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Emitter Location Using Bearing Measurements from a Moving Platform

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An algorithm is derived for determining for the location of a stationary emitter based upon a number of angular measurements from a moving platform. It is referred to as the asymptotic algorithm and is particularly useful when there is appreciable error in the angle measurements. It is restricted to platform motion which generates a base line small in comparison to the range from the platform to the emitter. Under there conditions better results are obtained than from the standard least

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20. ABSTRACT (Continued)

squares distance algorithm. This is accomplished with virtually the same amount of memory and computer logic.

An algorithm that obtains emitter velocity as well as position is also given in the report. This algorithm, which requires a nonlinear maneuver by the platform, gives useful results when the system has good direction-finding accuracy and a capability for recording the time of a measurement. Typical results for both algorithms are presented in tables from which it is possible to judge the merits of the method for a particular application.

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EMITTER LOCATION USING BEARING MEASUREMENTS FROM A MOVING PLATFORM

INTRODUCTION

This report presents a set of related mathematical algorithms for determining the location of an emitter based upon a number of angular measurements from a moving platform. For all except the last algorithm given, the emitter is stationary with respect to the bearing measurement platform. These algorithms are particularly useful when there is appreciable error in the angle measurements, but they are restricted to platform motion which generates a base line small in comparison to the range from the platform to the emitter. An alternative approach to this problem is given in a paper by Mahapatra [1].

Before presenting the new algorithms, another will be outlined for purposes of comparison. This is the standard algorithm for determining the best solution of an overdetermined system of linear equations [2]. It minimizes the sum of the squares of the distances to the measured bearing lines. Implementing this procedure does not require that all of the angle measurements be stored in memory but rather five sums are accumulated in memory as the various fixes are taken. Based upon this information an estimate of emitter location is obtained by solving a system of linear equations.

For the particular set of conditions one is concerned with here, the combined effect of large angle errors and short base line causes the standard algorithm to sometimes fail to even approximate the correct emitter position, whereas these new algorithms do work. The new algorithms approximate the condition that minimizes the sum of the squares of the angle errors. Although the exact imposition of this condition leads to numerical problems which are solvable only by time consuming iterative processes [3], the approximation presented herein gives a new algorithm comparable to the standard algorithm in that it also requires the accumulation in memory of only five sums. The estimate of the emitter location is again determined by a system of linear equations.

In the course of deriving this latter algorithm, an intermediate algorithm, referred to as the quadratic algorithm, will be arrived at whereby emitter location is determined by a simultaneous system consisting of a linear equation, a quadratic equation, and a sign condition. With further reckoning, this system is reduced to a pair of linear equations whereby an algorithm referred to as the asymptotic algorithm is determined. The last algorithm given is the result of extending the asymptotic algorithm so that it determines emitter velocity as well as position.

The general configuration being considered is that of an emitter along with a moving ESM platform from which angle measurements on the emitter are made at a number of positions indexed by i which runs from 1 to N. Notation used in presenting the algorithms referred to above is introduced as follows and is illustrated in Fig. 1:

- d_i — Perpendicular distance from estimated emitter position to the line of sight for the i th bearing measurement
- (p_i, q_i) — Position of ESM platform for the i th look at the emitter
- (x,y) — Cartesian coordinates of the estimated emitter position

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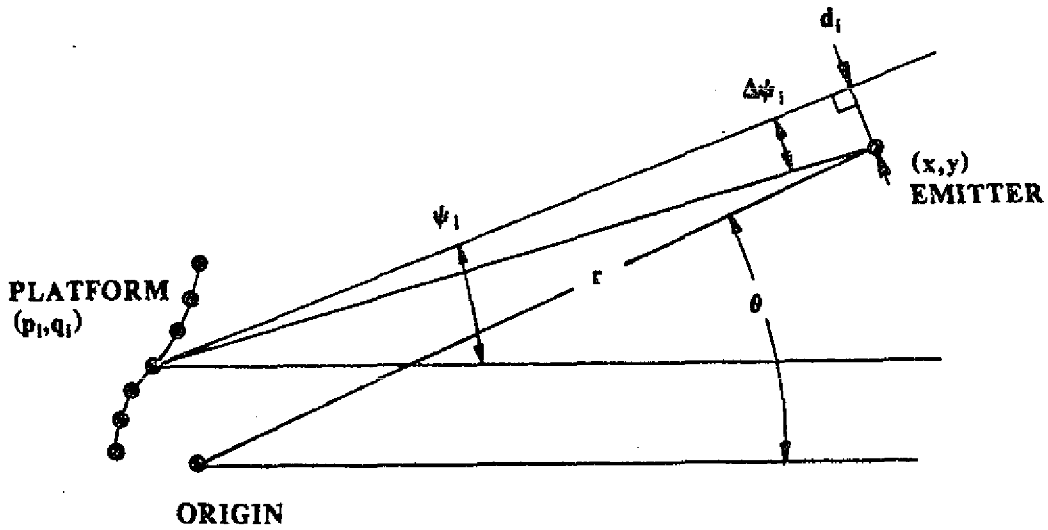

Fig. 1 — Geometric configuration of the emitter and moving platform illustrating the angle measurements

(r,θ)	Polar coordinates of the estimated emitter position
ψ_i	Emitter bearing as measured by the i th look from the ESM platform
$\Delta\psi_i$	Angular difference between i th measurement and calculated estimate of the emitter position

If the distances involved are such that the earth's curvature is significant, the above quantities are defined in a plane tangent to the earth's surface at a point near the platform position. The resulting emitter position is then projected from the plane onto the earth's surface.

THE DISTANCE LEAST SQUARES ALGORITHM

In this section, the algorithm that minimizes the sum of the squares of the distances to the measured bearing lines is derived. The distance to the i th bearing line is given by

$$d_i = |a_i x + b_i y - c_i|, \tag{1}$$

where

$$a_i = \sin\psi_i, \tag{2}$$

$$b_i = -\cos\psi_i, \tag{3}$$

and

$$c_i = p_i \sin\psi_i - q_i \cos\psi_i. \tag{4}$$

The quantity to be minimized is expressed as follows:

$$M = \sum_{i=1}^{N} d_i^2. \tag{5}$$

Taking partial derivatives of M with respect to x and y gives the following system of linear equations for these variables.

$$B_1x + Ay = E \tag{6}$$

$$Ax + B_2y = -D \tag{7}$$

Here, the following notation has been incorporated:

$$A = \sum_{i=1}^{N} a_i b_i, \tag{8}$$

$$B_1 = \sum_{i=1}^{N} a_i^2, \tag{9}$$

$$B_2 = \sum_{i=1}^{N} b_i^2, \tag{10}$$

$$D = -\sum_{i=1}^{N} b_i c_i, \tag{11}$$

and

$$E = \sum_{i=1}^{N} a_i c_i. \tag{12}$$

The simultaneous solution of Eqs. 6 and 7 is:

$$x = \frac{B_2 E + AD}{B_1 B_2 - A^2} \tag{13}$$

and

$$y = \frac{-B_1 D - AE}{B_1 B_2 - A^2} \tag{14}$$

This derivation determines the distance least squares algorithm for calculating the emitter position. It consists of the following equations to be evaluated in the order given: 2, 3, 4, 8, 9, 10, 11, 12, 13, and 14.

THE QUADRATIC ALGORITHM

In this section, an algorithm is devised that for small base line and large angle errors gives considerable improvement in the accuracy of the results. The algorithm approximately minimizes the sum of the squares of the angle errors $\Delta\psi_i$. This condition is expressed as follows:

$$M = \sum_{i=1}^{N} \Delta\psi_i^2, \tag{15}$$

where

$$\Delta\psi_i = \sin^{-1} \frac{d_i}{\sqrt{(x - p_i)^2 + (y - q_i)^2}}. \tag{16}$$

To arrive at a practical algorithm for determining the emitter position associated with this minimization, one must assume the following approximations: the ESM platform positions are all in the vicinity of the origin and the angle $\Delta\psi_i$ can be approximated by

$$\Delta\psi_i = \frac{d_i}{r}. \tag{17}$$

With these approximations the quantity to be minimized is given by

$$M = \sum_{i=1}^{N} \left(\frac{d_i}{r}\right)^2. \tag{18}$$

Expressed in polar coordinates d_i is given as

$$d_i = |a_i r \cos\theta + b_i r \sin\theta - c_i|. \tag{19}$$

Using this expression in Eq. 18 gives the following:

$$M = \sum_{i=1}^{N} \left(a_i \cos\theta + b_i \sin\theta - \frac{c_i}{r}\right)^2. \tag{20}$$

To minimize M, partial derivatives of the above expression are taken with respect to r and θ. The following expressions result.

$$\frac{\partial M}{\partial r} = 2 \sum_{i=1}^{N} \left(a_i \cos\theta + b_i \sin\theta - \frac{c_i}{r}\right)\left(\frac{c_i}{r^2}\right) \tag{21}$$

$$\frac{\partial M}{\partial \theta} = 2 \sum_{i=1}^{N} \left(a_i \cos\theta + b_i \sin\theta - \frac{c_i}{r}\right)\left(-a_i \sin\theta + b_i \cos\theta\right). \tag{22}$$

The partial derivatives represented by Eqs. 21 and 22 are set equal to zero. The resulting equations are multiplied by appropriate factors and expressed in terms of Cartesian coordinates. These steps result in the following system, a quadratic and a linear equation for x and y:

$$Ax^2 + Bxy - Ay^2 + Dx + Ey = 0 \tag{23}$$

$$Ex - Dy = G. \tag{24}$$

B and G are given as follows:

$$B = B_2 - B_1 \tag{25}$$

and

$$G = \sum_{i=1}^{N} c_i^2. \tag{26}$$

The coefficients A, B_1, B_2, D, and E have previously been defined. A variable u is introduced with the equation

$$Dx + Ey = u, \tag{27}$$

which is solved simultaneously with Eq. 24 to give

$$x = \frac{Du + EG}{E^2 + D^2} \tag{28}$$

and

$$y = \frac{Eu - DG}{E^2 + D^2}. \tag{29}$$

Substituting Eqs. 28 and 29 into Eq. 23 gives the following quadratic equation for u:

$$\Lambda u^2 + \Omega u + \Gamma = 0. \tag{30}$$

Here, the coefficients are given by the following expressions:

$$\Lambda = A(D^2 - E^2) + BDE, \tag{31}$$

$$\Omega = 4ADEG + BG(E^2 - D^2) + (E^2 + D^2)^2, \tag{32}$$

and

$$\Gamma = G^2(AE^2 - BDE - AD^2). \tag{33}$$

Using the quadratic formula, u is obtained as

$$u = \frac{-\Omega \pm \sqrt{\Omega^2 - 4\Lambda\Gamma}}{2\Lambda}. \tag{34}$$

To determine the $\pm$ sign in Eq. 34, one must use the following condition which assumes a minimum value of M:

$$\frac{\partial^2 M}{\partial\theta^2} > 0. \tag{35}$$

Carrying out the indicated partial differentiations results in the following condition:

$$Bx^2 - 4Axy - By^2 + Ex - Dy > 0. \tag{36}$$

Hence the $\pm$ sign in Eq. 34 is chosen to satisfy Eq. 36.

This derivation determines the quadratic algorithm for calculating the emitter position. It consists of the following equations to be evaluated in the order given: 2, 3, 4, 8, 9, 10, 25, 11, 12, 26, 31, 32, 33, 34, 28, 29, and 36.

THE ASYMPTOTIC ALGORITHM

In this section the quadratic algorithm of the previous section is simplified using an asymptotic approximation. Equation 23 is a hyperbola which may be approximated by one of its asymptotes. This leads to a new algorithm, which, for all practical purposes, gives nearly identical results to those of the quadratic algorithm.

Determination of the asymptote is accomplished in a coordinate system, referred to as the primed system, for which the axes have been rotated through an angle ω. In this system, Eqs. 23 and 24, after being transformed, retain the same form with the corresponding variables and parameters denoted by primes. The transformed equations are

$$A'x'^2 + B'x'y' - A'y'^2 + D'x' + E'y' = 0 \tag{37}$$

and

$$E'x' - D'y' = G'. \tag{38}$$

The primed quantities are expressed in terms of the unprimed quantities as follows:

$$x' = x\cos\omega + y\sin\omega \tag{39}$$

$$y' = -x \sin \omega + y \cos \omega \tag{40}$$

$$A' = A \cos^2 \omega + B \sin \omega \cos \omega - A \sin^2 \omega \tag{41}$$

$$B' = -4A \sin \omega \cos \omega + B(\cos^2 \omega - \sin^2 \omega) \tag{42}$$

$$D' = D \cos \omega + E \sin \omega \tag{43}$$

$$E' = -D \sin \omega + E \cos \omega \tag{44}$$

$$G' = G \tag{45}$$

The primed coordinate system is chosen so that the asymptotes are parallel to the coordinate axes. This is accomplished by choosing

$$A' = 0. \tag{46}$$

Imposing this condition on Eq. 41 gives

$$A \cos^2 \omega + B \sin \omega \cos \omega - A \sin^2 \omega = 0 \tag{47}$$

From this relation the sine and cosine of the angle of rotation are determined to be

$$\cos \omega = -2\alpha A, \tag{48}$$

and

$$\sin \omega = \alpha(\beta - B), \tag{49}$$

where

$$\alpha = \frac{1}{\sqrt{2\beta(\beta - B)}}, \tag{50}$$

and

$$\beta = \sqrt{4A^2 + B^2}. \tag{51}$$

Substituting Eqs. 48 and 49 into Eqs. 39 through 45 gives the following set of equations representing the coordinate transformation.

$$x' = \alpha[-2Ax + (\beta - B)y], \tag{52}$$

$$y' = \alpha[-(\beta - B)x + 2Ay], \tag{53}$$

$$B' = \beta, \tag{54}$$

$$D' = \alpha[-2AD + E(\beta - B)], \tag{55}$$

and

$$E' = \alpha[-D(\beta - B) - 2AE]. \tag{56}$$

Applying the condition of Eq. 46 to Eq. 37 gives, for the equation of the hyperbola in the primed system,

$$B'x'y' + D'x' + E'y' = 0. \tag{57}$$

This equation may be expressed as follows:

$$(B'x' + E')(B'y' + D') = D'E'. \tag{58}$$

Its asymptotes are

$$B'x' + E' = 0 \tag{59}$$

and

$$B'y' + D' = 0. \tag{60}$$

The condition that M be a minimum rather than a maximum makes Eq. 36 in the unprimed system become in the primed system

$$B'(x'^2 - y'^2) + E'x' - D'y' > 0. \tag{61}$$

This relation may also be expressed as

$$\left(x' + \frac{E'}{2B'}\right)^2 - \left(\frac{E'}{2B'}\right)^2 - \left(y' + \frac{D'}{2B'}\right)^2 + \left(\frac{D'}{2B'}\right)^2 > 0. \tag{62}$$

The asymptote represented by Eq. 60 satisfies this condition for values of x' far from the origin where the asymptote is a good approximation; minimum M is obtained for x' outside the interval $(-E'/B', 0)$. In the unprimed system this asymptote is given by

$$Rx + Sy = T, \tag{63}$$

where

$$R = \beta(\beta - B), \tag{64}$$

$$S = 2A\beta, \tag{65}$$

and

$$T = E(\beta - B) - 2AD. \tag{66}$$

The estimate of the target position is calculated by solving the simultaneous linear system consisting of Eqs. 24 and 63. This results in the following expressions for the target position:

$$x = \frac{GS + DT}{ES + DR} \tag{67}$$

and

$$y = \frac{ET - GR}{ES + DR}. \tag{68}$$

The algorithm resulting from this derivation consists of Eqs. 2, 3, 4, 8, 9, 10, 25, 11, 12, 26, 51, 64, 65, 66, 67, and 68, evaluated in that order.

The process followed in applying this algorithm may be summarized as follows. For each bearing measurement ψ_i it is necessary to know the platform position (p_i, q_i). Five sums are accumulated in memory: the quantities A, B, D, E, and G, which determine a pair of linear equations to be solved for the estimated emitter position.

SAMPLE RESULTS FOR THE ASYMPTOTIC ALGORITHM

Results are shown in Table 1 for the asymptotic algorithm under a number of assumed conditions. The true emitter range is 200 distance units. The ESM platform moves normal to the true emitter

Table 1 — Sample Results for the Asymptotic Algorithm (range = 200 units)

Looks	Base (units)	Sigma DF (deg)	Sigma Range (units)	Sigma Bear. (deg)	Ave. % Error Range
5	5	0.1	17.6	.04	8.81
5	10	0.1	8.6	.04	4.28
5	10	0.2	17.6	.09	8.81
5	20	0.1	4.2	.04	2.12
5	20	0.2	8.6	.09	4.28
5	50	0.1	1.7	.05	0.86
5	50	0.2	3.4	.09	1.72
5	50	0.5	8.7	.22	4.33
5	50	1.0	17.9	.45	8.93
10	5	0.1	13.0	.03	6.50
10	10	0.1	6.6	.03	3.28
10	10	0.2	13.0	.07	6.50
10	20	0.1	3.3	.03	1.65
10	20	0.2	6.6	.07	3.28
10	20	0.5	16.3	.17	8.13
10	50	0.1	1.3	.03	0.67
10	50	0.2	2.7	.07	1.34
10	50	0.5	6.6	.17	3.31
10	50	1.0	13.1	.35	6.57
20	5	0.1	11.1	.03	5.54
20	10	0.1	5.5	.03	2.77
20	10	0.2	11.1	.05	5.54
20	20	0.1	2.8	.03	1.39
20	20	0.2	5.5	.05	2.77
20	20	0.5	13.9	.13	6.95
20	50	0.1	1.1	.03	0.56
20	50	0.2	2.2	.05	1.12
20	50	0.5	5.6	.13	2.80
20	50	1.0	11.2	.27	5.60
50	5	0.1	6.6	.02	3.31
50	5	0.2	13.5	.03	6.73
50	10	0.1	3.3	.02	1.65
50	10	0.2	6.6	.03	3.31
50	10	0.5	17.0	.08	8.51
50	20	0.1	1.6	.02	0.82
50	20	0.2	3.3	.03	1.65
50	20	0.5	8.3	.08	4.16
50	20	1.0	17.0	.16	8.52
50	50	0.1	0.7	.02	0.33
50	50	0.2	1.3	.03	0.66
50	50	0.5	3.3	.08	1.66
50	50	1.0	6.7	.16	3.34
50	50	2.0	13.6	.32	6.81
100	5	0.1	5.1	.01	2.56
100	5	0.2	10.4	.02	5.21
100	10	0.1	2.6	.01	1.28

Table 1 — (Continued)

Looks	Base (units)	Sigma DF (deg)	Sigma Range (units)	Sigma Bear. (deg)	Ave. % Error Range
100	10	0.2	5.1	0.02	2.57
100	10	0.5	13.2	0.05	6.59
100	20	0.1	1.3	0.01	0.64
100	20	0.2	2.6	0.02	1.28
100	20	0.5	6.4	0.05	3.22
100	20	1.0	13.2	0.09	6.60
100	50	0.1	0.5	0.01	0.23
100	50	0.2	1.0	0.02	0.51
100	50	0.5	2.6	0.05	1.29
100	50	1.0	5.2	0.09	2.59
100	50	2.0	10.5	0.18	5.27
200	5	0.1	3.2	0.01	1.62
200	5	0.2	6.5	0.02	3.26
200	5	0.5	16.8	0.04	8.39
200	10	0.1	1.6	0.01	0.81
200	10	0.2	3.2	0.02	1.62
200	10	0.5	8.2	0.04	4.09
200	10	1.0	16.8	0.07	8.39
200	20	0.1	0.8	0.01	0.40
200	20	0.2	1.6	0.02	0.81
200	20	0.5	4.1	0.04	2.03
200	20	1.0	8.2	0.07	4.09
200	20	2.0	16.8	0.15	8.41
200	50	0.1	0.3	0.01	0.15
200	50	0.2	0.6	0.01	0.32
200	50	0.5	1.6	0.04	0.81
200	50	1.0	3.3	0.07	1.64
200	50	2.0	6.6	0.15	3.29
200	50	5.0	17.1	0.37	8.55
500	5	0.1	2.1	0.01	1.05
500	5	0.2	4.2	0.01	2.10
500	5	0.5	10.5	0.02	5.26
500	10	0.1	1.1	0.01	0.53
500	10	0.2	2.1	0.01	1.05
500	10	0.5	5.3	0.02	2.63
500	10	1.0	10.5	0.04	5.26
500	20	0.1	0.5	0.00	0.26
500	20	0.2	1.1	0.01	0.53
500	20	0.5	2.6	0.02	1.31
500	20	1.0	5.3	0.04	2.63
500	20	2.0	10.6	0.09	5.28
500	50	0.1	0.2	0.01	0.10
500	50	0.2	0.4	0.01	0.21
500	50	0.5	1.1	0.02	0.54
500	50	1.0	2.1	0.04	1.06
500	50	2.0	4.2	0.09	2.12
500	50	5.0	10.7	0.22	5.36

direction along base lines of 5, 10, 20, and 50 units. As the base line is traversed the number of equally spaced angular measurements is 5, 10, 20, 50, 100, 200, and 500. A Gaussian random error is introduced into the bearing ψ_i for each look. The standard deviations for these errors "Sigma DF" are 0.1°, 0.2°, 0.5°, 1°, 2°, and 5°. For each combination of the above parameters, 100 Monte Carlo runs were made. Standard deviations (over the 100 runs) were calculated for the estimated range and bearing. The average percent error was calculated from the ratio of range standard deviation to true range. Table 1 shows results of combinations for which the average percent error was less than 10%.

Consider the ellipse with semimajor axis given by the range standard deviation and semiminor axis given by the product of the range and the tangent of the bearing standard deviation. The probability of the true emitter position falling within this elipse is 0.393. This probability becomes one-half when the linear dimensions of the elipse are multiplied by the factor 1.177. For a circular error probability (CEP) of one-half, the radius is obtained by multiplying the range standard deviation by the factor 0.675.

A paper by Wegner [4] reports on the location accuracies that are possible using optimum estimation procedures. The errors indicated in Table 1 are in reasonable agreement with errors determined under like conditions from Wegner's paper, suggesting that the asymptotic algorithm is nearly optimal.

AN ALGORITHM FOR A MOVING TARGET

An algorithm that obtains emitter velocity as well as position is stated here. The algorithm will not yield an answer when measurements are made from a platform in straight line motion. The platform must make a nonlinear maneuver. For this algorithm in addition to obtaining at each look the angular fix ψ_i and the platform position (p_i,q_i), the time t_i is also recorded and utilized Equations 69 through 74 represent the calculations for a single look.

$$A_i = -\sin\psi_i \cos\psi_i \tag{69}$$

$$B_{1i} = \sin^2\psi_i \tag{70}$$

$$B_{2i} = \cos^2\psi_i \tag{71}$$

$$D_i = p_i \sin\psi_i \cos\psi_i - q_i \cos^2\psi_i \tag{72}$$

$$E_i = p_i \sin^2\psi_i - q_i \sin\psi_i \cos\psi_i \tag{73}$$

$$G_i = (p_i \sin\psi_i - q_i \cos\psi_i)^2 \tag{74}$$

Implementation of the algorithm requires the accumulation in memory of the 19 sums given by Eqs. 75 through 90.

$$P = \sum_{i=1}^{N} \cos\psi_i \tag{75}$$

$$Q = \sum_{i=1}^{N} \sin\psi_i \tag{76}$$

$$A = \sum_{i=1}^{N} A_i \tag{77}$$

$$A' = \sum_{i=1}^{N} t_i A_i \tag{78}$$

$$A'' = \sum_{i=1}^{N} t_i^2 A_i \tag{79}$$

$$B_k = \sum_{i=1}^{N} B_{ki}, \quad k = 1, 2 \tag{80}$$

$$B_k^I = \sum_{i=1}^{N} t_i B_{ki}, \quad k = 1, 2 \tag{81}$$

$$B_k^{II} = \sum_{i=1}^{N} t_i^2 B_{ki}, \quad k = 1, 2 \tag{82}$$

$$D = \sum_{i=1}^{N} D_i \tag{83}$$

$$D^I = \sum_{i=1}^{N} t_i D_i \tag{84}$$

$$D^{II} = \sum_{i=1}^{N} t_i^2 D_i \tag{85}$$

$$E = \sum_{i=1}^{N} E_i \tag{86}$$

$$E^I = \sum_{i=1}^{N} t_i E_i \tag{87}$$

$$E^{II} = \sum_{i=1}^{N} t_i^2 E_i \tag{88}$$

$$G = \sum_{i=1}^{N} G_i \tag{89}$$

$$G^I = \sum_{i=1}^{N} t_i G_i \tag{90}$$

When a position and velocity are desired, the next set of formulas to be implemented is given by Eqs. 91 through 98.

$$R = B_1 Q - AP \tag{91}$$

$$R^I = B_1^I Q - A^I P \tag{92}$$

$$R^{II} = B_1^{II} Q - A^{II} P \tag{93}$$

$$S = AQ - B_2 P \tag{94}$$

$$S^I = A^I Q - B_2^I P \tag{95}$$

$$S^{II} = A^{II} Q - B_2^{II} P \tag{96}$$

$$T = EQ + DP \tag{97}$$

$$T^I = E^I Q + D^I P \tag{98}$$

The initial position (x,y) and the velocity (v,w) are determined by solving the following linear system.

$$Ex - Dy + E^I v - D^I w = G \tag{99}$$

$$E^I x - D^I y + E^{II} v - D^{II} w = G^I \tag{100}$$

$$Rx + Sy + R^I v + S^I w = T \tag{101}$$

$$R^I x + S^I y + R^{II} v + S^{II} w = T^I \tag{102}$$

SAMPLE RESULTS FOR THE MOVING TARGET ALGORITHM

Listed in Table 2 is a number of sample calculations using the moving target algorithm. The emitter range, lengths and orientation of the base lines, errors in the angular measurements, and the

Table II — Sample Results for the Moving Emitter Algorithm (range = 200 units)

Looks	Base (Units)	Sigma DF (deg)	Sigma Range (units)	Sigma Bear. (deg)	% Error Range	% Error Total Displ.	% Error Radial Displ.	% Error Angular Displ.
20	50	.10	4.0	.12	2.01	10.28	18.24	2.38
50	50	.10	2.9	.08	1.45	7.10	12.69	1.61
50	50	.20	5.8	.16	2.91	14.12	25.45	3.23
100	20	.10	6.5	.09	3.27	10.12	20.32	1.25
100	50	.10	1.8	.05	0.92	4.45	7.97	1.12
100	50	.20	3.7	.11	1.83	8.76	15.95	2.25
200	20	.10	5.1	.06	2.57	7.92	15.88	0.83
200	50	.10	1.4	.04	0.72	3.43	6.23	0.75
200	50	.20	2.9	.07	1.45	6.78	12.48	1.50
500	10	.10	9.6	.07	4.78	9.19	19.32	1.00
500	20	.10	3.0	.04	1.50	4.91	9.39	0.50
500	20	.20	6.0	.08	3.00	9.77	18.76	1.00
500	50	.10	0.8	.02	0.42	2.08	3.72	0.47
500	50	.20	1.7	.05	0.84	4.16	7.44	0.92
500	50	.50	4.2	.12	2.11	10.37	18.59	2.32

number of runs are the same as those given in the section called "Sample Results for the Asymptotic Algorithm." Since a nonlinear maneuver is necessary, the platform is assumed in this case to traverse the base line first in one direction and then in the other. In the same length of time that it takes to make this maneuver, the emitter moving with a uniform velocity undergoes a displacement of 50 distance units. This motion makes a 45° angle with the initial emitter position vector. Results are presented in Table 2 for those cases where the percent error in the radial displacement is less than 30%. In addition to the quantities described in the section called "Sample Results for the Asymptotic Algorithm." Table 2 gives the percent errors for the total, radial, and angular emitter displacement as determined by the algorithm.

SUMMARY

The problem of determining the position and, in some cases the velocity, of an emitter from a moving platform equipped with passive direction finding equipment has been addressed. From a series of known platform positions spanned by a relatively small base line, a number of bearing measurements are made. When the angle errors are large, locating an emitter by geometric triangulation or the distance least squares method may not give acceptable accuracy. If so, the results may be brought up to an acceptable level of accuracy by using the asymptotic algorithm derived in this report. For a system with good direction finding accuracy and a capability for recording the time of a measurement, an algorithm has been presented for determining both the position and the velocity of the emitter. This algorithm requires a nonlinear maneuver by the platform. The methods given here have the virtue of requiring only a modest amount of memory and computational logic. Typical results have been presented in tables from which it is possible to judge the merits of the method for a particular application.

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