Naval Research Laboratory

Washington, DC 20375-5000

NRL Report 9047

UNCLASSIFIED

Incomplete Lipschitz-Hankel Integrals of Bessel Functions

ALLEN R. MILLER

Computer-Aided Design/Computer-Aided Manufacturing
Engineering Services Division

May 26, 1987

Approved for public release, distribution is unlimited.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release, distribution unlimited.
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Report 9047	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Research Laboratory — 6b. OFFICE SYMBOL *(If applicable)* Code 2303	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS *(City, State, and ZIP Code)* Washington, DC 20375-5000	7b. ADDRESS *(City, State, and ZIP Code)*
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Naval Research Laboratory — 8b. OFFICE SYMBOL *(If applicable)* Code 2303	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS *(City, State, and ZIP Code)* Washington, DC 20375-5000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*
Incomplete Lipschitz-Hankel Integrals of Bessel Functions

12. PERSONAL AUTHOR(S)
Miller, Allen R.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT *(Year, Month, Day)*	15. PAGE COUNT
Final	FROM 7/86 TO 1/87	1987 May 26	12

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
FIELD	GROUP	SUB-GROUP	Integrals of Bessel functions Hypergeometric functions

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Various representations for incomplete Lipschitz-Hankel integrals of Bessel functions have been given in terms of Kampé de Fériet double hypergeometric functions. Reduction formulas for the double series employed have been given in some cases.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL Allen R. Miller	22b. TELEPHONE *(Include Area Code)* (202) 767-2215 — 22c. OFFICE SYMBOL Code 2303

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete — SECURITY CLASSIFICATION OF THIS PAGE

☆U.S. Government Printing Office: 1985—507-047

UNCLASSIFIED

CONTENTS

INTRODUCTION 1

REPRESENTATIONS FOR $J^{\pm}_{e_{\mu,\nu}}(a, z)$, $J^{\pm}_{c_{\mu,\nu}}(a, z)$, $J^{\pm}_{s_{\mu,\nu}}(a, z)$ 2

REDUCTION FORMULAS FOR L, N, Q 6

SUMMARY 8

REFERENCES 8

UNCLASSIFIED

INCOMPLETE LIPSCHITZ-HANKEL INTEGRALS OF BESSEL FUNCTIONS

INTRODUCTION

The general incomplete Lipschitz-Hankel Integral of Bessel Functions of the first kind is defined by

$$J_{e_{\mu,\nu}}(a, z) \equiv \int_0^z e^{at} t^{\mu} J_{\nu}(t)\, dt \tag{1}$$

Here the symbol e denotes the presence of the exponential function, and μ, ν may be complex numbers. Analogously, we may define integrals that contain the functions sin (at) and cos (at) in place of exp (at):

$$J_{s_{\mu,\nu}}(a, z) \equiv \int_0^z \sin\,(\mathrm{at})\, t^{\mu} J_{\nu}(t)\, dt \tag{2}$$

$$J_{c_{\mu,\nu}}(a, z) \equiv \int_0^z \cos\,(\mathrm{at})\, t^{\mu} J_{\nu}(t)\, dt \tag{3}$$

To assure convergence of these integrals, it is necessary that $Re\,(1+\mu+\nu) > 0$. When $\mu = \nu$ we shall write, for example,

$$J_{e_{\mu,\mu}}(a, z) \equiv J_{e_{\mu}}(a, z) \tag{4}$$

We shall also define integrals of modified Bessel functions $I_{\nu}(t)$ or other cylindrical functions $C(t)$ by simply replacing J by I or C in the above definitions. In addition, we define $J^{+} \equiv J$, $J^{-} \equiv I$.

In Ref. 1 it is shown for the Bessel function of imaginary argument or MacDonald function K_0 that

$$K_{e_0}(a, z) = z\, K_0(z)\, A(a, z) + z^2 K_1(z)\, B(a, z)$$

where

$$A(a, z) \equiv L[\tfrac{1}{2}, 1; \tfrac{1}{2}, \tfrac{3}{2}; \tfrac{a^2z^2}{4}, \tfrac{z^2}{4}] + \tfrac{az}{2} Q[1, 1, 1; 1, 2, \tfrac{3}{2}; \tfrac{a^2z^2}{4}, \tfrac{z^2}{4}]$$

$$B(a, z) \equiv L[\tfrac{1}{2}, 1; \tfrac{3}{2}, \tfrac{3}{2}; \tfrac{a^2z^2}{4}, \tfrac{z^2}{4}] + \tfrac{az}{4} Q[1, 1, 1; 2, 2, \tfrac{3}{2}; \tfrac{a^2z^2}{4}, \tfrac{z^2}{4}]$$

Manuscript approved February 3, 1987.

Here L and Q are Kampé de Fériet double hypergeometric functions (defined below) of order three and four respectively. These functions are therefore non-Gaussian. Only members of the class of double Gaussian series of order two that consists of 34 distinct convergent forms have been given names [2, p. 54]. These 34 forms are sometimes referred to as Horn's list.

In this report we shall show that the functions L and Q may also be employed to give representations for Eqs. 1-4 for I and J. To this end we recall the definitions of the Kampé de Fériet functions L and Q:

$$Q[\alpha, \beta, \gamma; \mu, \nu, \lambda; x, y] \equiv F^{0:2;1}_{2:1;0}\left[\begin{matrix} - : \alpha, \beta ; \gamma ; \\ \mu, \nu : \quad \gamma ; - ; \end{matrix} x, y\right]$$

$$L[\alpha, \beta; \gamma, \delta; x, y] \equiv Q[\alpha, \lambda, \beta; \gamma, \delta, \lambda; x, y] \qquad |x| < \infty, |y| < \infty$$

We shall also introduce the third order function

$$N[\alpha; \beta, \gamma, \delta; x, y] \equiv F^{1:0;0}_{1:1;1}\left[\begin{matrix} \alpha : - ; - ; \\ \beta : \gamma ; \delta ; \end{matrix} x, y\right] \qquad |x| < \infty, |y| < \infty$$

REPRESENTATIONS FOR $\mathbf{J}^{\pm}_{e_{\mu,\nu}}(\mathbf{a}, \mathbf{z})$, $\mathbf{J}^{\pm}_{c_{\mu,\nu}}(\mathbf{a}, \mathbf{z})$, $\mathbf{J}^{\pm}_{s_{\mu,\nu}}(\mathbf{a}, \mathbf{z})$

Since

$$J^{\pm}_{\nu}(t) = \frac{(t/2)^{\nu}}{\Gamma(1+\nu)}\, {}_0F_1[-; 1+\nu; \mp t^2/4]$$

we easily find that

$$e^{at}\, t^{\mu} J^{\pm}_{\nu}(t) = \frac{1}{2^{\nu}\Gamma(1+\nu)} \sum_{n=0}^{\infty} \frac{a^n}{n!} \sum_{m=0}^{\infty} \frac{(\mp 1)^m\, t^{\mu+\nu+2m+n}}{2^{2m}(1+\nu)_m\, m!}$$

Now assuming that $Re(1+\mu+\nu) > 0$ we obtain, on integrating term by term with respect to t,

$$J^{\pm}_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}}{2^{\nu}\Gamma(1+\nu)} \sum_{m,n=0}^{\infty} \frac{(az)^n}{n!} \frac{(\mp z^2/4)^m}{m!} \frac{1}{(1+\nu)_m(1+\mu+\nu+2m+n)} \tag{5}$$

Substituting

$$\frac{1}{1+\mu+\nu+2m+n} = \frac{1}{1+\mu+\nu} \frac{\left(\dfrac{1+\mu+\nu}{2}\right)_m}{\left(\dfrac{3+\mu+\nu}{2}\right)_m} \frac{(1+\mu+\nu+2m)_n}{(2+\mu+\nu+2m)_n}$$

UNCLASSIFIED

into Eq. 5 then gives

$$J^{\pm}_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}}{2^{\nu}(1+\mu+\nu)\,\Gamma(1+\nu)} \sum_{m=0}^{\infty} \frac{\left(\frac{1+\mu+\nu}{2}\right)_m}{\left(\frac{3+\mu+\nu}{2}\right)_m (1+\nu)_m} \frac{(\mp z^2/4)^m}{m!}$$

$$\cdot\, {}_1F_1[1+\mu+\nu+2m;\, 2+\mu+\nu+2m;\, az] \tag{6}$$

Now using Kummer's first theorem

$${}_1F_1[a;\, c;\, z] = e^z\, {}_1F_1\,[c-a;\, c;\, -z]$$

we obtain from Eq. 6

$$J^{\pm}_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu} e^{az}}{2^{\nu}(1+\mu+\nu)\Gamma(1+\nu)} \sum_{m=0}^{\infty} \frac{\left(\frac{1+\mu+\nu}{2}\right)_m}{\left(\frac{3+\mu+\nu}{2}\right)_m (1+\nu)_m} \frac{(\mp z^2/4)^m}{m!}$$

$$\cdot\, {}_1F_1\,[1;\, 2+\mu+\nu+2m;\, -az] \tag{7}$$

Since

$$\frac{1}{(2+\mu+\nu+2m)_n} = \frac{2^{2m}\left(\frac{2+\mu+\nu}{2}\right)_m \left(\frac{3+\mu+\nu}{2}\right)_m}{(2+\mu+\nu)_{2m+n}}$$

we obtain from Eq. 7

$$J^{\pm}_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}\, e^{az}}{2^{\nu}(1+\mu+\nu)\Gamma(1+\nu)} \sum_{m,n=0}^{\infty} \frac{(\mp z^2)^m}{m!} \frac{(-az)^n}{n!}$$

$$\cdot\, \frac{(1)_n \left(\frac{1+\mu+\nu}{2}\right)_m \left(\frac{2+\mu+\nu}{2}\right)_m}{(1+\nu)_m\, (2+\mu+\nu)_{2m+n}} \tag{8}$$

Finally, noting that for any α

$$(2+\alpha)_{2m+2n} = 2^{2m} 2^{2n} \left(\frac{2+\alpha}{2}\right)_{m+n} \left(\frac{3+\alpha}{2}\right)_{m+n}$$

$$(2+\alpha)_{2m+2n+1} = (2+\alpha)2^{2m}2^{2n}\left(\frac{3+\alpha}{2}\right)_{m+n}\left(\frac{4+\alpha}{2}\right)_{m+n}$$

we obtain from Eq. 8 and the definition of Q given earlier

$$J^{\pm}_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}\, e^{az}}{2^{\nu}(1+\mu+\nu)\Gamma(1+\nu)} \tag{9}$$

$$\cdot\left\{Q[\frac{1+\mu+\nu}{2}, \frac{2+\mu+\nu}{2}, 1; \frac{2+\mu+\nu}{2}, \frac{3+\mu+\nu}{2}, 1+\nu; \frac{\mp z^2}{4}, \frac{a^2z^2}{4}]\right.$$

$$\left. - \frac{az}{2+\mu+\nu}\, Q[\frac{1+\mu+\nu}{2}, \frac{2+\mu+\nu}{2}, 1; \frac{3+\mu+\nu}{2}, \frac{4+\mu+\nu}{2}, 1+\nu; \frac{\mp z^2}{4}, \frac{a^2z^2}{4}]\right\}$$

On letting $\mu = \nu$ in Eq. 9 we have

$$J^{\pm}_{e_{\mu}}(a, z) = \frac{z(z^2/2)^{\mu}\, e^{az}}{(1+2\mu)\Gamma(1+\mu)}$$

$$\cdot\left\{L[\frac{1}{2}+\mu, 1; 1+\mu, \frac{3}{2}+\mu; \frac{\mp z^2}{4}, \frac{a^2z^2}{4}]\right.$$

$$\left. - \frac{az}{2(1+\mu)}\, L[\frac{1}{2}+\mu, 1; \frac{3}{2}+\mu, 2+\mu; \frac{\mp z^2}{4}, \frac{a^2z^2}{4}]\right\}$$

In addition we may use Eq. 5 and the definition of N to obtain

$$J^{\pm}_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}}{2^{\nu}\Gamma(1+\nu)}$$

$$\cdot\left\{\frac{1}{1+\mu+\nu}\, N[\frac{1+\mu+\nu}{2}; \frac{3+\mu+\nu}{2}, 1+\nu, \frac{1}{2}; \frac{\mp z^2}{4}, \frac{a^2z^2}{4}]\right.$$

$$\left. + \frac{az}{2+\mu+\nu}\, N[\frac{2+\mu+\nu}{2}; \frac{4+\mu+\nu}{2}, 1+\nu, \frac{3}{2}; \frac{\mp z^2}{4}, \frac{a^2z^2}{4}]\right\} \tag{10}$$

UNCLASSIFIED

For brevity we shall define the following parameter lists ∇_j:

$$\nabla_1 \equiv \frac{1+\mu+\nu}{2}, \frac{2+\mu+\nu}{2}, 1; \frac{2+\mu+\nu}{2}, \frac{3+\mu+\nu}{2}, 1+\nu$$

$$\nabla_2 \equiv \frac{1+\mu+\nu}{2}, \frac{2+\mu+\nu}{2}, 1; \frac{3+\mu+\nu}{2}, \frac{4+\mu+\nu}{2}, 1+\nu$$

$$\nabla_3 \equiv \frac{1+\mu+\nu}{2}; \frac{3+\mu+\nu}{2}, 1+\nu, \frac{1}{2}$$

$$\nabla_4 \equiv \frac{2+\mu+\nu}{2}; \frac{4+\mu+\nu}{2}, 1+\nu, \frac{3}{2}$$

$$\nabla_5 \equiv 1+\mu+\nu, \frac{1}{2}+\nu; 2+\mu+\nu, 1+2\nu$$

We may then obtain from Eqs. 9 and 10

$$J^{\pm}_{c_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}}{2^{\nu}(1+\mu+\nu)\Gamma(1+\nu)} \left\{\cos(az)\, Q[\nabla_1; \frac{\mp z^2}{4}, \frac{-a^2z^2}{4}]\right.$$

$$\left. + \frac{az}{2+\mu+\nu} \sin(az)\, Q[\nabla_2; \frac{\mp z^2}{4}, \frac{-a^2z^2}{4}]\right\}$$

$$= \frac{z^{1+\mu+\nu}}{2^{\nu}(1+\mu+\nu)\Gamma(1+\nu)} N[\nabla_3; \frac{\mp z^2}{4}, \frac{-a^2z^2}{4}] \tag{11}$$

$$J^{\pm}_{s_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}}{2^{\nu}(1+\mu+\nu)\Gamma(1+\nu)} \left\{\sin(az)\, Q[\nabla_1; \frac{\mp z^2}{4}, \frac{-a^2z^2}{4}]\right.$$

$$\left. - \frac{az}{2+\mu+\nu} \cos(az)\, Q[\nabla_2; \frac{\mp z^2}{4}, \frac{-a^2z^2}{4}]\right\}$$

$$= \frac{az^{2+\mu+\nu}}{2^{\nu}(2+\mu+\nu)\Gamma(1+\nu)} N[\nabla_4; \frac{\mp z^2}{4}, \frac{-a^2z^2}{4}]$$

And from these equations we obtain on letting $\mu = \nu$

$$J_{c_\mu}^{\pm}(a, z) = \frac{z^{1+2\mu}}{2^\mu(1+2\mu)\Gamma(1+\mu)} \left\{ \cos(az)\, L[\tfrac{1}{2}+\mu, 1; 1+\mu, \tfrac{3}{2}+\mu; \tfrac{\mp z^2}{4}, \tfrac{-a^2z^2}{4}] \right.$$

$$\left. + \frac{az}{2(1+\mu)} \sin(az)\, L[\tfrac{1}{2}+\mu, 1; \tfrac{3}{2}+\mu, 2+\mu; \tfrac{\mp z^2}{4}, \tfrac{-a^2z^2}{4}] \right\}$$

$$= \frac{z^{1+2\mu}}{2^\mu(1+2\mu)\Gamma(1+\mu)} N[\tfrac{1}{2}+\mu; \tfrac{3}{2}+\mu, 1+\mu, \tfrac{1}{2}; \tfrac{\mp z^2}{4}, \tfrac{-a^2z^2}{4}]$$

$$J_{s_\mu}^{\pm}(a, z) = \frac{z^{1+2\mu}}{2^\mu(1+2\mu)\Gamma(1+\mu)} \left\{ \sin(az)\, L[\tfrac{1}{2}+\mu, 1; 1+\mu, \tfrac{3}{2}+\mu; \tfrac{\mp z^2}{4}, \tfrac{-a^2z^2}{4}] \right.$$

$$\left. - \frac{az}{2(1+\mu)} \cos(az)\, L[\tfrac{1}{2}+\mu, 1; \tfrac{3}{2}+\mu, 2+\mu; \tfrac{\mp z^2}{4}, \tfrac{-a^2z^2}{4}] \right\}$$

$$= \frac{az^{2(1+\mu)}}{2^{1+\mu}(1+\mu)\Gamma(1+\mu)} N[1+\mu; 2+\mu, 1+\mu, \tfrac{3}{2}; \tfrac{\mp z^2}{4}, \tfrac{-a^2z^2}{4}]$$

Finally, noting that $I_\nu(z)$ may be represented by

$$I_\nu(z) = \frac{(z/2)^\nu}{\Gamma(1+\nu)} e^{\pm z}\, {}_1F_1[\tfrac{1}{2}+\nu; 1+2\nu; \mp 2z]$$

we readily obtain

$$I_{e_{\mu,\nu}}(a, z) = \frac{z^{1+\mu+\nu}}{2^\nu(1+\mu+\nu)\Gamma(1+\nu)} F^{1:1;0}_{1:1;0}\left[\begin{matrix} 1+\mu+\nu: & 1/2+\nu; & -; \\ 2+\mu+\nu: & 1+2\nu; & -; \end{matrix} \pm 2z, (a \mp 1)z\right] \tag{13}$$

REDUCTION FORMULAS FOR L, N, Q

In some instances $J_{e_{\mu,\nu}}^{\pm}(a, z)$ may be expressed in terms of generalized hypergeometric functions provided that we know a reduction formula for one of L, N, or Q. By using Ref. 3, p. 55, Eqs. 19, 20, and 21 respectively we find

$$N[\alpha; \beta, \gamma, \gamma; x, -x] = {}_2F_5[\tfrac{\alpha}{2}, \tfrac{\alpha+1}{2}; \tfrac{\beta}{2}, \tfrac{\beta+1}{2}, \gamma, \tfrac{\gamma}{2}, \tfrac{\gamma+1}{2}; \tfrac{-x^2}{4}]$$

UNCLASSIFIED

$$L[\alpha, \beta; \gamma, \delta; x, x] = {}_1F_2[\alpha + \beta; \gamma, \delta; x]$$

$$L[\alpha, \alpha; \gamma, \delta; x, -x] = {}_1F_4[\alpha; \frac{\gamma}{2}, \frac{\gamma+1}{2}, \frac{\delta}{2}, \frac{\delta+1}{2}; \frac{x^2}{16}]$$

Using Ref. 2, p. 28, Eqs. 33 and 34 respectively we find

$$N[\alpha; \beta, \gamma, \delta; x, x] = {}_3F_4[\alpha, \frac{\gamma+\delta-1}{2}, \frac{\gamma+\delta}{2}; \beta, \gamma, \delta, \gamma+\delta-1; 4x] \tag{14}$$

$$Q[\frac{-1/2+\nu}{2}, \frac{1/2+\nu}{2}, 1; \alpha, \beta, 1+\nu; x, x] = {}_2F_3[\frac{3+2\nu}{4}, \frac{5+2\nu}{4}; \alpha, \beta, 1+\nu; x]$$

Employing Eqs. 9 and 13, we easily deduce

$$Q[\nabla_1; \frac{z^2}{4}, \frac{z^2}{4}] = \frac{1}{2}\{e^z\, {}_2F_2[\nabla_5; -2z] + e^{-z}\, {}_2F_2[\nabla_5; 2z]\}$$

$$Q[\nabla_2; \frac{z^2}{4}, \frac{z^2}{4}] = \frac{2+\mu+\nu}{2z}\{e^z\, {}_2F_2[\nabla_5; -2z] - e^{-z}\, {}_2F_2[\nabla_5; 2z]\}$$

And finally, using Eqs. 11 and 12 we find

$$Q[\nabla_1; \frac{-z^2}{4}, \frac{-z^2}{4}] = \cos z\, N[\nabla_3; \frac{-z^2}{4}, \frac{-z^2}{4}]$$

$$+ \frac{1+\mu+\nu}{2+\mu+\nu} z \sin z\, N[\nabla_4; \frac{-z^2}{4}, \frac{-z^2}{4}]$$

$$Q[\nabla_2; \frac{-z^2}{4}, \frac{-z^2}{4}] = (2+\mu+\nu)\frac{\sin z}{z} N[\nabla_3; \frac{-z^2}{4}, \frac{-z^2}{4}]$$

$$- (1+\mu+\nu)\cos z\, N[\nabla_4; \frac{-z^2}{4}, \frac{-z^2}{4}]$$

Replacing z by iz in these equations then gives

$$Q[\nabla_1; \frac{z^2}{4}, \frac{z^2}{4}] = \cosh z\, N[\nabla_3; \frac{z^2}{4}, \frac{z^2}{4}] - \frac{1+\mu+\nu}{2+\mu+\nu} z \sinh z\, N[\nabla_4; \frac{z^2}{4}, \frac{z^2}{4}]$$

$$Q[\nabla_2; \frac{z^2}{4}, \frac{z^2}{4}] = (2+\mu+\nu)\frac{\sinh z}{z} N[\nabla_3; \frac{z^2}{4}, \frac{z^2}{4}] - (1+\mu+\nu)\cosh z\, N[\nabla_4; \frac{z^2}{4}, \frac{z^2}{4}]$$

where, on using Eq. 14,

$$N[\nabla_3; \frac{z^2}{4}, \frac{z^2}{4}] = {}_3F_4[\frac{1+\mu+\nu}{2}, \frac{1/2+\nu}{2}, \frac{3/2+\nu}{2}; \frac{3+\mu+\nu}{2}, \frac{1}{2}+\nu, 1+\nu, \frac{1}{2}; z^2]$$

$$N[\nabla_4; \frac{z^2}{4}, \frac{z^2}{4}] = {}_3F_4[\frac{2+\mu+\nu}{2}, \frac{3/2+\nu}{2}, \frac{5/2+\nu}{2}; \frac{4+\mu+\nu}{2}, \frac{3}{2}+\nu, 1+\nu, \frac{3}{2}; z^2]$$

SUMMARY

Various representations for incomplete Lipschitz-Hankel integrals of Bessel functions have been given in terms of Kampé de Fériet double hypergeometric functions. Reduction formulas for the double series employed have been given in some cases.

REFERENCES

1. A.R. Miller, "An Incomplete Lipschitz-Hankel Integral of K_0, Part II," NRL Report 9001, Feb. 1987.

2. H.M. Srivastava and P.W. Karlsson, *Multiple Gaussian Hypergeometric Series*, Halsted Press, 1985.

3. H.M. Srivastava and H.L. Manocha, *A Treatise on Generating Functions*, Halsted Press, 1984.