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Comparison of the Rough Surface Reflection Coefficient with Specularly Scattered Acoustic Data

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CONTENTS

INTRODUCTION 1

COMPARISON OF $R(g, \epsilon)$ WITH ACOUSTIC DATA 2

CONCLUSION 3

REFERENCES 3

APPENDIX — Integral Representations for $\Phi_1[\alpha, \beta; \gamma; x, y]$ 5

COMPARISON OF THE ROUGH SURFACE REFLECTION COEFFICIENT WITH SPECULARLY SCATTERED ACOUSTIC DATA

INTRODUCTION

Miller and Vegh [1] in treating reflection from the rough surface of the sea derived a one-parameter family of curves for the rough surface reflection coefficient or roughness factor R given by

$$
\begin{aligned}
R(g,\epsilon) &= \epsilon^2 \exp\,[-2\epsilon^2\eta^2(2\pi g)^2]\, I_0[2\epsilon^2\eta^2(2\pi g)^2] \\
&+ (1-\epsilon^2)^{1/2} \exp\,[-4\eta^2(2\pi g)^2] \\
&- \frac{1}{2}\epsilon^2(1-\epsilon^2)\,\Phi_1[\frac{3}{2}, 1; 2; \epsilon^2, -4\epsilon^2\eta^2(2\pi g)^2]
\end{aligned} \tag{1}
$$

where

$$g \equiv (\sigma/\lambda)\sin\psi$$

and

$$\eta \equiv [1 + \frac{\pi}{2}(1-\epsilon^2)]^{-1/2}$$

Here g is a measure of the effective surface roughness or simply surface roughness, $\epsilon\,(0 \leqslant \epsilon \leqslant 1)$ is the spectral width parameter, σ is the standard deviation of the water surface elevation, ψ is the grazing angle for specular reflection, λ is the wavelength of the incident radiation, and $I_0(x)$ is the modified Bessel function of order zero. The function $\Phi_1[\alpha, \beta; \gamma; x, y]$ is a confluent hypergeometric function in two variables first defined in 1920 by P. Humbert [2, p. 58]. In the Appendix we derive an integral representation for Φ_1 that may be used for numerical computation.

$R(g,\epsilon)$, given by Eq. (1), is essentially the Fourier transform of the probability density $D(y,\epsilon)$ for surface elevation y where

$$
\begin{aligned}
D(y,\epsilon) &= \frac{\epsilon}{2\pi^{3/2}\eta\sigma} \exp\,(\frac{-y^2}{8\epsilon^2\eta^2\sigma^2})\, K_0(\frac{y^2}{8\epsilon^2\eta^2\sigma^2}) \\
&+ \frac{(1-\epsilon^2)^{1/2}}{\pi^{3/2}\eta\sigma} \exp\,(\frac{-y^2}{4\eta^2\sigma^2})\,\{\cos^{-1}\epsilon + \epsilon\,(1-\epsilon^2)^{1/2} K_{e_0}(2\epsilon^2-1,\, y^2/8\epsilon^2\eta^2\sigma^2)\}
\end{aligned} \tag{2}
$$

Here $K_0(x)$ is the MacDonald function or Bessel function of imaginary argument of order zero. $K_{e_0}(a,x)$ is an incomplete Lipschitz-Hankel integral of $K_0(x)$ and may be written in closed form either in terms of incomplete cylindrical functions [3] or in various ways in terms of Kampé de Fériet functions [4,5]; e.g.

$$K_{e_0}(a,z) = z\,K_0(z)\,A_1(a,z) + z^2\,K_1(z)\,A_0(a,z)$$

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where

$$A_1(a,z) \equiv F\,{}^{0:1;1}_{2:0;0}\left[\begin{matrix} -:\ 1/2;\ 1; \\ 1/2,3/2:\ -;\ -; \end{matrix}\ \frac{a^2z^2}{4}, \frac{z^2}{4}\right]$$

$$+ \frac{1}{2}az\, F\,{}^{0:2;1}_{2:1;0}\left[\begin{matrix} -:\ 1,1;\ 1; \\ 1,2:\ 3/2;\ -; \end{matrix}\ \frac{a^2z^2}{4}, \frac{z^2}{4}\right]$$

$$A_0(a,z) \equiv F\,{}^{0:1;1}_{2:0;0}\left[\begin{matrix} -:\ 1/2;\ 1; \\ 3/2,3/2:\ -;\ -; \end{matrix}\ \frac{a^2z^2}{4}, \frac{z^2}{4}\right]$$

$$+ \frac{1}{4}az\, F\,{}^{0:2;1}_{2:1;0}\left[\begin{matrix} -:\ 1,1;\ 1; \\ 2,2:\ 3/2;\ -; \end{matrix}\ \frac{a^2z^2}{4}, \frac{z^2}{4}\right]$$

$D(y,\epsilon)$, given by Eq. (2), was derived in Ref. 1 by assuming that the water surface could be described locally by sinusoids with uniform phase distribution whose amplitude distribution is given by a density function derived by Rice [6] and by Cartwright and Longuet-Higgins [7]. Figure 1 gives graphs for $D(y,\epsilon)$, for various values of ϵ.

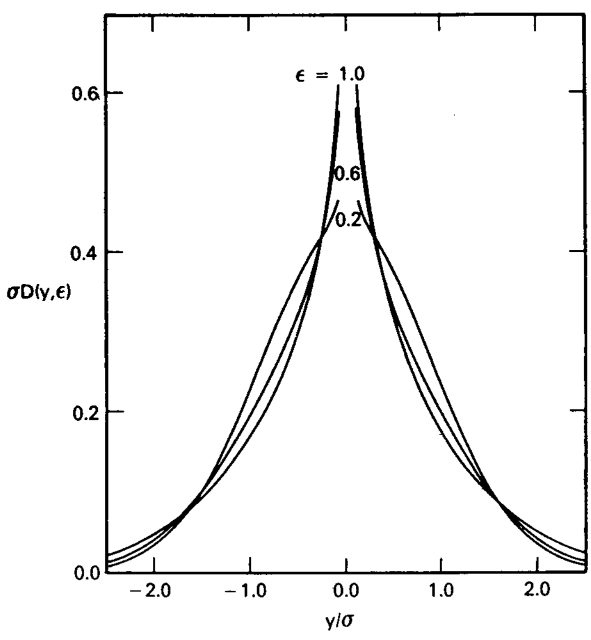

Fig. 1 — Density function $D(y,\epsilon)$ for various values of the spectral width parameter ϵ

COMPARISON OF $R(g,\epsilon)$ WITH ACOUSTIC DATA

In 1980 DeSanto [8, p. 70, Fig. 5] compared $R(g, 1)$ with acoustic data from Clay, Medwin, and Wright [9]. Although $R(g, 1)$ was first derived in 1974 [10], a mathematically rigorous derivation was not obtained until 1984 [11]. In view of Eq. (1), it now appears appropriate to compare $R(g,\epsilon)$ with the aforementioned data. Whereas $R(g, 1)$ takes into account only the standard deviation, σ, of surface elevation, $R(g, \epsilon)$ is dependent on ϵ also and hence on the moments of the frequency energy spectrum $\Phi(s)$ of the surface through the equations [12, p. 346]

$$\epsilon^2 = (m_0 m_4 - m_2^2)/m_0 m_4$$

$$m_\nu \equiv \int_0^\infty s^\nu \Phi(s)\, ds \quad (m_0 = \sigma)$$

Figure 2 compares $R^2(g, 1/3)$ with the data given by Clay et al. in Fig. 5 of Ref. 9. $R^2(g, 1/3)$ appears to be in better agreement with this data than the multiple scattering theoretical result given in Fig. 5 of Ref. 8.

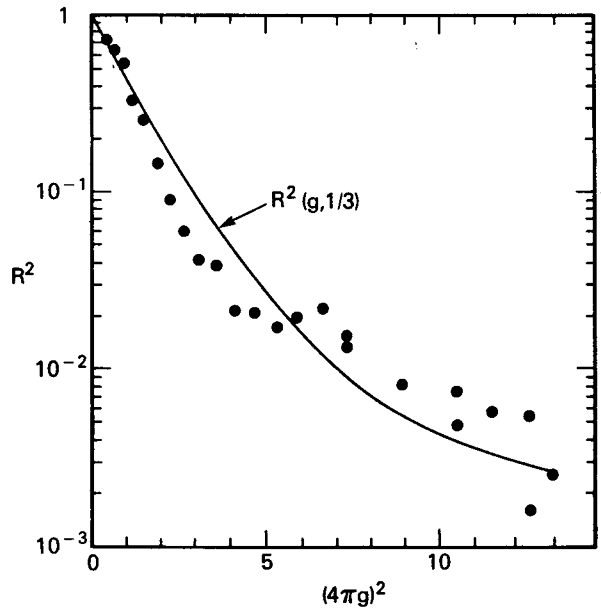

Fig. 2 — Comparison of the theoretical curve $R^2(g, 1/3)$ with experimental data

CONCLUSION

One of the family of rough surface reflection coefficients agrees with acoustic data reasonably well; at least as well as the curve given previously by the multiple scattering model.

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Appendix

INTEGRAL REPRESENTATIONS FOR $\Phi_1\,[\alpha,\beta;\gamma;x,y]$

The confluent double hypergeometric function Φ_1 is defined by

$$\Phi_1\,[\alpha,\beta;\gamma;x,y] \equiv \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}\,(\beta)_m}{(\gamma)_{m+n}}\,\frac{x^m}{m!}\,\frac{y^n}{n!} \qquad |x|<1, \quad |y|<\infty$$

The definition of Φ_1 given in Erdélyi et al. [A1, p. 225] and Gradshteyn et al. [A2, 9.261, Eq. 1] is incorrect.

By using Ref. A3, p. 266

$$\frac{(\alpha)_p}{(\gamma)_p} = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 t^{p+\alpha-1}\,(1-t)^{\gamma-\alpha-1}\,dt, \quad \mathrm{Re}\gamma > \mathrm{Re}\alpha > 0$$

with the definition of Φ_1 given above we obtain

$$\Phi_1\,[\alpha,\beta;\gamma;x,y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \int_0^1 t^{m+n+\alpha-1}\,(1-t)^{\gamma-\alpha-1}\,(\beta)_m\,\frac{x^m y^n}{m!n!}\,dt$$

Now interchanging the integral sign and double sum and noting that

$$\sum_{n=0}^{\infty} \frac{(ty)^n}{n!} = e^{ty}, \qquad \sum_{m=0}^{\infty} (\beta)_m\,\frac{(tx)^m}{m!} = (1-tx)^{-\beta}$$

we obtain for $\mathrm{Re}\gamma > \mathrm{Re}\alpha > 0$, $|x|<1$, $|y|<\infty$

$$\Phi_1\,[\alpha,\beta;\gamma;x,y] = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 e^{yt}\,(1-xt)^{-\beta}\,(1-t)^{\gamma-\alpha-1}\,t^{\alpha-1}\,dt$$

In particular,

$$\Phi_1\,[3/2,\,1;\,2;\,x,y] = \frac{2}{\pi}\int_0^1 \frac{e^{yt}\,t^{1/2}}{(1-xt)\,(1-t)^{1/2}}\,dt$$

Now making the transformation $t = \sin^2\theta$ and replacing x by ϵ^2 and y by $-\epsilon^2 y^2$ we obtain

$$\Phi_1[3/2,\,1;\,2;\,\epsilon^2,-\epsilon^2 y^2] = \frac{4}{\pi}\int_0^{\pi/2} \frac{\sin^2\theta\; e^{-\epsilon^2 y^2 \sin^2\theta}}{1-\epsilon^2\sin^2\theta}\,d\theta \qquad \text{(A1)}$$

For real ϵ, y the integrand here is nonnegative on the closed interval $[0, \pi/2]$ and has no singularities for $0 \leqslant \epsilon < 1$; the integral in Eq. (A1) is therefore suitable for numerical quadrature and Φ_1 may thereby be computed.

It may also be shown [1, Eq. 15] that

$$\Phi_1\,[3/2,\ 1;\ 2;\ \epsilon^2, -\epsilon^2 y^2] = \frac{2}{\epsilon^2(1-\epsilon^2)^{1/2}} \left\{ e^{-y^2} - 2\int_0^\infty te^{-t^2} J_0(2yt)\ erf\left[\frac{(1-\epsilon^2)^{1/2}}{\epsilon}\,t\right] dt \right\} \tag{A2}$$

from which it follows that

$$\lim_{\epsilon \to 1} \epsilon^2 (1-\epsilon^2)\, \Phi_1\,[3/2,\ 1;\ 2;\ \epsilon^2, -\epsilon^2 y^2] = 0$$

Hence Eq. (1) is valid in the limit for $\epsilon = 1$.

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