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Effects of Modulation Index on Telecommunications

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The relationship between modulation index and telecommunication link was studied for the Deep Space Program Science Experiment. Analytical aspects of modulation index are presented with a brief overview of modulation process in the relational format. Relations of modulation index with bandwidth, ranging/command/telemetry/carrier power losses, bit error rate, signal-to-noise ratio, and hardware simulation results of DSPSE communication links have been investigated with samples of graphical analysis to show optimum ranges of modulation indexes. It is noted that the influence of modulation indexes should not be overlooked in space communication link analysis.

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EFFECTS OF MODULATION INDEX ON TELECOMMUNICATIONS

1. INTRODUCTION

1.1 Background

The goal of all communication systems is to convey information-bearing signals or baseband signals through a communication channel separating the transmitter from the receiver. The form of the system may vary widely, from simple baseband information signals being transmitted over a transmission line (as in telephony and telegraphy), to complicated space communication links through satellites using radio waves. In any event, the efficient utilization of the wideband resources of these channels requires a shift of the range of baseband frequencies into other frequency ranges after reception. A shift of the range of frequencies in a signal is accomplished by using modulation which is defined as the process by which some characteristic of a carrier is varied in accordance with a modulating wave. The baseband signal is referred to as the modulating wave, and the result of the modulation process is referred to as the modulated wave.

There are various ways of producing modulated signals, but they break down essentially into three forms [1 to 9]:

1. Continuous-wave (CW) modulation, in which the amplitude, phase, or frequency of a specified sine wave (the carrier) is altered in accordance with the information being transmitted.

2. Pulse modulation, in which the height, the weight, or the position of a set of pulses (the carrier) is again altered in a definite pattern corresponding to the information to be transmitted.

3. Coded or Digital modulation, which involves changing a characteristic of a pulse train but is a markedly different method from pulse modulation. The method involves first sampling the information signal, quantizing the sample by rounding off to the closest of a number of discrete levels, and finally generating a prescribed number of pulses according to a code related to nearest discrete level. Frequently, a complete communication system will use combinations of the above three forms of modulation.

There are types of modulation that correspond to continuous-wave modulation of a carrier by either pulse or digital information-bearing signals, the latter being more representative of modern systems. In the digital case, amplitude shift keying (ASK), frequency shift keying (FSK), and phase shift keying (PSK) correspond to modulating amplitude, frequency, and phase of a carrier wave, respectively. Differential PSK (DPSK) is a variation of PSK that allows simplifications in receiving equipment. It is apparent that a large number of modulation types is available. Pulse modulation types are not presented here.

Many types of digital modulation have been developed. Pulse code modulation (PCM) is perhaps the most important. PCM may be either binary, where pulses have only two voltage levels or are m-ary, where pulses may take on m possible levels. Several important variations of binary PCM are delta

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modulation (DM), delta-sigma modulation (D-SM), adaptive delta modulation (ADM), and differential pulse code modulation (DPCM). These variations involve coding only information about changes in the information signal.

Speaking broadly, the primary reason for modulation is to convert the information signal to a more useful form. There are many specific reasons that may be divided into three main categories:

- for necessity or convenience,
- for performance increase, and
- for efficient spectrum utilization.

On the convenience side, modulation provides a mean whereby a number of signals may be added or multiplexed together for simultaneous transmission. Through multiplexing we are able to send many messages between two points while using a single communication channel.

It is possible to improve system performance by choice of modulation method. It is often necessary to use modulation to translate the useful band of frequencies up to a higher carrier frequency so that efficient electromagnetic radiation is possible from an antenna having reasonable size to operate. Efficiency requires antenna physical dimensions of about 1/10 wavelength or larger. Performance is limited by the presence of random noise and interference. The effect of these unwanted waveforms can be suppressed by using certain forms of modulation.

Finally, modulation serves as a means of efficient spectrum utilization. By choice of carrier frequency a designer could, in principle, elect to operate in any spectrum band, hoping that no one else would design for the same frequency. If someone else did, the two systems might hopelessly interfere with each other. To avoid such conditions and to assure efficient use of the radio spectrum, the Federal Communications Commission (FCC) and National Telecommunications & Information Administration (NTIA) control frequency allocations in the United States and cooperate with the International Telecommunication Union (ITU) for the orderly and efficient use of radio frequency spectrum within the frame of international radio and geostationary orbits, respectively.

Digital communications could not have occurred and experienced such growth in the 1970s without the basis of a concurrent technological growth in the modulator and demodulator devices in the modems of digital communication systems. System trade-offs are fundamental to all digital communication designs. The goals of the designer are [7]

(1) to maximize transmission bit rate;
(2) to minimize the probability of bit error;
(3) to minimize required bit energy to noise power spectral density;
(4) to minimize required system bandwidth;
(5) to maximize system utilization, that is to provide reliable service for a maximum resistance to interference; and
(6) to minimize system complexity, computational load, and system cost.

A good system designer seeks to achieve all these goals simultaneously. However, goals (1) and (2) are clearly in conflict with goals (3) and (4); they call for simultaneously maximizing bit rate, while minimizing the probability of bit error, bit energy to noise power spectral density, and bandwidth. Several constraints and theoretical limitations necessitate the trading off of any one system requirement with each of the others. Some of the constraints are [6-8]

- The Nyquist theoretical minimum bandwidth requirement;
- The Shannon-Hartley capacity theorem;
- Government regulations (e.g., frequency allocations);
- Technological limitations (e.g., state-of-art components);
- Other system requirements (e.g., satellite orbits).

Some of the reliable modulation and coding trade-offs can best be viewed as a change in operating point on one of several performance planes. These planes will be referred as a function of modulation index. For a smaller modulation index, we have essentially a carrier and one pair of significant sidebands. As the modulation index increases for phase modulation (PM) and frequency modulation (FM), the number of significant sidebands increases while the total average power of the sidebands plus the carrier remains constant. Since the magnitude of the modulation index defines the magnitude of the Bessel functions and, consequently, the amplitudes of the carrier and the sidebands, goals (1) through (4) above can be represented as a function of modulation index.

1.2 Purpose and Scope

The objective of this report is to investigate the sensitivity of modulation index in satellite communications as well as other telecommunications. A brief overview of continuous modulation (amplitude and angle modulation) and digital modulation is only introduced by emphasizing more theoretical analysis and computational development to derive a functional relationship between power, bandwidth, bit error rate (BER), signal-to-noise power ratio, and other parameters and modulation index. Specific discussion of pulse modulation is not presented in this report since it is covered in the digital modulation description. The comparison is based on telemetry performance as well as bandwidth efficiency. Most criteria are characterized in terms of the BER [10], loop bandwidth-to-data rate ratio, signal-to-noise ratio, ranging/command/telemetry/carrier power losses, and modulation index. Each relational development is thoroughly investigated based on the analytical relations presented in Section 2 with the test results of the satellite-ground station transmitter and receiver simulation for the Deep Space Program Science Experiment (DSPSE) [11, 12]. Additional reference materials are included in the report since they are directly related to modulation index derivation. Finally, a summary and conclusion of the study results are presented with references.

2. OVERVIEW OF MODULATION PROCESS

This section briefly reviews the modulation process, limiting discussion to the analytic as opposed to the systems aspects of the process. This limitation allows us to study the effects of modulation index in the communication linkage. The modulation process is essential in communication systems because

1. communication systems require a reduced bandwidth,
2. a channel is a bandpass while the signal is a baseband; hence, the signal must be shifted in frequency,
3. several baseband signals that occupy the same frequency band can be transmitted through a single broadband channel, if they are separated in frequency, by modulating each signal on a separate carrier frequency, and
4. the complexity and cost of a bandpass system may be less than a brute force baseband system.

Many other reasons can be listed in addition to the above mentioned. As pointed out in the previous section, this report only presents continuous modulation and digital modulation to focus on relational investigations between modulation index and other parameters of telecommunication systems links from ground to space.

2.1 Continuous Modulation

In continuous-wave modulation systems, a radio frequency (RF) sinusoid is used as a carrier in which the amplitude, phase, or frequency is modulated by the information-carrying signals or messages. The modulating signal may consist of one message only, or more often it is a composite of frequency- or time-division multiplexed messages. In a linear modulation process, the frequency components of the modulating signal are translated to occupy a different position in the spectrum. This is accomplished by effectively multiplying together the time functions that describe the modulating signal and the carrier. In a nonlinear modulation process (angle modulation), new components are generated in the resulting spectrum of the modulated signal that do not have a one-to-one correspondence in the original spectra involved. The linear modulation process further consists of amplitude modulation, vestigial sideband modulation, single-sideband and double-sideband modulation. Amplitude modulation is only briefly discussed here for theoretical analysis; we focus more on angle modulation, which consists of frequency modulation and phase modulation [1-9].

2.1.1 Amplitude Modulation

Amplitude modulation is one of the oldest forms of modulation. When we talk of sine-wave modulation, we imply that we have an available source of sinusoidal energy with an output voltage or a current of the form

$$v(t) = A_c(t)\cos(\omega_c t + \theta(t)), \tag{1}$$

where $A_c(t)$ is the carrier amplitude, ω_c is the carrier frequency rate, and $\theta(t)$ is the phase angle. For convenience, let us assume that the phase angle of the carrier wave is zero at $t = 0$. This implies that the amplitude only is assumed to be varied. This then means that we can talk of an envelope variation, or variation of the locus of the carrier peaks. Let $f(t)$ denote the baseband signal that carries the specification of the message. The carrier wave $v(t)$ is independent of $f(t)$. Amplitude modulation is defined as a process in which the amplitude of the carrier wave $v(t)$ is varied about a mean value, linearly with the baseband signal $f(t)$. An AM wave may thus be described as a function of time in the form

$$v(t) = A_c\,[1 + h_a f(t)]\cos(\omega_c t), \tag{2}$$

where h_a is called an amplitude modulation index. In order to maintain the envelope of $v(t)$ as the same shape as the baseband signal $f(t)$, two requirements are satisfied:

a). The amplitude of $h_a f(t)$ is always less than unity; that is

$$|haf(t)| < 1 \tag{3}$$

for all values of t. This condition ensures that the function $\{1 + h_a f(t)\}$ is always positive, and so we may express the envelope of the AM wave $v(t)$ of Eq. (2) as $A_c\,[1 + h_a f(t)]$. When condition (3) is not satisfied, the carrier wave becomes overmodulated, resulting in carrier phase reversals and, therefore, envelope distortion. It is therefore apparent that by avoiding overmodulation, a one-to-one relationship is maintained between the envelope of the AM wave and the modulating wave for all time values. In other words, the envelope is a replica of the modulating signal.

b). The carrier frequency f_c is much greater than the highest frequency component of $f(t)$; that is,

$$f_c >> W, \tag{4}$$

where f_c is the carrier frequency, and W is the message bandwidth. If this condition is not satisfied, an envelope cannot be visualized satisfactorily. Let us consider two types of modulating signals by using Eq. (2).

1). Periodic Modulating Signal: The modulating signal is bandlimited with no DC component,

$$f(t) = \Sigma_n C_n * \exp(jn\omega_m t) = 2*\Sigma_n |C_n| * \cos(n\omega_m t + \theta_n), \quad (5)$$

where $\omega_m = 2\pi/T_m$, T_m is the period of the modulating signal, and Σ_n is the summation with respect to n, $n\omega_m << \omega_c$. The output signal v(t) then becomes

$$v(t) = A_c \cos \omega_c t + h_a A_c * S\ |C_n| \{\cos[(\omega_c + n\omega_m)t + \theta_n] + \cos[(\omega_c - n\omega_m)t - \theta_n]\}\ . \quad (6)$$

The Fourier transform of Eq. (6) is given by

$$V(f) = A_c/2*[\delta(f - f_c) + \delta(f + f_c)] + h_a A_c/2*\Sigma\ [F(f - f_c) + F(f - f_c)], \quad (7)$$

where F(.) denotes the Fourier function of f(t).

Suppose that the baseband signal f(t) is bandlimited to the interval $-W_m < f_m < W_m$. Then the amplitude spectrum |V(f)| of the AM wave consists of two delta functions weighted by the factor $A_c/2$ and occurring at $\pm f_c$, and two versions of the baseband spectrum translated in frequency by $\pm f_c$ and scaled in amplitude by $h_a A_c/2$. The following observations are noted by analyzing Eq. (7).

a) The amplitude spectrum |V(f)| of an AM wave is symmetrical about the carrier frequency $\pm f_c$. For positive frequencies, the portion of the spectrum lying above f_c is referred to as the upper sideband, whereas the symmetrical portion below f_c is referred to as the lower sideband. For negative frequencies, the upper sideband is represented by the portion of the spectrum below $-f_c$ and the lower sideband by the portion above $-f_c$. The condition $f_c > W$ ensures that the sidebands do not overlap.

b) The transmission bandwidth B_T for an AM wave is exactly twice the message bandwidth W; that is

$$B_T = 2W \quad (8)$$

2). Nonperiodic Modulating Signal: Let f(t) represent a nonperiodic signal which is bandlimited to ω_m, where $\omega_m << \omega_c$. The modulated signal v(t) is given by Eq. (2) as

$$v(t) = A_c*[1 + h_a f(t)]*\cos \omega_c t \quad (9)$$

and the Fourier transform of the second term in Eq. (9) is

$$V(f) = h_a A_c/2*[F(f - f_c) + F(f + f_c)]. \quad (10)$$

F(.) is generally a complex quantity and is defined for negative ω also. The complete spectrum for v(t) must include the first term, cos $\omega_c t$, which is equivalent to discrete frequency components at $+\omega_c$ and $-\omega_c$.

From the previous brief discussion of amplitude modulation, the following conclusion and summary can be construed:

a) The modulation index h_a must be kept below unity to avoid envelope distortion due to overmodulation.
b) The ratio of the total sideband power to the total power in the modulated wave depends only on the modulation index h_a.
c) The carrier and modulating wave are in phase.
d) The amplitude of the AM wave is directly proportional to the modulation index h_a.
e) The transmission efficiency of an AM system is at best 33% [1].

Under the best condition, i.e., $h_a = 1$, 67% of the total power is expended in the carrier and represents wasted power vis a vis the transfer of information. If $h_a = 1$, that is, 100% modulation is used, the total power in the two side-frequencies of the resulting AM wave is only one-third of the total power in the modulated wave. This implies that the modulation index h_a of the AM wave controls the relative proportions of sideband to the carrier.

2.1.2 Angle Modulation [1 to 9]

A continuous-wave modulation sinusoidal signal can be varied by changing its amplitude and its phase angle. In Eq. (1) for the amplitude modulation, we kept the phase angle $\theta(t)$ constant and varied $A(t)$ proportional to $f(t)$. In this section, we are concerned with the opposite situation. Here A is maintained constant, while the angle $\theta(t)$, of the carrier varies in some manner according to the message waveform. Angle can be varied by phase or frequency changes. If phase is made to vary linearly as a function of the message $f(t)$, the carrier is said to have phase modulation imparted upon it. Frequency modulation corresponds to linearly varying instantaneous frequency with $f(t)$. FM and PM analyses are intimately related. Therefore, some detailed discussions are limited to FM, and the analogous PM developments are only outlined.

We can generalize Eq. (1) in an ordinary sinusoid form as

$$v(t) = A_C \cos(\omega_o t + \theta_o), \tag{11}$$

where ω_o and θ_o are constants. Let the angle $\theta(t) = \omega_o t + \theta_o$. The instantaneous frequency is then

$$\omega(t) = d\theta(t)/dt = \omega_o, \qquad \text{a constant.} \tag{12}$$

This presents two different cases: phase modulation and frequency modulation.

a). A sinusoid is said to be phase modulated if the instantaneous angle is a linear function of the information signal f(t). Thus,

$$\theta_{PM}(t) = \omega_o t + \theta_o + h_{PM} * f(t), \tag{13}$$

where ω_o and h_{PM} are the angular velocity and the phase modulation index, respectively, and are assumed to be positive constants, while θ_o is an arbitrary constant phase angle. Using Eq. (12), the instantaneous frequency of the PM waveform is

$$\omega_{PM}(t) = \omega_o + h_{PM} * (df(t)/dt), \tag{14}$$

while the PM waveform itself is

$$v_{PM}(t) = A_C \cos[\omega_o t + \theta_o + h_{PM} * f(t)]. \tag{15}$$

b). A sinusoid is said to be frequency modulated if instantaneous frequency is a linear function of the information signal f(t). Thus,

$$\omega_{FM}(t) = \omega_o + h_{FM} * f(t) \tag{16}$$

where ω_o and h_{FM} are assumed to be positive constants. The FM signal's instantaneous angle becomes

$$\theta_{FM}(t) = \omega_o t + \theta_o + h_{FM} * \int f(t)\, dt \tag{17}$$

while the FM signal itself is

$$v_{FM}(t) = A_C \cos[\omega_o t + \theta_o + h_{FM} * \int f(t)\, dt]. \tag{18}$$

c). As can be seen in Eq. (18), it may be viewed as a PM waveform if the modulating signal is taken to be $\int f(t)\, dt$. Similarly, the PM signal of Eq. (15) is equivalent to an FM waveform if the modulating signal is taken to be df(t)/dt.

2.1.2.1 Narrowband Angle Modulation

It is difficult to analyze an FM or PM signal to obtain a general expression of its Fourier spectrum. However, in the simplified problem where the maximum phase deviation due to modulation is kept small, the spectrum of the modulated signal may be found for an arbitrary information signal f(t). The effect of small phase deviation is to restrict bandwidth to a small value as compared with a large deviation case.

1. Narrowband frequency modulation (NBFM): Let's assume in Eq. (18) that the maximum phase deviation due to modulation is small. That is, assume

$$h_{FM} * \left| \int f(t)\, dt \right|_{max} << \pi/6. \tag{19}$$

With this restriction and using the trigonometric expansion of cos (A + B),

$$\begin{aligned} V_{NBFM}(t) &= A_C \cos(\omega_o t + \theta_o)\cos[h_{FM} \int f(t)\, dt] - A_C \sin(\omega_o t + \theta_o)\sin[h_{FM} \int f(t)\, dt] \\ &= A_C \cos(\omega_o t + \theta_o) - A_C h_{FM} [\int f(t)\, dt] \sin(\omega_o t + \theta_o) \end{aligned} \tag{20}$$

is the NBFM waveform. Here we have used the approximations cos (x) $\cong$ 1 and sin (x) $\cong$ x if x is small.

The spectrum of $v_{NBFM}(t)$ is easily found. If f(t) has the spectrum $F(\omega)$,

$$f(t) \leftrightarrow F(\omega) \tag{21}$$

then

$$\int f(t)\, dt \leftrightarrow F(\omega)/j\omega, \tag{22}$$

where F(0) is assumed to be zero. Using this result with Fourier transform pairs, one can obtain the spectrum of $v_{NBFM}(t)$ as

$$\begin{aligned} V_{NBFM}(\omega) = {} & \pi A_C [d(\omega - \omega_o) e^{j\theta_o} + \delta(\omega + \omega_o) e^{-j\theta_o}] \\ & + (A_C h_{FM} / 2) [F(\omega - \omega_o) e^{j\theta_o}/(\omega - \omega_o) - F(\omega + \omega_o) e^{-j\theta_o}/(\omega + \omega_o)]. \end{aligned} \tag{23}$$

There is considerable similarity between the narrowband FM signal spectrum and that of standard AM (i.e., Eq. (7)). Both spectra have identical carrier terms, corresponding to the impulses, and both have positive and negative frequency spectral components, due to modulation, that are centered about frequencies of $+\omega_o$ and $-\omega_o$, respectively. If f(t) is bandlimited to have a maximum spectral extent W_f, then these components in both standard AM and NBFM are constrained to a band $2W_f$. However, the NBFM components are seen to differ in form owing to the factors of $1/(\omega + \omega_o)$ and $1/(\omega - \omega_o)$. Another difference is reflected in the 180° phase reversal of the NBFM negative frequency component not present in the AM spectrum.

2. Narrowband phase modulation (NBPM): Assuming maximum phase deviation to be small, that is,

$$h_{PM} \left| f(t) \right|_{max} \ll \pi/6, \tag{24}$$

will allow Eq. (15) to be written as

$$V_{NBPM}(t) \cong A_C \cos(\omega_o t + \theta_o) - A_C h_{PM} f(t) \sin(\omega_o t + \theta_o). \tag{25}$$

The Fourier transform of Eq. (25) gives the NBPM spectrum as

$$V_{NBPM}(\omega) = \pi A_C[\delta(\omega - \omega_o)e^{j\theta_o} + \delta(\omega + \omega_o)e^{-j\theta_o}]$$

$$+ (jA_C h_{PM}/2)[F(\omega - \omega_o)e^{j\theta_o} - F(\omega + \omega_o)e^{-j\theta_o}]. \tag{26}$$

As pointed out in the NBFM case, this spectrum also resembles that of standard AM. A comparison with Eq. (7) reveals that the form and extent of the positive and negative frequency spectral components centered at $+\omega_o$ and $-\omega_o$ are identical. However, these components are phase shifted by $\pi/2$ and $-\pi/2$, respectively, as compared to standard AM.

2.1.2.2 Wideband FM with a Sinusoidal Signal

As already stated, general analysis of FM is quite difficult, and one tends to lose sight of the physical picture. However, to gain insight, let us first discuss in some detail the simplest possible wideband case, that of a single sinusoidal waveform. Thus,

$$f(t) = A_f \cos(\omega_f t) \tag{27}$$

is the assumed information signal. A_f and ω_f are constant amplitude and angular velocity, respectively. From Eq. (18), the FM waveform can be written as

$$V_{FM}(t) = A_f \cos[\omega_o t + \beta_{FM} \sin(\omega_f t)] \tag{28}$$

where Eq. (27) has been integrated, and define a quantity

$$\beta_{FM} = A_f h_{FM}/\omega_f = \Delta\omega/\omega_f \tag{29}$$

called the FM modulation index. It is the ratio of the maximum frequency deviation

$$\Delta\omega = h_{FM} \left| f(t) \right|_{max} = A_f h_{FM} \tag{30}$$

to the signal frequency ω_f. The modulation index is nothing more than a measure of how intense the modulation is relative to the rapidity of modulation. Using the well-known Jacobian equations as

$$\cos[\beta \sin(x)] = J_o(\beta) + 2\,\Sigma_k\, J_{2k}(\beta)\cos(2kx) \tag{31}$$

$$\sin[\beta \sin(x)] = 2\,\Sigma_k\, J_{2k-1}(\beta)\sin[(2k-1)x] \tag{32}$$

where β represents an arbitrary modulation index, and Σ_k the summation with respect to k. Using Eqs. (31) and (32), we may rewrite Eq. (28) as

$$v_{FM}(t) = A_f\,\Sigma_n\, J_n(\beta_{FM})\cos[(\omega_o + n\omega_f)t]. \tag{33}$$

The quantities $J_n(\beta)$ are coefficients called Bessel functions of the first kind of order n. They are constants in time but are functions of modulation index β. A property of Bessel functions used in obtaining Eq. (33) is

$$J_{-n}(\beta) = (-1)^n J_n(\beta). \tag{34}$$

Examination of Eq. (33) shows that $v_{FM}(t)$ contains an infinite number of frequencies. Thus, many frequencies are generated with FM that are not present in the information signal. Because of this, the FM waveform a much larger bandwidth than the signal. Before examining the FM spectrum, let us discuss some of the properties of the Bessel functions since the spectrum behavior depends on the Bessel coefficients and their behavior.

1). Figure 1 shows the first six Bessel functions. Data for other values of n and β may be found in many textbooks [13]. Figures 2 to 4 show interrelations among Bessel functions. Each function exhibits a cyclic or lobing behavior with decreasing peak values as β increases. As a function of n with β fixed,

$$J_n(\beta) \cong 0, \text{ for } n > \beta + 1 \tag{35}$$

where the approximation of Eq. (35) becomes more accurate as β becomes large relative to unity.

2). The sum of the squares of the Bessel functions equals unity as

$$\Sigma_n\, J_n^2(\beta) = 1 \text{ for all } \beta. \tag{36}$$

3). The Bessel functions are symmetrical, which satisfies the following criterion:

$$J_n(-\beta) = (-1)^n J_n(\beta). \tag{37}$$

These characteristics can be applied to the following communication system parameters.

1. FM Spectrum: The spectrum of $v_{FM}(t)$ is the Fourier transform of Eq. (33). Standard transformations yield

$$V_{FM}(\omega) = \pi\, A_f \Sigma_n\, J_n(\beta_{FM})[\delta(\omega - \omega_o - n\omega_f) + \delta(\omega + \omega_o + n\omega_f)]. \tag{38}$$

This spectrum has two main parts that are centered about frequencies of $+\omega_o$ and $-\omega_o$. The magnitudes of the parts are identical. The carrier level is proportional to $J_o(\beta_{FM})$, while the levels of the first sideband frequencies, at $\pm$-ω_f relative to the carrier, are proportional to $|J_1(\beta_{FM})|$. Similarly, the level of the nth sideband frequency is proportional to $|J_n(\beta_{FM})|$.

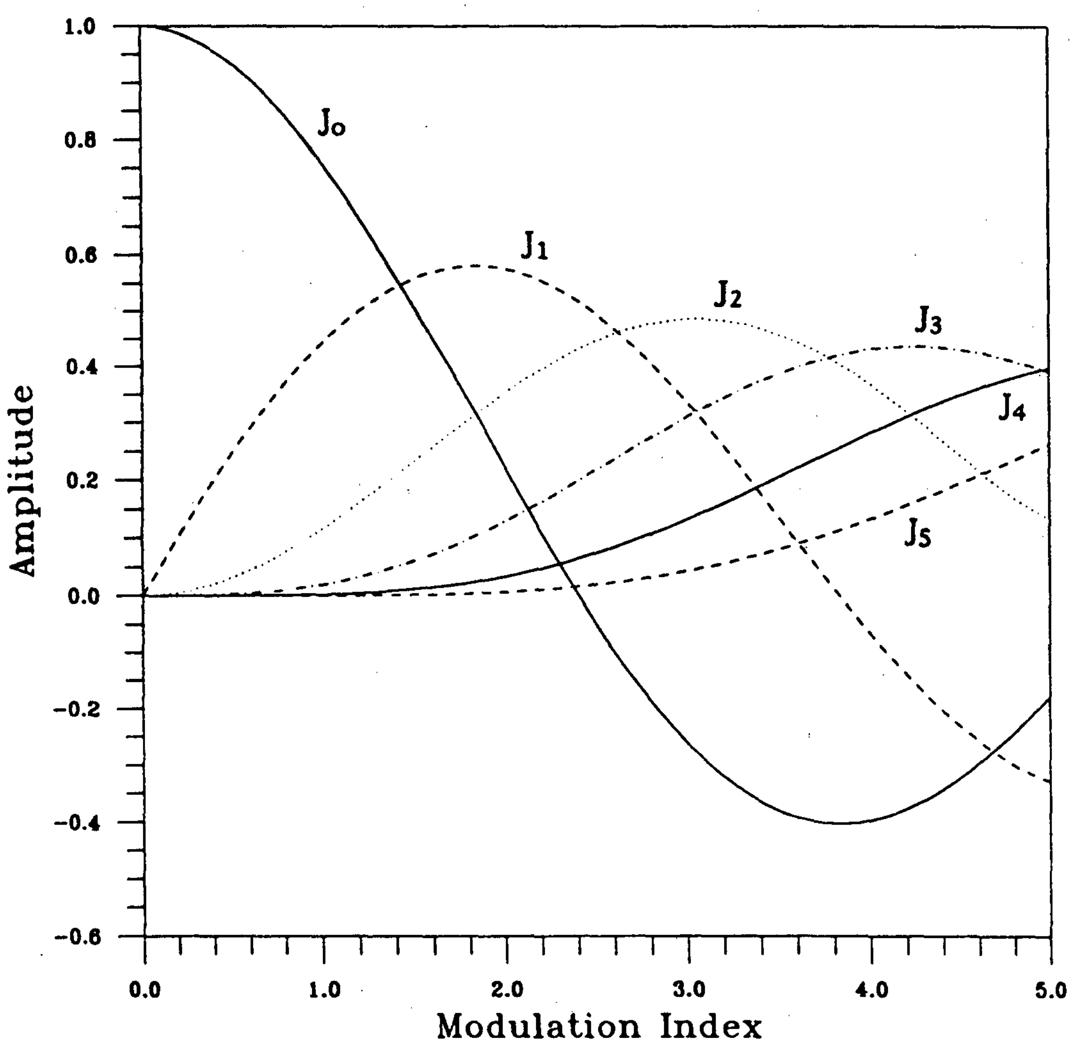

Fig. 1— Plots of first six Bessel functions vs modulation index

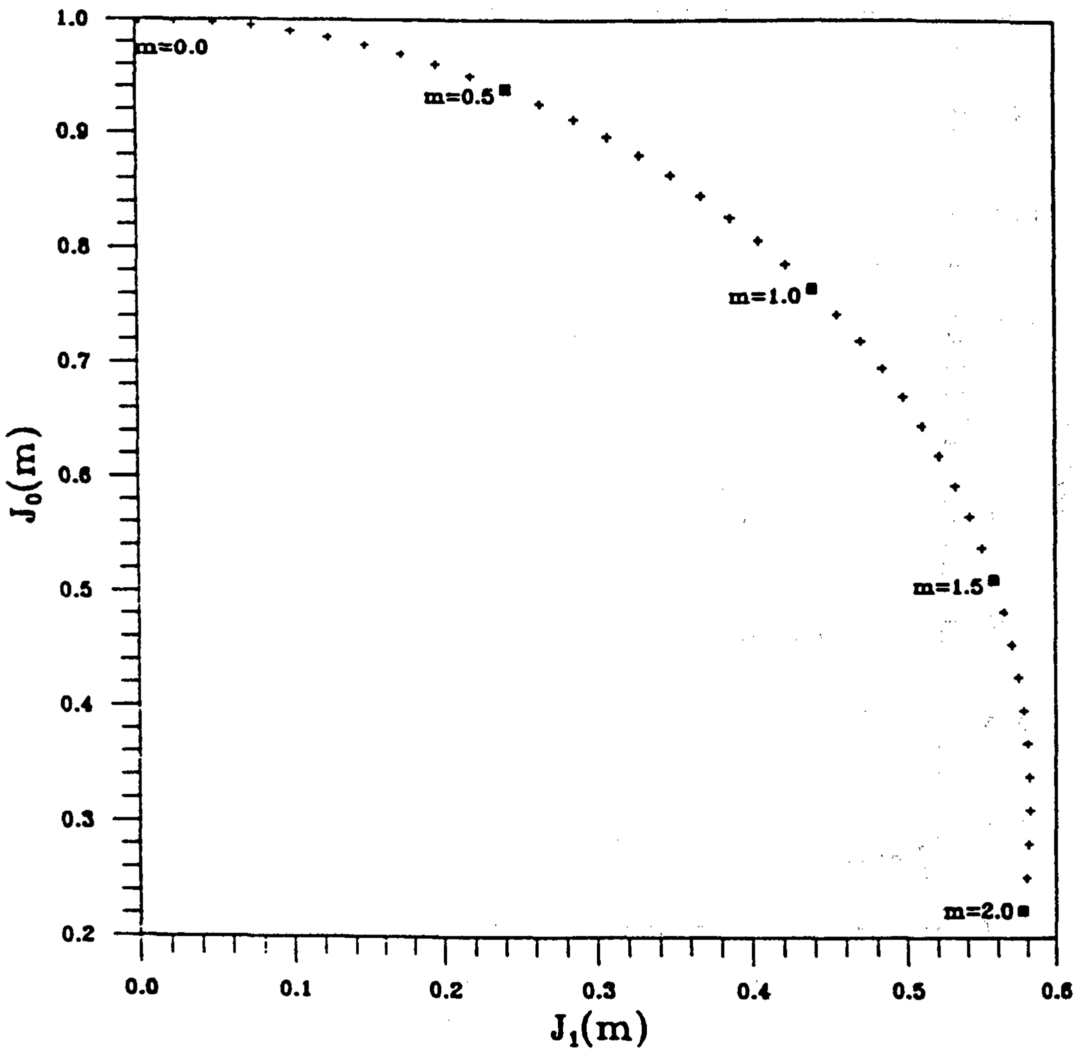

Fig. 2 — Zero-order and first-order Bessel functions as a function of modulation index

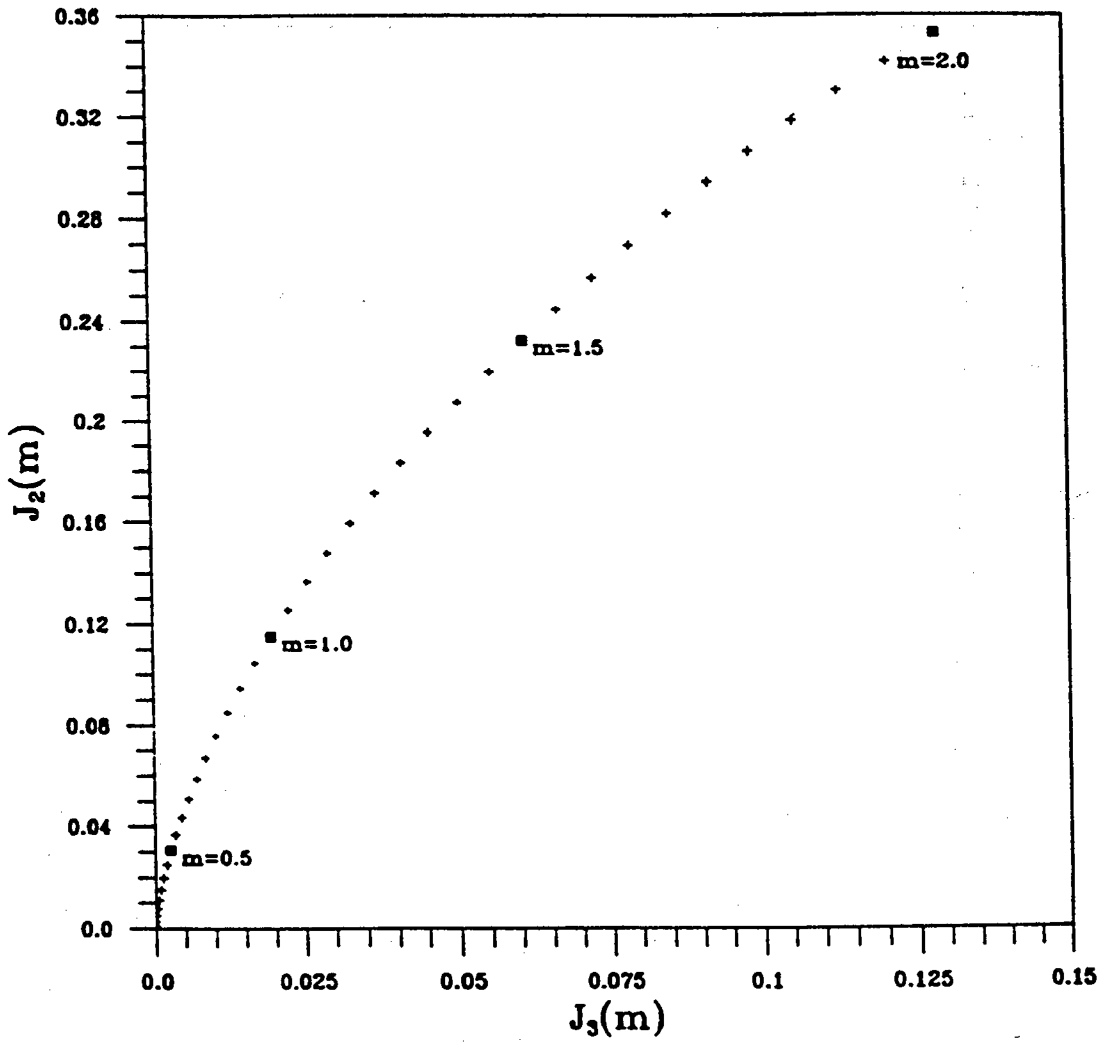

Fig. 3 — Second-order and third-order Bessel functions as a function of modulation index

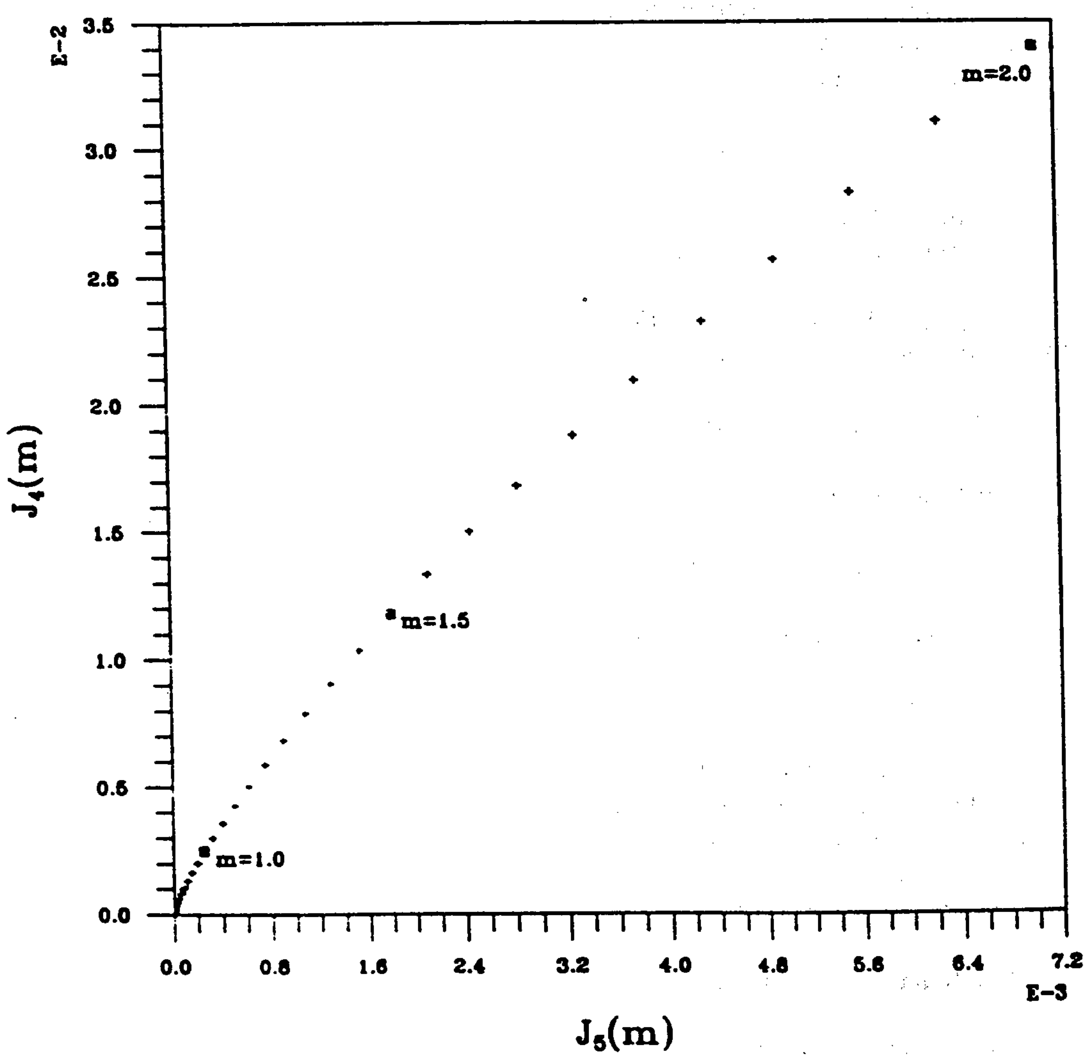

Fig. 4 — Fourth-order and fifth-order Bessel functions as a function of modulation index

2. Bandwidth: In terms of modulation index β_{FM} , we may write as

$$W_{FM} = 2*(\beta_{FM} + 1)\,\omega_f. \tag{39}$$

This result is a good rule-of-thumb for bandwidth in general for modulation by sinusoids. It can be justified by use of Eq. (35). Since the magnitude of the nth sideband frequency is proportional to $|J_n(\beta_{FM})|$, the frequencies for $|n| > \beta_{FM} + 1$ are negligible, and only $2n = 2(\beta_{FM} + 1)$ sideband frequencies are important. Since frequency separation is ω_f , Eq. (39) is derived as the bandwidth. When $\beta_{FM} << 1$, the bandwidth W_{FM} approaches $2\omega_f$ as expected (the narrowband case). If $\beta_{FM} >> 1$, which is the very broadband case,

$$W_{FM} = 2\beta_{FM}\,\omega_f = 2\Delta\omega. \tag{40}$$

In other words, the FM bandwidth for large modulation index equals twice the peak frequency deviation of the frequency excursions and is independent of the modulation frequency.

3. Spectral power distribution: Using Eq. (33), the autocorrelation function of an FM signal generated by modulation with a sinusoid is found to be

$$R_{FM}(\tau) = (A_f^2/2)*\Sigma_n J_n^2(\beta_{FM})\cos[(\omega_o + n\omega_f)\tau]. \tag{41}$$

The Fourier transform of Eq. (41) leads to the power density spectrum as

$$S_{FM}(\omega) = (\pi A_f^2/2)\,\Sigma_n J_n^2(\beta_{FM})[\delta(\omega - \omega_o - n\omega_f) + \delta(\omega + \omega_o + n\omega_f)]. \tag{42}$$

By integrating Eq. (42), we can obtain the total power P_{FM} in the FM signal as

$$P_{FM} = (1/2\pi)*\int S_{FM}(\omega)d\omega = (A_f^2/2)*\,\Sigma_n J_n^2(\beta_{FM}). \tag{43}$$

Since the sum is unity in Eq. (36) for all values of β_{FM}, the total power is $A_f^2/2$, which is independent of the FM modulation process. This result is not too surprising, since we might suspect that power would be related to signal amplitude, which is constant for FM, and not necessarily dependent on the signal's phase. Similar results can be derived for arbitrary signals such as periodic and random signals. No attempt is made to discuss those cases any further in this report.

2.1.2.3 Wideband Phase Modulation

Wideband PM is analogous to FM as found in the previous discussion. Since the procedures are quite similar to those used for FM, the main points are outlined here for comparison. For a sinusoidal signal of Eq. (27), the PM waveform is

$$V_{PM}(t) = A_p \cos[\omega_o t + \beta_{PM}\cos(\omega_f t)], \tag{44}$$

where A_p is the amplitude of the phase modulation signal, and the PM modulation index β_{PM} is defined as

$$\beta_{PM} = h_{PM} * A_f = \Delta\theta. \tag{45}$$

Here $\Delta\theta$ is the peak phase deviation. The peak frequency deviation is

$$\Delta\omega = \beta_{PM} * \omega_f. \tag{46}$$

In terms of a Bessel coefficient expansion, the PM signal may be written as

$$v_{PM}(t) = A_p^* \Sigma_n J_n(\beta_{PM}) \cos[(\omega_o + n\omega_f)t + n(\pi/2)]. \quad (47)$$

The spectrum of PM signal is the Fourier transform of Eq. (47):

$$V_{PM}(\omega) = \pi A_p \Sigma_n J_n(\beta_{PM}) [e^{jn(\pi/2)} \delta(\omega - \omega_o - n\omega_f) + e^{-jn(\pi/2)} \delta(\omega + \omega_o + n\omega_f)]. \quad (48)$$

As for the FM case, the amplitudes of the spectral components are proportional to Bessel functions. Indeed, the only difference between the form of the PM signal spectrum and that of the FM spectrum is the progressive phase shift of spectral lines by $\pi/2$, as can be seen by comparing Eq. (38) with Eq. (48). Since Eq. (35) again applies, only $2(\beta_{PM} + 1)$ side frequencies about $+\omega_o$ and $-\omega_o$ are significant. The bandwidth is thus again given by the right side of Eq. (39) with β_{FM} replaced by β_{PM}.

Although both FM and PM bandwidth expressions appear to be identical, there are some important differences. They may be compared by writing them as shown

$$W_{FM} = 2(\Delta\omega + \omega_f) \rightarrow 2\Delta\omega, \qquad \beta_{FM} >> 1, \quad (49)$$

$$W_{PM} = 2(\Delta\theta + 1)\omega_f \rightarrow 2\Delta\theta\omega_f, \qquad \beta_{PM} >> 1. \quad (50)$$

For fixed $\Delta\omega$ in wideband FM, bandwidth is approximately constant at $2\Delta\omega$ regardless of how ω_f varies. In PM with $\Delta\theta$ fixed, the bandwidth increases with increasing ω_f. On the other hand, if ω_f is fixed and modulation index is allowed to vary by changing $\Delta\omega$ or $\Delta\theta$, the behavior of both FM and PM spectra is similar. Both of these effects result from the modulation index's being independent of ω_f in PM but not in FM. As in FM, when the PM modulation index is small, $W_{PM} \cong 2\omega_f$. The PM power density spectrum is identical to that of FM in form and is given by Eq. (42) with β_{FM} replaced by β_{PM}. It is worthwhile to note here that phase modulation appears to be the most attractive for tracking and communication purposes [3]. This is because:

a). a carrier component is available for Doppler recovery (not true in FM),
b). a large amount of carrier power can be converted to sideband power,
c). nonlinear amplification does not distort the modulation waveform (not true in AM),
d). the ratio of peak power to average power is unity, providing a more efficient system (not true in AM), and
e). the modulation mechanism is simple and reliable (not true in AM).

2.2 Coded or Digital Modulation [6 to 9]

A method of transmitting an approximate, but adequate representation of a continuous signal wave, sending a succession of discrete numerical values was proposed by Rainey in 1926 (U. S. Patent 1608527, Nov. 30, 1926) [9]. A patent applying Rainey's principle to speech transmission was obtained by Reeves in 1939 (French Patent 853183, Oct. 23, 1939) [9]. The system, which has since come to be known as pulse code modulation (PCM), is basically quantized pulse amplitude modulation (PAM). Note that in pulse modulation systems such as PAM, pulse duration modulation (PDM), and pulse position modulation (PPM) systems, only time is quantized, whereas the respective modulation parameters (namely, pulse amplitude, duration, and position) are varied in accordance with the message in a continuous manner. On the other hand, in the PCM systems, instead of sending the actual values of the samples, a PCM transmitter selects the nearest approximations from an allowable discrete set of values. In the actual transmission, the resulting sequence of discrete numbers can be expressed in whatever notation is most convenient. The simplest and most widely used form is a binary representation. A binary code is a finite sequence of zeros and ones. Binary is the most commonly used form of PCM. The most important reasons for this choice are that

a). binary PCM is the most noise immune form [4] and
b). binary PCM can be processed quite easily using modern digital circuits.

PCM systems are considerably more complex than PAM, PDM, and PPM systems, in that the message signal is subjected to a greater number of operations. The essential operations in the transmitter of a PCM system are sampling, quantizing, and encoding. The quantizing and encoding operations are usually performed in the same circuit. The essential operations in the receiver are regeneration of impaired signals, decoding, and demodulation of the train of quantized samples. Regeneration usually occurs at intermediate points along the transmission route as necessary. When multiplexing is adopted, it becomes necessary to synchronize the receiver to the transmitter for the overall system to operate satisfactorily.

2.2.1 Sampling Process and Aliasing

The process of sampling leads a continuous data signal to select a finite number of points to represent it in the digital domain. To do this, the system captures small sections of the signal at periodic intervals to obtain amplitude-modulated, zero-width pulses whose envelope conforms to the analog signal. The sampling theorem tells how often you must take such samples: if a continuous bandwidth-limited signal contains no frequency components higher than f_c (Nyquist frequency), the original signal can be recovered without distortion if it is sampled at $2f_c$ samples/second or more. The sampling theorem shows the frequency spectrum of a continuous bandwidth-limited analog signal whose frequency components range as high as f_c, along with the spectrum resulting from sampling the signal at a frequency f_S ($f_S > f_c$). This latter spectrum reveals that sampling is a modulation process that replicates and shifts the original spectrum so that it and its copies are centered on f_S, $2f_S$, $3f_S$, and so on.

If f_S is not sufficiently large, a portion of the spectrum centered on f_S folds over (i.e., aliases) into the original signal spectrum. This process results in a signal component characterized by an alias frequency. Such alias frequencies can be substantially different from the original frequency. Once aliasing occurs, attempts to recover the original signal from the sample result in distortion caused by the folded part of the spectrum. Such distortion cannot be removed by filtering the recovered signal. Three methods can limit aliasing, however. One can use a higher sampling rate, a post filter, or a sharper filter that limits the analog signal's bandwidth. In the real world, though, some folding always occurs because of the original signal's high-frequency components and noises as well as the post filter's nonideal characteristics.

Besides sampling rate, a sampling analyst must consider the length of time. The sampling pulse is required to take a slice out of the analog input signal and produce the amplitude-modulated samples. Termed aperture time, this quantity has a maximum permissible value that depends on the analog input's time variation (slew rate) and the required accuracy. Aperture time is not usually precisely the same from sample to sample; the variation is termed aperture uncertainty.

2.2.2 Quantization Process and Performance Limitation

After a signal is sampled, a quantizer (A/D converter) must convert it to a digital format (usually a binary number). In an actual system, the quantizer accepts time-varying sampled inputs and produces a digital output code. The midriser characteristic has a threshold level with a zero value, and the midtread characteristic has a zero quantization level. The latter form is generally preferable because it furnishes an output insensitive to small input changes around zero. Both characteristics introduce a maximum error of $\Delta/2$, where Δ is the separation between adjacent threshold levels correspoding to the specified quantization level.

An important aspect of any quantizer is its resolution. As this resolution increases, the discrepancy between the applied analog input and the quantized output decreases. This quantization error is the minimum error the quantizer allows and can only be reduced by increasing the device's number of output states (resolution). The process of quantization leads to unavoidable error. Clearly, information is lost in the process of rounding off exact sample values to the nearest quantum level. Perfect recovery of the message by the receiver is therefore impossible in PCM, even if noise is absent. Thus, a quantization error exists in PCM and is the basic limitation in performance. Quantization and sampling produce the same result as sampling and quantizing.

The quantized message $f_q(t)$ is the sum of the message f(t) and an error $\varepsilon_q(t)$:

$$f_q(t) = f(t) + \varepsilon_q(t). \tag{51}$$

In the receiver, only $f_q(t)$ can be reconstructed from the PCM signal, so that $\varepsilon_q(t)$ is fundamental to overall performance. To illustrate this fact, the receiver error $\varepsilon_q(t)$ may be treated as a noise. The maximum possible output signal-to-noise ratio is then the ratio of $E[f^2(t)]$ to quantization error noise power $E[\varepsilon_q{}^2(t)]$:

$$(S_o/N_q)_{PCM} = E[f^2(t)]/E[\varepsilon_q{}^2(t)], \tag{52}$$

where E[.] denotes a sample mean. The evaluation of $E[\varepsilon_q{}^2(t)]$ involves averaging $\varepsilon_q{}^2(t)$ over all time range.

2.2.3 Bit Error Probability

The matched filter implementation of the PCM reconstruction stage leads to the optimum receiver. Such a receiver minimizes the number of errors made in deciding which pulse level was transmitted. Unfortunately, one cannot entirely eliminate errors, and a finite probability P_e exists that an incorrect level decision will be made. A wrong decision is called a bit error, and the corresponding probability is the bit error probability. To determine P_e, let's consider the two possible kinds of errors separately. Assume first that the symbol 0 was transmitted, corresponding to a level of zero volts. The received signal is then simply

$$v(t) = n(t). \tag{53}$$

Let V_t denote the random variable obtained by observing the random process V(t), of which v(t) is a sample function, at time t. The probability of error, in this case, is simply the probability that $V_t > A/2$. The random variable V_t is defined by

$$V_t = N_t, \tag{54}$$

where N_t is a Gaussian-distributed random variable with zero-mean and variance σ^2. It is apparent that V_t is also a Gaussian-distributed random variable with zero-mean and variance σ^2. Therefore, given that a 0 was transmitted, the conditional probability density function of V_t is given by

$$P(v_t/0) = (1/\sqrt{2\pi})*\exp(-v_t{}^2/2\sigma^2). \tag{55}$$

The probability of error is defined by the area under the $P(v_t/0)$ curve, from A/2 to infinity, which corresponds to the integral of the probability density function as

$$P_e = (1/2)*\mathrm{erfc}(V/\sqrt{2s}) \quad \text{for polar PCM} \tag{56}$$

and

$$P_e = (1/2)*\mathrm{erfc}(V/\sqrt{2s}) \quad \text{for unipolar PCM} \tag{57}$$

where erfc(.) is the complementary error function defined in a standard text [7]. The quantity inside the erfc function can be replaced by the signal-to-noise ratio of the bit energy transmitted.

2.2.4 Encoding

A quantized sample could be sent as a single pulse that would have certain possible discrete amplitudes, the resultant system being a modified form of PAM. However, if many allowed sample amplitudes are required, it would be difficult to make circuits to distinguish these one from another. On the other hand, it is very easy to make a circuit which will tell whether or not a pulse is present. Suppose then that several pulses are used as a code group to describe the amplitude of a single sample. In general, in an m-ary PCM system, any one quantized signal sample is coded into a group of n pulses each with m possible levels. The number of quantized amplitude levels the code group can express including zero level is given by

$$L = m^n. \tag{58}$$

Note here that the number L can be identified with the possible number of different messages of the information source, each message consisting of n samples using an alphabet of m symbols. For the purpose of encoding, one characterizes the L discrete amplitude levels by integers, the state numbers 0, 1, 2, ... (L-1). The state numbers are then considered to be one-digital numbers of a number system of base L. The process of encoding consists merely in encoding these numbers into another number system of base m.

2.2.5 Channel Bandwidth

A PCM system requires more bandwidth and less power than is required with direct transmission of the signal itself. In a sense, a bandwidth is exchanged with power. A good measure of the bandwidth efficiency is the information capacity of the system as compared with the theoretical limit for a channel of the same bandwidth and power. The information capacity of a system may be thought of as the number of independent symbols or discrete amplitude levels that can be transmitted without error in unit time. The simplest, most elementary character is a binary digit, and it is convenient to express the information capacity C as the equivalent number of binary digits per second that the channel can handle. A theorem due to Shannon states that by using sufficiently complicated encoding systems, it is possible to transmit binary digits at a rate given below in Eq. (59) with as small a frequency of errors as desired. Furthermore, it is not possible by any encoding method to send at a higher rate and have an arbitrarily low frequency of errors [14]. Thus,

$$C = B^* \log_2(1 + P/N) \text{ bits/second,} \tag{59}$$

where C is the channel capacity, the maximum rate of transmitting information, B is the system bandwidth, P the carrier power, and N the average noise power. Since the quantity $TB^*\log_2(1 + P/N)$ gives, for large T (sampling period), the number of bits that can be transmitted in time T, it can be considered as an exchange relation between the different parameters.

For example, one can increase the capacity of the system by increasing parameters T, B, and P, and N can be varied without changing the amount of information one can transmit, provided $TB*\log_2 (1 + P/N)$ is held constant. It should be pointed out, however, that these parameters cannot be varied at will without regard to the modulation system employed. In amplitude modulation (AM), for example, the channel bandwidth should be equal to the message bandwidth. For a given message, a greater channel bandwidth does not improve the transmission and does not permit the use of a lower carrier-to-noise ratio. A channel bandwidth narrower than the message bandwidth will inevitably result in loss of part of the information, and this loss cannot be compensated for by increasing the carrier-to-noise ratio.

3. RELATIONSHIPS BETWEEN PARAMETERS AND MODULATION INDEX

As discussed in the previous sections, a communication system's performance depends heavily on the goals of the designer and on the theoretical limitations imposed by several constraints that must trade off each system requirement against all of the others. This section investigates the sensitivity of the modulation index based on parameters such as transmitter/receiver system power, system bandwidth, bit error rate, and signal-to-noise power ratio. Most analysis results are based upon graphical traces obtained from the equations presented in Section 2. Each category of parameter dependency is briefly described with partial plots rather than comprehensive coverage of that category because of limited resources provided on this task. A result of a system simulation for the Deep Space Program Science Experiment (DSPSE) is also presented with respect to bit error rate, ranging, and telemetry performance.

3.1 System Bandwidth vs Modulation Index [15 to 21]

In most cases, the system bandwidth means 3-dB bandwidth. It is defined as the frequency interval over which the system transfer function magnitude $|H(\omega)|$ remains above $1/\sqrt{2}$ times its mid-band value. The 3-dB bandwidth of a signal is defined on a similar basis using the magnitude of the signal spectrum $|F(\omega)|$. There are several different definitions of bandwidth, such as fractional power containment bandwidth (99% of the signal power is inside the occupied bandwidth), bounded power spectral density, noise bandwidth, half-power bandwidth, Gabor bandwidth, null-to-null bandwidth, and FCC (Federal Communications Commission) spectrum envelope. Other classifications defined by the CCIR (International Radio Consultative Committee) are baseband bandwidth, necessary bandwidth [14], bandwidth expansion ratio, occupied bandwidth, x-dB bandwidth, and spectrum emitted outside the necessary bandwidth. The half-power bandwidth, noise bandwidth, and null-to-null bandwidth are widely accepted for continuous modulation while most of the bandwidth definitions are adopted for digital modulation. For an AM process in Eqs. (7) and (10) and for narrowband angle modulation in Eq. (26), the spectrum is directly proportional to the modulation index while the spectrum has a nonlinear relation with the modulation index as in Eqs. (38), (42), and (48) for the wideband angle modulation. For example, an important parameter of the system is the normalized bandwidth of the modulated signal v(t). The power spectrum for this digital random process is given by V(x) [14] as

$$\begin{aligned} V(x) &= (1/4)*\delta(x + \pi\beta_{FM}) + (1/4)*\delta(x - \pi\beta_{FM}) \\ &+ (2/R)[\beta_{FM}/(\pi(\beta_{FM}^2 - x^2))]^2*(1 - \cos(\pi\beta_{FM})\cos\pi x) \end{aligned} \tag{60}$$

where R denotes a bit rate, $x = 2(f - f_c)/R$ = normalized frequency variable, $\beta_{FM} = 2D/R$, the modulation index of the frequency modulation (or FSK) with D the peak deviation that equals half the difference between the maximum and minimum values of the instantaneous frequency, and R the data rate (bit rate). Figure 5 plots Eq. (60) with respect to modulation index. Power spectral density heavily depends on modulation index. Note here that a serious consideration should be made before

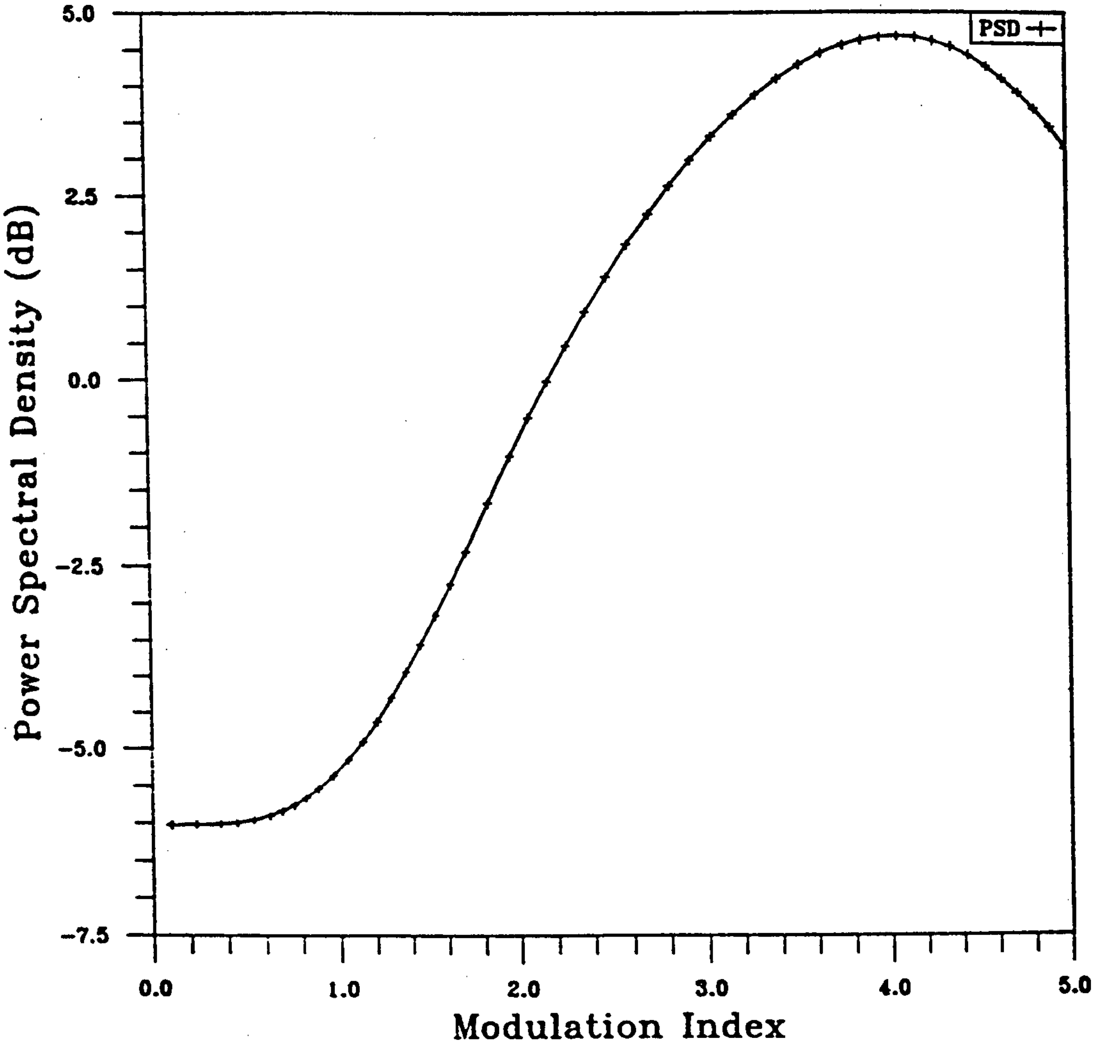

Fig. 5 — Power spectral density function of frequency-shift-keying as a function of modulation index

choosing maximum power spectral density 4.5 dB for b_{FM} = 3.8 from Fig. 5. A trade-off must be made with other link parameters for optimum applications of modulation index.

Figures 6 through 8 are plotted using Eq. (61) below, and show the relationship between the bandwidth (percentage necessary bandwidth) and modulation index for the different modulation index ranges from CCIR Recommendation 328-5 [24]. Necessary bandwidths are inversely varying with exponential functions. The formulas listed in CCIR Recommendation 328-5 for the necessary bandwidth of F1B modulation [24] are

$$
\begin{aligned}
&3.86D + 0.27R && \text{for } 0.3 < \beta < 1.0 \\
&2.6D + 0.55R && \text{for } 1.5 < \beta < 5.5 \\
&2.1D + 1.9R && \text{for } 5.5 < \beta < 20.0
\end{aligned}
\tag{61}
$$

F1B is specified in CCIR Record 328-5 [24] and Report 179-1. F1 modulation is a telegraphic radio telephone FSK emission using binary or quantized digital information.

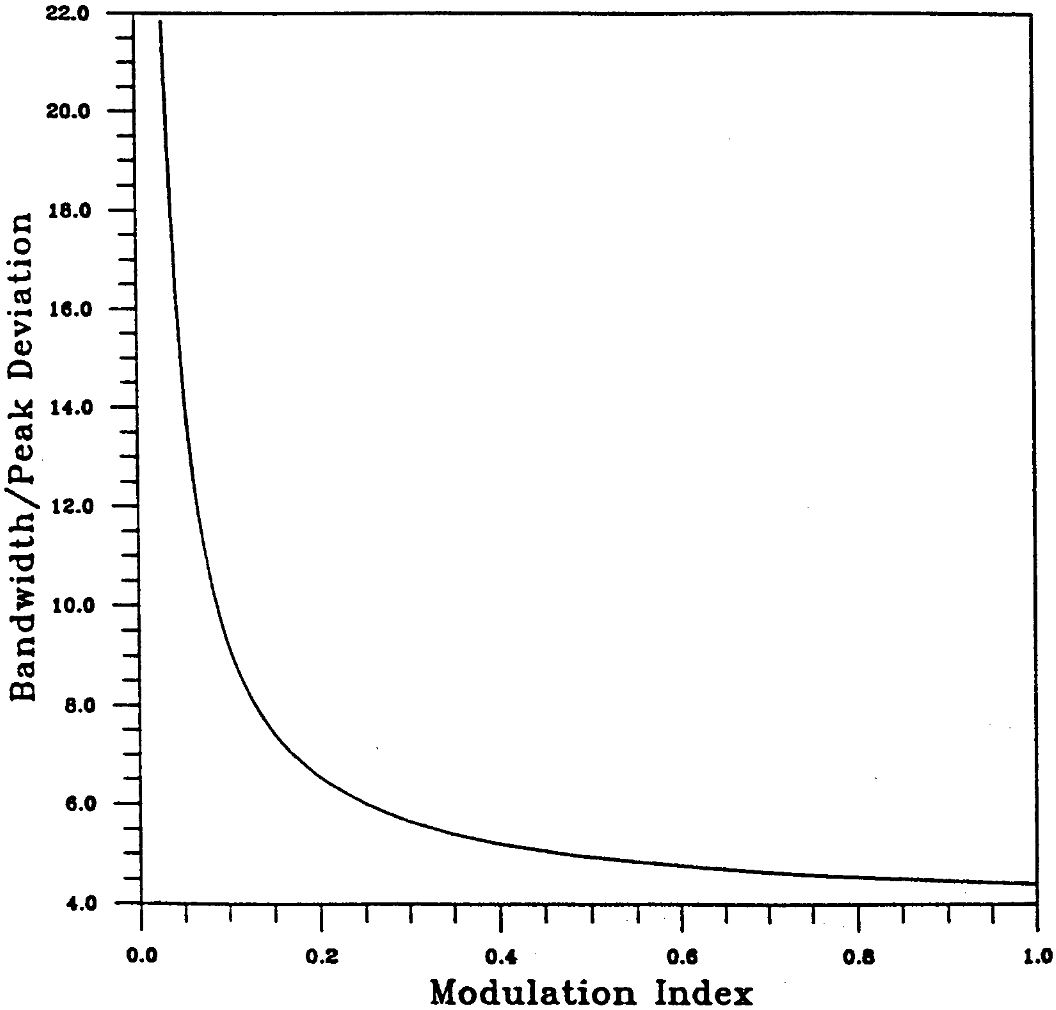

Fig. 6 — Necessary bandwidth plot for frequency-shift-keying as a function of modulation index (range 0.0 to 1.0)

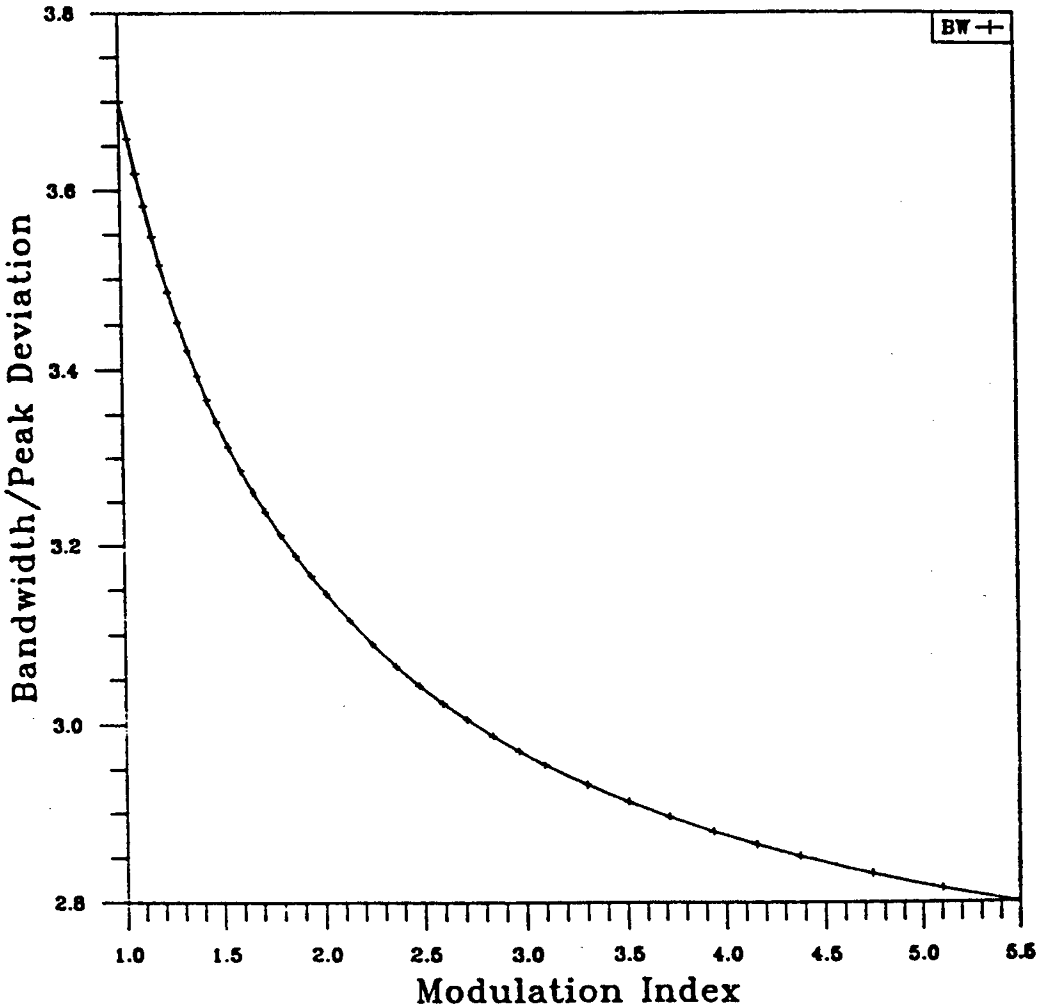

Fig. 7 — Necessary bandwidth plot for frequency-shift-keying as a function of modulation index (range 1.0 to 5.5)

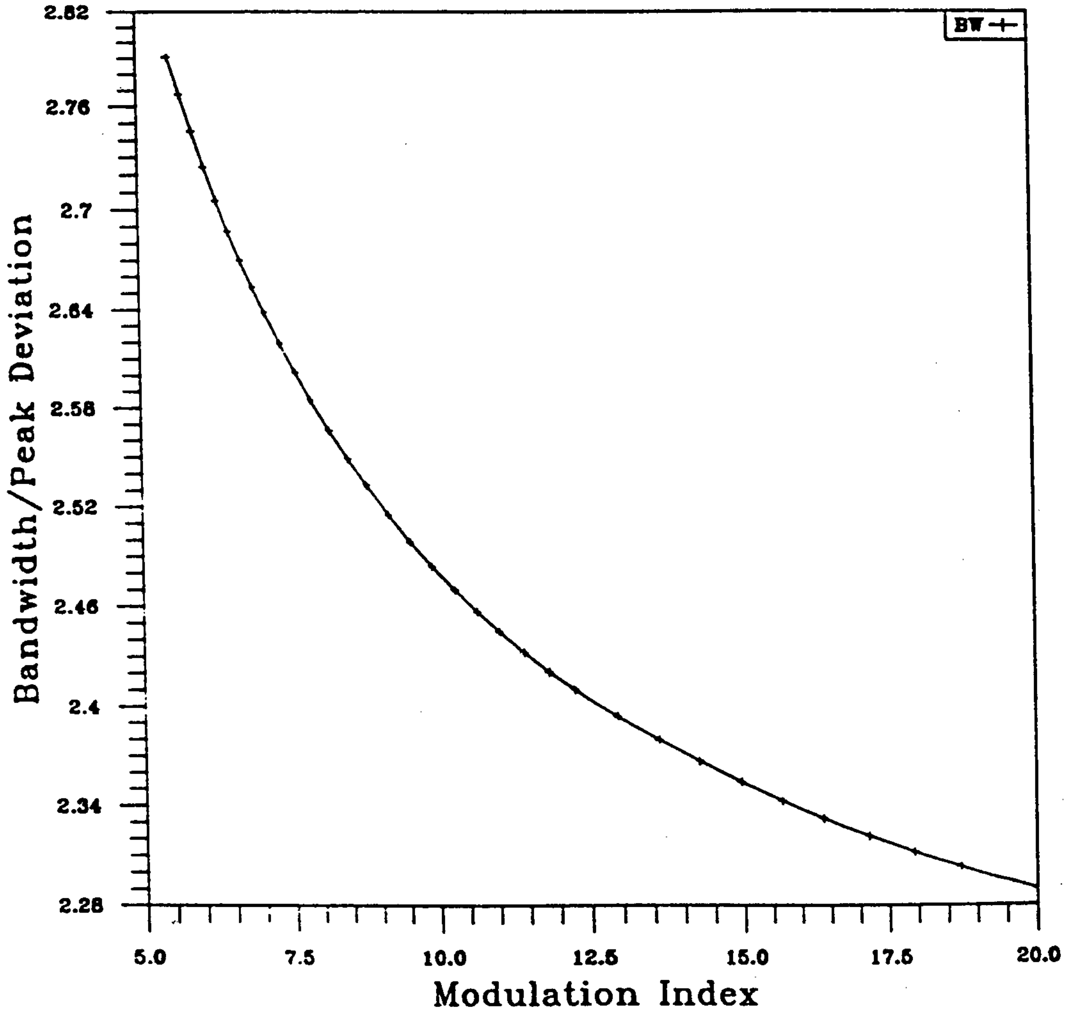

Fig. 8 — Necessary bandwidth plot for frequency-shift-keying as a function of modulation index (range 5.0 to 20.0)

Figures 9 and 10 show the two additional bandwidths [14] necessary for MSK and sinusoid FSK (SFSK), respectively, with respect to bit rates. As expected, necessary bandwidths are directly proportional to bit rate increases.

If the modulation index increases, the necessary bandwidth of the communication channel decreases inversely with exponential functions. Notice that increasing the amplitude of the modulating signal should increase the bandwidth occupied by the FM signal. Increasing the modulating signal amplitude corresponds to increasing the frequency deviation or the modulation index. Thus, one can conclude that the bandwidth of the FM wave depends on the modulation index. Also note that the average power in the frequency-modulated wave is independent of the modulating signal. This implies that increasing power in the sideband frequencies must be accompanied by a corresponding decrease in the power associated with the carrier. This accounts for the decrease in the carrier amplitude. Exact mathematical expressions for the relation between bandwidth and modulation index are given in Eqs. (42) and (48) for FM and PM, respectively.

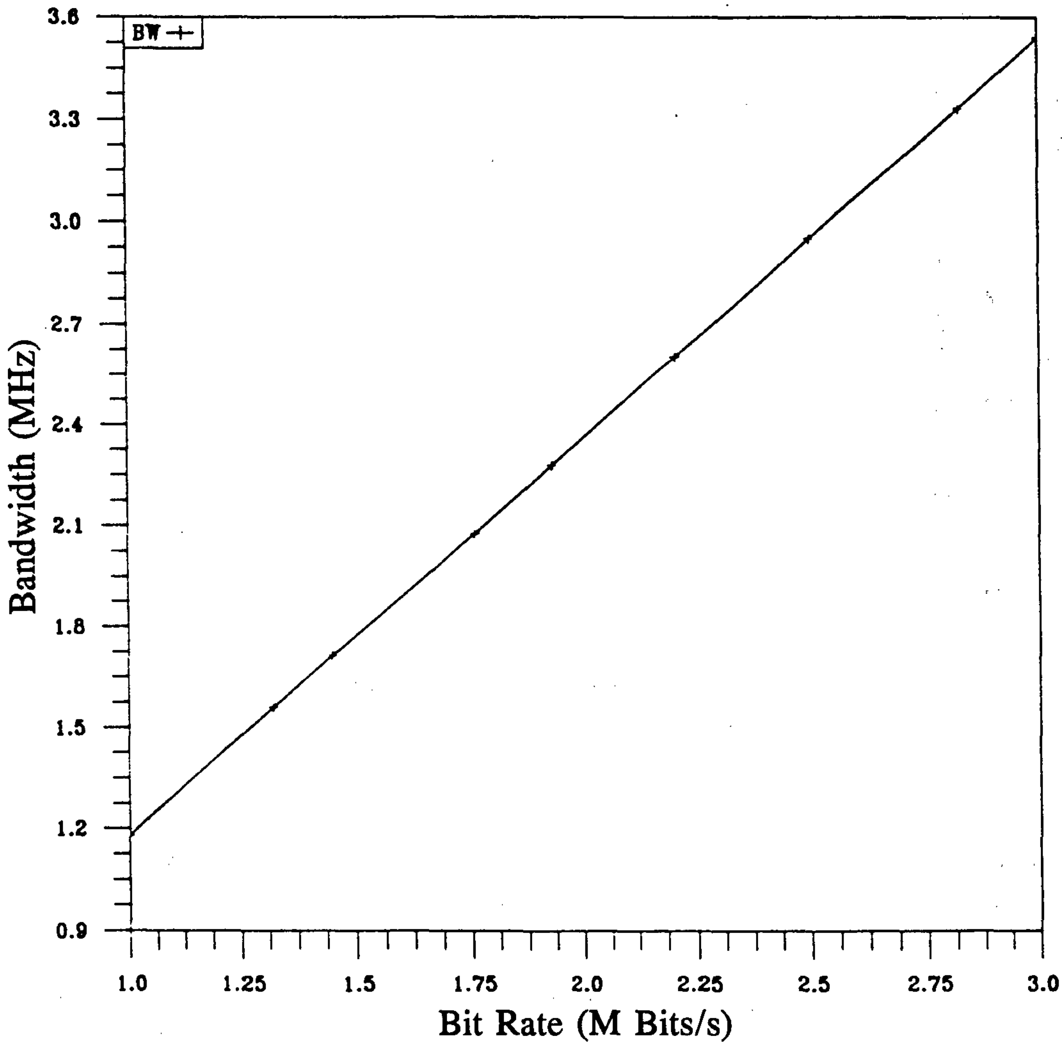

Fig. 9 — Necessary bandwidth plot for minimum-shift-keying

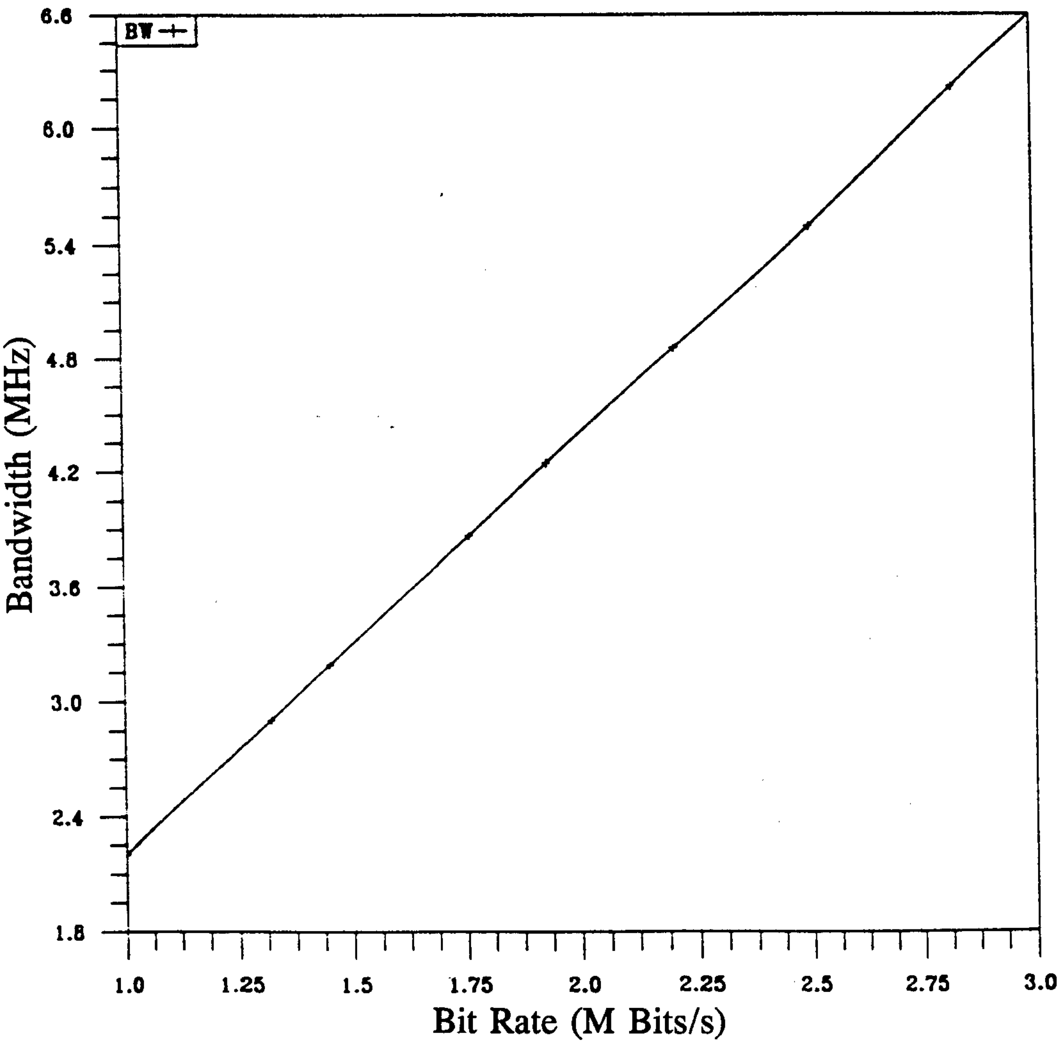

Fig. 10—Necessary bandwidth plot for sinusoidal frequency-shift-keying

3.2 Probability of Bit Error vs Modulation Index

The presence of Gaussian noise in a digital communications systems results in errors of bit transmission. An important measure of performance used for comparing digital modulation schemes is the probability of error P_e as defined in Eqs. (56) and (57). The bit error probability (or probability of bit error P_b, sometimes called the bit error rate) has been plotted against the signal-to-noise ratio (bit energy-to-noise ratio) in most communications texts [22] as shown in Fig. 11, for example. Since high-speed logic and the advent of fiber-optic data links enable increased baud rates that require bit error probabilities lower than 10^{-7}, Fig. 11 has been extended to signal-to-noise ratios greater than 12 dB unlike the conventional plots. Digital data is transmitted by any of several modulation schemes. The most common transmission is by frequency shift keying (FSK), or its variant, phase shift keying (PSK). FSK can either be coherent or incoherent. Pulse code modulation is frequently encountered and can be either polar or unipolar, as presented in Eqs. (56) and (57).

A modulated carrier, in cases of both PCM/PSK/PM and PCM/PM, may be (assuming nothing else modulates the carrier) represented mathematically by

$$v_T(t) = \sqrt{2P} * \sin[\omega_o t + \Sigma_i \beta_i s_i(t)] \tag{62}$$

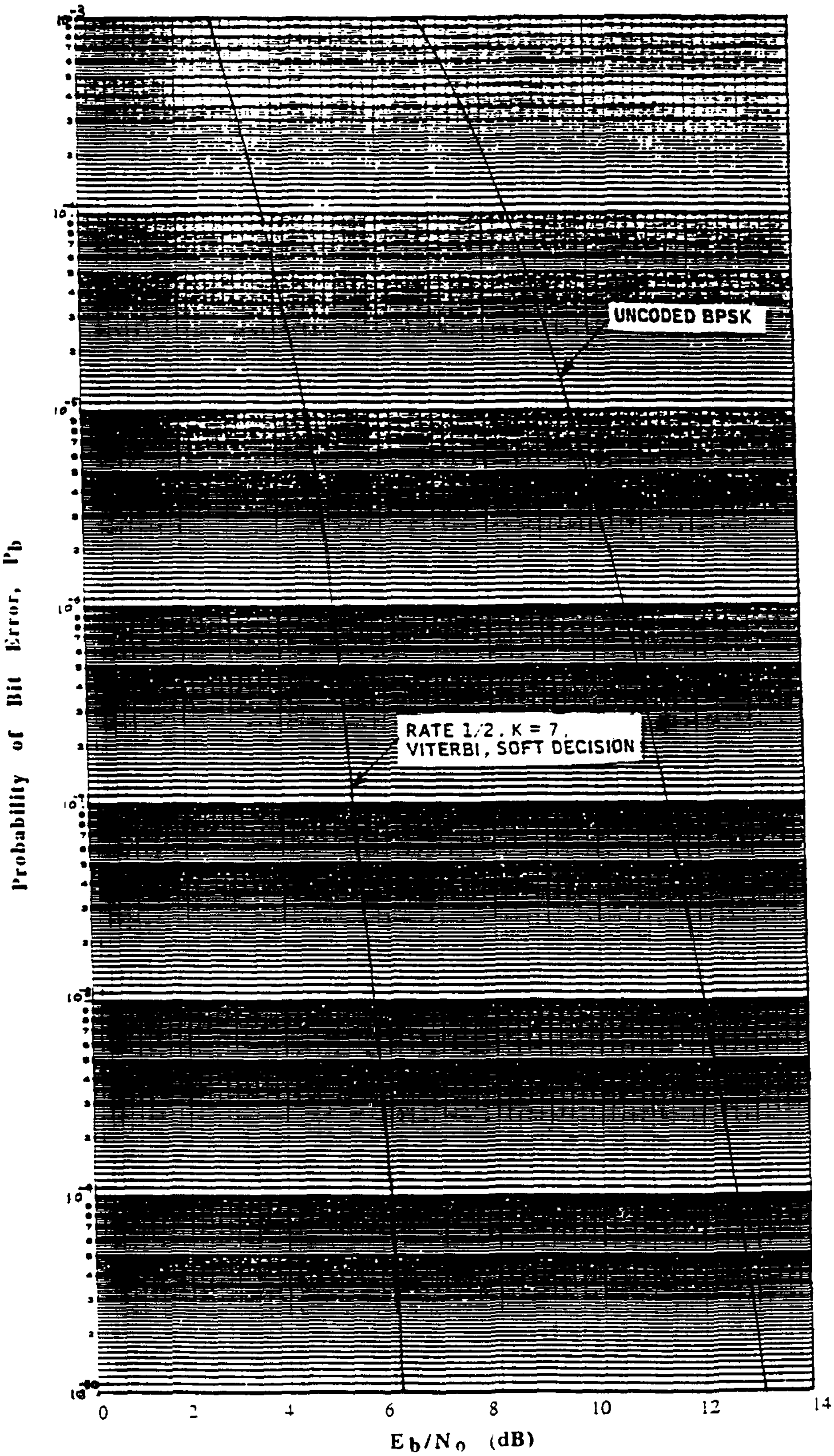

Fig. 11 — Bit error probability vs bit energy-to-noise power spectral density for Viterbi coded and uncoded BPSK

where P is the total power, β_i the modulation index for i = 1, 2, 3, ..., M, M the total number of subcarriers or data channels, and $s_i(t)$ represents a normalized data sequence (in case of PCM/PM) or a normalized modulated square-wave subcarrier (in case of PCM/PSK/PM). In the latter case, the telemetry is invariably phase reversal keyed (i.e., phase shifted keyed with signaling levels of $\pm 90^\circ$) onto the subcarrier. The equations used to calculate the bit error probability for those modulation methods are presented below without any effort to derive since they are almost standardized [10].

$$P_e = (1/2) * \mathrm{erfc}\,(\sqrt{S/4N}) \qquad \text{for FSK (coherent)/ASK/PCM} \tag{63}$$

$$P_e = (1/2) * \exp(-S/2N) \quad \text{for FSK (incoherent)} \tag{64}$$

$$P_e = (1/2) * \mathrm{erfc}\,(\sqrt{S/N}) \qquad \text{for PCM (polar)/PSK} \tag{65}$$

$$P_e = (1/2) * \mathrm{erfc}[(1/2)*\sqrt{S/N}] \qquad \text{for OOK (on-off keying)} \tag{66}$$

$$P_e = (1/2)*\exp(-S/2N)*[1 + 1/\sqrt{2\pi S/N}] \qquad \text{for ASK (incoherent)} \tag{67}$$

$$P_e = (1/2) * \exp(-S/N) \qquad \text{for DPSK} \tag{68}$$

$$P_e = (1/2) * \exp(-2S/N) \qquad \text{for BPSK} \tag{69}$$

where S/N is the signal-to-noise power ratio. If a signal power is replaced by a bit energy E_b, the probability of error can be represented with a bit energy-to-noise ratio without losing any generality. Figure 11 shows an example of probability of bit error vs bit energy per noise power spectral density for uncoded BPSK and coded DPSK (or Viterbi coded).

Simulation results of DSPSE communication links are presented based upon the relation of Eqs. (62) through (69) for both a high-gain antenna and an omni antenna. For the antennas of 26 m, 30 m, 34 m, and 70 m ground station, the probability of bit error has been plotted against modulation indexes as shown in Figs. 12 through 18. Each graph was plotted to cover the minimum bit error rate as a function of modulation index. During simulations, the bit energy-to-noise ratio was recorded for each different value of modulation index. The probability of error was calculated through Eqs. (62) through (69). As can be seen in the figures, the optimum choices of modulation indexes are in the range of 1.2 to 1.8 for the DSPSE program. It is assumed that the phase-lock-loop bandwidth can be set so that sufficient S/N exists in the carrier-recovery loop. The modulation index for the DSPSE program was set at 1.7.

3.3 Modulation Power Loss vs Modulation Index [8, 11, 12, 22, 23]

If we set M = 1 from Eq. (61) and expand using trigonometric identities, Eq. (61) can be rewritten as

$$v_T(t) = \sqrt{2P} * \cos b_1 \sin w_o t + \sqrt{2P} * s_1(t) \sin b_1 \cos w_o t. \tag{70}$$

If $0^\circ < \beta < 90^\circ$, this phase-modulated carrier comprises a pilot tone (residual tone) and a double-sideband (DSB) modulated carrier. A system with $\beta < 90^\circ$ is called a residual carrier system. A system with $\beta = 90^\circ$ is called a suppressed carrier system. A residual carrier receiver uses a phase-locked loop to track the pilot tone and provide a coherent reference for demodulating the DSB-modulated carrier.

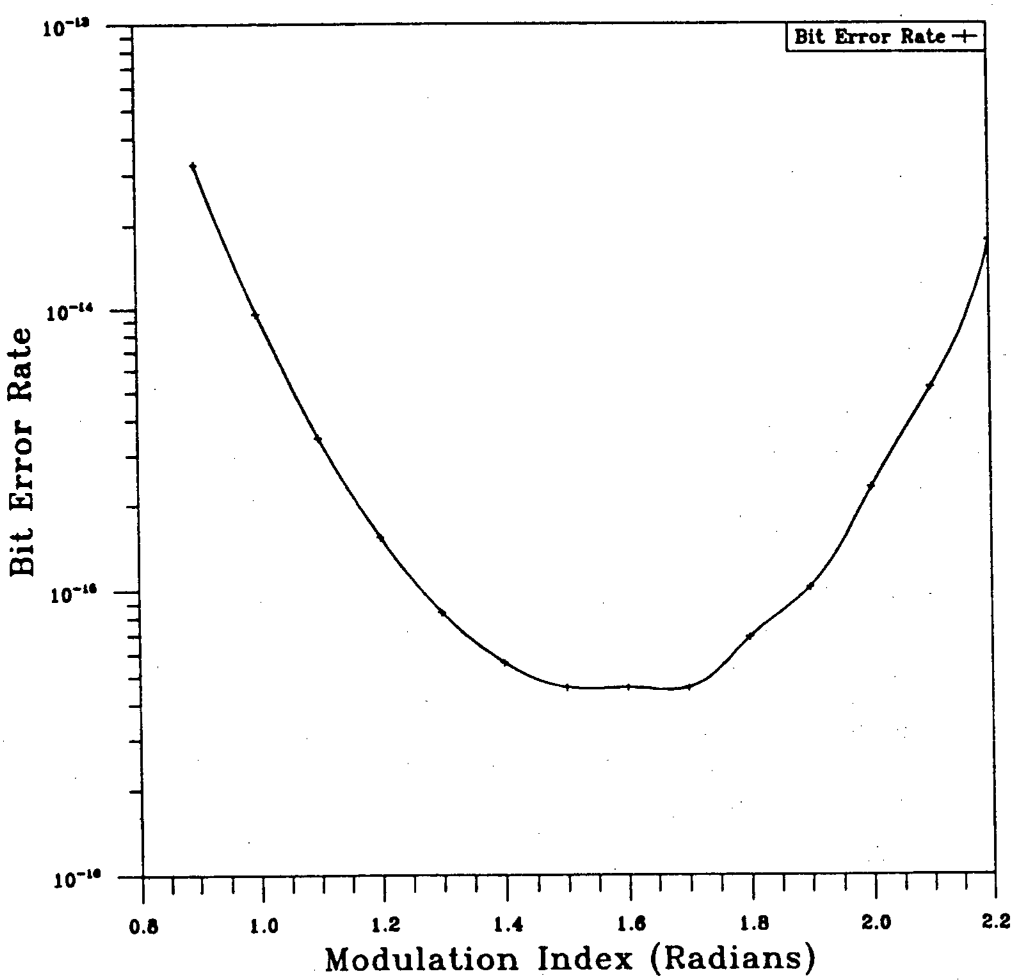

Fig. 12 — Bit error probability for DSPSE lunar mission high-gain antenna for 26 m ground as a function of modulation index

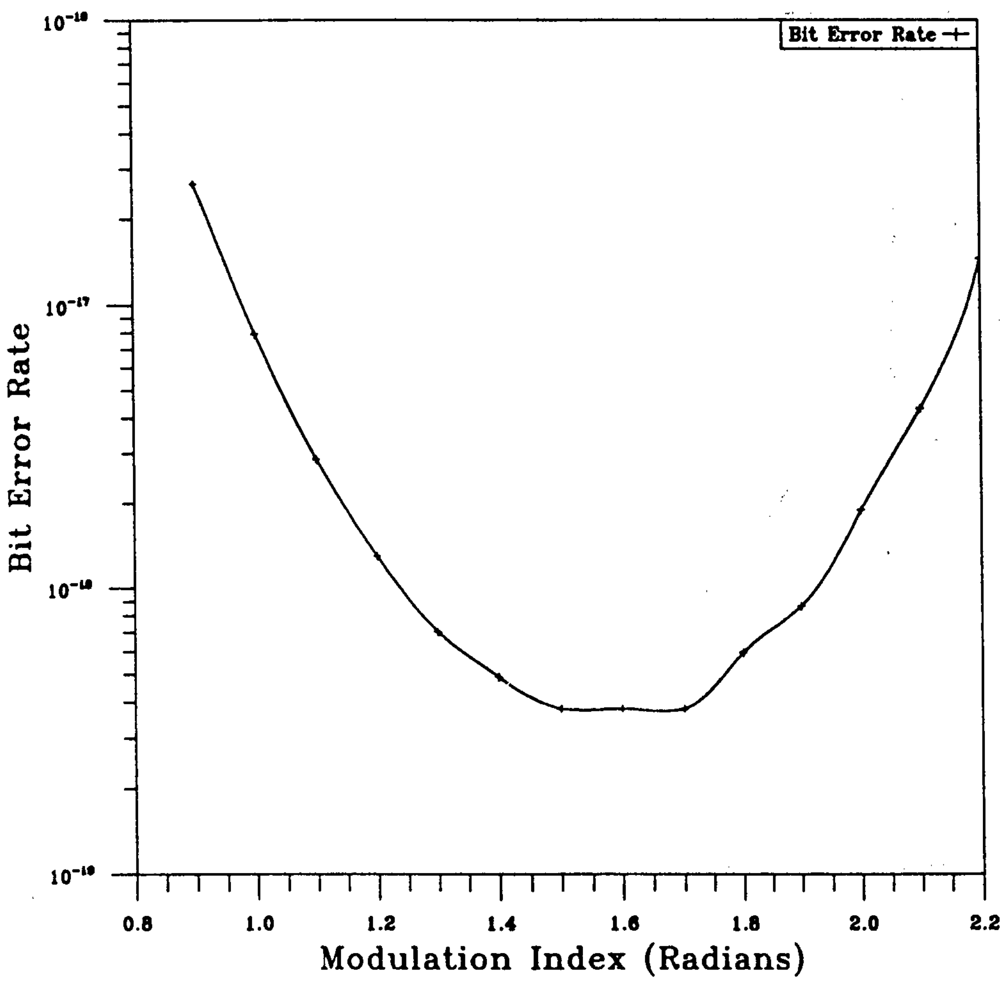

Fig. 13 — Bit error prolbability for DSPSE mission omni antenna for 30 m ground as a function of modulation index

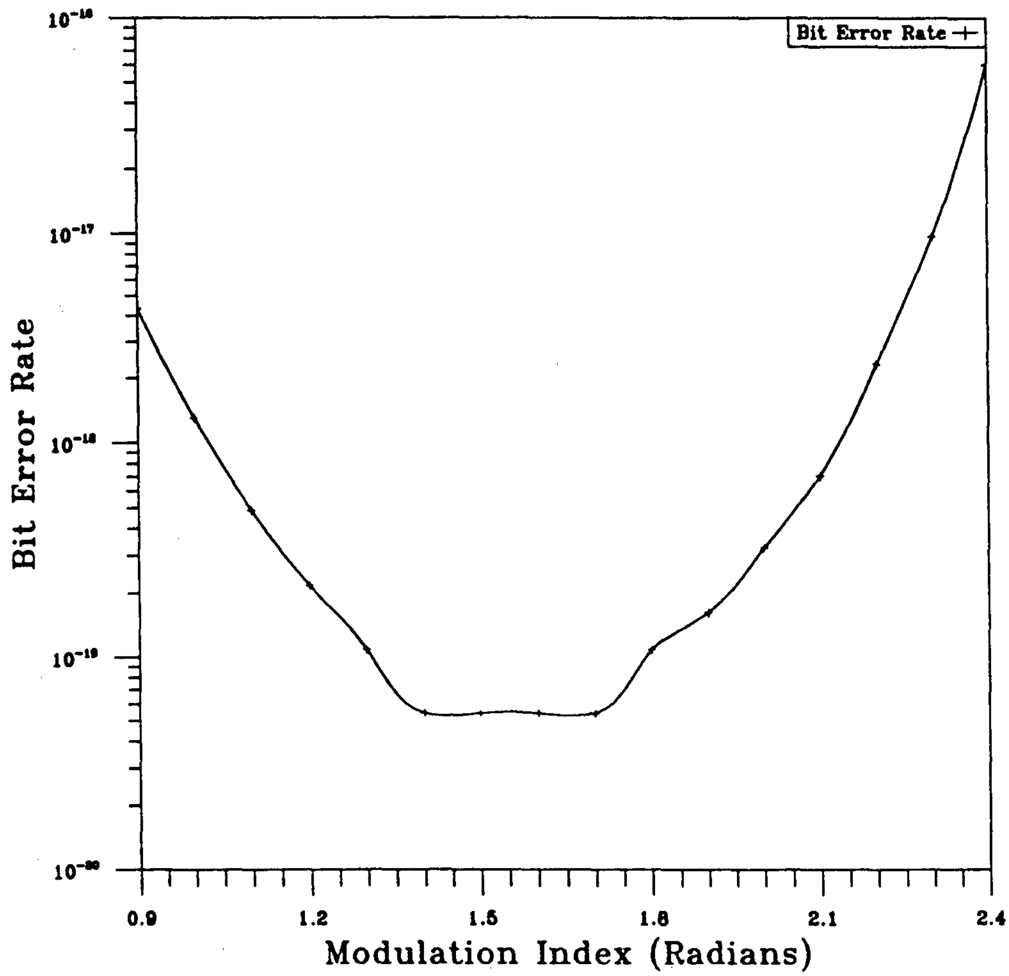

Fig. 14 — Bit error probability for DSPSE lunar mission high-gain antenna for 30 m ground as a function of modulation index

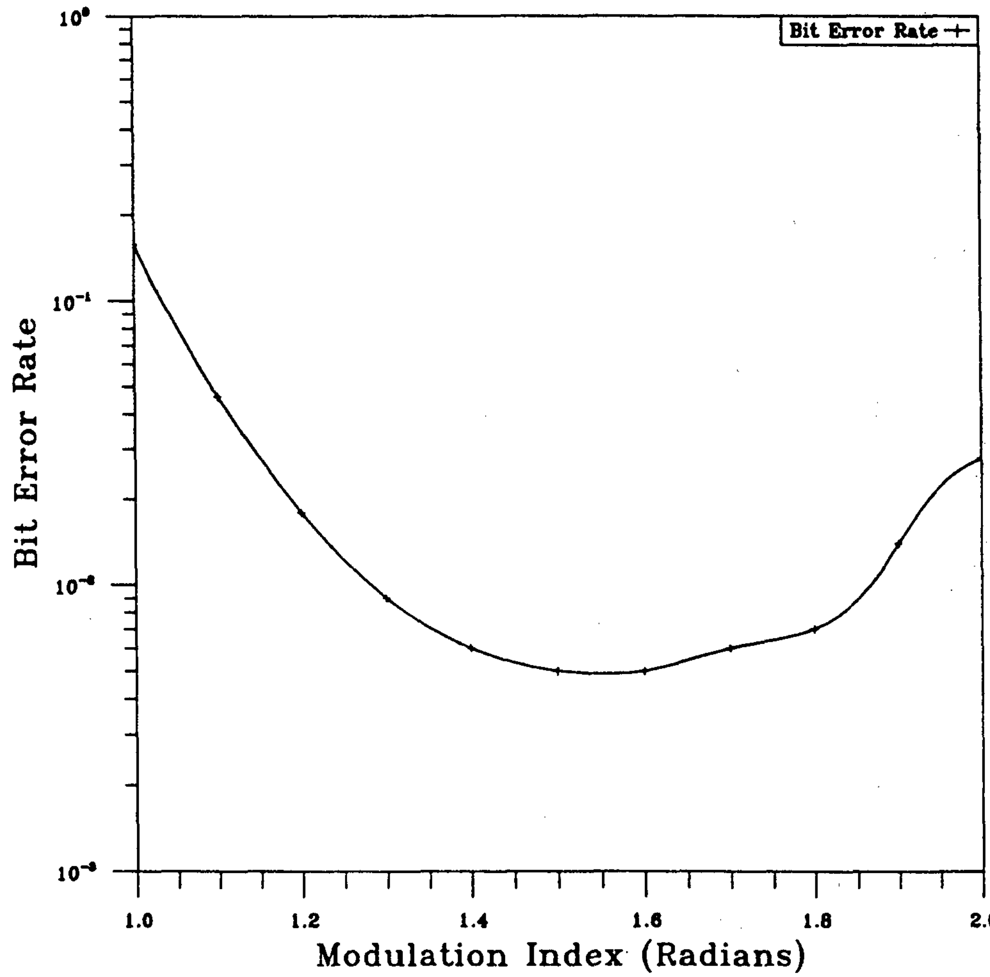

Fig. 15 — Bit error probability for DSPSE asteroid encounter omni antenna for DSN 34 m ground as a function of modulation index

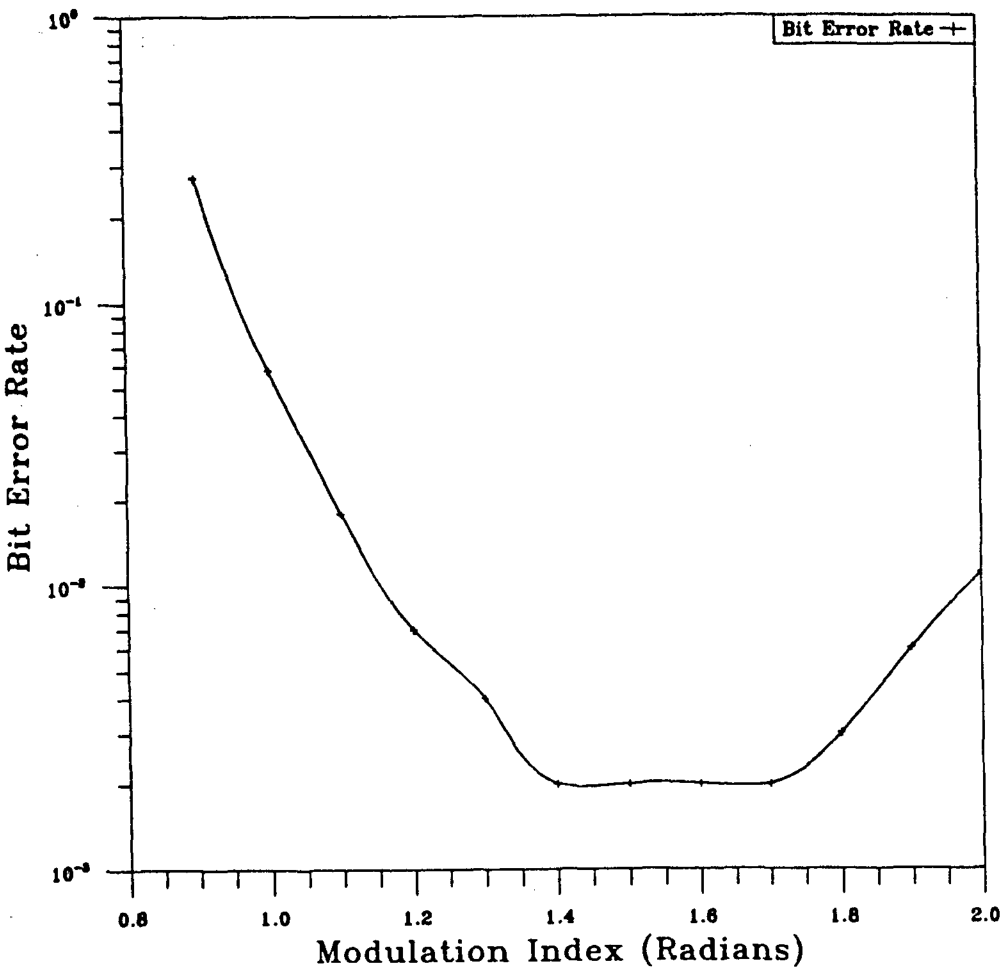

Fig. 16 — Bit error probability for DSPSE asteroid encounter high-gain antenna for DSN 34 m ground as a function of modulation

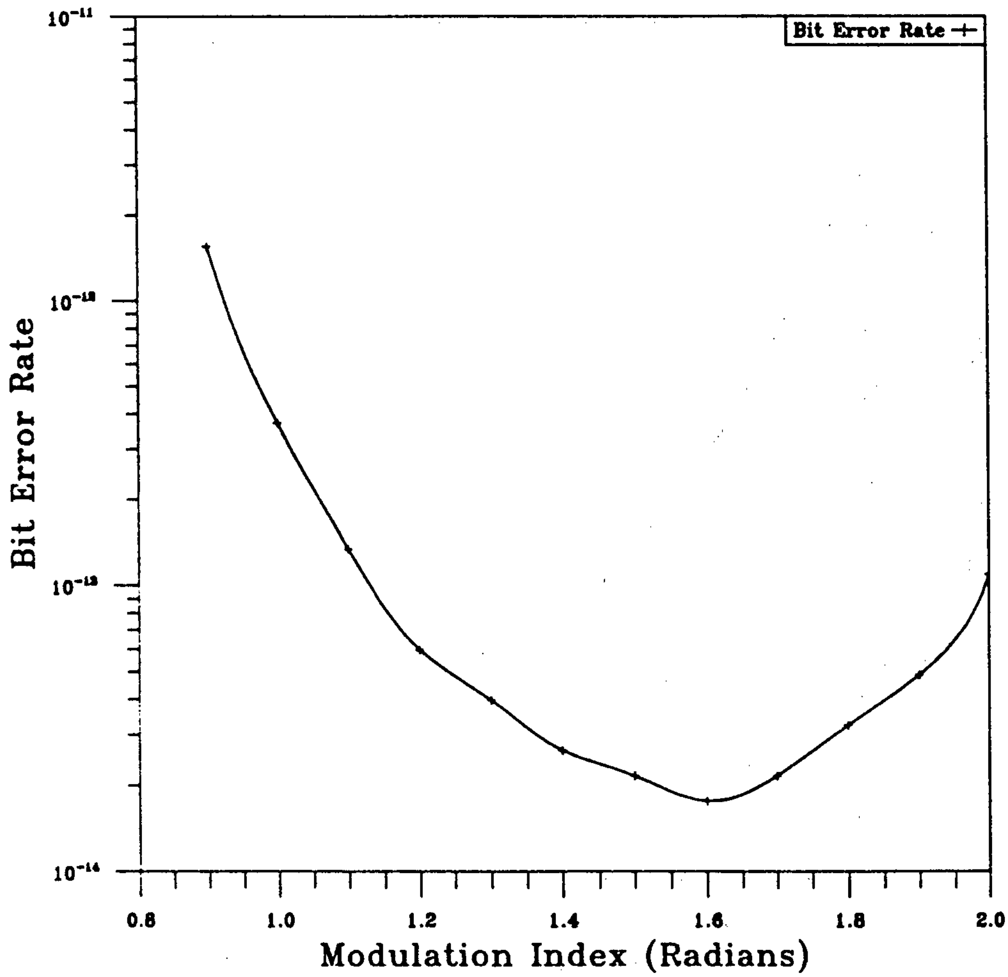

Fig. 17 — Bit error probability for DSPSE lunar mission omni antenna for DSN 70 m ground as a function of modulation index

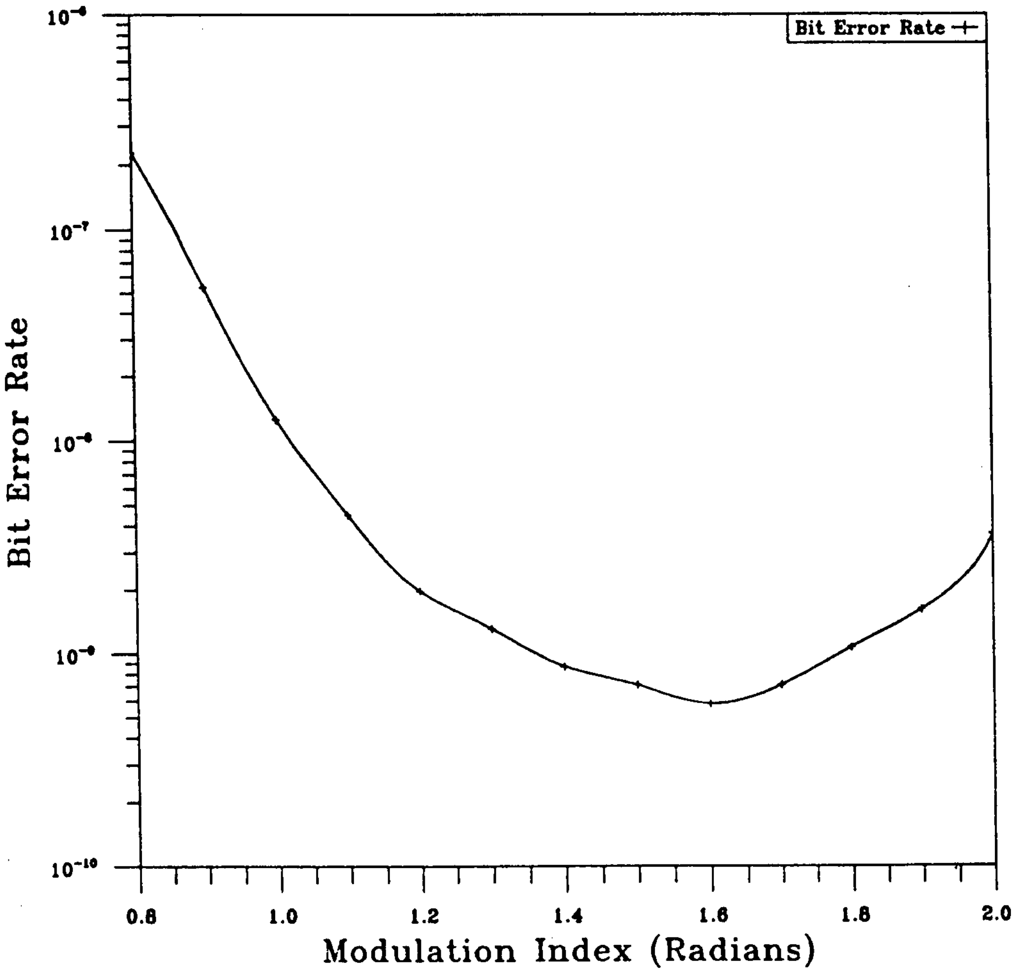

Fig. 18 — Bit error probability for DSPSE asteroid encounter high-gain antenna for DSN 70 m ground as a function of modulation index

A Costas loop may be used as a suppressed carrier receiver. In Eq. (70), the first term is the carrier while the second term is the data channel. Hence, the modulation index β_1 has allocated the total power P_T in the transmitted signal $v_T(t)$ to the carrier and to the data channel, where the carrier power and the data power are given respectively as,

$$P_C = P_T \cos^2 \beta_1, \quad P_D = P_T \sin^2 \beta_1. \tag{71}$$

When we have two data channels, Eq. (62) may be expanded as

$$v_T(t) = \sqrt{2P_T} \cos \beta_1 \cos \beta_2 \sin \omega_o t + \sqrt{2P_T}\, s_1(t) \sin \beta_1 \cos \beta_2 \cos \omega_o t$$
$$+ \sqrt{2P_T}\, s_2(t) \cos \beta_1 \sin \beta_2 \cos \omega_o t - \sqrt{2P_T}\, s_1(t)\, s_2(t) \sin \beta_1 \sin\beta_2 \sin \omega_o t. \tag{72}$$

The first term is the carrier component, the second term is the modulated subcarrier $s_1(t)$ component, the third term is the modulated subcarrier $s_2(t)$ component, and the fourth term is the cross-modulation loss component. Hence, the corresponding powers in these four components are allocated by the modulation indexes β_1 and β_2, with

$$\begin{aligned} P_C &= P_T \cos^2 \beta_1 \cos^2 \beta_2 \\ P_{D1} &= P_T \sin^2 \beta_1 \cos^2 \beta_2 \\ P_{D2} &= P_T \cos^2 \beta_1 \sin^2 \beta_2 \\ P_M &= P_T \sin^2 \beta_1 \sin^2 \beta_2. \end{aligned} \tag{73}$$

Similar interpretations can be given to the cases for $M \geq 3$. Here P_{D1} and P_{D2} are the modulated subcarrier 1 and 2 power, respectively. Finally, P_M is the cross-modulated power loss with P_C the carrier power. The powers presented are based on phase modulation for the telemetry data onto a radio frequency carrier, either directly (PCM/PM) or indirectly through the use of one or two phase-reversal-keyed squarewave subcarriers (PCM/PSK/PM) that have been adopted for the DSPSE project.

Similar relations can be derived for the command data as for the telemetry data. The RF carrier may be phase modulated at modulation indexes from 0.4 to 2.4 radians (peak) for sine wave subcarrier or 0.3 to 1.7 radians (peak) for squarewave subcarriers. The ratios of carrier-power suppression to data-power suppression as functions of modulation index angle are:

1). Sinewave subcarrier

$$P_C / P_T \text{ (dB)} = 10 \log J_o^2(\beta)$$

$$P_D / P_T \text{ (dB)} = 10 \log 2J_1^2(\beta) \qquad \text{(first upper and lower sidebands)} \tag{74}$$

2). Squarewave subcarrier

$$P_C / P_T \text{ (dB)} = 10 \log \cos^2 (\beta)$$

$$P_D / P_T \text{ (dB)} = 10 \log \sin^2 (\beta) \qquad \text{(for all sidebands)} \tag{75}$$

where J_o and J_1 are the zero-order and first-order Bessel functions, respectively. Based on Eqs. (73) to (75), the computation of the carrier, command, and telemetry and ranging margins requires an evaluation of the modulation losses.

The CCSDS (Cosultative Committee for Space Data Systems) [23] defines the modulation loss as a fraction of the total transmitted power allocated to a designated channel. CCSDS design control table (DCT) uses a statistical technique for analyzing the telecommunications link performance, for which every parameter in the DCT requires the specification of the design value with its favorable and adverse tolerances. The tolerances for the modulation losses are calculated based on the variations of the peak phase deviations (modulation indexes). Calculation of these tolerances can be tedious because it requires the designer to evaluate the modulation losses for all possible combinations of the modulation indexes. For the coherent turn-around ranging channel, assuming spacecraft on-board processing uses power-controlled automatic gain control (AGC) with simultaneous command and range on the uplink and simultaneous range and telemetry on the downlink, there are 128 possible combinations for computing the tolerances of the downlink telemetry modulation losses. Six possible combinations are presented here for both uplink and downlink modulation losses.

Expressions of carrier, telecommand, and ranging modulation losses for both uplink and downlink are presented below with respect to modulation indexes for both sinewave (sinusoid) and squarewave [23].

1). Uplink modulation losses

a). Carrier:

$$P_{C1} / P_T = \cos^2(\beta_R)\ J_o^2(\beta_{CD}) \quad \text{for squarewave}$$

$$= J_o^2(\beta_R)\ J_o^2(\beta_{CD}) \quad \text{for sinusoid} \tag{76}$$

b).Telecommand:

$$P_{CD} / P_T = 2\cos^2(\beta_R)\ J_1^2(\beta_{CD}) \quad \text{for squarewave}$$

$$= 2\,J_o^2(\beta_R)\ J_1^2(\beta_{CD}) \quad \text{for sinusoid} \tag{77}$$

c). Ranging:

$$P_{R1} / P_T = \sin^2(\beta_R)\ J_o^2(\beta_{CD}) \quad \text{for squarewave}$$

$$= 2\,J_1^2(\beta_R)\ J_o^2(\beta_{CD}) \quad \text{for sinusoid} \tag{78}$$

where P_{R1} is the uplink ranging power, P_{CD} the uplink telecommand power, P_T the total power, P_{C1} the carrier power losses, β_R and β_{CD} the modulation indexes for ranging and telecommand respectively, J_o, and J_1 the Bessel functions of the zero and first order.

2). Downlink modulation losses

a). Carrier:

$$P_{C2} / P_T = \cos^2(\beta_{TLM})\ J_o^2(\tau_1)\ J_o^2(\tau_2)\ \exp(-\tau_3^2) \quad \text{for squarewave}$$

$$= J_o^2(\beta_{TLM})\ J_o^2(\tau_1)\ J_o^2(\tau_2)\ \exp(-\tau_3^2) \quad \text{for sinusoid} \tag{79}$$

b). Telemetry:

$$P_{TLM} / P_T = \sin^2(\beta_{TLM})\, J_o^2(\tau_1)\, J_o^2(\tau_2)\exp(-\tau_3^2) \quad \text{for squarewave}$$

$$= 2\, J_1^2(\beta_{TLM})\, J_o^2(\tau_1)\, J_o^2(\tau_2)\exp(-\tau_3^2) \quad \text{for sinusoid} \qquad (80)$$

c). Ranging:

$$P_{R2} / P_T = 2\cos^2(\beta_{TLM})\, J_o^2(\tau_1)\, J_1^2(\tau_2)\exp(-\tau_3^2) \quad \text{for squarewave}$$

$$= 2\, J_o^2(\beta_{TLM})\, J_o^2(\tau_1)\, J_1^2(\tau_2)\exp(-\tau_3^2) \quad \text{for sinusoid} \qquad (81)$$

where P_{C2} shows the downlink carrier power losses, P_{R2} the downlink ranging power losses, P_{TLM} the downlink telemetry power losses, P_T the downlink total power losses, β_{TLM} the downlink modulation indexes, and τ_1, τ_2, τ_3 the noise-modified (actual) modulation indexes for feedthrough command (peak), ranging (peak), and noise (rms) respectively defined in CCSDS. Typical values on the DSPSE program for these are $\tau_1 = 0.42$, $\tau_2 = 0.26$, and $\tau_3 = 0.057$.

As shown in Eqs. (76) to (81), the computation of the modulation loss tolerances requires the use of Bessel, trigonometry, and exponential functions. Figure 19 presents functions of sinusoid squares and Bessel function squares since these are ingredients of power losses for both uplink and downlink modulation losses. Characteristics and interrelationships among those functions can be construed from Fig. 19. To avoid unnecessary increase in computational complexity, only maximum and minimum values of some of the modulation losses are plotted for brief references. Figures 20 through 25 show plots of Eqs. (76) to (78) for both sinusoid and squarewave ranging of uplink modulation losses. Note that Fig. 22 shows irregular pair of ranging modulation index since m_R = 2.4 is the minimum and symmetricity of the zero-order Bessel function of the first kind. The optimum range of modulation indexes can be observed between 1.0 to 2.8 from these plots.

Downlink modulation losses for both sinusoid and squarewave ranging are plotted in Figs. 26 through 31 for Eqs. (79) to (81). Each plot gives a unique relationship of modulation index vs carrier/command/ranging, and suggests some kind of trade-off has to be made for optimum applications with careful consideration of other communication link parameters.

3.4 Signal-To-Noise Ratio vs Modulation Index [22 to 25]

A common and useful measure of the fidelity of the received message is the output signal-to-noise ratio defined as

$$S/N = (\text{mean power of message})/(\text{mean power of noise}) \qquad (82)$$

at the receiver output. The signal-to-noise ratio (S/N or SNR) is unambiguous as long as the recovered message and noise at the demodulator output are additive. This requirement is satisfied exactly in the case of linear receivers that use coherent detection, and approximately in the case of nonlinear receivers (i.e., that use envelope detection) provided that the mean noise power is small compared with the carrier power. The output SNR depends, among other factors, on the type of modulation used in the transmitter and the type of demodulation adopted in the receiver. Thus, it

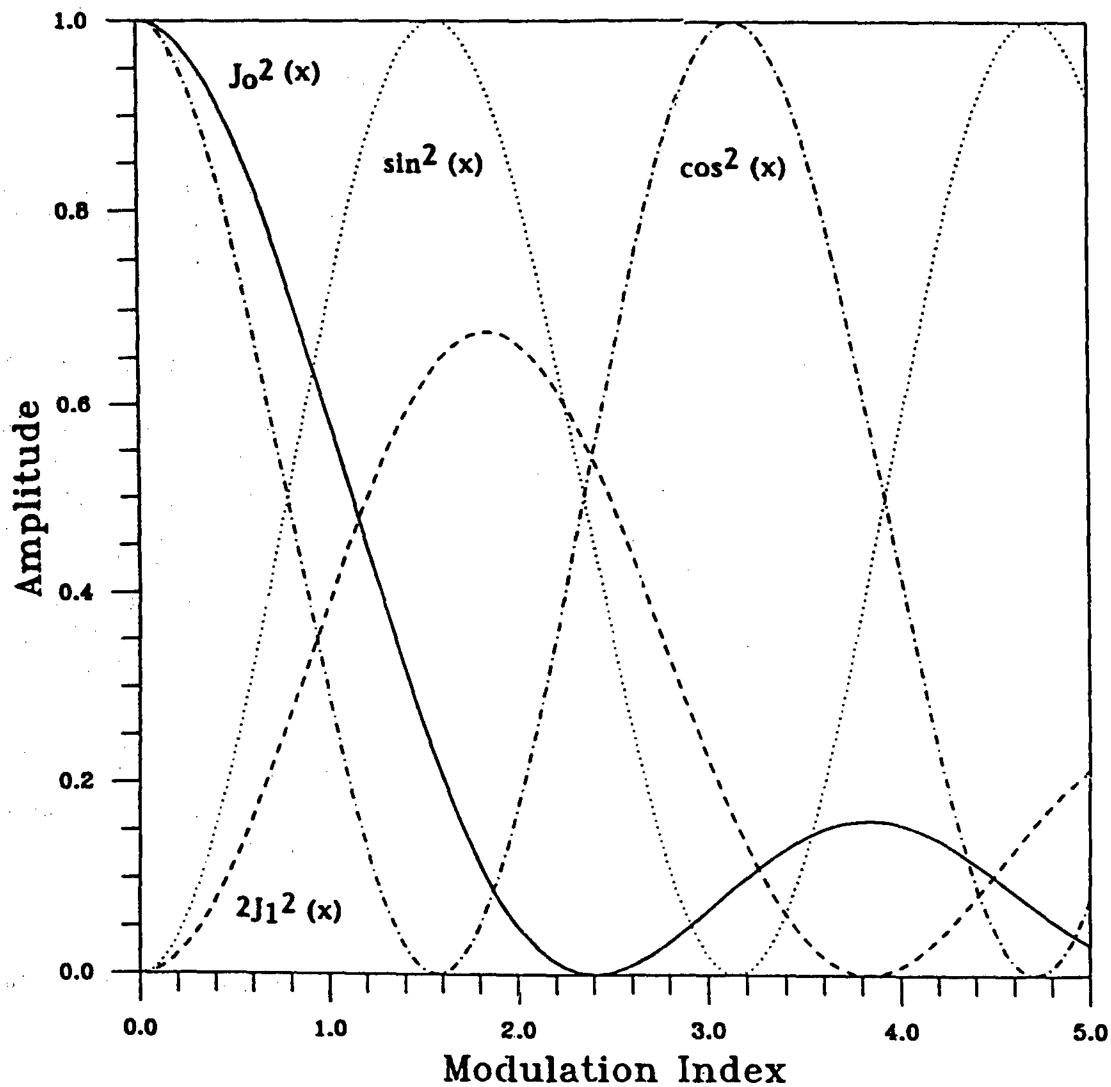

Fig. 19 — Squares of sinusoid and Bessel functions of zero- and first-order as a function of mulation index (range of 0.0 to 5.0)

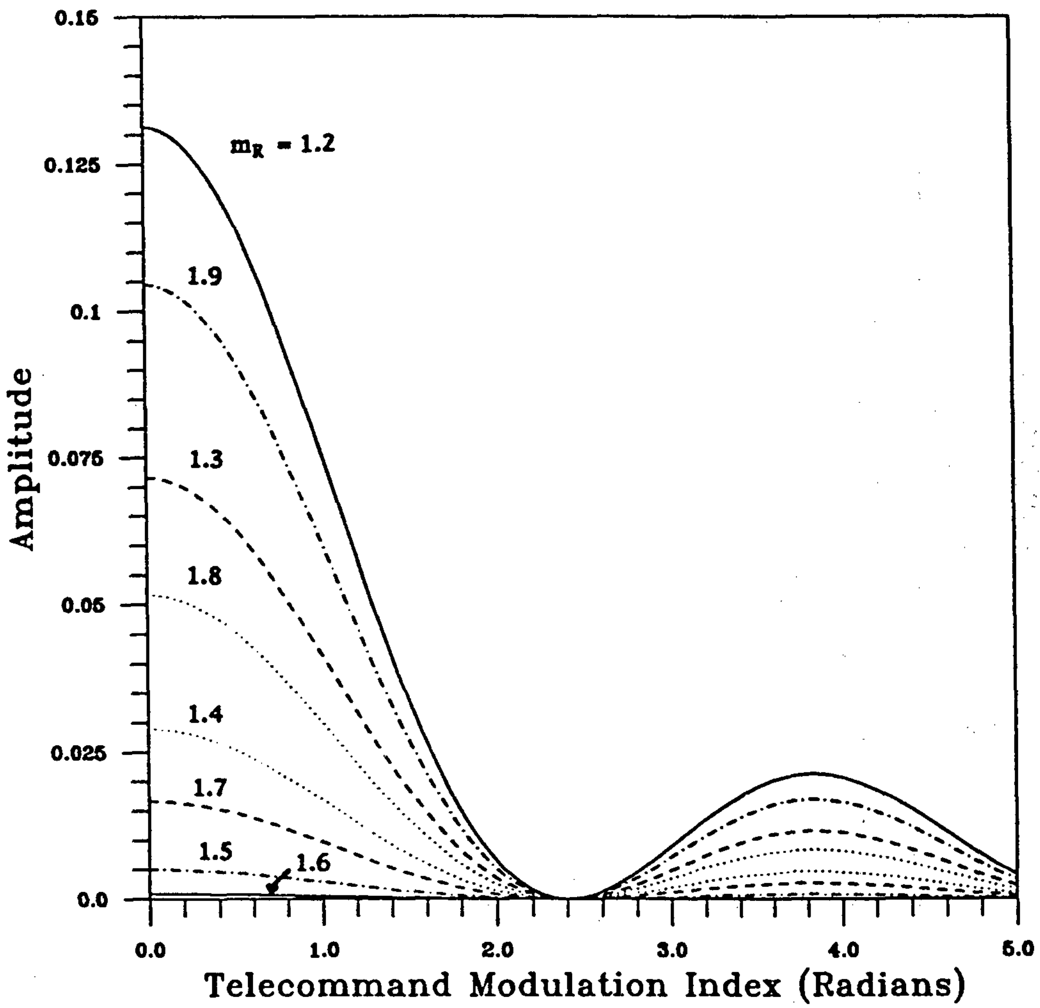

Fig. 20 — Uplink modulation loss for squarewave ranging vs carrier as a function of modulation index for ranging modulation index from 1.2 to 1.9

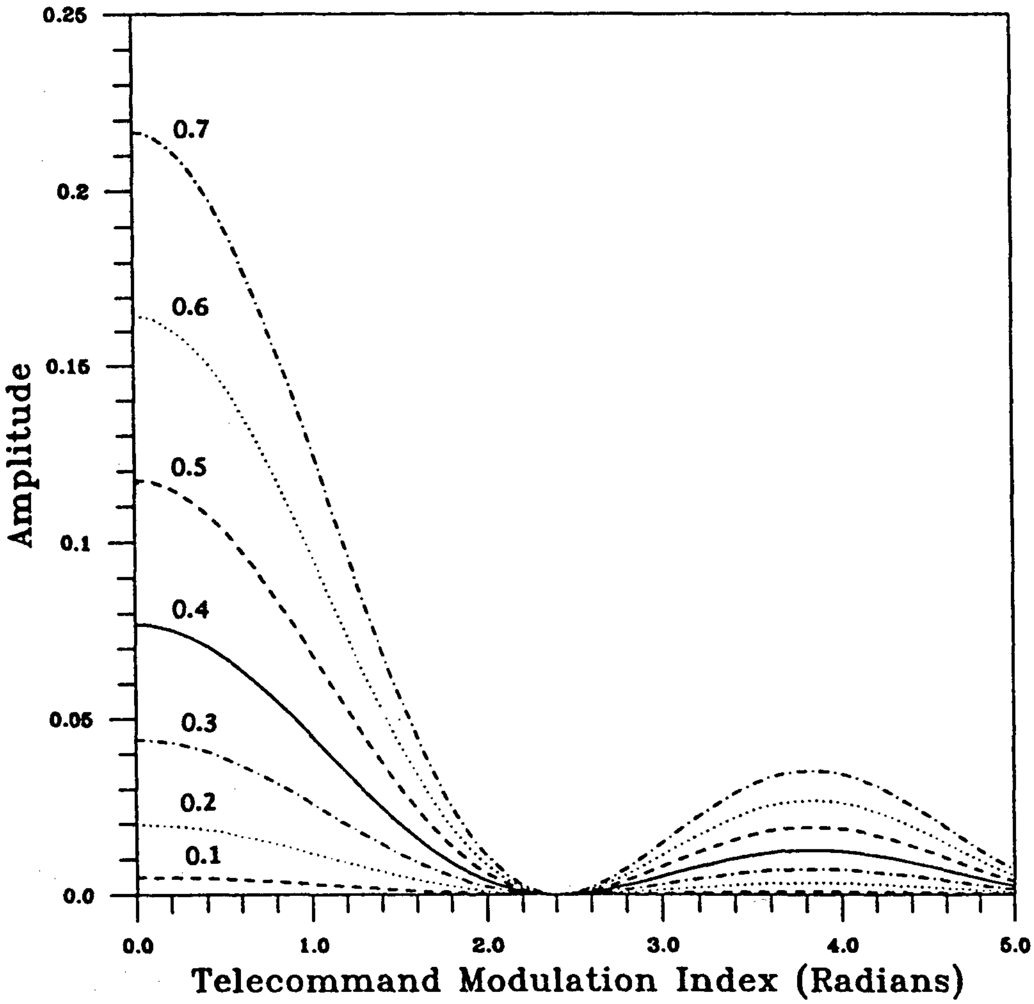

Fig. 21 — Uplink modulation loss for sinusoid ranging vs ranging as a function of modulation index for ranging modulation index from 0.0 to 0.7

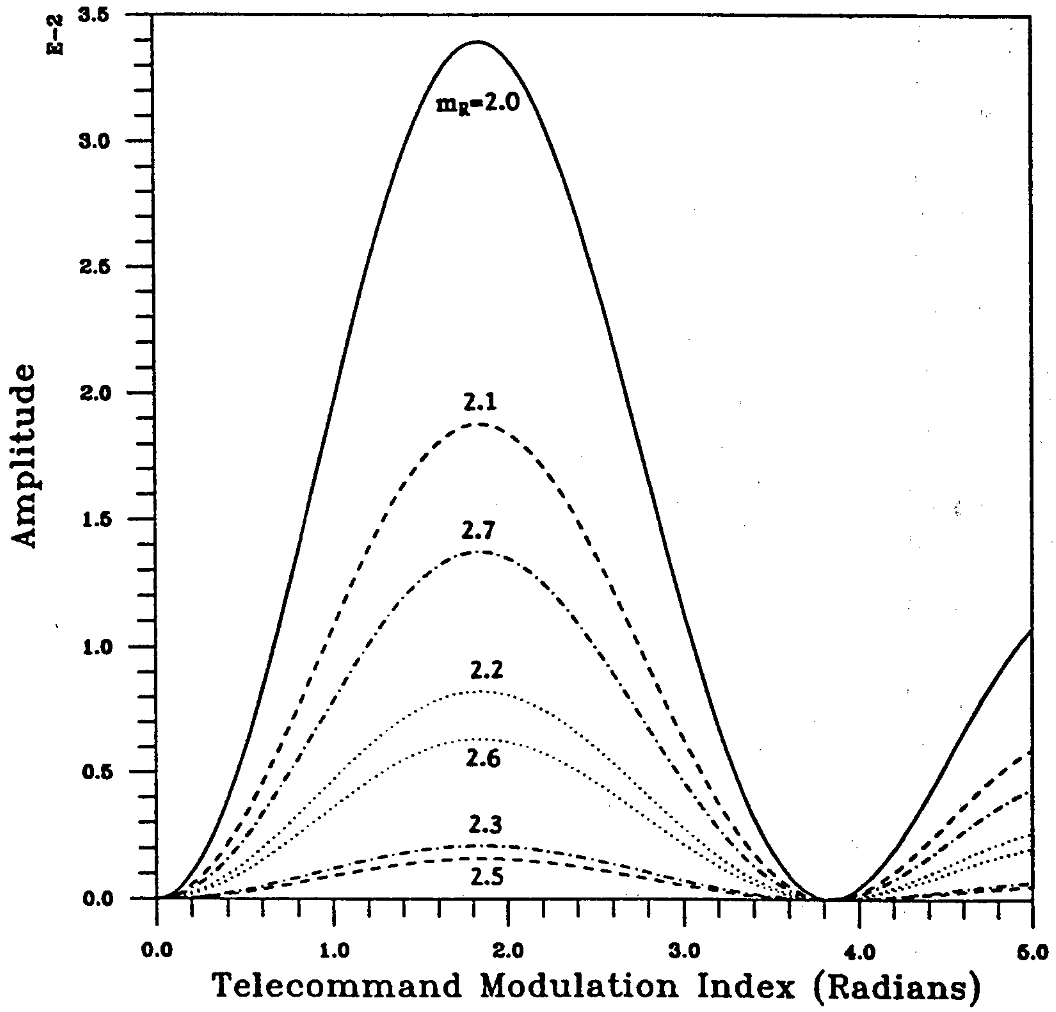

Fig. 22 — Uplink modulation loss for simusoid ranging vs telecommand as a function of modulation index for ranging modulation index from 2.0 to 2.7

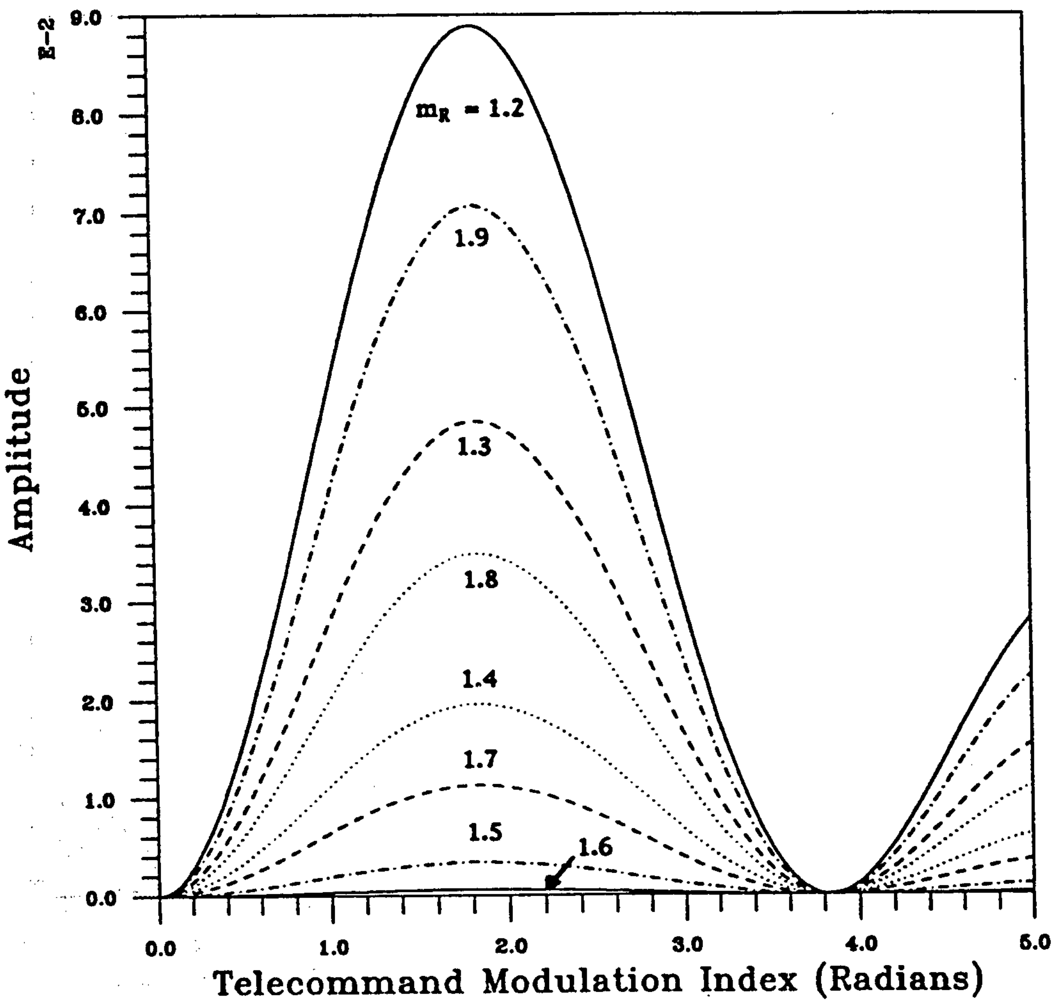

Fig. 23 — Uplink modulation loss for squarewave rangeing vs telecommand as a function of modulation index for ranging modulation index from 1.2 to 1.9

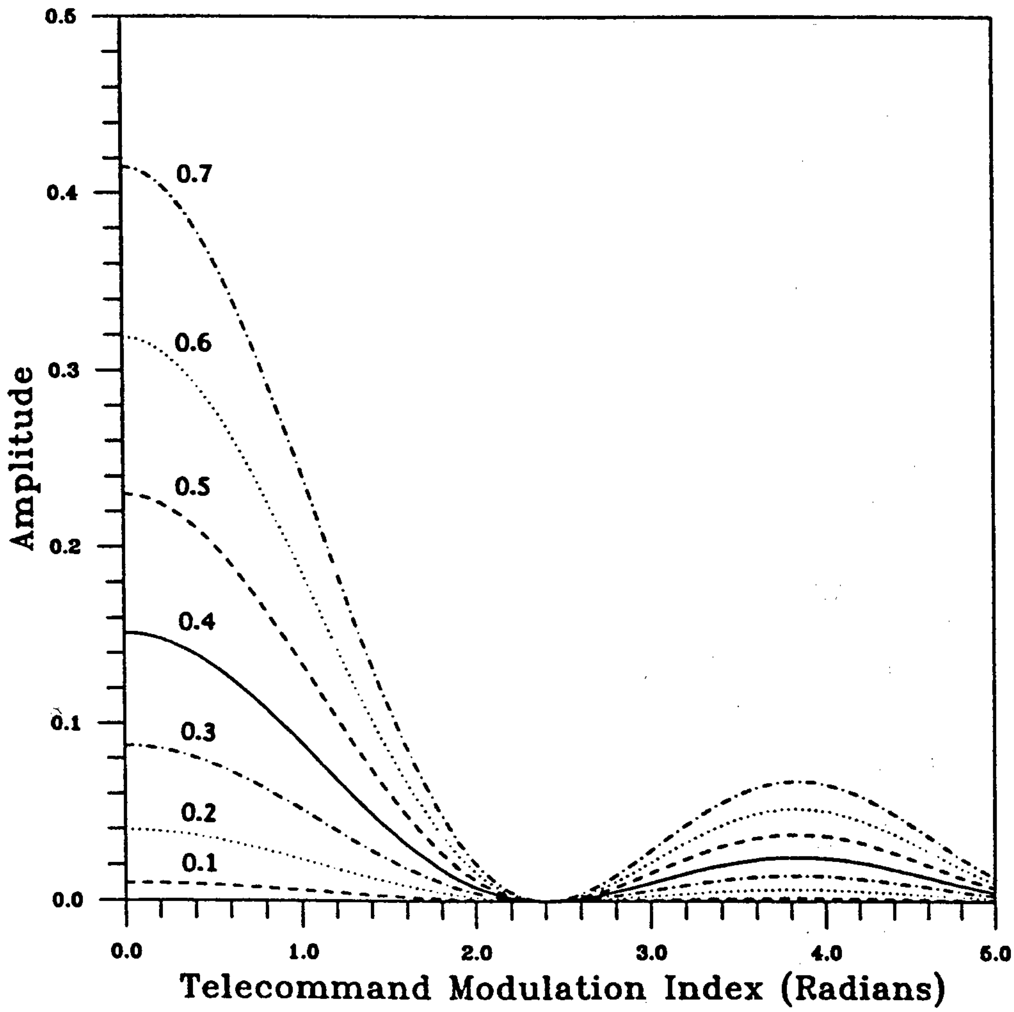

Fig. 24— Uplink moodultion loss for squarewave ranging vs ranging as a function of modulation index for ranging modulation index from 0.0 to 0.7

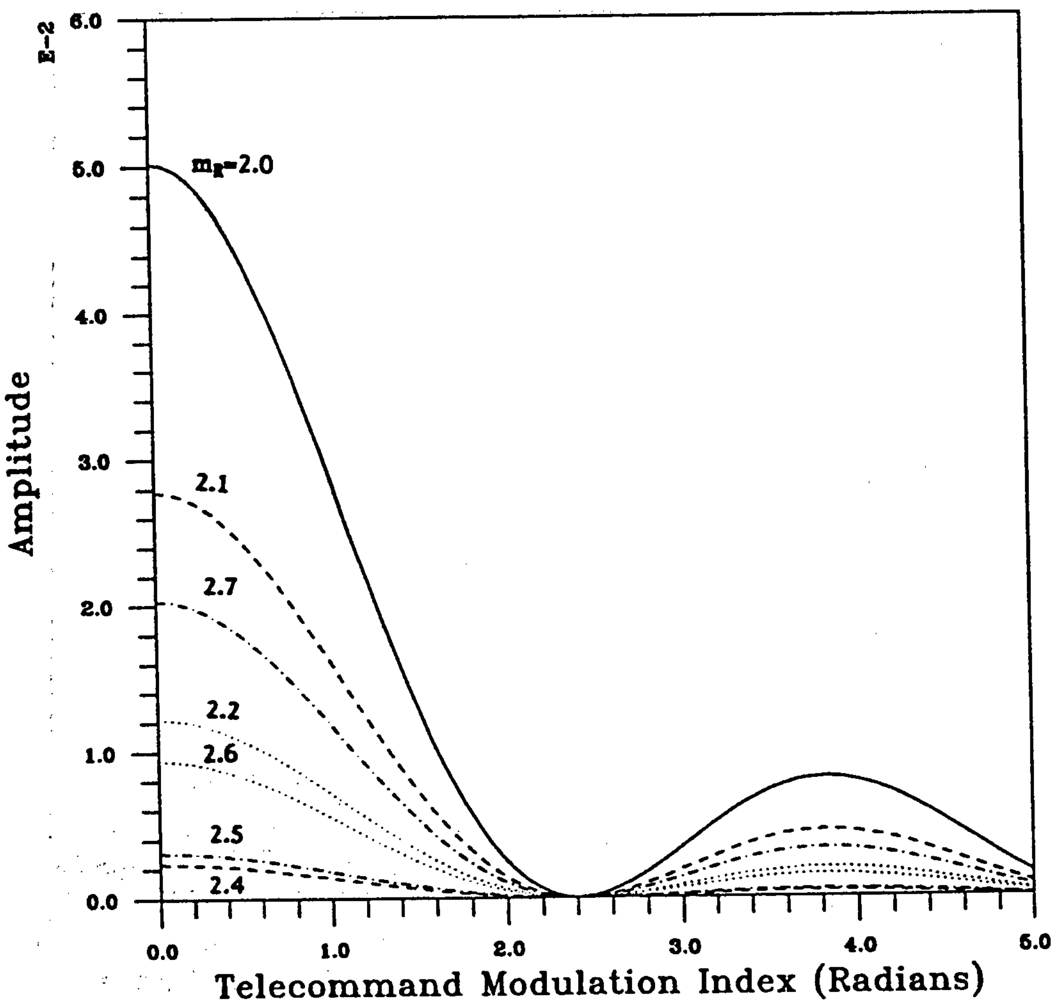

Fig. 25 — Uplink modulation loss for sinusoid ranging vs carrier as a function of modulation index for ranging modulation index from 2.0 to 2.7

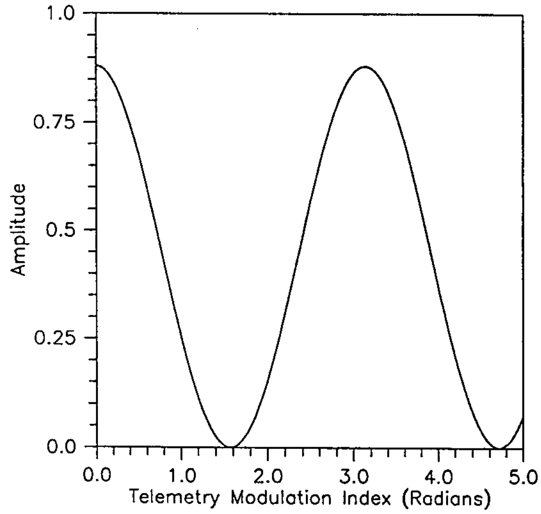

Fig. 26 — Downlink modulation loss for squarewave ranging vs carrier as a function of modulation index (range of 0.0 to 5.0)

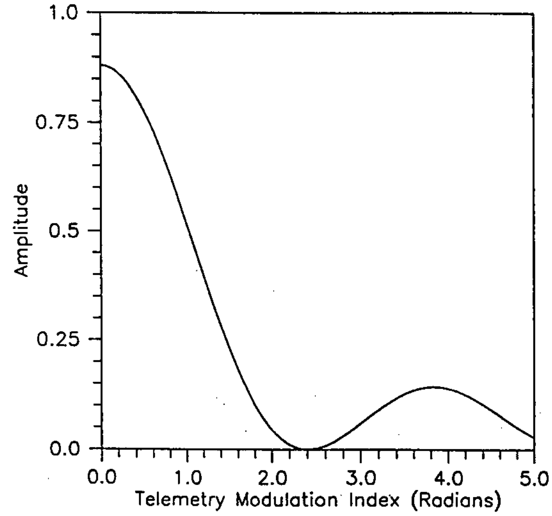

Fig. 27 — Downlink modulation loss for sinusoid ranging vs carrier as a function of modulation index (range of 0.1 to 5.0)

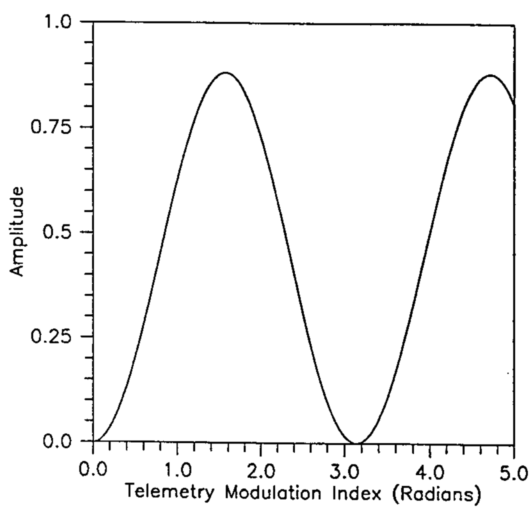

Fig. 28 — Downlink modultion loss for squarewave ranging vs telemetry as a function of modulation index index (range of 0.0 to 5.0)

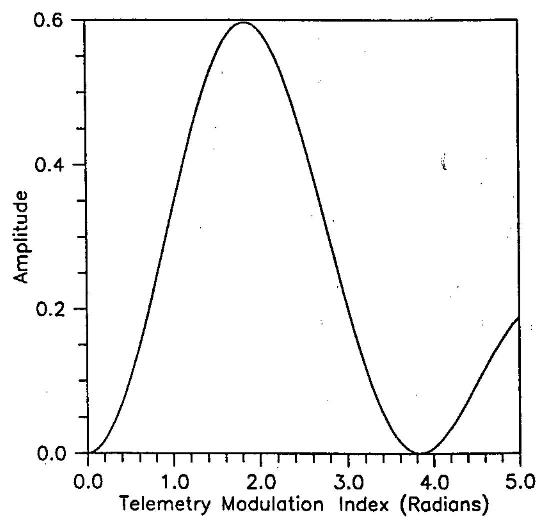

Fig. 29 — Downlink modulation loss for sinusoid ranging vs telemetry as a function of modulation index (range of 0.0 to 5.0)

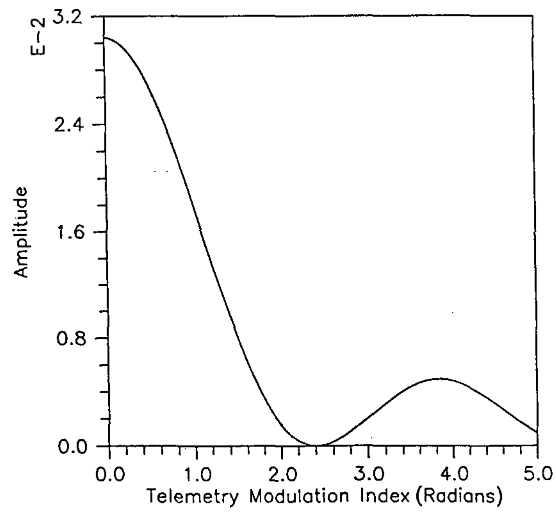

Fig. 30 — Downlink modulation loss for squarewave ranging vs ranging as a function of modulation index (range of 0.0 to 5.0)

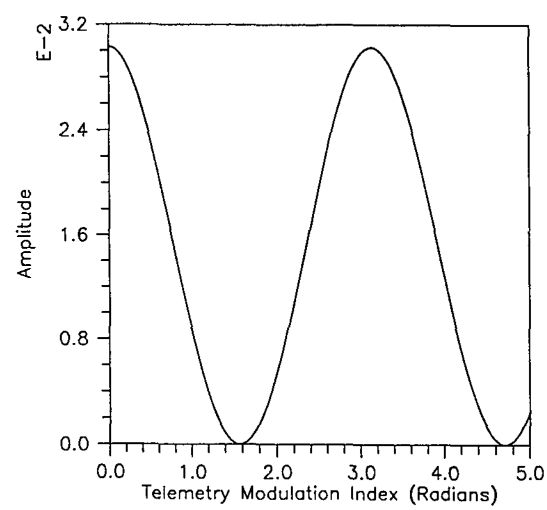

Fig. 31 — Downlink modulation loss for sinusoid ranging vs ranging as a function of modulation index (range of 0.0 to 5.0)

may be informative to compare the output SNRs of different modulation-demodulation systems. As a frame of reference to cope with the same noise power in the bandwidth of the message, the channel signal-to-noise ratio (C/N or CNR) is defined as

$$C/N = (\text{mean power of modulated message signal})/(\text{mean power of noise in message bandwidth}) \quad (83)$$

at the receiver input. This ratio may be viewed as the SNR that results from baseband or direct transmission of the message without modulation. Here, it is assumed that

a) the message power at the lowpass filter input is the same as the modulated message signal power and
b) the lowpass filter passes the message bandwidth and rejects out-of-band noise.

Let's consider both continuous and digital modulation SNR with minimal analytical efforts. Resulting relations are presented for different modulation processes.

1). Double-sideband suppressed carrier (coherent detection):

$$SNR = A_C^2 P_m / 2WN_o \qquad \text{for channel and output} \quad (84)$$

where P_m is the normalized mean power of the signal message, N_o the noise spectral density, and W the message bandwidth.

2). Single-sideband carrier (coherent detection):

$$SNR = A_C^2 P_m / 4WN_o \quad \text{for channel and output} \quad (85)$$

3). AM system using envelope detection:

$$SNR = A_C^2 (1 + h_a^2 P_m) / 2WN_o \qquad \text{for channel}$$
$$= A_C^2 h_a^2 P_m / 2WN_o \qquad \text{for receiver output} \quad (86)$$

4). FM system:

$$\begin{aligned} SNR &= A_C^2/2WN_o && \text{for channel} \\ &= 3A_C^2\, h_f^2\, P_m/2N_oW^3 && \text{for receiver output} \end{aligned} \tag{87}$$

where h_a and h_f are the modulation indexes for AM and FM, respectively. These expressions are, however, valid only if:

a). the noise at the receiver input is small with respect to the signal message, and

b). the modulation indexes are adjusted for a percentage modulation less than or equal to 100%.

Using Eqs. (84) and (85), one can obtain the figure of merit for AM as

$$\begin{aligned} (SNR)_o/(SNR)_C &\cong h_a^2\, P_m/(1 + h_a^2\, P_m) \\ &= 1/3 \qquad \text{for } h_a = 1 \text{ (100\% modulation).} \end{aligned} \tag{88}$$

Similarly from Eq. (86) for FM,

$$\begin{aligned} (SNR)_o/(SNR)_C &= 3\, h_f^2\, P_m/W^2 \\ &= 3\, \beta_{FM}^2/2 \quad \text{for sinusoid signal.} \end{aligned} \tag{89}$$

Under the same conditions of the carrier power and noise spectral density, the well-known output SNR relation between AM and FM can be derived as

$$(SNR)_{FM} = 3\, \beta_{FM}^2 (SNR)_{AM}. \tag{90}$$

Note here that for large modulation index (i.e., wide transmission bandwidth), one can presumably increase the output SNR significantly over the AM case. Alternatively, for the same SNR at the output in both receivers, the power of the FM carrier may be reduced 75 times. The RF signal-to-noise ratio for FM signals is shown by the following formula [25]:

$$(SNR)_{FM} = (CNR)_{FM} + 10\, \mathrm{Log}_{10}\, [1.5*\beta^2*B_{IF}/W_{FM}], \tag{91}$$

where CNR stands for the carrier-to-noise ratio or the RF signal-to-noise ratio, B_{IF} the receiver IF bandwidth. Figure 32 shows the plot of Eq. (91) for 6, 7, and 8 dB of CNR. Signal-to-noise ratio is uniformly proportional with exponential functions with respect to modulation index.

4. CONCLUSIONS AND SUMMARY

The performance of a telecommunications system depends on numerous communication link parameters. These include such parameters as received and transmitted power, antenna gain, noise power, carrier performance margin, telemetry and command performance margins, ranging performance margin, path loss, bit energy per noise power spectral density, and receiver sensitivity. Modulation index has not been generally emphasized in the link calculations. This study centered on determining the degree to which the modulation index affects telecommunication links in many different ways such as bandwidth, probability of bit error, power, and signal-to-noise ratio.

To present those effects, the overview of modulation process was included for the completion of analysis process and relational studies. From the analytical representations (presented in Section 2) of

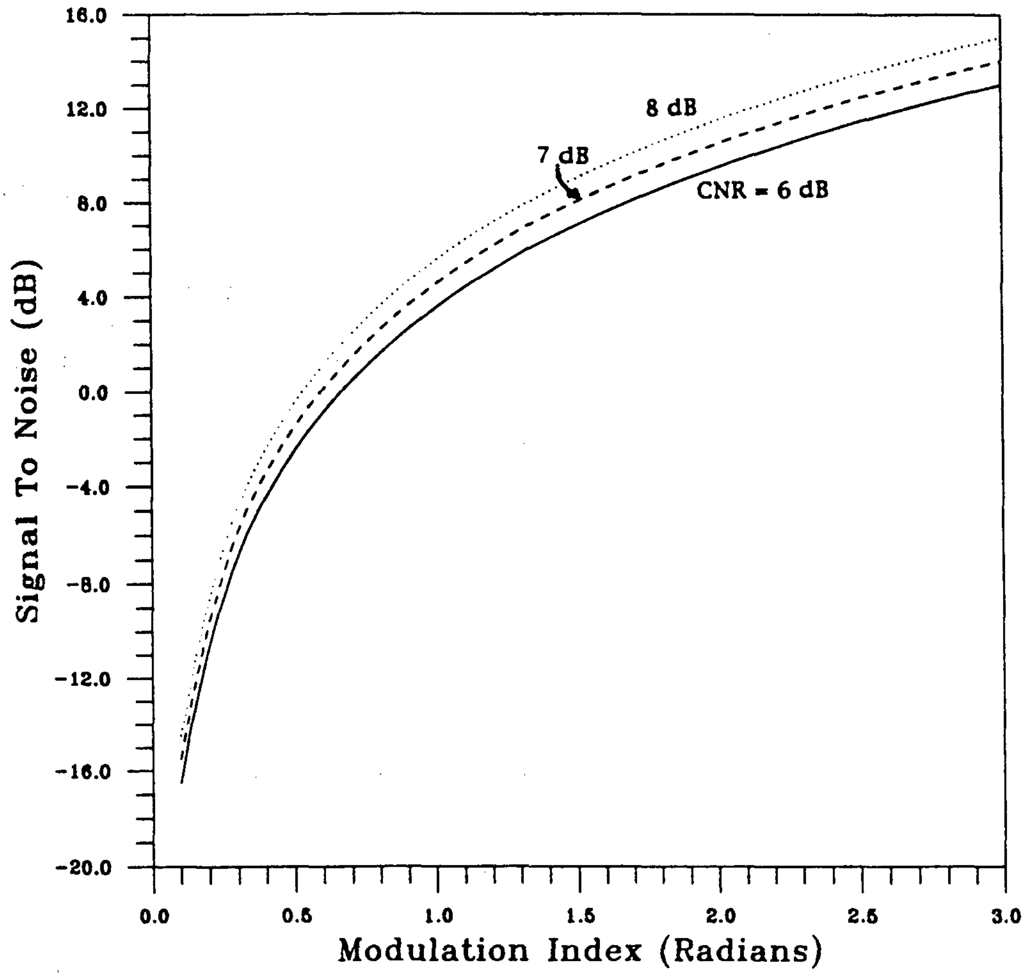

Fig. 32 — Radio frequency (RF) signal-to-noise ratio as a function of modulation index (range of 0.0 to 3.0)

relational examinations of modulation index with respect to ranging/command/telemetry power loss, bit error rate, signal-to-noise ratio, and bandwidth, and the additional reference graphics (Bessel functions, bit error vs bit energy per noise spectral density and other materials), it is apparent that modulation indexes exert a significant influence on telecommunication links. We determined that there is an optimum range of modulation index based on the DSPSE communication system link simulation test [26].

Further extensive study is required for completeness. Since this study was limited to current simulation efforts of the DSPSE satellite links, results presented are not comprehensive but rather constrained to limited examples and derivations. As previously noted, the objective of this report is to show the importance of modulation index influence on telecommunications. The authors believe this goal has been achieved in a limited sense. Extensions of this study should be pursued further if a future satellite mission requires a sensitive design criteria such as the DSPSE project.

UNCLASSIFIED

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