NRL Report 7740

The USRD Type F39A 1-kHz Underwater Helmholtz Resonator

T. A. HENRIQUEZ

Standards Branch
Underwater Sound Reference Division

10 April 1974

NAVAL RESEARCH LABORATORY
Underwater Sound Reference Division
P.O. Box 8337, Orlando, Fla. 32806

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER NRL Report 7740	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE USRD TYPE F39A 1-kHz UNDERWATER HELMHOLTZ RESONATOR		5. TYPE OF REPORT & PERIOD COVERED Final report on one part of the problem. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) T. A. Henriquez		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Underwater Sound Reference Division P. O. Box 8337, Orlando, Fla. 32806		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem S02-31 RF 11-121-403--4472
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Office of Naval Research Arlington, Va. 22217		12. REPORT DATE 10 April 1974 13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Electroacoustic transducers
Underwater sound source
Helmholtz resonator
Ferroelectric ceramics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A reliable, small, lightweight sound source capable of producing a sound pressure level of 170 dB re 1 μPa at 1 kHz using the principle of a Helmholtz resonator has been designed and built to satisfy a requirement of the Naval Ship Research and Development Center for a transducer to be used at 300-m depth in the ocean.

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THE USRD TYPE F39A 1-kHz UNDERWATER HELMHOLTZ RESONATOR

Introduction

The Naval Ship Research and Development Center needed a lightweight 1-kHz transducer for use in the ocean at depths of about 300 m. The peak acoustic pressure level required was at least 170 dB re 1 μPa at 1 m. Aside from this pressure level, the most critical requirement concerned total weight: The transducer was to be attached to a vertical hydrophone array to be dispersed from the deck of a ship and moored from free-floating buoys; therefore, it had to be light enough to be easily handled aboard ship.

No commercially available transducer would satisfy these requirements. Because of its experience in designing and building special transducers for unusual requirements, the Underwater Sound Reference Division was requested to undertake development of a new transducer. Preliminary analysis showed that a mechanically resonant transducer would be either too bulky and heavy or would have too high a Q, but previous experience with acoustically resonant transducers [1] seemed to warrant their consideration in this application. Equivalent circuit analysis showed that a design to meet all requirements was feasible with easily obtainable materials.

Six transducers designated USRD type F39A were built and tested. Users report that the transducers perform excellently in the field.

Design

Use of the Helmholtz resonator principle to increase the response of an underwater sound transducer has been described in an earlier report [1]. The resonator of the F39A consists of a thin-walled hollow piezoelectric sphere flooded with castor oil. For protection, the spherical piezoelectric ceramic element is enclosed in a cylindrical expanded stainless steel guard and suspended there by an equatorially mounted natural rubber band. Acoustic coupling between the sphere and the acoustic window is provided by electrical grade castor oil. Figure 1 shows the unassembled transducer. Figure 2 is a diagram of the basic component.

The one-piece spheres were provided by the manufacturer with a 1.9-cm-diam hole in the wall, which allows easy access for soldering the inner electrical lead. A Plexiglass plate with an orifice of the correct size

Fig. 1. Unassembled transducer USRD type F39A.

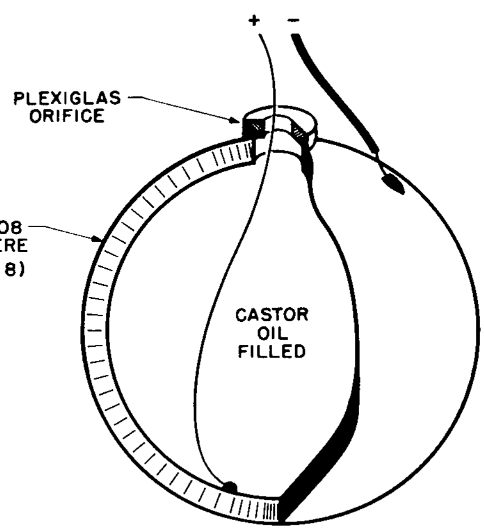

Fig. 2. Diagram of basic spherical ceramic element used in USRD type F39A transducer. Approximate dimensions: outside diameter, 4 in. (10.2×10^{-2} m); wall thickness, 0.250 in. (0.635×10^{-2} m).

is epoxied to the sphere to cover the larger hole. Use of the compliant Plexiglass reduces the chance that the orifice plate will shear from the ceramic because of thermal expansion or high drive conditions. Castor oil was used as the acoustic coupling fluid because its acoustic properties are well known for a large range of temperatures and pressures [2].

For protection against the salt water environment, all exposed metal parts are constructed of 316 stainless steel. Although 316 stainless is subject to pitting in sea water, the metal that is exposed to the water is thick enough to preclude any failure from this cause for some time. A special butyl rubber compound of low water permeability is used as the acoustic window material.

Analysis

The analysis of the transducer will be developed in three parts: (1) the equivalent circuit of the unloaded sphere, (2) the response of the water-loaded air-filled sphere, and (3) the oil-filled Helmholtz sphere in water. The analysis is in the form of equivalent circuits based on the voltage-force analogy, which were solved by means of a time-sharing library program. Physical parameters used in the analysis are given in Table 1; piezoelectric parameters are given in Table 2.

Table 1. Physical parameters for the piezoelectric sphere.

Total mass	1.317 kg
Outside diameter (average)	10.130×10^{-2} m
Internal volume	370.7 cm^3
Inner radius (average)	4.456×10^{-2} m
Wall thickness (average)	0.609×10^{-2} m
Mean radius (a_m)	4.760×10^{-2} m
Density of material	7.66×10^3 kg/m^3

Part I. The Acoustically Unloaded Sphere

The lumped-constant equivalent circuit for a thin-walled ceramic sphere vibrating radially is given in reference [3] and shown in Fig. 3. The circuit elements are obtained from the relation $N = 4\pi a_m d_{31}/s_C^E$ for the electromechanical transformer ratio, where a_m is the mean radius of the sphere, d_{31} is the piezoelectric charge constant, $s_C^E = \frac{1}{2}(s_{11}^E + s_{12}^E)$ is the mechanical compliance constant at constant electrical field for the radial vibration mode, and s_{11}^{E} and s_{12}^{E} are compliance constants whose values are given in Table 2.

Table 2. Physical constants for ceramic material Type III, MIL-STD-1376(SHIPS).

Free relative dielectric constant K_{33}^{T}	1000
Loss tangent	0.003
Coupling factor k_p	0.60
Piezoelectric charge constant d_{31}	-80×10^{-12} C/N
Mechanical quality factor Q_M	1200
Piezoelectric voltage constant g_{33}	22×10^{-3} V·m/N
Piezoelectric voltage constant g_{31}	-9.0×10^{-3} V·m/N
Compliance constant s_{11}^{E}	11.1×10^{-12} m^2/N
Compliance constant s_{12}^{E}	-3.7×10^{-12} m^2/N
Compliance constant s_{C}^{E}	3.7×10^{-12} m^2/N

The clamped electrical capacitance is given by $C_0 = 4\pi a_m \varepsilon_{33}^{T}(1 - k_p^{2})/t$, where $\varepsilon_{33}^{T} = K_{33}^{T}\varepsilon_0$, $\varepsilon_0 = 8.85\times10^{-12}$ F/m is the permeability of free space, K_{33}^{T} is the free relative dielectric constant given in Table 2, t is the wall thickness of the sphere, and k_p is the planar coupling factor.

The mechanical compliance is given by $C_M = s_C^{E}/4\pi t$. The equivalent mechanical mass is $m_M = 4\pi a_m^{2} t\rho$, where ρ is the density of the ceramic material. Mechanical losses in the material are represented by $R_M = (\rho s_C^{E})^{\frac{1}{2}} a_m/C_M Q_M$, where Q_M is the mechanical quality factor in Table 2. The values computed for the circuit elements are given in Table 3.

Removing the transformer in the circuit of Fig. 3 results in the circuit shown in Fig. 4.

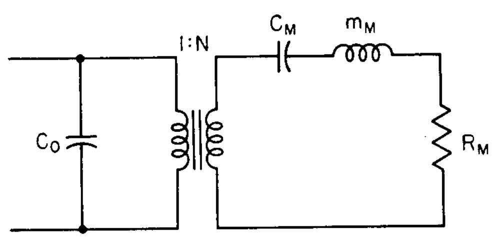

Fig. 3. Lumped-constant equivalent circuit for a thin-walled ceramic sphere vibrating radially without load.

Table 3. Computed values of circuit elements of Fig. 3.

C_0	0.0384×10^{-6} F
N	16.5
C_M	0.4835×10^{-10} m/N
m_M	1.33 kg
R_M	138 N•s/m

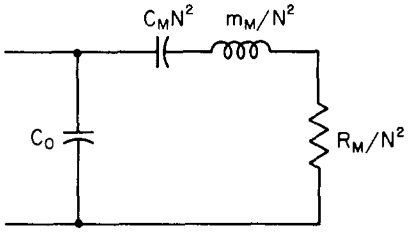

Fig. 4. Lumped-constant equivalent circuit for a thin-walled ceramic sphere vibrating radially without load (result of removing transformer from Fig. 3).

Part II. Equivalent Circuit of the Sphere Radiating into an Unbounded Medium

For a spherical source executing radial vibration and radiating into an unbounded medium, the radiation impedance is

$$Z_R = \rho_0 c_0 S_o \left[\frac{(ka_o)^2}{1 + (ka_o)^2} + j \frac{(ka_o)}{1 + (ka_o)^2} \right], \tag{1}$$

where c_0 is the speed of sound in water, $S_o = 4\pi a_o^{\ 2}$, a_o is the outer radius of the sphere, ρ_0 is the density of water, $k = 2\pi f/c_0$, and f is frequency. This impedance can be added to the equivalent circuit of the free sphere as a parallel resistance and inductance. The resistance part is $R_R = S_o \rho_0 c_0$ and the equivalent inductance is $M_R = S_o \rho_0 a_o$.

The equivalent circuit with the added radiation load is shown in Fig. 5 with a constant current generator and with the nodes numbered to

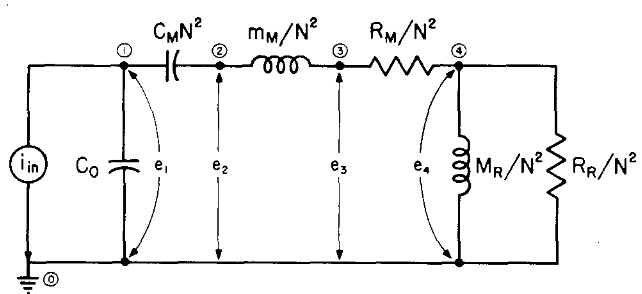

Fig. 5. Equivalent circuit for thin-walled ceramic sphere vibrating radially with radiation load and constant current generator. Nodes numbered to correspond to time-sharing computer program.

conform to the time-sharing computer program used to evaluate the circuit. The program gives the output voltage at any node for a given input voltage or current. If an input current of 1 A is specified, the output voltage at node 1 is equivalent to the input impedance. The acoustic pressure generated at 1 m can be determined from the power dissipated in the radiation resistance:

$$p = (1/r)(P_a \rho_0 c_0 R_\theta / 4\pi)^{\frac{1}{2}}, \tag{2}$$

where p is the acoustic pressure at distance r from the center of the sphere, P_a is the acoustic power output, and R_θ is the directivity factor, which is unity for an omnidirectional source. The acoustic power is obtained from

$$P_a = e_4{}^2 N^2 / R_R, \tag{3}$$

where e_4 is the voltage at node 4. If the distance r is set at 1 m, substituting Eq. (3) into Eq. (2) results in the transmitting current response

$$S = e_4 N (\rho_0 c_0 R_\theta / 4\pi R_R)^{\frac{1}{2}}. \tag{4}$$

The MKS units (pascals per ampere) of Eq. (4) can be converted to standard acoustical units (micropascals per ampere) by multiplying by the factor 10^6.

The transmitting current response for the circuit is plotted in Fig. 6. The response shown has been modified by the factor $D_f = [1 + (ka_o)^2]^{\frac{1}{2}}$ to account for the effects of diffraction [4].

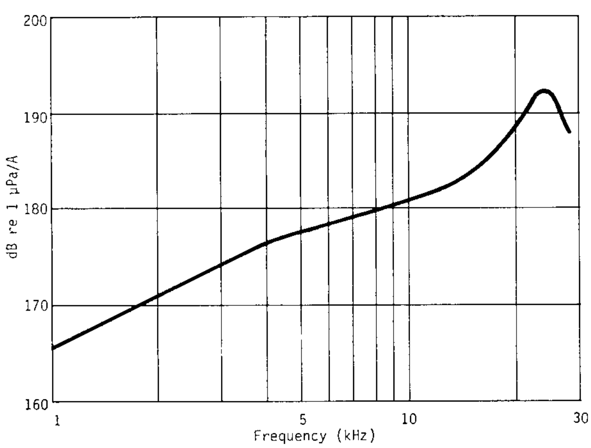

Fig. 6. Transmitting current response of circuit shown in Fig. 5.

Part III. Oil-Filled Helmholtz Sphere in Water

A slightly different approach will be used to analyze the response of the oil-filled sphere in water. Previously, the transmitting response was obtained from the equivalent circuit of the hollow sphere radiating into the water. Now we have an oil-filled sphere from which sound is being radiated from the radially vibrating shell and also from the orifice in which the plug of fluid acts as a small piston. These two motions are 180 degrees out of phase, which fact complicates the analysis for obtaining the transmitting response by means of the time-sharing computer program. Because the transducer is reciprocal, we can modify the circuit to obtain a free-field voltage sensitivity by inserting voltage generators for the sound pressure incident on the shell and the orifice, one the negative of the other, to give the desired phase relationship. The output voltage then can be converted by means of the free-field reciprocity parameter to obtain the desired transmitting current response [5].

Determination of the correct orifice size to give the desired resonance at 1000 Hz is detailed in Appendix A.

The lumped elements that represent the oil-filled sphere are shown in Fig. 7. The transformer ratio S_i:1 is due to the internal area S_i of the sphere acting on the contained fluid.

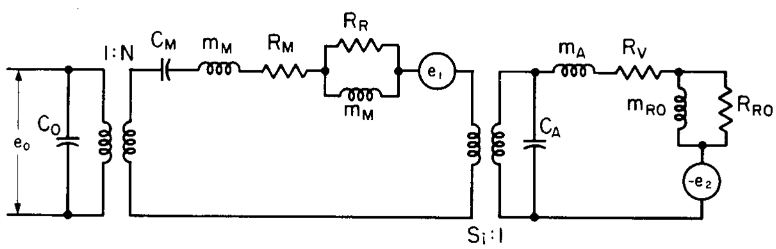

Fig. 7. Equivalent circuit of oil-filled sphere in water.

The acoustic elements are obtained from the following:

1. Acoustic compliance of the contained fluid:

$$C_A = V_i/\rho c^2,$$

where V_i is the volume of enclosed fluid, ρ is the density of the fluid, and c is the speed of sound in the fluid.

2. Acoustic mass of the fluid in the orifice:

$$m_A = \rho\ell/S_H,$$

where ℓ is the length of the orifice and S_H is the cross-sectional area of the orifice.

3. Viscous loss in the fluid in the orifice [6]:

$$R_V = 4R_s(1 + 0.5\ell/a_H),$$

where $R_s = \frac{1}{2}(2\eta\rho\omega)^{\frac{1}{2}}$, $\omega = 2\pi f$ is the angular frequency, and η is the dynamic viscosity of the fluid. The value of R_V is computed at 1000 Hz and assumed to be constant.

4. Total radiation mass of the fluid in the orifice acting as a small piston:

$$m_{RO} = 1.46\rho/S_H.$$

5. Total radiation resistance of the fluid in the orifice [7]:

$$R_{RO} = 0.459\rho_0 c_0/a_H^2,$$

where a_H is the radius of the orifice.

The computed values of the circuit elements are given in Table 4.

Table 4. Computed values of circuit elements of Fig. 5.

Symbol	Acoustical	Electrical
C_A	1.645×10^{-13} m^3/Pa	0.0718×10^{-6} F
m_A	70.6×10^3 kg/m^4	0.161 H
R_V	126×10^6 kg/m^{4}s	288 Ω
R_{RO}	2.47×10^{10} kg/m^{4}s	0.0565×10^{-6} Ω
m_{RO}	85.0×10^3 kg/m^4	0.194 H

The force on the sphere is represented by the voltage generator e_1 and the equivalent voltage is the area of the sphere times the sound pressure 1 μPa. The force at the orifice is the same sound pressure times the area of the orifice; it is represented by the voltage generator $-e_2$. The resulting output from the equivalent circuit is shown in Fig. 8 as a free-field open-circuit voltage sensitivity; transmitting current response

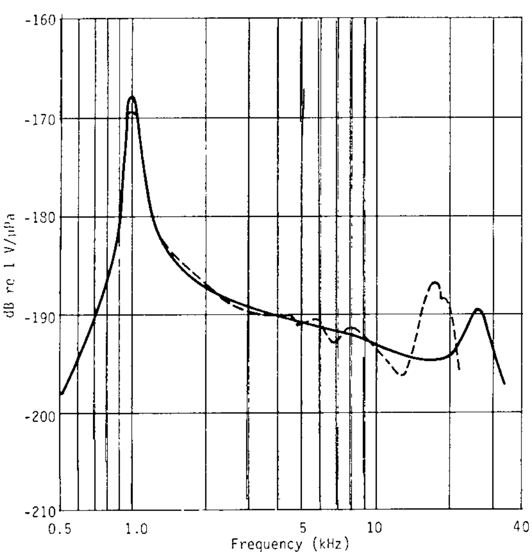

Fig. 8. Free-field voltage sensitivity of USRD type F39A transducer. Solid line: computed from circuit shown in Fig. 7. Dashed line: measured.

as obtained by applying the reciprocity parameter is shown in Fig. 9. The curves are shown together with measured responses. Irregularities in the measured response in the frequency range between 4 kHz and 10 kHz are due to normal modes in the fluid in the sphere.

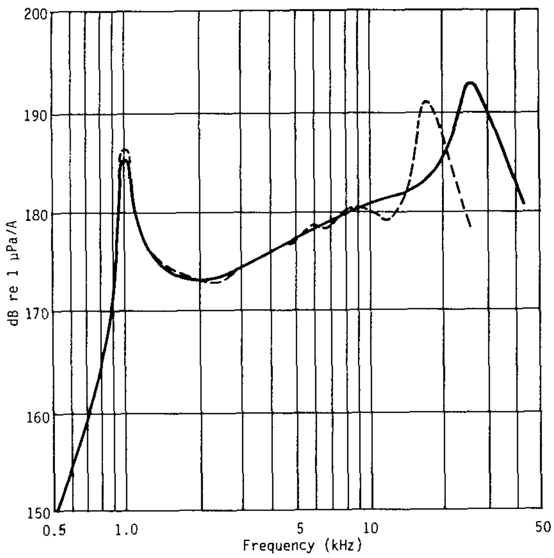

Fig. 9. Transmitting current response of USRD type F39A transducer. Solid line: computed from circuit shown in Fig. 7. Dashed line: measured.

Conclusion

The USRD type F39A transducer is a completely pressure-compensated transducer that provides as much as 10 dB gain at the Helmholtz resonance. Simple construction insures the ruggedness and dependability necessary for at-sea missions. The choice of materials insures long life in the open-ocean environment.

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Appendix A

Calculation of Orifice Size for 1 kHz

The undamped resonance frequency for the Helmholtz resonator is given [3] by

$$\omega_0^2 = 1/m_A C_A, \tag{A1}$$

where m_A is the acoustic mass of the fluid in the orifice and C_A is the acoustic compliance of the fluid in the sphere.

The acoustic mass is made up of the plug of fluid vibrating in the orifice and the end effect on either side of the orifice. The total acoustic mass is

$$m_A = (\rho/\pi a_H^2)(\ell + 1.46 a_H). \tag{A2}$$

Substituting Eq. (A2) into (A1) produces

$$a_H^2 - 1.46(\omega_0^2/\pi)\rho C_A a_H - \omega_0^2 \rho(\ell/\pi) C_A = 0. \tag{A3}$$

For a 1000-Hz resonance, $\omega_0 = 2\pi \times 10^3$. Solving Eq. (A3) for a_H, using $\ell = 6.35 \times 10^{-3}$ m, gives $a_H = 5.24 \times 10^{-3}$ m = 0.206 in.