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Radar-ESM Correlation Decision Procedure

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The problem considered is how to correlate an ESM signal with one or none of *m* possible radar tracks when there are an unequal number of measurements associated with each radar track. It is suggested that the decision should be based on the probabilities of obtaining at least the measured squared-errors between the ESM measurements and each of the radar tracks. Using these probabilities, one of five possible decisions is made by comparing the largest probability to various thresholds and to the second largest probability. The accuracy of the method was investigated via computer simulations.

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RADAR-ESM CORRELATION DECISION PROCEDURE

INTRODUCTION

The problem considered in this report is how to correlate an Electronic Support Measures (ESM) signal with one or none of m possible radar tracks. Previous work [1,2] was concerned only with the angular measurements (i.e., it excluded all classification information pertaining to the type of radiating platform) and always assumed that the ESM signal went with one of two radar tracks. Furthermore, it was assumed that the number of measurements associated with both radar tracks was the same. Reference 1 was concerned with several least-squared error discriminants and compared their performance via simulation. Reference 2 was concerned with the problem that radar and ESM measurements are not coincident in time, so that measurements from one or both sources must be smoothed to common times. The present report again is only concerned with angular measurements, but it considers the additional complexities of more than two tracks and unequal numbers of measurements. The possibility of the ESM signal not belonging to any one of the radar tracks will also be considered.

LEAST-SQUARED ERROR DEVELOPMENT

Assume that n independent and unbiased Gaussian-distributed ESM measurements are made with a variance of σ^2. Coleman [2] has shown that the probability of correct association is only slightly dependent on the radar measurement accuracy if the radar accuracy is good compared to the ESM accuracy. Thus, for simplicity we will initially assume that the radar measurements have zero measurement error (the case of nonperfect radar measurements is considered in the last section), and we will furthermore assume that the radar measurements are coincident in time with the ESM measurements. Then the probability of error can be minimized [1] by choosing the radar track with the smallest squared error,

$$d_j = \sum_{i=1}^{n} [\theta_i - \beta_i(j)]^2, \quad j = 1, \ldots, m, \tag{1}$$

where θ_i is the i th ESM measurement and $\beta_i(j)$ is the true position of the j th radar target at the time of the i th ESM measurement. This decision rule assumes that the m possible associations are, a priori, equally likely.

Let us now consider the problem of unequal numbers of radar measurements. In its simplest form, we are given two ESM measurements, θ_1 and θ_2, two positions for radar track 1, $\beta_1(1)$ and $\beta_2(1)$, but only one position for radar track 2, $\beta_1(2)$. Since the corresponding hypothesis-testing problem is

H_1: $E\{\theta_1\} = \beta_1(1)$ and $E\{\theta_2\} = \beta_2(1)$

H_2: $E\{\theta_1\} = \beta_1(2)$ and $E\{\theta_2\} =$ anything,

no uniform most powerful test* exists, that is, no optimal test exists with respect to the probability of correct decision.

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* A uniform most powerful test maximizes the probability of detection for any point in the composite hypothesis H_2. The absence of such a test indicates that there is no optimal test and suboptimal tests must be used.

One way of proceeding is the generalized likelihood ratio test. This implies substituting maximum likelihood estimates for the unknown parameters. If this were done, any unknown $\beta_i(j)$ would simply be replaced by θ_i. Thus, if there were n_j radar measurements for the j th radar track, the decision rule (1) would reduce to choosing the smallest squared error

$$d_j = \sum_{i=1}^{n_j} [\theta_i - \beta_i(j)]^2, \quad j = 1, \ldots, m. \tag{2}$$

The decision rule given by (2) biases the decision in favor of the radar track with the smallest number of measurements. However, (2) is not the only option. One obvious variation of it is

$$d_j = \frac{1}{n_j} \sum_{i=1}^{n_j} [\theta_i - \beta_i(j)]^2, \quad j = 1, \ldots, m, \tag{3}$$

and since the expected value of the summand is σ^2 when j is the correct choice, another variation is

$$d_j = (n - n_j)\sigma^2 + \sum_{i=1}^{n_j} [\theta_i - \beta_i(j)]^2, \quad j = 1, \ldots, m, \tag{4}$$

where $n = \max\{n_1, n_2, \ldots, n_m\}$.

To consider the applicability of these various discriminants, consider the following two extreme examples in which there are 100 samples from radar track 1 and only one sample from radar track 2. In the first example, let the corresponding squared errors [as given by (2)] be $90\,\sigma^2$ and $0.8\,\sigma^2$, respectively. In this case discriminants (2) and (3) choose track 2 and discriminant (4) chooses track 1. Given the fact that $100\sigma^2$ is the expected value of the squared error (if the correct association is with radar track 1), we believe the decision should be made to associate the ESM signal with track 1. This is because the probability of obtaining a squared error of at least $90\sigma^2$ from 100 measurements from the correct track is near 1, and this is larger than the probability of obtaining a squared error of at least $0.8\sigma^2$ from one measurement from the correct track. In the second example, the corresponding squared errors are $104\sigma^2$ and $3\sigma^2$. In this case discriminants (2) and (4) choose track 2 and discriminant (3) chooses track 1. The same probabilistic argument again indicates that track 1 is the correct track. Thus, in the two examples none of the three discriminants consistently yielded the "correct" track. Rather, we believe that the decision should be based on the largest probability of obtaining a squared error greater than the measured value, given that the radar target under consideration is the correct one.

PROBABILITY STATISTICS

The suggested procedure is as follows: for each radar track, calculate the probability of obtaining at least the observed squared error under the assumption that the ESM signal comes from the radar track. This is accomplished by first normalizing the squared error; i.e.,

$$S_j = \sum_{i=1}^{n_j} [\theta_i - \beta_i(j)]^2/\sigma^2. \tag{5}$$

Then, if the association is the correct association (i.e., $E\{\theta_i\} = \beta_i(j)$, $i = 1, \ldots, n_j$), S_j has a chi-square density with n_j degrees of freedom. Thus, the desired probability P_j is given by

$$P_j = \Pr\{z \geqslant S_j\}, \quad \text{where } z \sim \chi^2(n_j).$$

It is shown in the appendix that

$$\Pr\{z \geqslant S\} = I_k, \quad z \sim \chi^2(n_j), \tag{6}$$

where I_k is obtained by iteration,

$$I_k = F_k + I_{k-1}, \tag{7}$$

$$F_k = F_{k-1}\,S/2k,$$

where the initial values are

$$I_0 = F_0 = e^{-S/2} \tag{8}$$

and the even integer n is related to k by

$$k = \frac{n}{2} - 1.$$

Now define P_{max} as the largest P_j and P_{next} as the second largest P_j in the set of P_j's, $j = 1 \ldots m$. From this set of P_j's, one of five possible decisions is made:

(1) ESM signal goes with track corresponding to P_{max};

(2) ESM signal probably goes with track corresponding to P_{max};

(3) ESM signal probably goes with some track;

(4) ESM signal probably does not go with any track; or

(5) ESM signal does not go with any track.

We arrive at these decisions by defining three probability thresholds, T_H (high), T_M (middle), and T_L (low), and a probability ratio R. The corresponding decision rules are

Category 1: $P_{max} \geqslant T_H$ and $P_{max} \geqslant R\, P_{next}$;

Category 2: $T_H > P_{max} \geqslant T_M$ and $P_{max} \geqslant R\, P_{next}$;

Category 3: $P_{max} \geqslant T_M$ but $P_{max} < R\, P_{next}$;

Category 4: $T_M > P_{max} > T_L$; and

Category 5: $T_L \geqslant P_{max}$.

Furthermore, if for any P_i both $P_i < T_L$ and $R\, P_i < P_{max}$ are true, the i th radar track is dropped from further consideration for association with the ESM signal. It should be noted that if the number of radar measurements is the same for each track, the outlined procedure reduces to picking the radar track with the minimum squared error. The problem which will now be considered is how to set the three thresholds, T_L, T_M, and T_H.

THRESHOLD SETTINGS

The easiest threshold to set is T_L. This threshold determines the probability that the correct radar track (i.e., the one associated with the ESM signal) will be incorrectly rejected from further consideration. The key element in setting the threshold is noting that the probability P_j for the correctly associated radar track is uniformly distributed between zero and one. This can be shown as follows: First, note that P_j is given by

$$P_j = F(S_j) \triangleq \int_{S_j}^{\infty} p_{\chi^2}(z)\, dz, \tag{9}$$

where $p_{\chi^2}(z)$ is the probability density function of a chi-square random variable with n_j degrees of freedom. The value of P_j is obviously bounded between zero and one. The cumulative distribution of P_j can then be written as

$$\Pr\{P_j \leqslant P\} = \Pr\{F(S_j) \leqslant P\} = \Pr\{S_j \geqslant F^{-1}(P)\}, \tag{10}$$

where the direction of the inequality changes because $F(\cdot)$ is a monotonic decreasing function of its argument. When target j is the correct association, S_j [defined by (5)] has the density $p_{\chi^2}(S_j)$ mentioned in Eq. 9. Therefore,

$$\Pr\{S_j \geqslant z\} = F(z) \tag{11}$$

from (9). Substituting (11) into (10) yields

$$\Pr\{P_j \leqslant P\} = F[F^{-1}(P)] = P,$$

and thus, P_j is uniformly distributed. Consequently, if one desires a rejection rate of P_R, one can obtain this by setting $T_L = P_R$.

In a similar vein of thought the threshold T_H is set by P_{FA}, the probability of falsely associating a track with an ESM signal when the signal does not belong with any of the tracks. Then S_j [defined by (5)] has a noncentral chi-square density with noncentrality parameter

$$F_{\lambda_j} = \sum_{i=1}^{n_j} [E\{\theta_i\} - \beta_i(j)]^2/\sigma^2. \tag{12}$$

Then corresponding to (11) one has

$$\Pr\{S_j \geqslant z\} = F_{\lambda_j}(z), \tag{13}$$

where $F_\lambda(\cdot)$ is the noncentral chi-square distribution. Substituting (13) into (10) we obtain

$$\Pr\{P_j \leqslant P\} = F_{\lambda_j}[F^{-1}(P)]. \tag{14}$$

The threshold T_H is now determined by

$$P_{FA} = \Pr\{P_j \geqslant T_H\} = 1 - F_{\lambda_j}[F^{-1}(T_H)]. \tag{15}$$

Unfortunately, (15) cannot be inverted to yield an analytic expression for T_H.

Obviously, the threshold T_H is a function of λ_j, which is related to the difference between the true (ESM) position and the radar track under consideration. This difference was denoted by μ,

$$\mu = E\{\theta_i\} - \beta_i(j),$$

and the threshold T_H was found for arbitrarily chosen values of μ by simulation techniques. The results for $P_{FA} = 0.01$ and $\mu = 1.0\sigma$ and 1.5σ are shown in Fig. 1. For each value of n_j $(2, 4, 6, \ldots, 100)$ 3000 repetitions were performed. The threshold is near one for small values of n and drops to zero as n approaches infinity. Since the threshold is still high for large n's when $\mu = 1.0\sigma$, the curve for $\mu = 1.5\sigma$ was used in later simulations. Of course, the value that should be used in a real system would depend on the expected target separation divided by the measurement accuracy in a realistic scenario.

The middle threshold divides the "tentative" region into a tentative track region and a tentative no track region. The rationale in setting the threshold is to set both of these probabilities equal for a particular separation; i.e.,

$$\Pr\{P_j \leqslant T_M \mid \text{correct match}\} = \Pr\{P_j \geqslant T_M \mid \text{incorrect match}\}.$$

Thus the equation specifying T_M is

$$T_M = 1 - F_{\lambda_j}\{F^{-1}(T_M)\}.$$

Again, the threshold T_M is a function of λ_j and must be found by simulation techniques. The previous simulation used to find T_H was also used to find T_M; the results are shown in Fig. 2. Again, the curve for $\mu = 1.5\sigma$ was used for further simulation results.

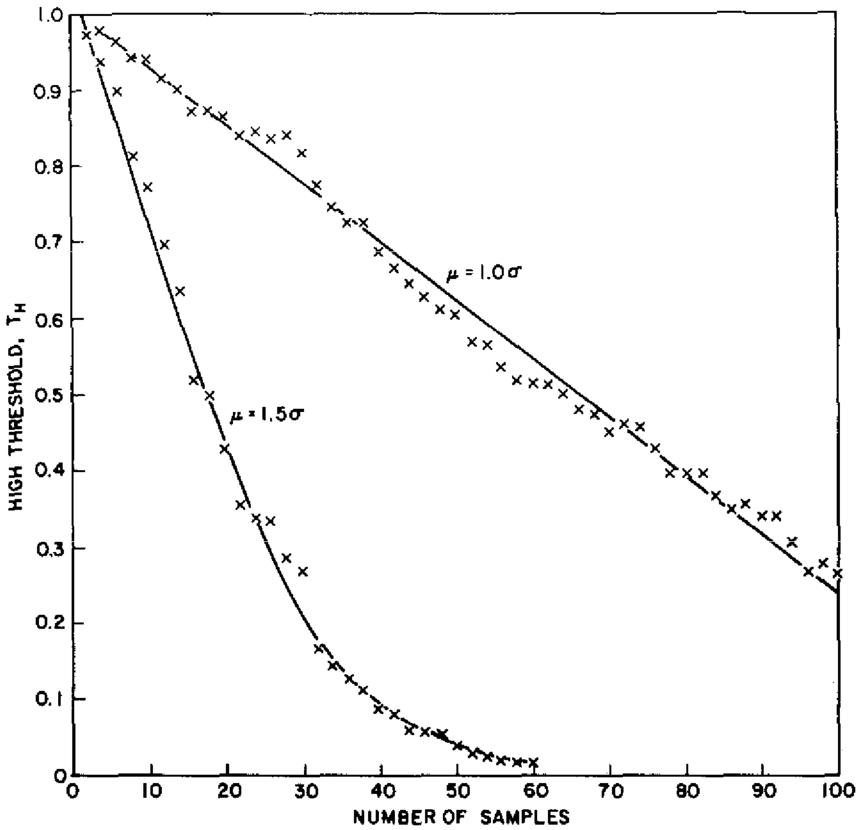

Fig. 1 — High threshold vs number of samples for two different target separations

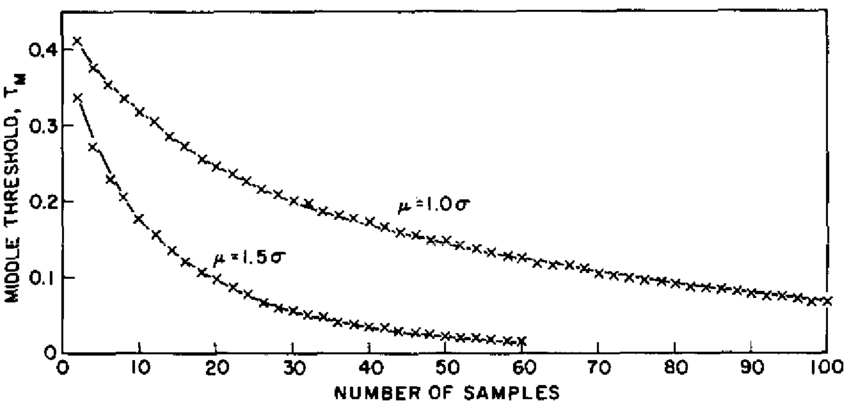

Fig. 2 — Middle threshold vs number of samples for two different target separations

The only parameter that remains to be set is the probability ratio R, which is used to inhibit a firm track decision if P_{max} is not greater than R times P_{next}. No methodology has been developed for setting R. In the simulation R has been set to 9. (Some of the simulation results suggest that this value may be too large.)

SIMULATION RESULTS

All simulations involved an attempt to associate an ESM signal with one of three possible radar tracks whose angular positions are known to be μ_1, μ_2, and μ_3, that is, $\beta_i(j) = \mu_j$ for all measurements. First, two ESM angles of arrival were generated from a zero-mean Gaussian density with unit variance. The discriminant probabilities P_1, P_2, and P_3 were then calculated for $n = 2$ using the iterative relation given by (6), (7), and (8). The highest probability P_{max} was found and compared to the various thresholds to determine to which of the five possible categories the ESM signal should be assigned. In all simulations the lower threshold was set to 0.001, the upper threshold was set by the $\mu = 1.5\sigma$ curve in Fig. 1, the middle threshold was set by the $\mu = 1.5\sigma$ curve in Fig. 2, and the probability ratio was set at 9.0. Next, two more ESM measurements were generated and the procedure just described was repeated for $n = 4$ measurements. This addition of two ESM measurements with each repetition continued until 50 ESM measurements were generated. The whole set of measurements comprises a single radar-ESM correlation trial. To obtain significant statistics, 1000 repetitions were generated for each case.

For case 1, $\mu_1 = 0$, $\mu_2 = 1$, and $\mu_3 = -1$; the results of 1000 repetitions are shown in Table 1. Initially, when only 2 ESM measurements are present, approximately 80% of the ESM signals are tentatively associated with no radar track. The number of tentative associations with no track falls below 10% with 16 ESM measurements, and it eventually falls below 2% with 50 ESM measurements. The number of cases in which a firm "no track" decision is made is always less than 0.5%. While originally 80% of the ESM signals are tentatively associated with unknown tracks, it is not until one receives 26 ESM measurements that 50% of the ESM signals are firmly correlated with the correct track, track 1. This is caused by two facts: First, the high threshold is 0.29 for 26 ESM measurements. Second, the difference between 71% (which is the percentage above the high threshold) and the actually obtained 50% firm decisions is due to the probability ratio, $R = 9$.

Table 1—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 0$, $\mu_2 = 1$, and $\mu_3 = -1$)

Number of ESM	Firm Correlation			Tentative Correlation			Tentative	Tentative	Firm
Measurements	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3	Unknown Track	No Track	No Track
2	0	0	0	0	3	10	818	169	0
4	0	0	0	0	6	6	786	201	1
6	0	0	0	0	2	4	804	190	0
8	0	0	0	0	2	4	840	154	0
10	0	0	0	23	2	2	836	137	0
12	0	0	0	106	1	1	787	105	0
14	23	0	0	188	2	0	674	113	0
16	100	1	0	240	0	0	562	96	1
18	191	1	0	232	0	1	484	91	0
20	279	0	1	218	0	0	421	81	0
22	379	1	0	192	1	0	355	71	1
24	433	0	0	191	1	0	312	62	1
26	512	0	0	166	1	0	264	54	3
28	597	0	0	149	0	0	206	45	3
30	660	0	0	123	0	0	172	42	3
32	705	0	0	112	0	0	141	39	3
34	765	0	0	88	0	0	106	38	3
36	796	0	0	70	0	0	95	36	3
38	828	0	0	55	0	0	84	29	4
40	858	0	0	40	0	0	69	29	4
42	874	0	0	41	0	0	58	22	5
44	895	0	0	38	0	0	40	23	4
46	912	0	0	27	0	0	35	21	5
48	933	0	0	18	0	0	24	21	4
50	936	0	0	20	0	0	22	18	4

For case 2, with $\mu_1 = 2$, $\mu_2 = 1$, and $\mu_3 = -1$, the results of 1000 repetitions are shown in Table 2. In this case the ESM signal does not belong to any radar track but lies between radar tracks 2 and 3. Initially, about 75% of the ESM signals are tentatively associated with radar tracks, and after 8 ESM measurements this number is reduced to 50%. It is not until one has slightly less than 50 ESM measurements that 90% of the ESM signals are either tentatively or firmly declared not to be associated with any radar track. It should be noted that while initially 75% of the ESM signals were tentatively associated with radar tracks, at no time was the number of firm associations more than 5%.

Table 2—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 2$, $\mu_2 = 1$, and $\mu_3 = -1$)

Number of ESM	Firm Correlation			Tentative Correlation			Tentative	Tentative	Firm
Measurements	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3	Unknown Track	No Track	No Track
2	0	0	11	0	0	183	545	261	0
4	0	0	11	0	65	156	388	378	2
6	0	11	16	0	141	158	206	468	0
8	0	11	24	0	152	143	169	500	1
10	0	20	11	0	130	153	124	559	3
12	0	19	14	0	144	166	109	539	9
14	0	25	18	0	138	150	71	579	19
16	0	20	19	0	142	127	58	601	33
18	0	24	16	0	133	113	45	627	42
20	0	25	14	0	115	110	40	635	61
22	0	28	17	0	102	107	29	628	89
24	0	23	18	0	112	102	23	605	117
26	0	26	23	0	93	89	18	608	143
28	0	24	21	0	87	76	11	603	178
30	0	25	24	0	78	66	9	582	216
32	0	24	23	0	62	53	8	599	231
34	0	24	20	0	60	54	11	555	276
36	0	24	21	0	54	50	6	535	310
38	0	25	21	0	52	45	6	502	349
40	0	30	20	0	47	40	5	462	396
42	0	29	21	0	40	32	2	428	448
44	0	27	21	0	32	27	4	393	496
46	0	28	21	0	31	20	2	375	523
48	0	29	22	0	23	20	2	343	561
50	0	31	21	0	24	20	1	304	599

In both cases, the results improve dramatically as the number of ESM measurements increases. However, the probability of correct decision does not reach 90% until about 50 measurements are received. It should be noted, however, that the situation encountered in cases 1 and 2 is rather difficult: there is an incorrect track one standard deviation away from the true track. Consequently, cases 3 and 4 were generated. In case 3, $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$, and the results are given in Table 3. If we compare Tables 1 and 3, while the number of tentative no track decisions is slightly smaller for Table 3 (2σ separation), the major difference is that the ESM signals are associated with track 1 very quickly. In particular, the number of associations with "unknown" tracks falls to 0.2% after only 10 ESM measurements. In case 4, $\mu_1 = 4$, $\mu_2 = 2$, and $\mu_3 = -2$, and the results are given in Table 4. Here, after 10 ESM measurements, 90% of the ESM signals are correctly designated as not associating with any radar track, and at 22 ESM measurements, all ESM signals are correctly designated as not associating with any radar track. Thus, when the nearest incorrect radar track is two standard deviations away from the correct track, correct decisions are arrived at very rapidly.

One of the major reasons for basing the decision on the probability statistic P_i is the resulting ability to handle a different number of radar measurements for each radar track. To test this behavior,

Table 3—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$)

Number of ESM Measurements	Firm Correlation			Tenatative Correlation			Tentative Unknown Track	Tentative No Track	Firm No Track
	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3			
2	26	0	0	371	10	14	339	240	0
4	83	0	1	536	4	4	99	272	1
6	139	0	0	576	0	2	34	249	0
8	216	0	0	586	0	0	12	184	2
10	276	0	0	565	0	0	2	156	1
12	334	0	0	539	0	0	1	125	1
14	409	0	0	469	0	0	0	120	2
16	458	0	0	439	0	0	0	100	3
18	520	0	0	384	0	0	0	95	1
20	581	0	0	335	0	0	0	82	2
22	646	0	0	278	0	0	0	75	1
24	683	0	0	251	0	0	0	64	2
26	729	0	0	211	0	0	0	59	1
28	777	0	0	173	0	0	0	49	1
30	813	0	0	139	0	0	0	47	1
32	829	0	0	129	0	0	0	41	1
34	860	0	0	100	0	0	0	39	1
36	885	0	0	77	0	0	0	37	1
38	907	0	0	61	0	0	0	30	2
40	925	0	0	43	0	0	0	30	2
42	931	0	0	43	0	0	0	23	3
44	936	0	0	38	0	0	0	24	2
46	948	0	0	27	0	0	0	22	3
48	958	0	0	18	0	0	0	22	2
50	959	0	0	20	0	0	0	19	2

Table 4—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 4$, $\mu_2 = 2$, and $\mu_3 = -2$)

Number of ESM Measurements	Firm Correlation			Tentative Correlation			Tentative Unknown Track	Tentative No Track	Firm No Track
	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3			
2	0	0	0	0	53	67	0	876	4
4	0	0	1	0	19	13	0	827	140
6	0	0	0	0	3	5	0	473	519
8	0	0	0	0	0	0	0	222	778
10	0	0	0	0	1	0	0	96	903
12	0	0	0	0	0	0	0	44	956
14	0	0	0	0	0	0	0	16	984
16	0	0	0	0	0	0	0	7	993
18	0	0	0	0	0	0	0	3	997
20	0	0	0	0	0	0	0	1	999
22	0	0	0	0	0	0	0	0	1000
24	0	0	0	0	0	0	0	0	1000
26	0	0	0	0	0	0	0	0	1000
28	0	0	0	0	0	0	0	0	1000
30	0	0	0	0	0	0	0	0	1000
32	0	0	0	0	0	0	0	0	1000
34	0	0	0	0	0	0	0	0	1000
36	0	0	0	0	0	0	0	0	1000
38	0	0	0	0	0	0	0	0	1000
40	0	0	0	0	0	0	0	0	1000
42	0	0	0	0	0	0	0	0	1000
44	0	0	0	0	0	0	0	0	1000
46	0	0	0	0	0	0	0	0	1000
48	0	0	0	0	0	0	0	0	1000
50	0	0	0	0	0	0	0	0	1000

cases 5 and 6 were generated. Both cases are similar to case 3 in that $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$. The difference is that in case 5 one starts off with 12 ESM measurements from radar track 1 but with only 2 radar measurements each from radar tracks 2 and 3, and in case 6 one starts off with 12 ESM measurements and 12 radar measurements each from radar tracks 2 and 3, but only 2 radar measurements from radar track 1. The results for cases 5 and 6 are given in Tables 5 and 6, respectively. If we compare Tables 3, 5, and 6, the following conclusions can be drawn:

(1) The number of associations with unknown tracks is primarily determined by the number of measurements on incorrect tracks; i.e., after a reasonable number of measurements is obtained on incorrect tracks, they can be recognized as not being associated with the ESM signal. (Note: 12 ESM measurements in Table 3, 22 ESM measurements in Table 5, and 12 ESM measurements in Table 6 all correspond to 12 radar measurements on tracks 2 and 3.)

(2) The number of tentative "no track" decisions is mainly a function of the number of measurements on the correct track, track 1. (The 12, 12, and 22 ESM measurements in Tables 3, 5, and 6, respectively, all correspond to 12 measurements on track 1.)

(3) The numbers of firm and tentative decisions to associate the ESM signal with the correct track, track 1, are again mainly functions of the number of radar measurements on the correct track.

Thus, it appears that the proposed method yields good results when the number of measurements associated with each radar track is different.

Table 5—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$; initial number of measurements on each radar track is 12, 2, and 2, respectively)

Number of ESM	Firm Correlation			Tentative Correlation			Tentative	Tentative	Firm
Measurements	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3	Unknown Track	No Track	No Track
12	225	0	0	169	8	6	455	137	0
14	365	0	0	375	1	0	140	119	0
16	455	0	0	384	0	0	52	109	0
18	496	0	0	393	0	0	12	96	3
20	588	0	0	316	0	0	3	91	2
22	629	0	0	290	0	0	1	76	4
24	671	0	0	259	0	0	1	65	4
26	728	0	0	210	0	0	1	58	3
28	779	0	0	162	0	0	0	56	3
30	802	0	0	153	0	0	0	42	3
32	838	0	0	119	0	0	0	39	4
34	858	0	0	107	0	0	0	33	2
36	889	0	0	74	0	0	0	35	2
38	902	0	0	70	0	0	0	27	1
40	923	0	0	49	0	0	0	27	1
42	929	0	0	42	0	0	0	26	3
44	940	0	0	35	0	0	0	22	3
46	952	0	0	27	0	0	0	18	3
48	964	0	0	18	0	0	0	15	3
50	968	0	0	17	0	0	0	13	2

NONPERFECT RADAR MEASUREMENTS

In the previous development it was assumed that the radar measurements were perfect. We will now assume that the radar makes noisy measurements, that $\beta_i\,(j)$ is the predicted azimuth of the j th radar track at the time of the i th ESM measurement and is Gaussian distributed, and that the means and variances of θ_i and $\beta_i\,(j)$ are

Table 6—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$; initial number of measurements on each radar track is 2, 12, and 12, respectively)

Number of ESM	Firm Correlation			Tentative Correlation			Tentative	Tentative	Firm
Measurements	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3	Unknown Track	No Track	No Track
12	27	0	0	650	0	0	0	321	2
14	75	0	0	658	0	0	0	266	1
16	160	0	0	610	0	0	0	230	0
18	201	0	0	601	0	0	0	198	0
20	292	0	0	535	0	0	0	172	1
22	349	0	0	507	0	0	0	140	4
24	415	0	0	451	0	0	0	131	3
26	482	0	0	393	0	0	0	123	2
28	536	0	0	349	0	0	0	113	2
30	612	0	0	288	0	0	0	98	2
32	660	0	0	260	0	0	0	78	2
34	672	0	0	253	0	0	0	73	2
36	727	0	0	212	0	0	0	60	1
38	758	0	0	183	0	0	0	57	2
40	804	0	0	151	0	0	0	43	2
42	828	0	0	123	0	0	0	47	2
44	862	0	0	99	0	0	0	37	2
46	883	0	0	84	0	0	0	30	3
48	906	0	0	66	0	0	0	25	3
50	916	0	0	62	0	0	0	20	2

$$E\{\theta_i\} = E\{\beta_i(j)\}, \quad j = \text{true track},$$
$$\mathrm{Var}\{\theta_i\} = \sigma_E^2,$$

and

$$\mathrm{Var}\{\beta_i(j)\} = \sigma_{ij}^2.$$

Now, if one sets

$$\sigma^2 = \sigma_E^2 + \sigma_{ij}^2,$$

the statistic S_j given by (5) still has a chi-square density for the correct association. Thus, it follows that all the thresholds should have the same value, whether or not the radar measurements are noisy. There will be some difference in the multitarget performance because of the different dependence between the squared errors $\{S_j : j = 1, \ldots, m\}$ due to the radar variances. As shown in [2], the degree of smoothing of the radar data (or degree of dependency between radar measurements) can also affect performance. However, in this simulation the measurements were unsmoothed (independent).

To show that the performance difference is small, case 3 with $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$ was repeated with $\sigma_E^2 = 0.06$ and $\sigma_{ij}^2 = 0.94$, and the results are shown in Table 7. Comparing the results in Table 7 with those in Table 3, one sees very little difference. Thus, the total variance is the important quantity: it makes little difference with this algorithm how the uncertainty is divided between the radar and the ESM measurements.

SUMMARY

When the numbers of measurements associated with different radar tracks are not the same, the squared error association criterion should not be used. Rather, the decision should be based on the conditional probability P_i of obtaining at least the observed squared error given that the association is the correct one. Using this probability, five possible decisions were formulated:

Table 7—Simulation Results for Five-Decision Association Rule as a Function of the Number of ESM Measurements (Target separations are $\mu_1 = 0$, $\mu_2 = 2$, and $\mu_3 = -2$; measurement accuracies are $\sigma_E^2 = 0.06$ and $\sigma_R^2 = 0.94$

Number of ESM	Firm Correlation			Tentative Correlation			Tentative	Tentative	Firm
Measurements	Track 1	Track 2	Track 3	Track 1	Track 2	Track 3	Unknown Track	No Track	No Track
2	25	0	1	336	3	2	330	303	0
4	62	0	0	549	0	2	112	275	0
6	136	0	0	583	0	0	28	252	1
8	201	0	0	566	0	0	11	221	1
10	251	0	0	559	0	0	5	185	0
12	301	0	0	535	0	0	1	163	0
14	373	0	0	477	0	0	1	149	0
16	444	0	0	418	0	0	0	138	0
18	491	0	0	401	0	0	1	106	1
20	551	0	0	346	0	0	0	103	0
22	601	0	0	315	0	0	0	84	0
24	646	0	0	292	0	0	0	62	0
26	695	0	0	239	0	0	0	66	0
28	753	0	0	186	0	0	0	61	0
30	799	0	0	144	0	0	0	57	0
32	831	0	0	113	0	0	0	56	0
34	863	0	0	89	0	0	0	48	0
36	872	0	0	90	0	0	0	38	0
38	900	0	0	63	0	0	0	37	0
40	922	0	0	48	0	0	0	29	1
42	937	0	0	30	0	0	0	33	0
44	941	0	0	30	0	0	0	29	0
46	942	0	0	31	0	0	0	27	0
48	950	0	0	23	0	0	0	26	1
50	959	0	0	21	0	0	0	19	1

ESM signal goes with a particular track;

ESM signal probably goes with a particular track;

ESM signal probably goes with some track;

ESM signal probably does not go with any track; and

ESM signal does not go with any track.

These decisions are reached by comparing the largest P_i to various thresholds and to the next largest P_i. Various simulations were run in an attempt to quantify the performance in several situations. The performance was generally good when targets were separated by twice the ESM measurement accuracy (standard deviation).

There are several problems still remaining concerning the radar-ESM correlation algorithms. Among these are:

(1) Should one continually accumulate the squared error of all the samples or should one weight the past squared errors differently after a fixed number of samples has been accumulated?

(2) If one has declared that an ESM signal belongs to a particular radar track, how long should one keep this association if a new radar track with a small S_j and corresponding large P_j appears?

(3) How should the algorithms be modified to take into account equipment deficiencies like bias and quantized measurements?

(4) How should nonangular information (i.e., radar-track parameters such as velocity and elevation and ESM parameters such as signal type and equipment identification) be included in the discriminant function?

Future work will be concerned with answering these questions and testing the algorithms with recorded experimental data.

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Appendix
CALCULATION OF CHI-SQUARE PROBABILITY

The probability that z is greater than s when z is chi-squared distributed with n degrees of freedom is given by

$$\Pr\{z \geqslant s\} = \int_s^\infty \frac{z^{(n/2)-1}\, e^{-z/2}}{2^{n/2}\Gamma\,(n/2)}\, dz.$$

Assuming that n is an even integer and defining k by $k = (n/2) - 1$, we get

$$I_k = \Pr\{z \geqslant s\} = \int_s^\infty \frac{z^k\, e^{-z/2}}{2^{k+1}\, k!}\, dz.$$

Integrating by parts yields

$$I_k = \left. \frac{-z^k\, e^{-z/2}}{2^k\, k!} \right|_s^\infty + \int_s^\infty \frac{z^{k-1}\, e^{-z/2}}{2^k\,(k-1)!}\, dz,$$

which can be simplified to

$$I_k = \frac{s^k\, e^{-s/2}}{2^k\, k!} + I_{k-1}.$$

Thus, I_k can be calculated interatively by

$$I_k = F_k + I_{k-1},$$

$$F_k = \frac{s}{2k}\, F_{k-1},$$

where

$$I_0 = F_0 = e^{-s/2}.$$