as both future and past temporal redundancy are used. B-frames are never used as reference points. I-frames are the least compressed but are completely independent and are isolated from error propagation.

5. UNEQUAL ERROR PROTECTION FOR VIDEO TRANSMISSION

The strategy of unequal error protection signaling for video transmission is to weight some of the data within an I-frame as important. Lower frequency coefficients will be denoted as important since errors in these coefficients will cause greater visual distortion than errors in higher coefficients. Prioritizing lower frequency coefficients has been demonstrated to have significant visual effect for noisy transmission of MPEG data (R. Srinivasan, 1993).

Among a group of a BBP sequence of frames, the P-frame is marked as important data. Since P-frames are used as reference frames and B-frames are not, this strategy will reduce the propagation of errors. For example, if an error occurred in the P-frame, the error will propagate to neighboring B-frames. On the other hand, errors in the B-frame will have no effect on any other frame. In a highly noisy environment, minimizing the propagation of errors will improve the visual quality of the transmitted video.

MPEG data for various scenes of high motion to low motion have been tabulated (L. Jackson, 1994). The order of frame types done in this study is I-BBP-BBP-BBP-BBP-BB, which is a standard sequence. The statistics vary significantly depending on the degree of motion. On average, the ratio of bits in a P-frame to the number of bits in a B-frame is approximately 2:1. Video data

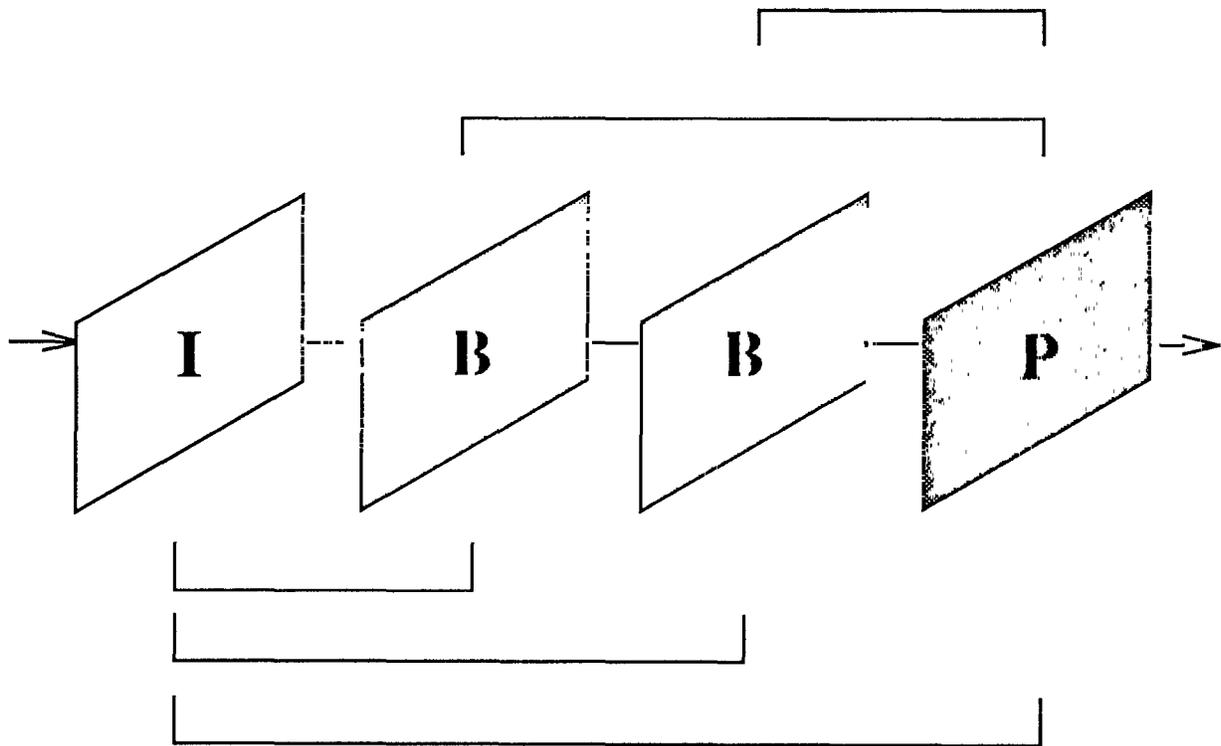


Fig. 9 — Typical MPEG frame sequence

are typically categorized as high motion, medium motion, and low motion. For scenes of the low motion category, the ratio of P:B bits are at most two. A common example of video scenes with low activity is video conferencing. Thus, in a group of BBP, the ratio of P bits to the total number of B bits is 1:1.

To apply unequal error protection for MPEG transmission, we first define a block of data to be 1/2 important and 1/2 less important. To transmit an I-frame, the lower 1/2 frequencies are labeled as important. To transmit a sequence of BBP, the P-frame is labeled as important. This strategy for MPEG transmission is easily implemented and requires minimal overhead. The transmission of the BBP sequence is required to be scanned across all three frames. Therefore, at the receiver, no decoding can be completed until all three frames have been transmitted. But this is the case in typical transmission, since the B-frames require both forward and backward frames for referencing (R. Srinivasan, 1993), (I. Tam, et al. 1992).

6. PROPAGATION OF ERRORS

We compute a reasonable upper bound on the probability of error with the described one-dimensional signal constellation. Define the error energy of post detection as σ^2 . An error is declared if the decoded signal point is more than distance d away from the transmitted signal.

Figure 10 shows the probability of error varying as a function of the SNR under an equal error protection scheme. Figures 11 and 12 show the probability of error as a functions of the ratio of the distances in the multiplexing constellations. Several plots are made for different values of SNR. For the these graphs, we can show that, for a small increase in the number of less important errors, we can effectively eliminate errors in the important data. For MPEG, this means we will have no propagatable errors.

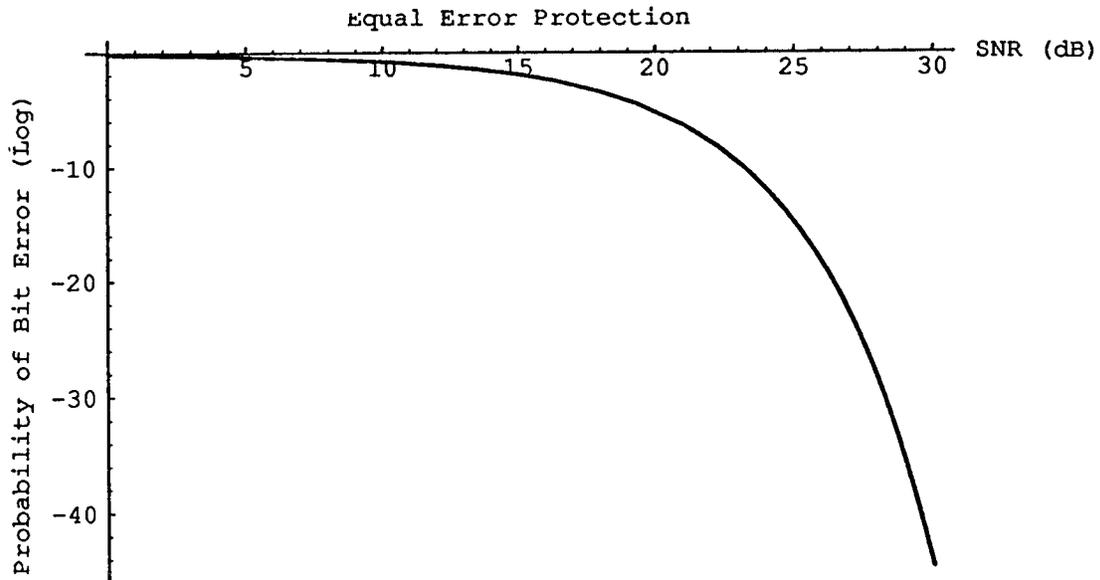


Fig. 10 — Probability of error versus SNR for equal error protection

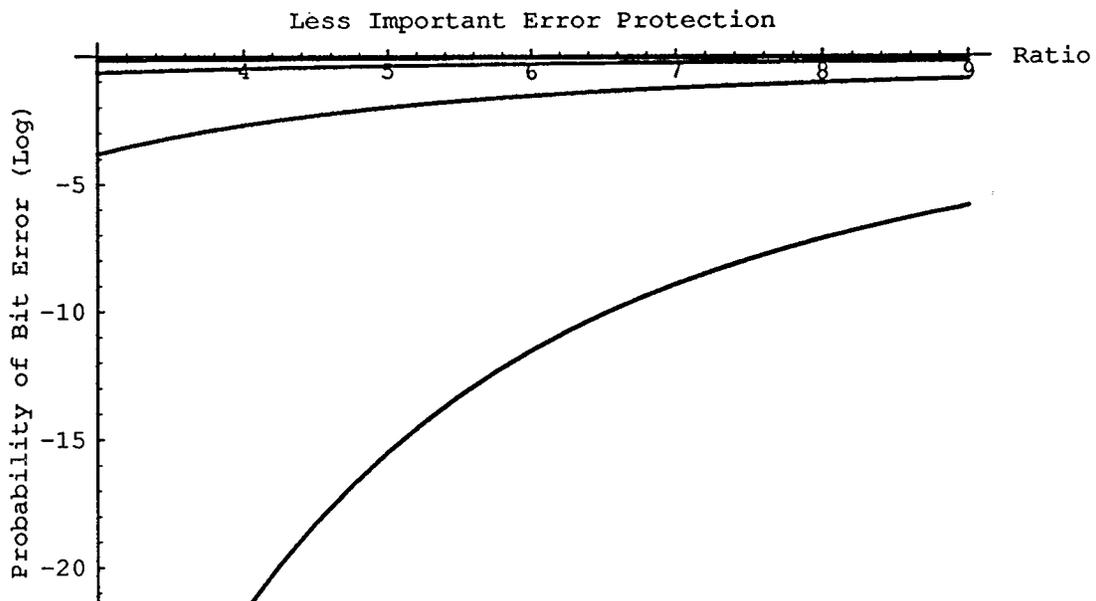


Fig. 11 — Less important probability of error vs multiplexing ratio for several different SNRs

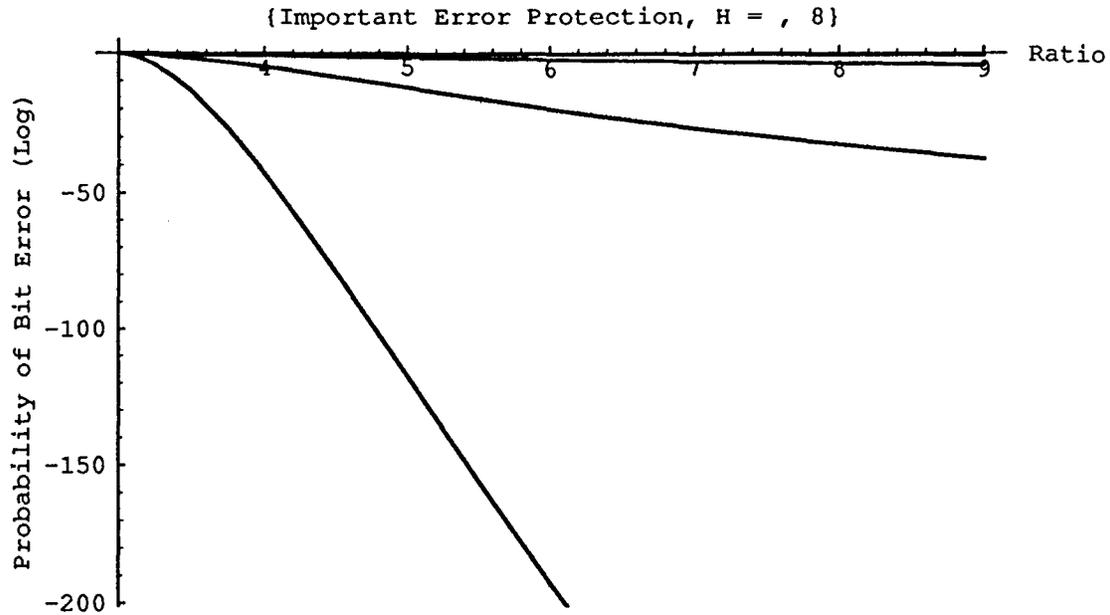


Fig. 12 — Important probability of error versus multiplexing ratio for several different SNRs

7. CONCLUSIONS

We compare two methods for unequal error protection and show that the multiplexing scheme can achieve improved performance versus the embedded scheme. This result is primarily due to the fact the multiplexing scheme is a function of the Hamming distance of the multiplexing rule. This can be increased without additional power penalty, and thus an overall improvement can be obtained. There is no comparable mechanism for the embedded scheme.

We described scheme for applying unequal error protection for MPEG video transmission. A characteristic of video data is that lower frequency coefficients are more important in terms of visual quality. Another aspect of video data is the susceptibility to error propagation. Here we presented a scheme for implementing unequal error protection for MPEG data to prioritize lower frequency coefficients and describe how this scheme simultaneously decreases the probability of error propagation.

8. FUTURE WORK

In this paper, we compared the performance of two methods for unequal error protection and determined some theoretical results. A natural extension of this is to simulate the two schemes and the performance of signal transmission in a noisy environment. In addition, a simulation of the unequal error protection scheme with actual MPEG video data input may provide additional insight into the visual effects of the transmitted data. In the scheme for the video transmission we described, a ratio of 2:1 for the number of bits in a P-frame to the number of bits in a B-frame is assumed. This ratio can vary depending on the content of the scene. There may be interesting



ways to take advantage in the case where the ratio is less than 2:1; i.e., there are fewer important data bits than allocated. The opposite case, where the ratio is greater than 2:1, poses a difficulty, and there may be novel approaches to handle this situation.

The comparison of the embedding and multiplexing schemes completed in this paper is restricted to signal constellations of one dimension. An obvious extension is to consider two-dimensional constellations. This degree of freedom leads to various ways of positioning the signal points nonuniformly and provides the ability to rotate entire regions. In the analysis for propagation of errors, the noise is assumed to be Gaussian. The situation where noise is non-Gaussian may lead to interesting results. In addition, the comparison of the two methods for unequal error protection in the presence of non-Gaussian noise is potential area of research.

For the comparison of embedded and multiplexing methods, no additional coding is implemented. By using Trellis-Coded Modulation, considerable coding gain can be achieved. The effect of adding TCM to both schemes may lead to interesting comparisons. It has been demonstrated that unequal error protection can improve the joint transmission rates for broadcast scenarios. Cover has shown that for an embedded scheme, there is an increase in the capacity region (T. Cover, 1972). An information theoretic research question is to determine if the capacity region for the multiplexing rule is the same, and if not, how does it differ.

9. ACKNOWLEDGMENT

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