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Radar Frequency Selection Strategies

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Strategies for improving the radar detection of weak targets are examined. These strategies are based on transmitting a few pulses, selecting a frequency which obtains the highest signal-to-noise ratio (S/N) on a fluctuating target whose position is not exactly known, and transmitting the remaining pulses during mainbeam illumination at that frequency. Some improvement over random frequency selection is obtained on the selected target while a small degradation occurs for other targets located in the beam.

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CONTENTS

RADAR FREQUENCY SELECTION STRATEGIES

INTRODUCTION

Consider a radar that obtains n pulses on a target during mainbeam illumination. Furthermore, consider the radar to have pulse-to-pulse frequency agility such that the signal-to-noise ratio (S/N) on a selected target fluctuates with frequency changes but maintains the same amplitude for fixed frequency pulse transmissions over the n pulses. These two cases are characterized by the Swerling case II and case I models, when the fluctuations are Rayleigh distributed [1]. Fluctuations can be caused by the target being a complex scattering surface, by the external interference field (jamming, multipath) being frequency sensitive, or by both.

We pose the following:

- Transmit m pulses, where $m < n$, whose frequencies are chosen randomly and store the signals obtained over a selected range interval which is thought to contain a weak target.
- During the m pulse transmissions, one of these pulses might obtain a significantly higher energy on the target than the others due to the S/N fluctuations per pulse.
- If the remaining $(n - m)$ pulses are transmitted at the frequency which yielded the highest energy out of the first m pulses, we expect to obtain a higher signal-to-noise on these than expected from random frequency selection.

The question is, are there strategies for enhancing the probability of detection over random frequency selection on a chosen weak target?

If the frequency selection procedure is successful, a radar system would be envisioned to operate as follows. One target per beam position would be selected for applying the frequency selection strategy. Examples of targets considered would be a new target detection on the previous scan for which no track yet exists, a track that has a low probability of detection (blip-scan-ratio) per scan, a high interest target, etc. In all these cases an approximate range and azimuth is known. After the target is selected, we gather data with m frequency agile pulse transmissions, select a frequency, and transmit the remaining pulses at this frequency. If successful, we will enhance the detectability of the selected target and will pay a slight penalty in detectability for other targets in the beam with respect to random frequency selections.

We begin our discussion by defining the problem. Next, we perform a simplified analysis for a fixed sample size strategy and a sequential strategy. Finally, we obtain the performance of these strategies by simulation and describe the results.

PROBLEM DESCRIPTION

Baseband samples of the received echoes from a pulsed radar at a given range containing a target are represented by a sequence of complex numbers

$$r_i = S_i + N_i$$

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for $i = 1, 2, \dots n$ pulses. We assume that N_i is a zero mean, Gaussian noise sample whose real and imaginary parts (representing the in phase and quadrature signal components) are independent with a standard deviation of σ_n. We will use a Rayleigh fluctuating target model which is independent of the noise. For this case, S_i is also a zero mean, Gaussian signal whose real and imaginary components are independent with a standard deviation of σ_s^2. The S/N ratio per pulse is then conventionally defined by

$$(S/N) = \sigma_s^2/\sigma_n^2.$$

For those range samples which do not contain targets or echoes, $\sigma_s = 0$.

The problem considered in this report is that given pulse returns from random frequencies that provide target decorrelation, how many random frequencies m should be transmitted before the frequency with the maximum returned energy is selected and transmitted on the remaining $(n - m)$ pulses? The received signal can then be written as

$$\begin{aligned} r_i &= S_i + N_i \qquad \text{for } i = 1, \dots m \\ r_i &= S_j + N_i \qquad \text{for } i = (m + 1), \dots n \end{aligned}$$

where the signal amplitude of S_j, denoted by $|S_j|$, is the same as one of the signal amplitudes $|S_i|$ for $i = 1, \dots m$. At a fixed frequency, it is assumed that the target does not decorrelate over the time frame of the n-pulses but that its phase will change due to doppler effects. The signal S_j is obtained by repeating one of the first m frequency selections for the remaining $n - m$ pulses. The selection of the frequency denoted by the index "j" proceeds as follows: The signal energy from the first m pulses using random frequency selection is examined, and the pulse with the maximum energy is chosen. This can be written as

$$|r_j|^2 = \begin{array}{c}\text{max} \\ \text{of the} \\ \text{set}\end{array} \left\{ |r_1|^2,\ |r_2|^2,\ \dots\ |r_m|^2 \right\},$$

where $|r_j|^2$ is the maximum energy of any of the m pulses and j is the associated index which indicates the frequency used.

In many cases we do not know the exact signal location in range, for instance, when a target is being tracked with a search radar. However, we often know the vicinity of the target after an initial detection. In these cases, we need to examine the signals over k range cells where k may be typically 10 to 20. In this case, the selection process is as follows. We compute the total energy

$$\sum_{i=1}^{m} |r_i|^2$$

in the m pulses for each of the k range cells and choose the range cell that has the largest energy. We then find the frequency denoted by index j that yielded the largest energy out of the m pulses for this selected range cell as described previously.

Two different frequency selection criteria are used. The first is based on fixing the value of m pulses. If $m = 1$ (all pulses at a fixed randomly selected frequency), the detection performance is that of Swerling case I and for $m = n$, the detection performance is that of Swerling case II. We examine the performance for other fixed values of m both analytically and by simulation using the selection criterion described previously. The analytic analysis is simplified by considering the noise-free case, $\sigma_n = 0$, and the range cell of the target is known a priori, $k = 1$. The analysis is based on computing the expected energy for various values of m and choosing an m that maximizes the expected energy. For notational convenience we define $x_i = |r_i|^2 = |S_i|^2$ and $y = |r_j|^2 = |S_j|^2$ for $\sigma_n = 0$ and we note x_i is exponentially distributed

$$P_x(x_i) = \frac{1}{2\sigma_s^2}\, e^{-x_i/2\sigma_s^2}. \tag{1}$$

The second frequency selection criterion is based on a sequential hypothesis test. In this case m is not fixed but is a random variable. As we show later, after each pulse i, we make the comparison

$$\frac{|r_j|^2}{2(\sigma_s^2 + \sigma_n^2)} > T(i).$$

If the statistic is greater than the threshold, we set $i = m$ and transmit the remaining $(n - m)$ pulses at the frequency associated with the index j. If the statistic is less than the threshold, another random frequency is chosen for the next pulse transmission. Again the analytic is simplified by considering the noise-free case and the range cell containing the target is known a priori. The analysis is based on computing expected energies for the two cases of selecting a previously used frequency at the ith pulse or selecting a random frequency. The strategy yielding the largest expected energy at the ith step is chosen. We now consider the two criteria by using analysis, followed by the simulation performance results.

FIXED SAMPLE SIZE SELECTION ANALYSIS

The average total energy return E_f for the fixed sample size m selection process is given by

$$E_f(m) = m\, E\{x\} + (n - m)\, E\{y\}, \tag{2}$$

where $E\{\cdot\}$ is the expected value operator. The $E\{x\}$ is given by

$$E\{x\} = \int_0^\infty \frac{x}{2} \exp\,(-x/2)\, dx = 2, \tag{3}$$

where σ_s has been set to one for convenience. To calculate $E\{y\}$ we first need to calculate the density of y. Since y is the maximum value of m identically distributed random variables, its density is given by

$$P_y(y) = m \left[\int_0^y P_x(x)\, dx\right]^{m-1} P_x(y). \tag{4}$$

Substituting Eq. (1) into Eq. (4) and performing the indicated integration yields

$$P_y(y) = \frac{m}{2}\left[1 - e^{-y/2}\right]^{m-1} e^{-y/2}. \tag{5}$$

The expected value of y is

$$\begin{aligned} E\{y\} &= \int_0^\infty y\, P_y(y)\, dy \\ &= 2m \sum_{i=0}^{m-1} \binom{m-1}{i} \frac{(-1)^i}{(i+1)^2}. \end{aligned} \tag{6}$$

The value of $E\{y\}$ is given in Table 1 for several values of m. The optimal value of m (call m_{opt}) for a given n is found by evaluating $E_f(m)$ for all m

Table 1 — The Expected Value of y

m	1	2	3	4	5	6	7
$E\{y\}$	2.0	3.0	3.666	4.166	4.567	4.9	5.186

and selecting the largest $E_f(m)$. The optimal values of m for several n are given in Table 2. The associated signal-to-noise improvement I is given by

$$I = 10 \log\,[E_f(m_{\text{opt}})/E_f(n)] \tag{7}$$

and is also shown in Table 2.

Table 2 — Optimal Value of m

n	4	5	8	10	16
m_{opt}	2	3	4	4	6
I	0.97	1.25	1.88	2.17	2.80

Note that we do not always select the proper frequency because the returned signal is corrupted with noise. Since the actual improvement will be less than that shown in Table 2, the actual improvement for this model is found by simulation.

SEQUENTIAL SELECTION ANALYSIS

In this section we consider a sequential selection process. After receiving m pulses we either

(a) transmit the frequency which had the maximum return,

or

(b) transmit a new frequency.

Strategy (a) was considered in the previous section, and its expected energy return conditioned on $x_1, \ldots x_m$ is given by

$$E_f(m) = \sum_{i-1}^{m} x_i + (n - m)\, y. \tag{8}$$

The expected energy produced by strategy (b) is calculated next, and we will use whichever strategy has the highest energy return.

First, let P be the probability that the new sample z is larger than y; i.e., P is the probability that z is the largest of the $(m + 1)$ samples. Then the expected energy return for strategy (b) again conditioned on $x_1, \ldots, x_m$ is

$$E_s(m) = \sum_{i=1}^{m} x_i + E\{z\} + P(n - m - 1)\; E\{z|z \geqslant y)\}$$

$$+ (1 - P)\; (n - m - 1)y. \tag{9}$$

The first term is the previous returns, the second term is the expected value of the energy in the new return, the third term is the expected value of the remaining samples when z is larger than y, and the fourth term is the value of the remaining samples when z is less than y. The value of P is

$$P = \int_y^\infty \frac{1}{2\sigma_s}\; e^{-x_i/2\sigma_s^2}\; dx_i = e^{-y/2\sigma_s^2}. \tag{10}$$

However, since the value of σ_s^2 is unknown, we will replace it by its maximum likelihood estimate. Specifically, let

$$2\hat{\sigma}_s^2 = \bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i. \tag{11}$$

Substituting Eq. (11) into Eq. (10) yields

$$P \approx e^{-y/\bar{x}}. \tag{12}$$

The expected value of z is $2\sigma_s^2$ is approximated by $\bar{x}$. The density of z given that $z \geqslant y$ is just

$$P(z|z \geqslant y) = \begin{cases} \dfrac{1}{P}\,\dfrac{1}{2\sigma_s^2}\, e^{-z/2\sigma_s^2} & z \geqslant y \\ 0 & z < y \end{cases} \quad . \tag{13}$$

Then

$$E\{z|z \geqslant y\} = \int_y^{\infty} \frac{z}{P}\,\frac{1}{2\sigma_s^2}\, e^{-z/2\sigma_s^2}\, dz. \tag{14}$$

Substituting Eqs. (11) and (12) into Eq. (14) and integrating by parts yields

$$E\{z|z \geqslant y\} \approx y + \bar{x}. \tag{15}$$

Next, substituting Eqs. (12) and (15) into Eq. (9) yields

$$E_s(m) = \sum_{i=1}^{m} x_i + \bar{x} + e^{-y/\bar{x}}\,(n - m - 1)\,(y + \bar{x}) \tag{16}$$

$$+ (1 - e^{-y/\bar{x}})\,(n - m - 1)\,y$$

which simplifies to

$$E_s(m) = \sum_{i=1}^{m} x_i + \bar{x} + (n - m - 1)\,y + e^{-y/\bar{x}}\,(n - m - 1)\,\bar{x}. \tag{17}$$

Finally, one should transmit a new frequency if Eq. (17) is greater than Eq. (8). Equivalently, one should transmit a new frequency when

$$(n - m - 1)\,e^{-y/\bar{x}} + 1 \geqslant y/\bar{x}. \tag{18}$$

This decision rule can be put in the form of

$$y/\bar{x} \leqslant T(m). \tag{19}$$

The threshold values for $n = 4$, 8, and 16 are given in Table 3. Since the sample y is included in $\bar{x}$ (See Eq. 11)), the ratio $y/\bar{x}$ is bounded by

$$\frac{y}{\bar{x}} \leqslant m. \tag{20}$$

Consequently, when $n = 16$, the threshold $T(m)$ cannot be exceeded until $m = 3$.

SIMULATION RESULTS

Most of the simulation effort was centered on the fixed sample size selection criteria and then the sequential selection criteria was compared to it. Two detectors were considered. One detector integrated the square of the amplitude or energy for the n pulses and compared it to a threshold. This operation is defined by

$$\sum_{i=1}^{n} |r_i|^2 > T_f(n),$$

and a detection is declared when the threshold is exceeded. Recall the first m samples use random frequency selection while the remaining $(n - m)$ pulses use a frequency selected from these first m pulses. The thresholds $T_f(n)$ are obtained from Ref. 2 and the values used for a probability of false alarm (P_{fa}) of 10^{-6} are given by

$$T_f(n) = 2\,\alpha(n)\,\sigma_n^2$$

where values of $\alpha(n)$ are given in Table 4.

Table 3 — Threshold Value for $y/\bar{x}$ when $n = 16$, 8, and 4

	$n = 16$	$n = 8$	$n = 4$
m	$T(m)$	$T(m)$	$T(m)$
1	2.347	1.899	1.463
2	2.301	1.815	1.278
3	2.257	1.718	1.0
4	2.208	1.603	
5	2.157	1.463	
6	2.101	1.278	
7	2.040	1.0	
8	1.973		
9	1.899		
10	1.815		
11	1.718		
12	1.603		
13	1.463		
14	1.278		
15	1.000		

Table 4 — Threshold Values for Square Law Integration, Probability of False Alarm of 10^{-6}, and an Underlying Gaussian Noise Process

n	$\alpha(n)$
1	13.8155
2	16.6884
3	19.1292
4	21.3505
5	23.4315
6	25.4126
7	27.3177
8	29.1622
16	42.6158

The second detector considered uses a coherent integration after a frequency is selected and is maintained over the remaining $(n - m)$ pulse transmissions. We assume we know the phase change from pulse to pulse due to doppler perfectly. Generally this is not true and there is some loss associated with a doppler filter bank when the target does not exactly match the filter. This loss is ignored in this analysis. However, this loss can be avoided if all the energy in the last $(n - m)$ pulses is placed in one pulse at the selected frequency. Whether this last procedure can be accomplished or not depends on the transmitter design. The second detector performs the operation

$$\sum_{i=1}^{m} |r_i|^2 + \frac{1}{(n-m)} \left| \sum_{i=m+1}^{n} w_i r_i \right|^2 > T_f(m+1),$$

and a detection is declared when the threshold is exceeded. For complete coherent integration we set $m = 0$. The doppler filter weights are

$$w_i = S_i^* / |S_i|,$$

where (*) is the complex conjugate. If only one large pulse is used, the second term is replaced by its received signal energy. The thresholds $T_f(m+1)$ are found in the same manner as the thresholds for the noncoherent detector case.

The simulation generated the signals as described earlier, passed them through the detectors and the percentage of threshold crossings, which is an estimate of the probability of detection (P_d) is found. The probability of detection is plotted versus S/N per pulse for $P_{fa} = 10^{-6}$ for all the performance curves given. We begin by observing the performance using the fixed sample size selection criteria for $n = 16$, 8, and 4 in Figs. 1, 2, and 3. In all cases $k = 1$ which means that the range cell containing the target is known a priori. In each figure $m = n$, $m = 0$, and $m = m_{opt}$ random frequencies are used. The $m = n$ cases used the noncoherent detector, the $m = 0$ cases used the coherent detector, and $m = m_{opt}$ used both the coherent an noncoherent detectors. The $m = n$ and $m = 0$ curves are the standard square law detection curves for pulse-to-pulse fluctuating targets and scan-to-scan fluctuating targets for a pulse burst of length n embedded in thermal noise. The typical characteristic of $m = n$ is better at high S/N and $m = 0$ is better at low S/N. For $m = m_{opt}$ the noncoherent detection curve is slightly worse than the coherent detection curve. In all three cases, $n = 16$, 8, or 4, and $m = m_{opt}$, we find the detection performance is close to the $m = 0$ case for low S/N and near the $m = n$ case for high S/Ns. We observe that the $m = m_{opt}$ strategy provides improved detection performance over random frequency selection. Also, the performance improvement increases with the number of pulses n that are transmitted. For a P_d of 0.5, we find a 2.4 dB, 1.4 dB, and 0.6 dB improvement in S/N for m_{opt} over $m = n$ for $n = 16$, 8, and 4.

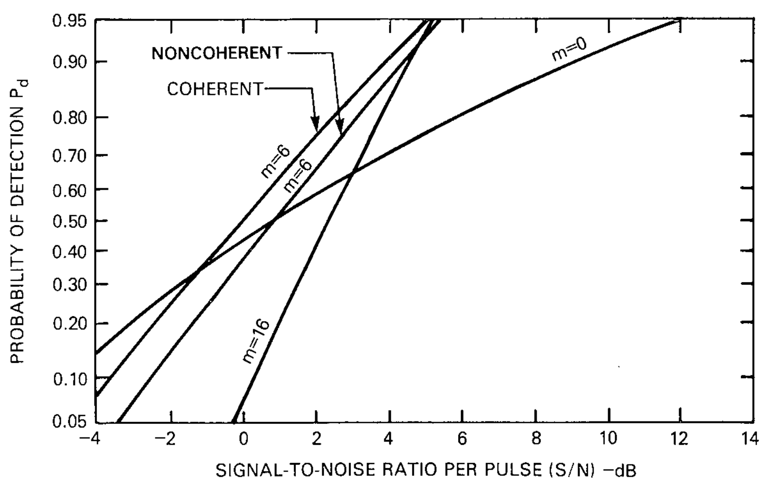

Fig. 1 — Performance characteristics for $n = 16$ pulses, $k = 1$ range cells, probability of false alarm of 10^{-6} for the cases of (1) random frequency selection $m = 16$, (2) fixed frequency selection using coherent integration $m = 0$, (3) fixed sample size using coherent integration $m = m_{opt} = 6$, and (4) fixed sample size using noncoherent integration $m = m_{opt} = 6$.

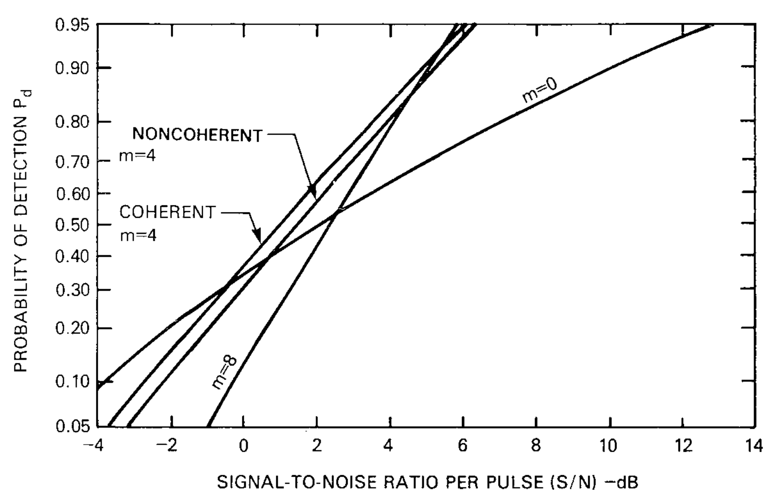

Fig. 2 — Performance characteristics for $n = 8$ pulses, $k = 1$ range cells, probability of false alarm of 10^{-6} for the cases of (1) random frequency selection $m = 8$, (2) fixed frequency selection using coherent integration $m = 0$, (3) fixed sample size using coherent integration $m = m_{opt} = 4$, and (4) fixed sample size using noncoherent integration $m = m_{opt} = 4$.

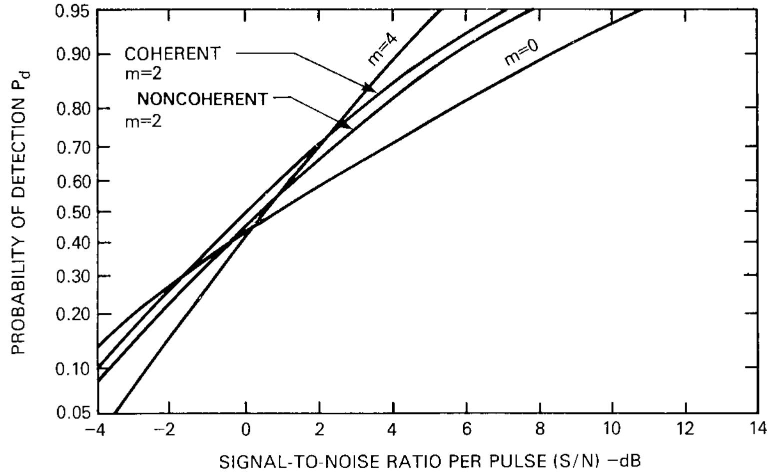

Fig. 3 — Performance characteristics for $n = 4$ pulses, $k = 1$ range cells, probability of false alarm of 10^{-6} for the cases of (1) random frequency selection $m = 4$, (2) fixed frequency selection using coherent integration $m = 0$, (3) fixed sample size using coherent integration $m = m_{opt} = 2$, and (4) fixed sample size using noncoherent integration $m = m_{opt} = 2$.

We next examined the sensitivity of the performance to m. Figure 4 shows the noncoherent detection results for $m = (m_{opt} - 1)$, m_{opt}, and $(m_{opt} + 1)$, with $n = 16$, 8, and 4, and $k = 1$. We find that the detection performance is not very sensitive to m except for small n where small changes in m become large percentage changes with respect to n.

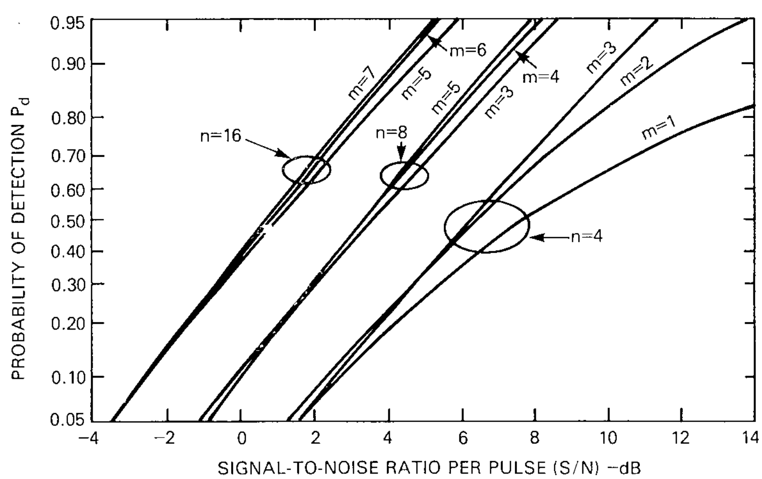

Fig. 4 — Comparison of performance for $m = (m_{opt} - 1)$, m_0, and $(m_{opt} + 1)$, $n = 16$, 8, and 4. In all cases, the fixed sample size selection criteria and the noncoherent integration were used for $k = 1$ range cells and a probability of false alarm of 10^{-6}.

We next examined the losses due to not knowing exactly the range cell of the target. Figure 5 shows the probability of detection for the noncoherent detector for $m = m_{opt}$, with $n = 16$, 8, and 4, and $k = 1$ and 10. Not knowing the range of the target exactly is not detrimental to the detection performance. It appears that in most cases we chose correctly the range cell for selecting the frequency for the remaining $(n - m)$ pulses transmissions.

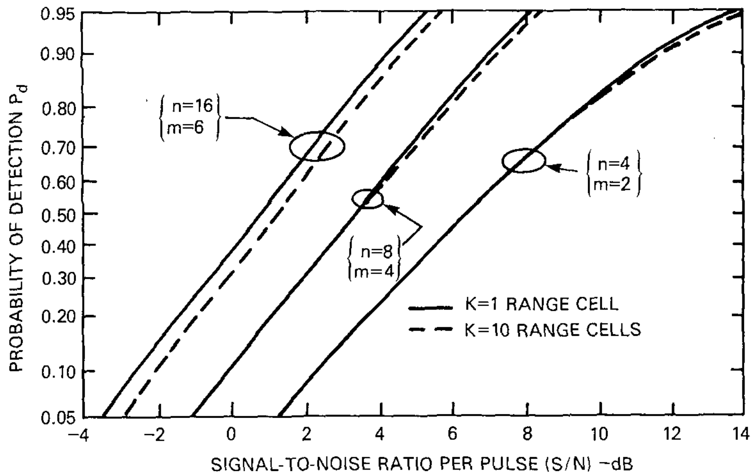

Fig. 5 — Comparison of performance for $k = 1$ and 10 using $n = 16$, 8, and 4, and $m = m_{opt}$. The fixed sample size selection criteria and the noncoherent integration were used for a probability of false alarm of 10^{-6}.

We next examine some performance bounds. Figure 6 shows the probability of detection for three cases each with $n = 4$ and 16 and $k = 1$. One case is the performance for noncoherent integration using the fixed sample size criteria with $m = m_{opt}$. The second case shows the performance if the frequency selected after m pulses was chosen from noise-free samples rather than noisy samples. The third case involves knowing a priori which frequency out of the n should be transmitted. We do not pay a large penalty for selecting the best frequency out of the first m noisy samples over that of being able to select the frequency in the absence of noise. However, if we knew which frequency to use a priori, large performance gains can be obtained. It is interesting to note that the performance is not limited much by having to choose a range cell and frequency from noisy samples. Rather, most of the performance gains are taken away by having to use a significant amount of energy (m pulses) in making a frequency selection and then not always making a good choice because the first m pulses may not contain a good frequency.

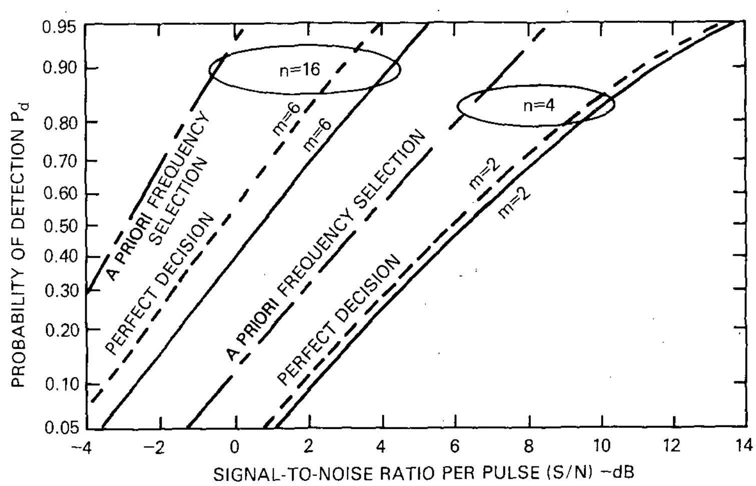

Fig. 6 — Comparison of performance for the cases of (1) fixed sample-size selection, (2) noise free or perfect fixed sample-size selection, (3) a priori best frequency selection for $n = 4$ and 16. In all cases $k = 1$, the probability of false alarm is 10^{-6}, and noncoherent integration is used.

If the radar selects a frequency based on one target in the radar beam, what happens to the detection performance for other targets which might also be in the radar's beam? Figure 7 examines this detection performance using the noncoherent detector for $n = 16$, 8, and 4, and $k = 1$. The dashed curve is for the other targets where the first m pulses uses random frequency selection and the remaining $(n - m)$ pulses use one of the first m pulses' frequencies but is randomly selected. The solid curves are the $m = n$ curves which are valid for either target. We find that at high S/Ns there is some loss over random selection, but at low S/N there is even a little gain over random selection. We conclude that the performance for the other targets is not as good as the target for which the frequency is being selected.

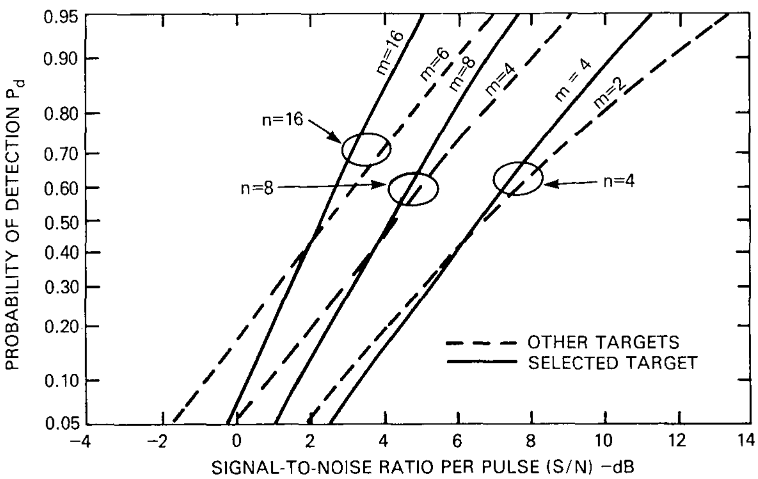

Fig. 7 — Comparison of performance of target of opportunity using fixed sample-size selection for another chosen target to that of random selection for $n = 16$, 8, and 4, and $m = m_{\text{opt}}$. In all cases $k = 1$, the probability of false alarm is 10^{-6} and noncoherent integration is used.

Finally, we observe the detection performance for the sequential hypothesis selection criteria. Figure 8 shows the probability of detection for both the sequential hypothesis and fixed sample size selection criteria for $n = 16$, 8, and 4, $k = 1$, m_{opt}, and using the noncoherent detector. The sequential criteria was slightly better than the fixed sample criteria, however, we used a perfect estimate of $\bar{x} = 0.5/(\sigma_s^2 + \sigma_n^2)$ which would yield some loss when estimated from the data. Since the performance of the sequential test was not much better than the fixed test, we did not examine it in the same detail.

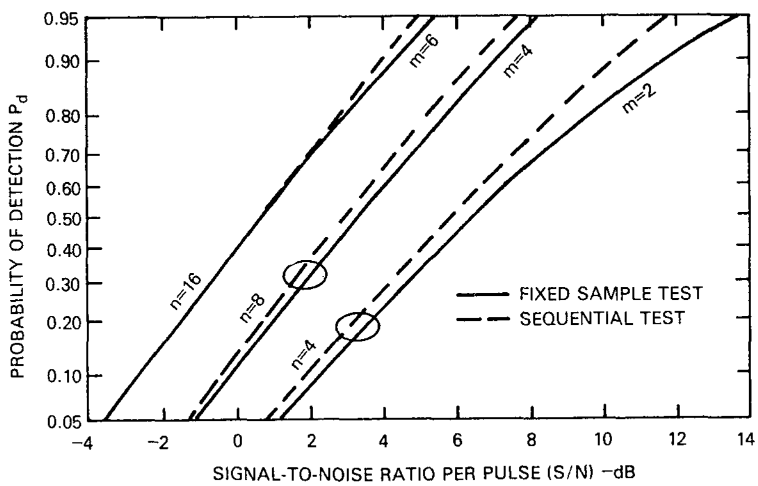

Fig. 8 — Comparison of performance of sequential selection criteria to the fixed sample-size selection criteria for $m = m_{\text{opt}}$ using $n = 16$, 8, and 4. In all cases $k = 1$, the probability of false alarm is 10^{-6} and noncoherent integration is used.

SUMMARY

Several strategies for selecting the frequencies of the transmitted radar pulses were considered for the purpose of improving the radar's detection performance for selected targets. In general, we found that the detection performance could be improved over random frequency selection by a small amount. The performance improvement depended mostly on the S/N and the number of pulses n. For a probability of detection of 0.5, the performance improvement over random frequency selection using the fixed sample selection criteria for $m = m_{\text{opt}}$, $k = 1$, $P_{fa} = 10^{-6}$, is 2.4 dB, 1.4 dB, and 0.6 dB for $n = 16$, 8, and 4.

Using the fixed sample size selection criteria, we found that the performance was not very sensitive to not knowing the range of the target exactly, to m the number of random frequencies, to the type of detector (whether coherent or noncoherent), or to whether the decision was made from noisy samples or noiseless samples. We found that the losses with respect to random frequency selection were not very large for other targets in the radar beams and that there was even a small improvement in performance at low S/N. Finally, we found that the sequential frequency selection criteria yielded nearly the same results as the fixed sample size criteria.

The results presented involved four frequency selection criteria: random frequency, fixed sample size, sequential, and fixed frequency. Only one target and noise model was used, and it was assumed that frequency agility yielded independent target samples. Undoubtedly, there are other selection criteria, target and noise models, and system characteristics which could be used and evaluated. The results to date show that small improvements in radar performance can be obtained by using a good frequency selection criteria.

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