NRL Report 7876

Model for the Calculation of Multiply Scattered Fields

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June 23, 1975

NAVAL RESEARCH LABORATORY
Washington, D.C.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

UNCLASSIFIED

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Report 7876	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) A MODEL FOR THE CALCULATION OF MULTIPLY SCATTERED FIELDS		5. TYPE OF REPORT & PERIOD COVERED An interim report on a continuing NRL problem. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(*s*) Laura C. Davis		8. CONTRACT OR GRANT NUMBER(*s*)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Problem R02-88 Project 5W3427001
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Naval Air Systems Command Washington, D.C. 20360		12. REPORT DATE June 23, 1975 13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

Electromagnetic field
Multipath
Scattered field
Computer model
Helmholtz integral

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

This report is a description of a computer model developed to calculate the field *multiply* scattered by a collection of flat polygonal metallic reflectors. Given a radiating dipole and the vertices of a configuration of flat reflecting polygons, the model produces the total field at any number of specified point locations. The program incorporates a recently developed finite-sum representation of the standard far-field approximation to the Helmholtz integral. The replacement of a double integral by a finite sum significantly reduces computation time.

CONTENTS

A MODEL FOR THE CALCULATION OF MULTIPLY SCATTERED FIELDS

INTRODUCTION

This report is a description of a model developed to calculate the field *multiply* scattered by a collection of flat metallic polygonal reflectors. A Fortran listing of the corresponding computer algorithms, programmed for the CDC 3800, is presented as Appendix A. Given a radiating dipole and the vertices of a collection of flat polygonal reflectors, the model produces the field received at any specified sample point (receiver). The field at the receiver consists of the direct ray (transmitter to receiver) combined with indirect rays resulting from multiple scatterings from the polygonal reflectors. The model traces the transmitted radiation through all possible paths in the maze of reflectors, calculates the value of the field after scattering by each reflector encountered, and then sums all contributions to arrive at the total field. In the doppler system the transmitting array elements are energized so as to impart a "virtual velocity" to each transmission, with the result that the signal received at a given point within the beam is shifted in wavelength by an amount proportional to sin θ, where θ is the angle of the point off the beam center. The model provides for the specification of a single virtual-velocity vector to be associated with each array transmission, separates field-component calculations by wavelength, and produces an energy *spectrum* for each field point sampled.

The program is modular in structure, facilitating the interchange of a number of methods for the calculation of the field scattered from each reflector. At present the program incorporates the standard far-field approximation to the fields scattered by flat plates. The results produced by these calculations may provide only a gross description of system behavior in the near-field and Fresnel regions. More exact scattering formulas that are valid in all three regions are currently under development. Meanwhile the current far-field approach is adequate for program verification and preliminary investigations and will prove valuable in validating the results produced by the model when more sophisticated methods are incorporated.

In the current version of the model, great economy of calculation has been achieved by the use of certain "finite-sum" representations developed by W. B. Gordon [1] for the far-field formulas. The far-field approximation to the field scattered by a flat plate is given by a certain double integral over the reflecting surface [Eq. (1)], and a direct calculation of this integral would involve evaluating the integrand at a large number of points. In the finite-sum representation the evaluation of this integral over a flat, N-sided polygon is reduced to a sum $T_1 + T_2 + ... + T_N$, where each T_i is a certain complex quantity evaluated at the ith vertex of the polygon.

The discussion to follow first considers the calculation of the field scattered from a single reflector. Then the framework within which such calculations are made, the ray-

Note: Manuscript submitted February 19, 1975.

tracing techniques from reflector to reflector, is developed, and the total field spectrum is calculated. Finally, expansions of the model currently underway and areas for future refinement are discussed.

FINITE-SUM REPRESENTATION OF THE STANDARD FAR-FIELD APPROXIMATION TO THE HELMHOLTZ INTEGRAL

Consider a plane wave incident on a flat-plate reflector S of finite dimension. Let $\mathbf{f}$ represent the incident field, with wave number $k = 2\pi/\lambda$.

Let x_1, x_2, x_3 be rectangular coordinates, and let S lie in the x_1x_2 plane. Define R, θ, φ to be spherical coordinates, where

$$x_1 = R \sin\theta \cos\varphi$$

$$x_2 = R \sin\theta \sin\varphi$$

$$x_3 = R \cos\theta .$$

Assume P to be a fixed sample field point in space. If the origin of this coordinate system is located in the interior of S, then $\mathbf{g}_p$, the standard far-field approximation of the scattered field at P, is given by

$$\mathbf{g}_p = \left[\frac{-ik\,(1 + \cos\theta)\, e^{ikR}}{4\pi R} \int_S e^{-ik\mathbf{x}\cdot\boldsymbol{\nu}}\, dx_1 dx_2 \right] \mathbf{f}_r, \tag{1}$$

where

$\mathbf{f}_r$ = incident field $\mathbf{f}$ reflected from and evaluated at S,
$\mathbf{x} = [x_1, x_2]$,

and (2)

$$\boldsymbol{\nu} = [\nu_1, \nu_2] = [\sin\theta \cos\varphi, \sin\theta \sin\varphi].$$

(See Refs. 2 and 3.)

For the finite-sum representation, let S be a flat polygon with N vertices $\mathbf{a}_1, ..., \mathbf{a}_N$, where each vertex $\mathbf{a}_i$ is a 2-vector in the x_1x_2 plane. Set $\mathbf{a}_{N+1} = \mathbf{a}_1$, and define $\Delta\mathbf{a}_n = \mathbf{a}_{n+1} - \mathbf{a}_n$, $1 \leqslant n \leqslant N$. Define $\boldsymbol{\eta}$ to be the unit normal to the plane of S, pointing into the half space containing P. Let the unit normal to the incident plane wave be represented by the "ray" $\boldsymbol{\xi}_i$ and the unit normal to the reflected wave by $\boldsymbol{\xi}_r$, which is given by

$$\boldsymbol{\xi}_r = \boldsymbol{\xi}_i - 2(\boldsymbol{\xi}_i \cdot \boldsymbol{\eta})\boldsymbol{\eta}. \tag{3}$$

Define $\mathbf{R}$ to be the vector from a fixed point in S, such as the centroid, to the field point P, so that $R = |\mathbf{R}|$ represents a "mean" distance from S to P, and let $\boldsymbol{\rho}$ be the projection of $\mathbf{R}$ onto the plane of S:

$$\boldsymbol{\rho} = \mathbf{R} - (\mathbf{R} \cdot \boldsymbol{\eta})\boldsymbol{\eta}. \tag{4}$$

Define $\mathbf{w}$ to be the projection of $[\boldsymbol{\xi} - (\boldsymbol{\rho}/R)]$ onto the plane of S. Since the projections of $\boldsymbol{\xi}_r$ and $\boldsymbol{\xi}_i$ onto the plane of S are equal,

$$\mathbf{w} = \boldsymbol{\xi}_r - (\boldsymbol{\xi}_r \cdot \boldsymbol{\eta})\boldsymbol{\eta} - \frac{\boldsymbol{\rho}}{R} = \boldsymbol{\xi}_i - (\boldsymbol{\xi}_i \cdot \boldsymbol{\eta})\boldsymbol{\eta} - \frac{\boldsymbol{\rho}}{R}. \tag{5}$$

Then Gordon has shown in Ref. 1 that the standard far-field approximation to the Helmholtz integral, given in Eq. (1), can be reduced to the following forms involving no intergrations at all:

(a) for $\mathbf{w} \neq 0$,

$$\mathbf{g}_p \doteq \frac{\left(\frac{\mathbf{R}}{R} - \boldsymbol{\xi}_r\right) \cdot \boldsymbol{\eta}}{\mathbf{w} \cdot \mathbf{w}} \, \frac{e^{ikR}}{4\pi R} (T_1 + \ldots + T_N)\mathbf{f}_r, \tag{6}$$

where, for $1 \leqslant n \leqslant N$,

$$T_n = [(\mathbf{w} \times \boldsymbol{\eta}) \cdot \Delta\mathbf{a}_n] \, \frac{\sin\left[\frac{k}{2}\mathbf{w} \cdot \Delta\mathbf{a}_n\right]}{\left[\frac{k}{2}\mathbf{w} \cdot \Delta\mathbf{a}_n\right]} \exp\left[\frac{ik}{2}\mathbf{w} \cdot (\mathbf{a}_n + \mathbf{a}_{n+1})\right], \tag{7}$$

and

(b) for $\mathbf{w} = 0$,

$$\mathbf{g}_p = -ik\left[\frac{\mathbf{R}}{R} \cdot \boldsymbol{\eta}\right] \frac{e^{ikR}}{2\pi R} A\mathbf{f}_p, \tag{8}$$

where A is the area of S. Note that geometrically $\mathbf{w} = 0$ implies $\boldsymbol{\xi}_r = \mathbf{R}/R$, which means that the specular reflection of the incident wave is in the direction of the field point P.

The following sections illustrate the application of this result in the case of the single and then multiple reflector configurations.

MODEL DEFINITIONS AND PRELIMINARY CALCULATIONS

Let

$\mathbf{E}_T = [E_{Tx}, E_{Ty}, E_{Tz}]$ = transmitted radiation (indicating polarization),

λ = transmitting wavelength,

and

$\mathbf{R}_T = [X_T, Y_T, Z_T]$ = coordinates of transmitter, assuming a right-hand coordinate system.

Define the reflector S by specifying

N = number of vertices of the reflecting polygon

and

$\mathbf{V}_n = [V_{nx}, V_{ny}, V_{nz}]$ = coordinates of the nth vertex, $1 \leqslant n \leqslant N$.

The vertices are taken in a positive direction around the reflector, which is defined to be in counterclockwise order as viewed from the transmitter.

Let $\mathbf{VC}$ be the centroid of the reflecting plate S,

$$\mathbf{VC} = \frac{\mathbf{V}_1 + \mathbf{V}_2 + \ldots + \mathbf{V}_N}{N} \tag{9}$$

and $\boldsymbol{\eta}$ the unit normal to the plate,

$$\boldsymbol{\eta} = \frac{\pm\,(\mathbf{V}_1 - \mathbf{VC}) \times (\mathbf{V}_2 - \mathbf{VC})}{|\mathbf{V}_1 - \mathbf{VC}|\,|\mathbf{V}_2 - \mathbf{VC}|}, \tag{10}$$

where the sign of $\boldsymbol{\eta}$ is chosen so that $\boldsymbol{\eta}$ points into the half space containing the transmitter,

$$(\mathbf{R}_T - \mathbf{VC}) \cdot \boldsymbol{\eta} \geqslant 0.$$

The unit normal to the incident plane wave, $\boldsymbol{\xi}_i$, is given by

$$\boldsymbol{\xi}_i = \frac{\mathbf{VC} - \mathbf{R}_T}{|\mathbf{VC} - \mathbf{R}_T|}, \tag{11}$$

and the unit normal to the reflected wave, $\boldsymbol{\xi}_r$, which is obtained by reflecting $\boldsymbol{\xi}_i$ about $\boldsymbol{\eta}$, is given by

$$\xi_r = \xi_i - 2\,(\xi_i \cdot \eta)\eta. \tag{12}$$

Fig. 1 illustrates a typical single-reflector situation.

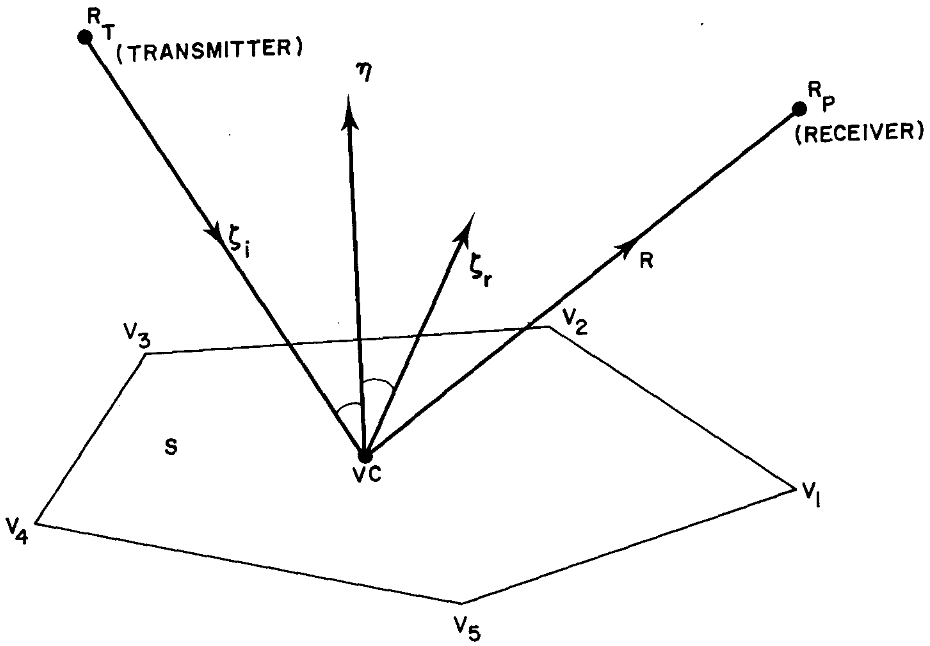

Fig. 1 — Geometry of a typical single-reflector case

The area A of the reflecting polygon can be calculated by dividing the polygon into N triangles of area Δ_n, each with a vertex at **VC** (Fig. 2),

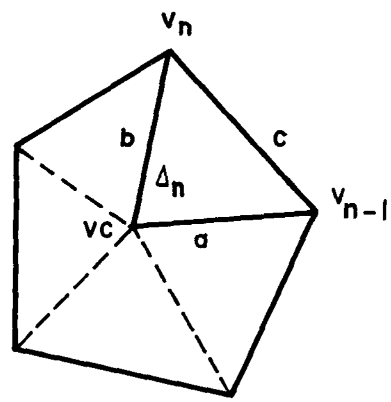

Fig. 2 — Division of the reflecting polygon for the area calculation

so that

$$\Delta_n = \frac{1}{4}[2(a^2b^2 + b^2c^2 + c^2a^2) - (a^4 + b^4 + c^4)]^{1/2}, \tag{13}$$

where

$$a = |\mathbf{V}_{n-1} - \mathbf{VC}|, \ b = |\mathbf{V}_n - \mathbf{VC}|, \ c = |\mathbf{V}_n - \mathbf{V}_{n-1}|. \tag{14}$$

Thus it follows that

$$A = \sum_{n=1}^{N} \Delta_n. \tag{15}$$

FIELD CALCULATION FOR A SINGLE REFLECTOR

Let $\mathbf{R}_p = [X_p, Y_p, Z_p]$ represent the position of the field sample point P at which the total field is to be evaluated. Assuming a dipole radiator, the direct-ray contribution to the total field at P is given by

$$\mathbf{E}_D(\mathbf{R}_P) = \left\{ \mathbf{E}_T - \left[\frac{\mathbf{E}_T \cdot (\mathbf{R}_P - \mathbf{R}_T)}{|\mathbf{R}_P - \mathbf{R}_T|^2} \right] (\mathbf{R}_P - \mathbf{R}_T) \right\} \frac{\exp\,(ik|\mathbf{R}_P - \mathbf{R}_T)}{|\mathbf{R}_P - \mathbf{R}_T|}. \tag{16}$$

The scattered-ray component of the total field consists of the radiation reflected from the surface S and scattered in the direction of P. The direct field at the centroid **VC** of plate S is

$$\mathbf{F} = \frac{\mathbf{E}_T - \left[\frac{\mathbf{E}_T \cdot (\mathbf{VC} - \mathbf{R}_T)}{|\mathbf{VC} - \mathbf{R}_T|^2} \right] (\mathbf{VC} - \mathbf{R}_T),}{|\mathbf{VC} - \mathbf{R}_T|,} \tag{17}$$

which, on reflection from S, becomes

$$\mathbf{F}_r = [2(\mathbf{F} \cdot \eta)\eta - \mathbf{F}] \quad \exp\,[ik|\mathbf{VC} - \mathbf{R}_T|], \tag{18}$$

where the first factor represents the optical reflection of the incident field and the second factor contributes the proper phase.

The next step is to apply the finite-sum representation of the far-field approximation to the Helmholtz integral, as stated in Eqs. (6) through (8), to the reflected field $\mathbf{F}_r$ to obtain the scattered field at $\mathbf{R}_p$. However, if the source $\mathbf{R}_T$ of the incident field and the location $\mathbf{R}_p$ of the sample point are on opposite sides of the reflector, there is no scattered-field component at $\mathbf{R}_p$, since shadowing is not currently considered in this model. In this case, no further calculations are made. Assume therefore that $\mathbf{R}_P$ and $\mathbf{R}_T$ are on the same side of the plane containing the reflector. As in the second section, define $\mathbf{R}$ to be the vector from the centroid of S to the sample field point, so that

$$\mathbf{R} = \mathbf{R}_P - \mathbf{VC}, \tag{19}$$

and

$$R = |\mathbf{R}|. \tag{20}$$

The projection of $\mathbf{R}$ onto the plane of S is given by

$$\boldsymbol{\rho} = \mathbf{R} - (\mathbf{R} \cdot \boldsymbol{\eta})\boldsymbol{\eta}, \tag{21}$$

and the projection of $[\boldsymbol{\xi} - (\boldsymbol{\rho}/R)]$ onto the same plane is given by

$$\mathbf{w} = \boldsymbol{\xi}_r - (\boldsymbol{\xi}_r \cdot \boldsymbol{\eta})\boldsymbol{\eta} - \frac{\boldsymbol{\rho}}{R} = \boldsymbol{\xi}_i - (\boldsymbol{\xi}_i \cdot \boldsymbol{\eta})\boldsymbol{\eta} - \frac{\boldsymbol{\rho}}{R}, \tag{22}$$

since the projections of $\boldsymbol{\xi}_r$ and $\boldsymbol{\xi}_i$ onto S are equal. Figure 3 illustrates the geometry involved.

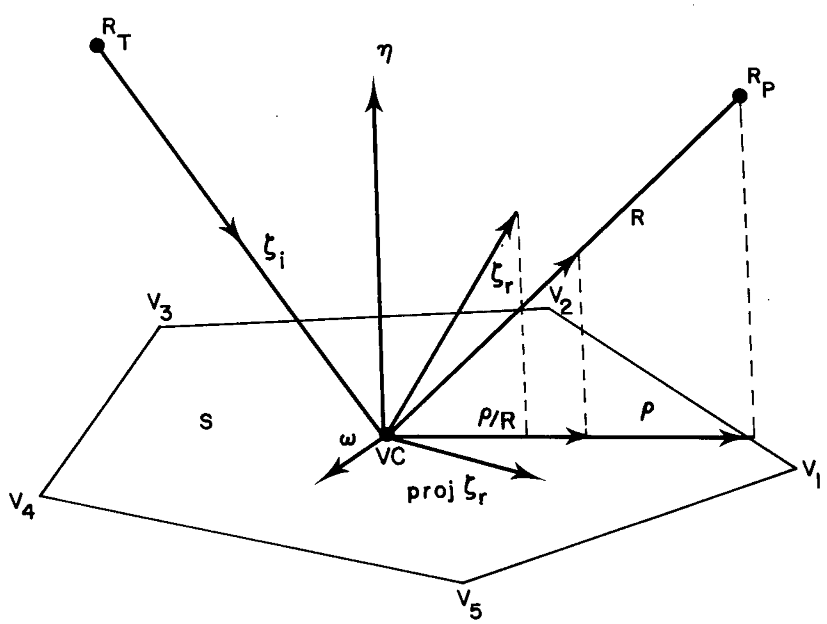

Fig. 3 — Geometry for the calculation of w

In the case where $\mathbf{w} = 0$, indicating that the specular reflection of the incident wave is in the direction of the field point P, so that $\xi_r = \mathbf{R}/R$, the scattered-field component is given by Eq. (8), yielding

$$\mathbf{E}_{SC}(\mathbf{R}_P) = \left\{ -ik \left[\frac{\mathbf{R}}{R} \cdot \eta \right] \frac{e^{ikR}}{2\pi R} A \right\} \mathbf{F}_r. \tag{23}$$

If $\mathbf{w} \neq 0$, then by Eqs. (6) and (7), the scattered field component is

$$\mathbf{E}_{SC}(\mathbf{R}_P) = \left[\frac{\left(\frac{\mathbf{R}}{R} \quad \xi_r \right) \cdot \eta}{\mathbf{w} \cdot \mathbf{w}} \frac{e^{ikR}}{4\pi R} \sum_{n=1}^{N} T_n \right] \mathbf{F}_r, \tag{24}$$

where for each vertex n, $1 \leqslant n \leqslant N$,

$$T_n = [(\mathbf{w} \times \eta) \cdot \Delta\mathbf{V}_n] \frac{\sin\left[\frac{k}{2} \mathbf{w} \cdot \Delta\mathbf{V}_n\right]}{\left[\frac{k}{2} \mathbf{w} \cdot \Delta\mathbf{V}_n\right]} \exp\left[\frac{ik}{2} \mathbf{w} \cdot (\mathbf{V}_n + \mathbf{V}_{n+1})\right] \tag{25}$$

in which

$$\begin{aligned} \Delta\mathbf{V}_n &= \mathbf{V}_{n+1} - \mathbf{V}_n, \\ \mathbf{V}_{N+1} &= \mathbf{V}_1. \end{aligned} \tag{26}$$

The total field at P is then the sum of the direct and scattered fields evaluated at $\mathbf{R}_p$:

$$\mathbf{E}(\mathbf{R}_P) = \mathbf{E}_D(\mathbf{R}_P) + \mathbf{E}_{SC}(\mathbf{R}_p). \tag{27}$$

FIELD CALCULATION FOR MULTIPLE REFLECTORS

When two or more reflectors are involved, it becomes necessary to keep track of rays bouncing back and forth between them while calculating the scattering taking place at each surface. The contributions from all these multiple scatterings, along with the direct ray, then constitute the total field seen at any given sample point. This section describes the approach used within the model to systematically exhaust all possible ray paths among a given set of reflectors and arrive at the total scattered field. The preliminary calculations initially parallel those of the single-reflector case, with the addition of indices to keep track of individual reflectors.

Consider a configuration with M flat polygonal reflectors $S_1, S_2, \ldots, S_M$, where $2 \leqslant M < \infty$. As in the case of a single reflector, let

$$\mathbf{E}_T = (E_{Tx}, E_{Ty}, E_{Tz})$$

be the transmitted radiation, and let

$$\mathbf{R}_T = [X_T, Y_T, Z_T]$$

the position of the transmitter. As a logical extension of the earlier case, define N_i to be the number of vertices of the ith reflector S_i, $1 \leqslant i \leqslant M$ and define

$$\mathbf{V}_{ik} = [V_{ikx}, V_{iky}, V_{ikz}]$$

to be the coordinates of the kth vertex of S_i, $1 \leqslant k \leqslant N_i$. Then

$$\mathbf{VC}_i = \frac{(\mathbf{V}_{i1} + \mathbf{V}_{i2} + \ldots + \mathbf{V}_{iN_i})}{N_i} \tag{28}$$

is the centroid of the ith reflecting plate, and

$$\boldsymbol{\eta}_i = \pm \frac{(\mathbf{V}_{i1} - \mathbf{VC}_i) \times (\mathbf{V}_{i2} - \mathbf{VC}_i)}{|\mathbf{V}_{i1} - \mathbf{VC}_i|\, |\mathbf{V}_{i2} - \mathbf{VC}_i|}, \tag{29}$$

where again the sign of $\boldsymbol{\eta}_i$ is taken such that

$$(\mathbf{R}_T - \mathbf{VC}_i) \cdot \boldsymbol{\eta}_i \geqslant 0,$$

is the unit normal to S_i.

Let A_i represent the area of the ith plate, so that

$$A_i = \sum_{n=1}^{N_i} \Delta_{in}, \tag{30}$$

where

$$\Delta_{in} = \frac{[2(a^2b^2 + b^2c^2 + c^2a^2) - (a^4 + b^4 + c^4)]^{1/2}}{4},$$

$$a = |\mathbf{V}_{i(n-1)} - \mathbf{VC}_i|,$$

$$b = |\mathbf{V}_{in} - \mathbf{VC}_i|,$$

and

$$c = |\mathbf{V}_{in} - \mathbf{V}_{i(n-1)}|.$$

Define $\mathbf{R}_{ij}$ to be the unit direction vector from the centroid of reflector S_i to the centroid of reflector S_j. Thus

$$\mathbf{R}_{ij} = \frac{\mathbf{VC}_j - \mathbf{VC}_i}{|\mathbf{VC}_j - \mathbf{VC}_i|} \tag{31}$$

for $0 \leqslant i \leqslant M$, $1 \leqslant j \leqslant M$, $i \neq j$.

Note that $\mathbf{R}_{0j} = (\mathbf{VC}_j - \mathbf{R}_T)/|\mathbf{VC}_j - \mathbf{R}_T|$ represents the unit vector from the transmitter $\mathbf{R}_T$ (indicated by subscript 0) to the centroid of S_j, which is the quantity defined as ξ_i in Eq. (11) for the single-reflector case. Thus each $\mathbf{R}_{ij}$ gives the direction of radiation originating from reflector S_i (radiation actually being scattered from S_i when $i \neq 0$) incident on reflector S_j.

The preliminary quantities defined and calculated so far have followed closely those of the single-reflector case, as would be expected. In review, associated with each reflector S_i, $1 \leqslant i \leqslant M$, are the centroid $\mathbf{VC}_i$, the normal η_i to the plane containing S_i, the area A_i, and a set of unit direction vectors $\mathbf{R}_{ij}$, $1 \leqslant j \leqslant M$, $j \neq i$, which point from S_i to each of the other reflectors S_j. Note that the indices i and j as used here denote reflectors; when used together, such as in $\mathbf{R}_{ij}$, index i refers to the "source" reflecting plate and index j to the scattering plate. The index k serves as a reflector-vertex index, where $1 \leqslant k \leqslant N_j$, $1 \leqslant j \leqslant M$.

With the preliminary calculations completed, the actual scattered-field computations for field point P begin with the selection of an initial reflector S_i and the corresponding computation of the direct field at $\mathbf{VC}_i$ due to transmission from R_T, minus the phase factor, given by

$$\mathbf{F}_0(\mathbf{VC}_i) = \frac{\mathbf{E}_T - \left[\dfrac{\mathbf{E}_T \cdot (\mathbf{VC}_i - \mathbf{E}_T)}{|\mathbf{VC}_i - \mathbf{R}_T|^2}\right](\mathbf{VC}_i - \mathbf{R}_T)}{|\mathbf{VC}_i - \mathbf{R}_T|}. \tag{32}$$

If the magnitude of $\mathbf{F}_0$ falls below the threshold,

$$|\mathbf{F}_0(\mathbf{VC}_i)| < \frac{\epsilon A_i |\mathbf{E}_D(\mathbf{R}_P)|}{A_1 + A_2 + \ldots + A_M}, \tag{33}$$

where the weighing factor ϵ is selected by the user and $\mathbf{E}_D(\mathbf{R}_P)$ is the direct field at P, then the field contribution of $\mathbf{F}_0$ scattered from S_i to the total field at P is considered negligible, and S_i is replaced as an initial reflector. However, if $\mathbf{F}_0(\mathbf{VC}_i)$ exceeds the threshold in Eq. (33), then $|\mathbf{F}_0(\mathbf{VC}_i)|$ is reflected from S_i, with the appropriate change in phase, so that the reflected $\mathbf{F}_{0i}(\mathbf{VC}_i)$, evaluated at the centroid $\mathbf{VC}_i$ is given by

$$\mathbf{F}_{0i}(\mathbf{VC}_i) = [2\mathbf{F}_0(VC_i) \cdot \eta_i)\eta_i - \mathbf{F}_0(VC_i)] \cdot \exp(ik|\mathbf{VC}_i - \mathbf{R}_T|). \tag{34}$$

Note that the subscripts of $\mathbf{F}$ trace the history of the radiation involved; thus $0i$ denotes radiation from R_T reflected from S_i.

Let $\mathbf{G}_i$ $(\mathbf{R}_Q)$ represent the reflected field $\mathbf{F}_{0i}$ *scattered* by S_i, evaluated at the arbitrary point Q. By Eqs. (6) and (7),

$$\mathbf{G}_i(\mathbf{R}_Q) = \frac{1}{\mathbf{w} \cdot \mathbf{w}} \left[\left(\frac{\mathbf{R}_Q - \mathbf{VC}_i}{|\mathbf{R}_Q - \mathbf{VC}_i|} - \mathbf{R}_{0i} \right) \cdot \eta_i \frac{e^{ik|\mathbf{R}_Q - \mathbf{VC}_i|}}{4\pi |\mathbf{R}_Q - \mathbf{VC}_i|} \sum_{n=1}^{N} T_n \right] \mathbf{F}_{0i}, \tag{35}$$

where

$$\mathbf{w} = [\mathbf{R}_{0i} - (\mathbf{R}_Q - \mathbf{VC}_i)] - \left\{ [\mathbf{R}_{0i} - (\mathbf{R}_Q - \mathbf{VC}_i)] \cdot \eta_i \right\} \eta_i \tag{36}$$

and

$$T_n = [(\mathbf{w} \times \eta_i) \cdot \Delta \mathbf{V}_n] \frac{\sin\left[\frac{k}{2} \mathbf{w} \cdot \Delta \mathbf{V}_n\right]}{\left[\frac{k}{2} \mathbf{w} \cdot \Delta \mathbf{V}_n\right]} \exp\left[\frac{ik}{2} \mathbf{w} \cdot (\mathbf{V}_n + \mathbf{V}_{n+1})\right], \tag{37}$$

with

$$\Delta \mathbf{V}_n = \mathbf{V}_{i(n+1)} - \mathbf{V}_{i(n)}$$

and (38)

$$\mathbf{V}_{i(N_i+1)} = \mathbf{V}_{i1}$$

for $1 \leqslant n \leqslant N_i$. Thus in particular the value of the field at P resulting from the scattering of $\mathbf{F}_{0i}$ from S_i is given by $\mathbf{G}_i$ $(\mathbf{R}_P)$, and the value of the scattered field at the centroid of each of the reflecting plates other than S_i by

$$\mathbf{G}_i\ (\mathbf{VC}_j),\ 1 \leqslant j \leqslant M, j \neq i.$$

Consider a second reflector S_j, $j \neq i$. If, as in Eq. (33),

$$|\mathbf{G}_i(\mathbf{VC}_j)| < \frac{\epsilon A_j |\mathbf{E}_D\ (\mathbf{R}_P)|}{A_1 + A_2 + \ldots + A_M}, \tag{39}$$

then the contribution of any further scattering of this ray path is taken to be insignificant, and the path (R_T to S_i to S_j) is abandoned. Consequently a new second reflector S_k, $k \neq i$, $k \neq j$, is selected, and the Eq. (39) threshold check is repeated.

Assume, however, that the condition of Eq. (39) is not met, that is the value of the scattered field at the centroid of reflector S_j exceeds the specified threshold. The path

(R_T to S_i to S_j) is then pursued. By Eq. (34) the field due to the reflection of $\mathbf{F}_{0i}$ at the centroid of S_j is given by

$$\mathbf{F}_{0ij} = [(2\mathbf{G}_i \cdot \eta_j)\eta_j - \mathbf{G}_i]. \tag{40}$$

Let $\mathbf{G}_{ij}$ represent the *scattering* of $\mathbf{F}_{0ij}$ from S_j, as given by Eq. (35). Then $G_{ij}(R_P)$ is the field contribution of the ray path (R_T to S_i to S_j to R_P) to the total field at P. Likewise, $\mathbf{G}_{ij}(VC_k)$ is the value of this scattered field at the centroid of S_k for all $1 \leqslant k \leqslant M$, $k \neq j$. Third-level reflection, where a reflector S_k, $k \neq j$, is selected, is treated in the same way as the second-level reflection just discussed. Additional levels of reflection are considered along each path until the path is terminated by either failing to meet the threshold at a given reflector and finding that all other reflectors for this path at this level have been considered or exceeding a predetermined maximum on the number of reflection levels allowed. A limit is set on the number of "bounces" between reflectors a field ray may take to facilitate bookkeeping within the computer implementation of the model. In practice the bounce or reflection-level limit is set high enough that paths are invariably terminated by failing to meet the threshold.

It is evident then that when one path is terminated at a particular reflector on a given level of reflection, a new reflector is to be selected at the same reflection level, subject only to the conditions that the chosen reflector is different from that of the preceding level (insuring a distinct bounce) and has not been previously considered for the current path base (as defined up through the preceding level). If all reflectors have been exhausted on this level, attention is returned to the previous level, a new path is defined by selecting a new reflector at this now current level, and the whole process moves forward again. Thus all paths are exhausted when, in the process of stepping down level by level in an attempt to define a new path, all reflectors are found to have been already considered on each level, down through the initial level of reflection.

An example may clarify the steps outlined previously in the generation of all possible ray paths. Consider a configuration consisting of three reflectors S_i, $1 \leqslant i \leqslant 3$, a transmitter at R_T, and field sample point at R_P. For purposes of illustration assume that no reflector shadows another, impose a bounce limit of three on any single ray path, and ignore any threshold restrictions. Figure 4 shows all possible ray paths for this situation. These paths are generated, as indicated previously, by starting at the initial level or first tier of reflection in the diagram, directly above R_T, with the lowest indexed available reflector S_1. Thus path 1 is simply (R_T to S_1 to R_P). Note that in Fig. 4 the last leg of each path (from the reflector to field point R_P) is indicated by a dashed rather than solid line for easier identification. For the next path the lowest indexed available (different from S_1 and not previously selected) reflector in the second tier, S_2, is chosen, so that path 2 is (R_T to S_1 to S_2 to R_P). Moving along to the third level, path 3 becomes (R_T to S_1 to S_2 to S_1 to R_P) through the selection of S_1 on tier 3. Since a bounce limit of three has been adopted for this case (illustrated in the figure by the existence of only three reflection tiers), path 4 requires a step down in the

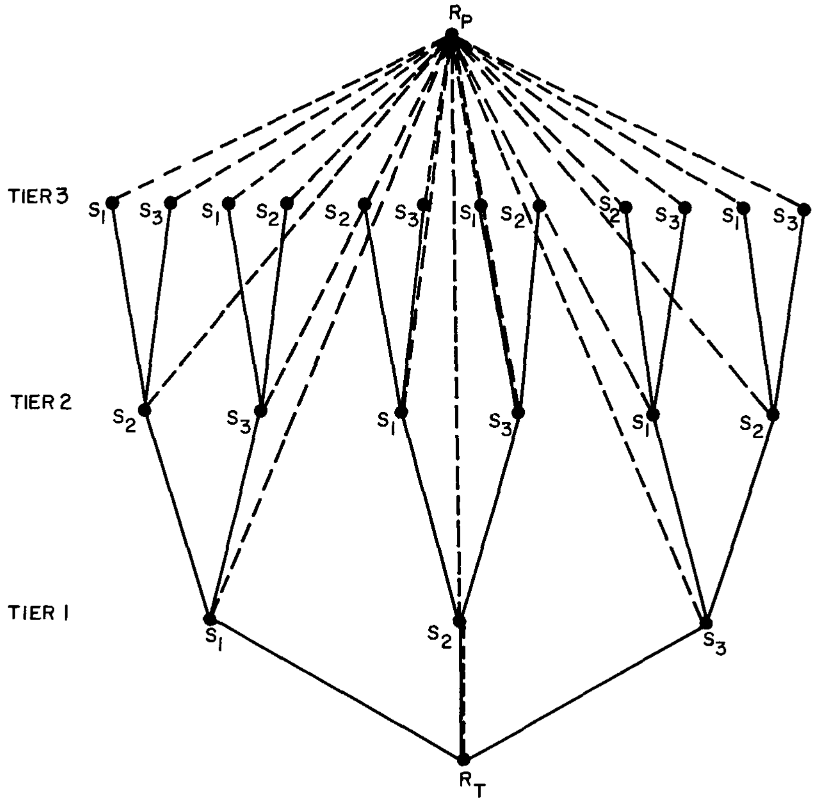

Fig. 4 — Ray paths

diagram back to the second tier, to S_2, and a reevaluation of the reflectors still available at the third level. Reflector S_3 on level 3 is the only available reflector not yet used on the current path base (R_T to S_1 to S_2), so the selection of S_3 on level 3 defines path 4 as (R_T to S_1 to S_2 to S_3 to R_P). Note reflector S_2 is ineligible for consideration at this level for path base (R_T to S_1 to S_2) since it would not constitute a distinct bounce. Again attention is returned to level 2 at S_2 since the third level allows no further reflections, but now all available third-tier reflectors for path base (R_T to S_1 to S_2) have been selected in previous paths. Thus a second step back, to S_1 on level one, is made. From the vantage point of S_1, with path base (R_T to S_1), S_3 remains as the only available reflector on level two, so path 5 becomes (R_T to S_1 to S_3 to R_P). Paths 6, as (R_T to S_1 to S_3 to S_1 to R_P), and 7, as (R_T to S_1 to S_3 to S_2 to R_P), follow in the same way. Stepping down now to S_1 on level 1 reveals that all level 2 alternatives for path base (R_T to S_1) have been used (namely S_2 and S_3), so the next lowest indexed reflector on level one, reflector S_2, is selected. Thus path 8 is (R_T to S_2 to R_P), and the whole process described previously for S_1 is repeated. When attention is returned to the initial level for the third time, the remaining reflector S_3 is used, the preceding procedure is reiterated, and the generation of all possible ray paths for this configuration is completed.

Note that Fig. 4 is composed of three vertical sections, one for each reflector, as enumerated in the first tier. Using the diagram, the paths described can be traced by starting from the leftmost section and, within that section, moving along the leftmost solid lines from tier to tier, working toward the right. At each reflector in a given tier, a dotted line directly to R_P represents the completion of a path. Table 1 lists all possible paths and corresponding field contributions for this example in the order of their generation by the previously outlined steps.

Table 1
Paths and Corresponding Field Contributions

Path Number	Path	Field Contributions at R_P
0	R_T to R_P	$\mathbf{E}_D(R_P)$
1	R_T to S_1 to R_P	$\mathbf{G}_1(R_P)$
2	R_T to S_1 to S_2 to R_P	$\mathbf{G}_{12}(R_P)$
3	R_T to S_1 to S_2 to S_1 to R_T	$\mathbf{G}_{121}(R_P)$
4	R_T to S_1 to S_2 to S_3 to R_P	$\mathbf{G}_{123}(R_P)$
5	R_T to S_1 to S_3 to R_P	$\mathbf{G}_{13}(R_P)$
6	R_T to S_1 to S_3 to S_1 to R_P	$\mathbf{G}_{131}(R_P)$
7	R_T to S_1 to S_3 to S_2 to R_P	$\mathbf{G}_{132}(R_P)$
8	R_T to S_2 to R_P	$\mathbf{G}_2(R_P)$
9	R_T to S_2 to S_1 to R_P	$\mathbf{G}_{21}(R_P)$
10	R_T to S_2 to S_1 to S_2 to R_P	$\mathbf{G}_{212}(R_P)$
11	R_T to S_2 to S_1 to S_3 to R_P	$\mathbf{G}_{213}(R_P)$
12	R_T to S_2 to S_3 to R_P	$\mathbf{G}_{23}(R_P)$
13	R_T to S_2 to S_3 to S_1 to R_P	$\mathbf{G}_{231}(R_P)$
14	R_T to S_2 to S_3 to S_2 to R_P	$\mathbf{G}_{232}(R_P)$
15	R_T to S_3 to R_P	$\mathbf{G}_3(R_P)$
16	R_T to S_3 to S_1 to R_P	$\mathbf{G}_{31}(R_P)$
17	R_T to S_3 to S_1 to S_2 to R_P	$\mathbf{G}_{312}(R_P)$
18	R_T to S_3 to S_1 to S_3 to R_P	$\mathbf{G}_{313}(R_P)$
19	R_T to S_3 to S_2 to R_P	$\mathbf{G}_{32}(R_P)$
20	R_T to S_3 to S_2 to S_1 to R_P	$\mathbf{G}_{321}(R_P)$
21	R_T to S_3 to S_2 to S_3 to R_P	$\mathbf{G}_{323}(R_P)$

The general expression for the total field at the sample point P is thus

$$\mathbf{E}_T(\mathbf{R}_P) = \mathbf{E}_D(\mathbf{R}_P) + \sum_{i=1}^{M} \left[\mathbf{G}_i(\mathbf{R}_P) + \sum_{\substack{j=1\\ j\neq i}}^{M} \left[\mathbf{G}_{ij}(\mathbf{R}_P) + \sum_{\substack{k=1\\ k\neq j}}^{M} \left[\mathbf{G}(\mathbf{R}_P)_{ijk} + \dots + \sum_{\substack{n=1\\ n\neq n\text{-}1}}^{M} \left[\mathbf{G}(\mathbf{R}_P)_{ijk\,\dots\,n}\right] \dots \right]\right]\right], \tag{41}$$

where $\mathbf{E}_D(\mathbf{R}_P)$ is the direct field at P, $\mathbf{G}_{ijk}(\mathbf{R}_P)$ is the field contribution at P due to path (R_T to S_i to S_j to S_k to R_P), and n is the maximum reflection level.

DOPPLER SYSTEM

In the doppler system the transmitting-array elements are activated sequentially, giving the effect of a "virtual velocity" to each transmission. Consequently the signal received at a given point within the beam is shifted in wavelength by an amount proportional to sin θ, where θ is the angle of the point off the beam center. In the model described here, doppler transmissions are accommodated by specifying a "virtual velocity" vector **VV** to be associated with each transmission. Then all field components *initally* reflected from a given reflector S_i have wavelength λ_i given by

$$\lambda_i = \lambda - \frac{\lambda}{c}\left[\frac{(\mathbf{VC}_i - \mathbf{R}_T)\cdot \mathbf{VV}}{|\mathbf{VC}_i - \mathbf{R}_T|}\right],$$

where c is the speed of light.

Let $E_i(\mathbf{R}_P)$ be the component of the field at P due to all transmissions originating at $\mathbf{R}_T$ initially scattered from reflector S_i (the field component at P with wavelength λ_i). Then

$$\mathbf{E}_i(\mathbf{R}_P) = \mathbf{G}_i(\mathbf{R}_P) + \sum_{\substack{j=1\\ j\neq i}}^{M} \left[\mathbf{G}_{ij}(\mathbf{R}_P) + \sum_{\substack{k=1\\ k\neq j}}^{M} \left[\mathbf{G}_{ijk}(\mathbf{R}_P) + \dots + \sum_{\substack{n=1\\ n\neq n\text{-}1}}^{M} [\mathbf{G}_{ijk\,\dots\,n}] \dots \right]\right], \tag{42}$$

and the total field at P as given by Eq. (41) becomes

$$\mathbf{E}_T(\mathbf{R}_P) = \mathbf{E}_O(\mathbf{R}_P) + \sum_{i=1}^{M} \mathbf{E}_i(\mathbf{R}_P), \tag{43}$$

where $|\mathbf{E}_i(\mathbf{R}_P)|$ represents the line in the energy spectrum at P corresponding to wavelenth λ_i.

CONCLUSIONS

The model described herein calculates the standard far-field approximation to the Helmholtz integrals representing the field multiply scattered by flat polygonal metallic reflectors. The new representation of the far-field approximation to the Helmholtz integral as a finite sum, as developed in Ref. 2, has been employed to achieve a great economy in calculation. More generally, it is known that the Helmholtz integral has an exact closed-form representation as a line integral evaluated over the boundary of the reflecting surface. An expansion of the model is being developed to include the treatment of curved reflecting surfaces, along with the exact line-integral representations of the Helmholtz integral, applicable to plane-wave, spherical-wave, and dipole radiation. Also under investigation is the possibility of developing a closed-form expression for the Helmholtz integral which would involve no integrations at all, similar to that obtained and used for the far-field approximation to the exact Helmholtz integral.

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1. W.B. Gordon, "Far-Field Approximations to the Kirchhoff-Helmholtz Representations of Scattered Fields," to appear in *IEEE Trans. Antennas Propag.*, July, 1975.

2. W.B. Gordon, "Evaluating the Helmholtz Integral: Part I - Basic Theory," NRL Report 7792, Aug. 21, 1974.

3. S. Siver, editor, *Microwave Antenna Theory and Design*, Dover, New York, 1965, p. 173.

UNCLASSIFIED

Appendix A

SOURCE PROGRAM LISTING

```
      PROGRAM FIELD
C
C
C     PROGRAM FIELD CALCULATES THE ELECTROMAGNETIC FIELD DUE TO MULTIPLE
C     SCATTERING BY A COLLECTION OF FLAT METALLIC POLYGONAL REFLECTORS.
C
C     GIVEN A SET OF RADAR PARAMETERS, THE TRANSMITTER POSITION, AND THE
C     VERTICES OF A COLLECTION OF FLAT POLYGONAL REFLECTORS, THE FIELD
C     PRODUCED AT ANY SPECIFIED SAMPLE POINT (RECEIVER) CONSISTS OF THE
C     DIRECT RAY COMBINED WITH INDIRECT RAYS RESULTING FROM MULTIPLE
C     SCATTERINGS FROM THE REFLECTORS.
C
C     THE PROGRAM TRACES THE TRANSMITTED RADIATION THROUGH ALL POSSIBLE
C     PATHS AMONG THE REFLECTORS, CALCULATES THE VALUE OF THE FIELD
C     AFTER SCATTERING BY EACH REFLECTOR ENCOUNTERED, AND THEN SUMS ALL
C     CONTRIBUTIONS TO ARRIVE AT THE TOTAL FIELD.
C
C     FOR DOPPLER SYSTEMS, A USER SPECIFIED VIRTUAL VELOCITY VECTOR IS
C     ASSOCIATED WITH EACH ARRAY TRANSMISSION, PRODUCING AN ENERGY
C     SPECTRUM BY WAVELENGTH OR FREQUENCY FOR EACH FIELD POINT SAMPLED.
C
C
C         JANUARY, 1975
C
      COMMON/CONST/PI,TWOPI,CM
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1           NV(50),V(3, 4,50)
      COMMON/PRELIM/M,WAVENM,VC(3,50),VNORM(3,50),VA(50),VASUM,XLAM(50),
     1           XK(50),R(3,31,30)
      DIMENSION RP(3), PRTL(50)
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM, PROJ) AND
C        UP TO 4 (INPUT, PROJ) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
      PI = 3.1415927
      TWOPI = 6.2831853
C     SPEED OF LIGHT IN METERS/SEC
      CM = 2.998E08
C
C     READ IN DATA
      CALL READIN
C     DETERMINE GEOMETRICAL QUANTITIES NEEDED FOR SCATTERING
C     CALCULATIONS
      CALL SETUP
C     INITIALIZE FIELD POINT GENERATOR
      NP = 0
   10 CALL FPGEN(NP,RP,I)
      IF(NP .LT. 0) GO TO 50
C     CALCULATE DIRECT FIELD AT RP
      CALL DIRECT(RP)
C     CALCULATE SCATTERED FIELD AT RP
      CALL REFLEC(RP,PRTL,NBMAX)
C     PRINT DIRECT AND SCATTERED FIELDS AT RP
      CALL PRNT(RP,PRTL,1)
      GO TO 10
```

```
C
   50 CONTINUE
      END
```

```
                                              IDENT      FIELD
PROGRAM LENGTH                     00146
ENTRY POINTS        FIELD          00070
BLOCK NAMES
                    CONST          00003
                    INPUT          01243
                    PRELIM         06253
EXTERNAL SYMBOLS
                    Q8QENTRY
                    Q8QDICT.
                    READIN
                    SETUP
                    FPGEN
                    DIRECT
                    REFLEC
                    PRNT
   00053 SYMBOLS
```

```
      SUBROUTINE FPGEN(NP,RP,ID)
C
C     THIS SUBROUTINE TO BE FURNISHED BY THE USER
C
C     SUBROUTINE FPGEN GENERATES THE FIELD POINT RP AT WHICH THE DIRECT
C        AND SCATTERED FIELDS ARE TO BE EVALUATED
C
C     PARAMETERS
C        NP = FIELD POINT STATUS       (NP = 0 FOR INITIAL ENTRY)
C        RP = GENERATED FIELD POINT
C        ID = DATA SET ID
C
      COMMON/CONST/PI,TWOPI,CM
      DIMENSION RP(3)
   10 IF(NP .NE. 0) GO TO 20
C
C     INITIALIZE GENERATOR
C
C     THIS EXAMPLE WILL CALCULATE THE 10 SAMPLE POINTS
C        RP(1) = 100.*COS(N*PI/180.)
C        RP(2) = 0.
C        RP(3) = 15.*SQRT(3.) + 100.*SIN(N*PI/180.)
C     FOR N FROM 1 THROUGH 10 ON 10 SUCCESSIVE ENTRIES
C
      NMAX = 10
      CONST = 15.0*SQRT(3.0)
      RP(2) = 0.
      NP = 1
      N = 0
   20 N = N + 1
      IF(N .GT. NMAX) GO TO 30
      FN = N - 1
      ARG = FN*PI/180.
      RP(1) = 100.*COS(ARG)
      RP(3) = CONST + 100.*SIN(ARG)
      RETURN
   30 NP = -1
      RETURN
      END
```

```
                                        IDENT      FPGEN
PROGRAM LENGTH                     00136
ENTRY POINTS        FRGEN          00003
BLOCK NAMES
                    CONST          00003
EXTERNAL SYMBOLS
                    Q8QDICT.
                    SQRTF
                    SINF
                    COSF
   00046 SYMBOLS
```

```
      SUBROUTINE READIN
C
C     SUBROUTINE READIN INPUTS AND PRINTS THE FOLLOWING DATA
C        ITITLE    = 80 CHARACTER RUN IDENTIFICATION
C        XLMDA     = TRANSMITTING WAVELENGTH (METERS)
C        VV(3)     = VIRTUAL VELOCITY (METERS/SEC)
C        ET(3)     = TRANSMITTED RADIATION
C        RT(3)     = TRANSMITTER POSITION COORDINATES
C        D(3)      = DIRECTION OF MAIN BEAM
C
C        NBOUND    = MAX. NO. OF REFLECTED BOUNCES ALLOWED
C        MP        = NUMBER OF REFLECTING PLATES
C        NV(I)     = NUMBER OF VERTICES OF PLATE I
C        V(K,J,I)  = COORDINATE K OF VERTEX J OF PLATE I
C
C
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1             NV(50),V(3,4,50)
      INTEGER WT
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM, PROJ) AND
C        UP TO 4 (INPUT, PROJ) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
C     ASSIGN INPUT, OUTPUT LOGICAL TAPE UNITS
      IN = 60
      WT = 61
      READ(IN,100)  (ITITLE(I),I=1,10),XLMDA,(VV(I),I=1,3),(ET(I),I=1,3)
     1              (RT(I),I=1,3),  (D(I),I=1,3),NBOUND,MP
  100 FORMAT(10A8/8F10.3/5F10.3/2I5)
      WRITE(WT,200)  (ITITLE(I),I=1,10),XLMDA,(VV(I),I=1,3),(ET(I),I=1,3)
     1              ,(RT(I),I=1,3),  (D(I),I=1,3),NBOUND,MP
  200 FORMAT(1H1/10A8,//5X,10HWAVELENGTH,16X,F10.3//5X,16HVIRTUAL VELOCI
     1TY,10X3F10.3//5X,21HTRANSMITTED RADIATION,5X,3F10.3//5X,23HTRANSMI
     2TTER COORDINATES,3X,3F10.3//5X,19HMAIN BEAM DIRECTION,7X,3F10.3//
     35X,16HMAX. NO. BOUNCES,10X,I10//5X,21HNO. REFLECTING PLATES,5X,
     4I10)
C
      READ(IN,110)  (NV(I),I=1,MP)
  110 FORMAT(16I5)
      DO 130 I = 1,MP
         N = NV(I)
         READ(IN,120)  ((V(K,J,I),K=1,3),J=1,N)
  120    FORMAT(8F10.3)
      WRITE(WT,210)  I, N,  ((V(K,J,I),K=1,3),J=1,N)
  210 FORMAT(///5X,5HPLATE,I10,5X,12HNO. VERTICES,I10//1X,12F10.3)
  130 CONTINUE
C
C
      RETURN
      END

                                      IDENT     READIN
 PROGRAM LENGTH                   00510
 ENTRY POINTS       READIN        00134
 BLOCK NAMES
                    INPUT         01243
 EXTERNAL SYMBOLS
                    THEND.
                    Q8QDICT.
                    TSH.
                    STH.
                    QNSINGL.
     00104 SYMBOLS
```

```
      SUBROUTINE SETUP
C
C     SUBROUTINE SETUP MAKES PRELIMINARY GEOMETRICAL CALCULATIONS
C        (COMMON PRELIM) DEPENDENT UPON INPUT REFLECTING PLATE DATA
C        AND ALSO COMPUTES THE DOPPLER WAVELENGTH AND WAVENUMBER
C        ASSOCIATED WITH EACH REFLECTING PLATE
C
      COMMON/CONST/PI,TWOPI,CM
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1            NV(50),V(3, 4,50)
      COMMON/PRELIM/M,WAVFNM,VC(3,50),VNORM(3,50),VA(50),VASUM,XLAM(50),
     1            XK(50),R(3,31,30)
      DIMENSION CS(3),V1V(3),V2V(3)
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM, PROJ) AND
C        UP TO 4 (INPUT, PROJ) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
C     SET NO. OF EFFECTIVE PLATES EQUAL TO NO. OF PLATES INPUT
      M = MP
C
C     CALCULATE CENTROID VC OF EACH PLATE
      DO 35 J = 1,M
         CS(1) = 0.
         CS(2) = 0.
         CS(3) = 0.
         N = NV(J)
         DO 25 I = 1,N
            DO 20 K = 1,3
               CS(K) = CS(K) + V(K,I,J)
   20       CONTINUE
   25    CONTINUE
         FNV= FLOAT(N)
         DO 30 K = 1,3
            VC(K,J) = CS(K)/FNV
   30    CONTINUE
   35 CONTINUE
C
C
C     CALCULATE UNIT NORMAL VNORM TO EACH PLATE
      DO 70 J = 1,M
         VS1 = 0.
         VS2 = 0.
         DO 55 K = 1,3
            V1V(K) = V(K,1,J) - VC(K,J)
            V2V(K) = V(K,2,J) - VC(K,J)
            VS1 = VS1 + V1V(K)*V1V(K)
            VS2 = VS2 + V2V(K)*V2V(K)
   55    CONTINUE
         VM = SQRT(VS1)*SQRT(VS2)
         VNORM(1,J) = (V1V(2)*V2V(3) - V1V(3)*V2V(2))/VM
         VNORM(2,J) = (V1V(3)*V2V(1) - V1V(1)*V2V(3))/VM
         VNORM(3,J) = (V1V(1)*V2V(2) - V1V(2)*V2V(1))/VM
C     CHECK SIGN AND NORMALIZE
         VNM = SQRT(VNORM(1,J)*VNORM(1,J) + VNORM(2,J)*VNORM(2,J) +
```

```
     1              VNORM(3,J)*VNORM(3,J))
         DOT = 0.
         DO 60 K = 1,3
            DOT = DOT + VNORM(K,J)*(VC(K,J) - RT(K))
   60    CONTINUE
         IF(DOT .GT. 0.) VNM = -VNM
         DO 65 K = 1,3
            VNORM(K,J) = VNORM(K,J)/VNM
   65    CONTINUE
   70 CONTINUE
C
C
C     CALCULATE AREA VA OF EACH PLATE
      VASUM = 0.
      DO 85 J = 1,M
         AREA = 0.
         N = NV(J)
         DO 80 I = 1,N
            A = 0.
            B = 0.
            C = 0.
C           COMPUTE SIDES OF TRIANGLE J
            II = I + 1
            IF (II .GT. N) II = 1
            DO 75 K = 1,3
               A = A + (V(K,I,J) - VC(K,J))**2
               B = B + (V(K,II,J) -  VC(K,J))**2
               C = C + (V(K,II,J) - V(K,I,J))**2
   75       CONTINUE
C           CALCULATE AREA OF TRIANGLE J
            DELA = SQRT(2.0*(A*B + B*C + C*A) - (A*A + B*B + C*C))/4.0
            AREA = AREA + DELA
   80    CONTINUE
         VA(J) = AREA
         VASUM = VASUM + AREA
   85 CONTINUE
C
C
C     CALCULATE PLATE (I-1) TO PLATE (J) UNIT DIRECTION VECTORS R(K,I,J)
C        WITH RESPECT TO PLATE CENTROIDS VC
C     NOTE 1.LE.I.LE.M+1 SO THAT I REFERS TO PLATE I-1, WHERE PLATE 0
C        IS THE TRANSMITTER
C
C     CALCULATE TRANSMITTER (PLATE 0) TO PLATE J UNIT DIRECTION VECTORS
      DO 115 J = 1,M
         VM = 0.
         DO 105 K = 1,3
            V1V(K) = VC(K,J) - RT(K)
            VM = VM + V1V(K)*V1V(K)
  105    CONTINUE
         VM = SQRT(VM)
         DO 110 K = 1,3
            R(K,1,J) = V1V(K)/VM
  110    CONTINUE
  115 CONTINUE
C
```

```
C     CALCULATE REMAINING DIRECTION VECTORS
      DO 135 I = 1,M
         II = I + 1
         DO 130 J = 1,M
            IF(J .EQ. I) GO TO 130
            VM = 0.
            DO 120 K = 1,3
               V1V(K) = VC(K,J) - VC(K,I)
               VM = VM + V1V(K)*V1V(K)
  120       CONTINUE
            VM = SQRT(VM)
            DO 125 K = 1,3
               R(K,II,J) = V1V(K)/VM
  125       CONTINUE
  130    CONTINUE
  135 CONTINUE
C
C
C
C     CHECK FOR DOPPLER
      VVM = 0.
      DO 138 K = 1,3
         VVM = VVM + VV(K)*VV(K)
  138 CONTINUE
      VVM = SQRT(VVM)
      IF(VVM .EQ. 0.0) GO TO 150
C     CALCULATE DOPPLER WAVELENGTH XLAM AND WAVENUMBER XK AS SEEN
C        BY EACH PLATE
      DO 145 J = 1,M
         VS1 = 0.
         VS2 = 0.
         DO 140 K = 1,3
            CS(K) =  VC(K,J) - RT(K)
            VS1 = VS1 + CS(K)*CS(K)
            VS2 = VS2 + VV(K)*CS(K)
  140    CONTINUE
      VS1 = SQRT(VS1)
         XLAM(J) = XLMDA*(1.0 - VS2/(CM*VS1))
         XK(J) = TWOPI/XLAM(J)
  145 CONTINUE
      GO TO 160
C
C     NO DOPPLER
C
C     CALCULATE WAVE NUMBER
  150 WAVENM = TWOPI/XLMDA
      DO 155 J = 1,M
         XLAM(J) = XLMDA
         XK(J) = WAVENM
  155 CONTINUE
C
  160 RETURN
      END
```

```
                                     IDENT     SETUP
PROGRAM LENGTH               00740
ENTRY POINTS      SETUP      00014
BLOCK NAMES
                  CONST      00003
                  INPUT      01243
                  PRELIM     06253
EXTERNAL SYMBOLS
                  QBQDICT.
                  SQRTF
   00203 SYMBOLS
```

```
      SUBROUTINE DIRECT(RP)
C
C     SUBROUTINE DIRECT CALCULATES THE DIRECT FIELD COMPONENT ED FROM
C     TRANSMITTER RT TO FIELD POINT RP
C
      COMMON/CONST/PI,TWOPI,CM
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1             NV(50),V(3, 4,50)
      COMMON/PRELIM/M,WAVENM,VC(3,50),VNORM(3,50),VA(50),VASUM,XLAM(50),
     1              XK(50),R(3,31,30)
      COMMON/FIELD/ED(3),ES(3,50)
      COMPLEX ED, ES
      COMPLEX FAC
      DIMENSION RP(3), WS(3)
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM, PROJ) AND
C        UP TO 4 (INPUT, PROJ) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
C     DIPOLE
      DOT = 0.
      DO 10 K = 1,3
         WS(K) = RP(K) - RT(K)
         DOT = DOT + ET(K)*WS(K)
   10 CONTINUE
      WM2 = WS(1)*WS(1) + WS(2)*WS(2) + WS(3)*WS(3)
      WM = SQRT(WM2)
      DW = DOT/WM2
C     CALCULATE WAVE NUMBER (INCLUDING DOPPLER IF PRESENT)
      DO 12 K = 1,3
         DOT = DOT + WS(K)*VV(K)
   12 CONTINUE
      FLAM = XLMDA*(1.0 - DOT/(CM*WM))
      WAVENM = TWOPI/FLAM
      WK = WAVENM*WM
      FAC = CMPLX(COS(WK),SIN(WK))
      DO 15 K = 1,3
         ED(K) = (ET(K) - DW*WS(K))*FAC/WM
   15 CONTINUE
C     PRINT
      CALL PRNT(RP,RP,-1)
      RETURN
C
      END
```

```
                                                  IDENT       DIRECT
PROGRAM LENGTH                          00205
ENTRY POINTS        DIRECT              00006
BLOCK NAMES
                    CONST               00003
                    INPUT               01243
                    PRELIM              06253
                    FIELD               00462
EXTERNAL SYMBOLS
                    Q1Q04330
                    Q1Q05310
                    Q8QDICT.
                    PRNT
                    SQRTF
                    SINF
                    COSF
    00076 SYMBOLS
```

```
      SUBROUTINE REFLEC(RP,PRTL,NBMAX)
C
C     SUBROUTINE REFLEC CALCULATES THE REFLECTED (SCATTERED) FIELD
C        COMPONENT ES(K,J) FOR EACH WAVELENGTH XLAM(J) AT POINT RP
C
      COMMON/CONST/PI,TWOPI,CM
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1             NV(50),V(3, 4,50)
      COMMON/PRELIM/M,WAVENM,VC(3,50),VNORM(3,50),VA(50),VASUM,XLAM(50),
     1             XK(50),R(3,31,30)
      COMMON/FIELD/ED(3),ES(3,50)
      COMPLEX ED,ES,CW
      DIMENSION VJT(3), FOJ(3),PRTL(50)
      DIMENSION NB(50)
      COMPLEX F,G
      DIMENSION NP(10),F(3,10),G(3)
      DIMENSION RR(3)
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM) AND
C        UP TO 4 (INPUT) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
C     INITIALIZE SCATTERED FIELD COMPONENTS TO ZERO
      DO 15 J = 1,M
         DO 10 K = 1,3
            ES(K,J) = 0.
   10    CONTINUE
   15 CONTINUE
C
C     INITIALIZE BOUNCE COUNTERS
      DO 20 J = 1,M
         NB(J) = 0
   20 CONTINUE
C
C     SET EPSILON FOR FIELD STOPPING RULE
      EPS = 0.01
C     CALCULATE FACTOR FOR FIELD STOPPING RULE AT GIVEN PLATE
      FAC = EPS*CVMAG(ED)/VASUM
C
C     *****************************************************************
C
      DO 200 J = 1,M
C
C     CALCULATE DIRECT FIELD AT PLATE J (SCAT PROVIDES 1/R FACTOR)
C     DIPOLE
      DOT = 0.
      DO 30 K = 1,3
         VJT(K) = VC(K,J) - RT(K)
         DOT = DOT + ET(K)*VJT(K)
   30 CONTINUE
      VJTM2 = VJT(1)*VJT(1) + VJT(2)*VJT(2) + VJT(3)*VJT(3)
      VJTM = SQRT(VJTM2)
      DV = DOT/VJTM2
      DO 33 K = 1,3
         FOJ(K) = (ET(K) - DV*VJT(K))
```

```
   33 CONTINUE
C
C     CHECK DIRECT FIELD AGAINST STOPPING RULE
      FMIN = VA(J)*FAC
      FMAG = SQRT(FOJ(1)*FOJ(1) + FOJ(2)*FOJ(2) + FOJ(3)*FOJ(3))
      IF (FMAG .LT. FMIN) GO TO 200
C     CHECK BOUNCE COUNTER AGAINST MAXIMUM ALLOWED
      IF (NB(J) .GE. NBOUND) GO TO 200
C
C     INCREMENT BOUNCE COUNTER
      NB(J) = NB(J) + 1
C
C     REFLECT FOJ WITH RESPECT TO PLATE J
      DOT = 0.
      DO 45 K = 1,3
         DOT = DOT + FOJ(K)*VNORM(K,J)
   45 CONTINUE
      DO 48 K = 1,3
         F(K,1) = 2.0*DOT*VNORM(K,J) - FOJ(K)
   48 CONTINUE
C
C     COMPUTE FIELD COMPONENT AT RP DUE TO SCATTER OF F(1) FROM PLATE J
      CALL SCAT(0,J,XK(J),F(1,1),RP,G)
C
C     ADD TO SCATTER FIELD COMPONENT FOR XLAM(J)
      DO 55 K = 1,3
         ES(K,J) = ES(K,J) + G(K)
   55 CONTINUE
C
C     ********************************************************************
C
C
C     CALCULATE TOTAL SCATTERED FIELD COMPONENT AT RP WITH WAVELENGTH
C        XLAM(J)
C     NB(J) = MAX. NO. BOUNCES INVOLVED IN COMPUTATION OF FIELD
C        COMPONENT WITH WAVELENGTH XLAM(J)
C     NP(N) = CURRENT PLATE NO. FOR NTH BOUNCE
C         N = CURRENT BOUNCE NO.
C
      NP(1) = J
      DO 60 N = 2,NBOUND
         NP(N) = [illegible]
   60 CONTINUE
C     INITIALIZE BOUNCE SEQUENCE
   70 N = 2
   75 NP(N) = NP(N) + 1
      IF(NP(N) .GT. M) GO TO 95
C     CHECK FOR DISTINCT BOUNCE
   78 IF (NP(N) .EQ. NP(N-1)) GO TO 75
C     COMPUTE F(N), THE VALUE OF FIELD F(N-1) SCATTERED BY PLATE NP(N-1)
C     AT THE POINT VC(NP(N))
      NP1 = 0
      IF(N .GT. 2) NP1 = NP(N-2)
      NP2 = NP(N-1)
      CALL SCAT(NP1,NP2,XK(J),F(1,N-1),VC(1,NP(N)),F(1,N))
      FMIN = VA(NP2)*FAC
      FMAG = CVMAG(F(1,N))
```

```
      IF(FMAG .LT. FMIN) GO TO 75
C
C     REFLECT F WITH RESPECT TO PLATE NP2
   80 DOT = 0.
      DO 82 K = 1,3
         DOT = DOT + F(K,N)*VNORM(K,NP2)
   82 CONTINUE
      DO 85 K = 1,3
         F(K,N) = 2.0*DOT*VNORM(K,NP2) - F(K,N)
   85 CONTINUE
C
C     CALCULATE FIELD COMPONENT AT RP DUE TO SCATTER OF F(N) FROM
C     PLATE NP(N)
      CALL SCAT(NP2,NP(N),XK(J),F(1,N),RP,G)
C
C     ADD TO CURRENT SCATTERED FIELD COMPONENT
      DO 90 K = 1,3
         ES(K,J) = ES(K,J) + G(K)
   90 CONTINUE
C
C     UPDATE MAX. NO. OF BOUNCES FOR THIS WAVELENGTH, IF APPROPRIATE
      IF (N .GT. NB(J)) NB(J) = N
C
C     STEP UP BOUNCE LEVEL
      N = N + 1
      IF (N .LE. NBOUND) GO TO 75
C
C     END OF BOUNCE STRING
C     STEP BACK ONE BOUNCE LEVEL AND INCREMENT
   95 N = N - 1
      IF (N .GT. 1) GO TO 75
C
C
C     FINISHED WITH WAVELENGTH XLAM(J) CONTRIBUTION TO FIELD
  200 CONTINUE
C
C     ******************************************************************
C
C     SET UP PRINT FREQUENCY ARRAY
      DO 205 J = 1,M
         PRTL(J) = CM/XLAM(J)
  205 CONTINUE
C
C     CHECK FOR DISTINCT SCATTER FREQUENCIES
      MM = M - 1
      DO 230 I = 1,MM
         II = I + 1
         IF(PRTL(I) .EQ. -1.0) GO TO 230
         DO 225 J = II,M
      WS = ABSF(PRTL(I) - PRTL(J))
      IF( WS .GT. 1000.) GO TO 225
            DO 220 K = 1,3
               ES(K,I) = ES(K,I) + ES(K,J)
  220       CONTINUE
            PRTL(J) = -1.0
  225    CONTINUE
```

```
  230 CONTINUE
C
C      DETERMINE MAXIMUM NUMBER OF BOUNCES
       NBMAX = 0
       DO 235 J = 1,M
          IF(NB(J) .GT. NBMAX) NBMAX = NB(J)
  235 CONTINUE
C
       RETURN
       END
```

```
                                           IDENT     REFLEC
PROGRAM LENGTH                    01157
ENTRY POINTS       REFLEC         00207
BLOCK NAMES
                   CONST          00003
                   INPUT          01243
                   PRELIM         06253
                   FIELD          00462
EXTERNAL SYMBOLS
                   Q1Q02330
                   Q1Q04310
                   Q1Q02310
                   Q1Q03330
                   Q8QDICT.
                   CVMAG
                   SCAT
                   SQRTF
   00257 SYMBOLS
```

```
      SUBROUTINE SCAT(I,J,FK,F,RP,G)
C
C     SUBROUTINE SCAT CALCULATES THE VALUE OF THE SCATTERED FIELD (FIELD
C        F SCATTERED FROM PLATE J) AT POINT R
C
C     PARAMETERS
C        I  = SOURCE PLATE NUMBER
C        J  = REFLECTOR PLATE NUMBER
C        FK = WAVE NUMBER
C        F  = REFLECTED FIELD AT CENTROID OF PLATE J WHOSE SOURCE IS
C             THE CENTROID OF PLATE I (COMPLEX)
C        RP = POINT AT WHICH SCATTERED FIELD IS TO BE EVALUATED
C        G  = FIELD AT POINT R ARISING FROM THE SCATTER OF FIELD F
C             FROM PL.TE J
C
      COMMON/CONST/PI,TWOPI,CM
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1             NV(50),V(3, 4,50)
      COMMON/PRELIM/M,WAVENM,VC(3,50),VNORM(3,50),VA(50),VASUM,XLAM(50),
     1             XK(50),R(3,31,30)
      DIMENSION VIJ(3), W(3), WN(3)
      DIMENSION F(3), FP(3), RP(3), G(3), RV(3), DELV(3)
      COMPLEX F, FP, G, CF, XFACT, T, TN
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM) AND
C        UP TO 4 (INPUT) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
      EPS = 0.001
C
C
C
C     CALCULATE FIELD WITH PHASE SHIFT
C     CHECK FOR TRANSMITTER  (I = 0)
      IF( I .EQ. 0) GO TO 3
      DO 2 K = 1,3
         VIJ(K) = VC(K,J) - VC(K,I)
    2 CONTINUE
      GO TO 5
    3 DO 4 K = 1,3
         VIJ(K) = VC(K,J) - RT(K)
    4 CONTINUE
    5 VIJM = SQRT( VIJ(1)*VIJ(1) + VIJ(2)*VIJ(2) + VIJ(3)*VIJ(3) )
      ARG = FK*VIJM
      CF = CMPLX(COS(ARG)/VIJM,SIN(ARG)/VIJM)
C
      DO 6 K = 1,3
         FP(K) = CF*F(K)
    6 CONTINUE
C
C     CHECK IF FIELD SOURCE AND FIELD RECEIVER RP ARE ON SAME SIDE OF
C        PLATE J
      DO 7 K = 1,3
         RV(K) = RP(K) - VC(K,J)
    7 CONTINUE
```

```
      RVM = SQRT(RV(1)*RV(1) + RV(2)*RV(2) + RV(3)*RV(3))
      DOT = 0.
C
      DRVN = 0.
      DO 10 K = 1,3
         DRVN = DRVN + RV(K)*VNORM(K,J)
   10 CONTINUE
      IF(DRVN .GT. 0.0) GO TO 25
      DO 20 K = 1,3
         G(K) = (0.,0.)
   20 CONTINUE
      GO TO 95
C
C     NORMALIZE VIJ AND RV VECTORS
C     INITIATE W VECTOR
   25 DO 27 K = 1,3
         VIJ(K) = VIJ(K)/VIJM
         RV(K) = RV(K)/RVM
         W(K) = VIJ(K) - RV(K)
   27 CONTINUE
C
C
      DOT = 0.
      DO 28 K = 1,3
         DOT = DOT + W(K)*VNORM(K,J)
   28 CONTINUE
C
      WM2 = 0.
      DO 30 K = 1,3
         W(K) = W(K) - DOT*VNORM(K,J)
         WM2 = WM2 + W(K)*W(K)
   30 CONTINUE
C
C     CHECK FOR SPECULAR REFLECTION
C
      IF (WM2 .GT. 0.0) GO TO 45
C
C     SPECULAR REFLECTION IN DIRECTION OF RP
      WS = (FK*DRVN*VA(J))/TWOPI
      ARG = FK*RVM
      CF = CMPLX(WS*SIN(ARG),-WS*COS(ARG))
C
      DO 40 K = 1,3
         G(K) = CF*FP(K)
   40 CONTINUE
      GO TO 95
C
C     BEGIN EQ. 4.6
   45 DOT = 0.
      DO 50 K = 1,3
         DOT = DOT + (RV(K) - VIJ(K))*VNORM(K,J)
   50 CONTINUE
C
      WS = DOT/(WM2*2.0*TWOPI*RVM)
      ARG = FK*RVM
      XFACT = CMPLX(WS*COS(ARG),WS*SIN(ARG))
```

```
C
      T = (0.,0.)
      NN = NV(J)
      DO 85 N = 1,NN
         N1 = N + 1
         IF(N1 .GT. NN) N1 = 1
         DO 55 K = 1,3
            DELV(K) = V(K,N1,J) - V(K,N,J)
   55    CONTINUE
C
         DOT = 0.
         DO 60 K = 1,3
            DOT = DOT + W(K)*DELV(K)
   60    CONTINUE
      IF(DOT .NE. 0.0) GO TO 65
         S = 1.0
         GO TO 70
C
   65    WS = FK*DOT
         S = SIN(WS)/WS
C
C        WN = (W)X(VNORM)
   70    WN(1) = W(2)*VNORM(3,J) - W(3)*VNORM(2,J)
         WN(2) = W(3)*VNORM(1,J) - W(1)*VNORM(3,J)
         WN(3) = W(1)*VNORM(2,J) - W(2)*VNORM(1,J)
C
         DOT = 0.
         DO 75 K = 1,3
            DOT = DOT + WN(K)*DELV(K)
   75    CONTINUE
         WS = S*DOT
C
C
         DOT = 0.
         DO 80 K = 1,3
            DOT = DOT + W(K)*(V(K,N,J) + V(K,N1,J))
   80    CONTINUE
C
         ARG = FK*DOT/2.
C
         TN = CMPLX(WS*COS(ARG),WS*SIN(ARG))
         T = T + TN
C
   85 CONTINUE
C
C        SCATTERED FIELD
      DO 90 K = 1,3
         G(K) = T*XFACT*FP(K)
   90 CONTINUE
   95 RETURN
C
      END
```

```
                                        IDENT     SCAT
PROGRAM LENGTH                00765
ENTRY POINTS        SCAT      00030
BLOCK NAMES
                    CONST     00003
                    INPUT     01243
                    PRELIM    06253
EXTERNAL SYMBOLS
                    Q1Q04330
                    Q1Q02330
                    Q8QDICT.
                    SQRTF
                    SINF
                    COSF
   00232 SYMBOLS
```

UNCLASSIFIED

```
      FUNCTION CVMAG(A)
C
C     FUNCTION CVMAG COMPUTES THE MAGNITUDE OF THE COMPLEX
C        THREE-DIMENSIONAL VECTOR A
C
      COMPLEX A,SUM
      DIMENSION A(3)
C
      SUM = 0.
      DO 10 I = 1,3
         SUM = SUM + A(I)*CONJG(A(I))
   10 CONTINUE
      CVMAG = SQRT(SUM)
      RETURN
      END
```

```
                                        IDENT    CVMAG
PROGRAM LENGTH                   00111
ENTRY POINTS      CVMAG          00003
EXTERNAL SYMBOLS
                  Q1Q04330
                  Q1Q02330
                  Q8QDICT.
                  SQRTF
   00031 SYMBOLS
```

```
      SUBROUTINE PRNT(RP,PRTL,KEY)
C
C     SUBROUTINE PRNT PRINTS THE DIRECT AND SCATTERED FIELD SPECTRUM
C        EVALUATED AT POINT RP
C
      COMMON/CONST/PI,TWOPI,CM
      COMMON/INPUT/ITITLE(10),XLMDA,VV(3),ET(3),RT(3),D(3),NBOUND,MP,
     1              NV(50),V(3, 4,50)
      COMMON/PRELIM/M,WAVENM,VC(3,50),VNORM(3,50),VA(50),VASUM,XLAM(50),
     1               XK(50),R(3,31,30)
      COMMON/FIELD/ED(3),ES(3,50)
      COMPLEX ED,ES
      INTEGER WT
      DIMENSION RR(1), PRTL(1)
      COMMON/FLAG/IFLAG
C
C     CURRENT DIMENSIONS ALLOW UP TO 30 REFLECTORS (PRELIM, PROJ) AND
C        UP TO 4 (INPUT, PROJ) VERTICES PER REFLECTOR
C        INPUT IS DIMENSIONED FOR 50 REFLECTORS
C
      WT = 61
C     CHECK FOR INITIAL ENTRY
      IF(KEY .GE. 0) GO TO 10
C     WRITE HEADING
      GO TO (5,6), IFLAG
    5 EDP = CABS(ED(1))**2
      GO TO 7
    6 CONTINUE
C     Y COMPONENT POWER
      EDP = CABS(ED(2))**2
    7 CONTINUE
      XFREQ = WAVENM*CM/TWOPI
      WRITE (WT,110) (RP(K) ,K=1,3), XFREQ, EDP
  110 FORMAT(//8X,11HFIELD POINT,5X,4HX = ,E11.3,3X,4HY = ,E11.3,3X,
     1         4HZ = ,E11.3////21X,12HDIRECT FIELD/13X,10HFREQUENCY ,10X,
     2         5HPOWER// 8X,E15.7,8X,E10.2 //19X,15HSCATTERED FIELD/13X,
     3         10HFREQUENCY ,10X,5HPOWER,15X,3HN/S,15X,7HLOG N/S//)
      LINECT = 5
      RETURN
C
   10 CONTINUE
      GO TO (15,20), IFLAG
   15 DO 18 I = 1,M
         IF(PRTL(I) .EQ. -1.) GO TO 18
         ESP = CABS(ES(1,I))**2
         RSD = ESP/EDP
         RSDL = ALOG10(RSD)
      WRITE (WT,120) PRTL(I), ESP, RSD, RSDL
      LINECT = LINECT + 1
      IF(LINECT .LT. 60) GO TO 18
      WRITE(WT,130)
      LINECT = 1
   18 CONTINUE
      RETURN
   20 DO 30 I = 1,M
```

```
      IF(PRTL(I) .EQ. -1.) GO TO 30
      ESP = CABS(ES(2,I))**2
      RSD = ESP/EDP
      RSDL = ALOG10(RSD)
   WRITE (WT,120) PRTL(I), ESP, RSD, RSDL
120 FORMAT(8X,E15.7,7X,F10.2,9X,E10.3,10X,E10.3/)
   LINECT = LINECT + 1
   IF(LINECT .LT. 60) GO TO 30
   WRITE(WT,130)
   LINECT = 1
130 FORMAT(1H1/)
 30 CONTINUE
C
C
   RETURN
   END
```

```
                                          IDENT      PRNT
    PROGRAM LENGTH                  00451
    ENTRY POINTS     PRNT           00114
    BLOCK NAMES
                     CONST          00003
                     INPUT          01243
                     PRELIM         06253
                     FIELD          00462
                     FLAG           00001
    EXTERNAL SYMBOLS
                     THEND.
                     Q8QDICT.
                     ALOG10
                     CABS
                     STH.
                     QNSINGL.
         00145 SYMBOLS
LOAD
RUN,2,3500
```