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A Two-Pole Filter for Use With a Scanning Radar

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A two-pole filter was proposed as a detector for a scanning radar. The optimum values of the filter coefficients were found and are approximated by a simple expression. The optimum two-pole filter requires a 0.15-dB increase in signal-to-noise ratio in order to provide the same detection capability as the optimum detector and yields azimuth estimates whose standard deviation is within 15% of the Cramér-Rao lower bound. The estimator is simple to implement since it avoids the storage requirements of the moving window detector and the bias complications of the feedback integrator.

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CONTENTS

A TWO-POLE FILTER FOR USE WITH A SCANNING RADAR

INTRODUCTION

The data processor for a scanning radar usually performs two functions. The first is target detection and the second is estimation of angular position. The detection problem has been studied by Marcum and Swerling [1], Blake [2-4], Hall [5], Cooper and Griffiths [6], Palmer and Cooper [7], Hansen [8], and Trunk [9]. The detection results are summarized as follows:

1. The optimum weighting* is directly proportional to the 4th power of the antenna beam pattern.

2. Uniform weighting with the optimal number of pulses is 0.5 dB less efficient than the optimum weighting.

3. The feedback integrator (a single delay-loop integrator) is about 1.0 dB less efficient than optimum weighting.

4. The double delay-loop integrator (a two-pole filter, which has a multiple pole) is about 0.3 dB less efficient than optimum weighting.

Since the double delay-loop integrator is nearly optimum and requires little storage, further detection studies are not required, either to improve the signal-to-noise (S/N) performance or to simplify the system.

The problem of estimating angular position has not been studied as thoroughly as the detection problem. Swerling [10] has calculated the maximum angular accuracy (using the Cramér-Rao lower bound), Hansen [8] calculated the performance of the moving window (uniform weighting of the pulses), and Trunk [11] calculated the performance of the feedback integrator. Using either the moving window or the feedback integrator, one can obtain angular estimates whose standard deviations (SD) are 15% greater than the optimum calculated by Swerling. The difficulty with estimating angular position using the moving window is that all the pulses must be saved. The difficulty with the feedback integrator, using a threshold-crossing estimation technique, is that the expected value of the angular position is a function of the S/N which must be estimated. If the maximum of the feedback integrator is used as an estimator, there is no changing bias. However, the SD of the estimates are 100% greater than the lower bound.

Note: Manuscript submitted January 14, 1974.

*The optimum weighting is one which maximizes the output signal-to-noise ratio. This definition is used because it reduces the complexity of the necessary calculations and has been shown [9] in the large sample case to be equivalent to maximizing the probability of detection.

Although the angular accuracy of the double delay-loop integrator was not calculated in Ref. 6, it would be suspected that the estimator using the maximum value, while not having a changing bias, would yield poor angular estimates. This is because its weighting function is of the form te^{-t} and its tail is higher than the feedback integrator's which is e^{-t}. To test this conjecture about the performance of the double delay-loop integrator, a Monte Carlo simulation (described in a later part of this report) was run. The results are shown in Fig. 1. The SD of the estimates obtained are 50 to 100% greater than the Cramér-Rao lower bound.

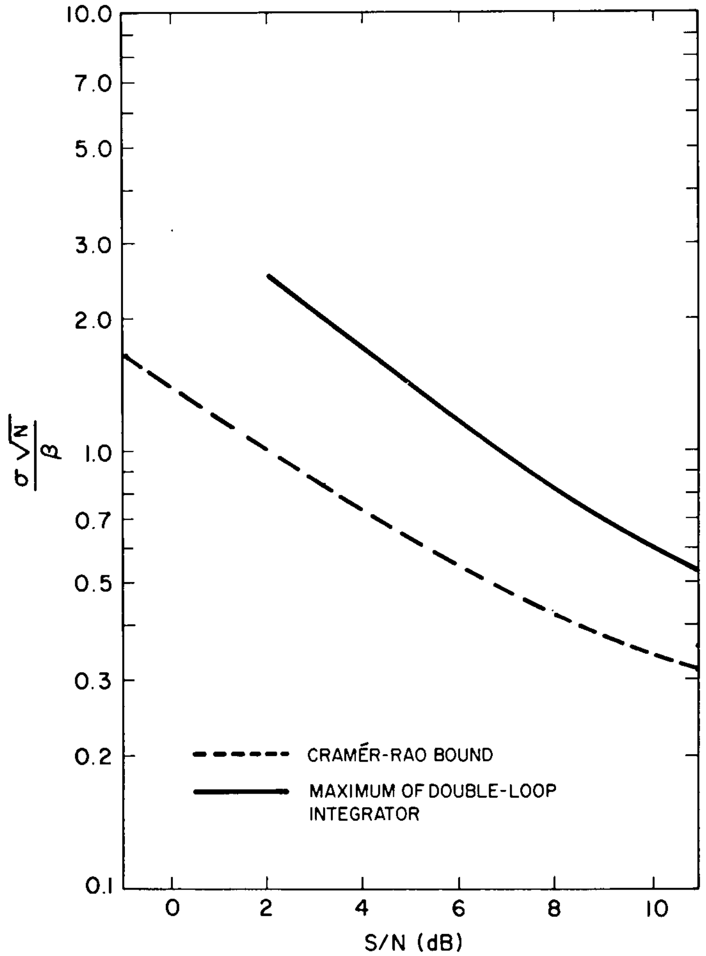

Fig. 1—Comparison of angular estimate with the Cramér-Rao lower bound. σ is the standard deviation of the estimation error and N is the number of pulses within the 3-dB beamwidth, which is 2β.

In order to find a detector which is simple and yields both good detection and estimation results, a two-pole integrator was investigated. This integrator differs from the double delay-loop integrator by *not* requiring the two poles of the filter to be at the same location.

TWO-POLE FILTER

The proposed two-pole filter is shown in Fig. 2. The state equations are

$$\mathbf{Y}(i) = \Theta \mathbf{Y}(i\text{-}1) + \Gamma x(i), \tag{1}$$

where

$$\Theta = \begin{bmatrix} 0 & -k_2 \\ 1 & k_1 \end{bmatrix}$$

$$\mathbf{Y}(i) = \begin{bmatrix} y_1(i) \\ y_2(i) \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

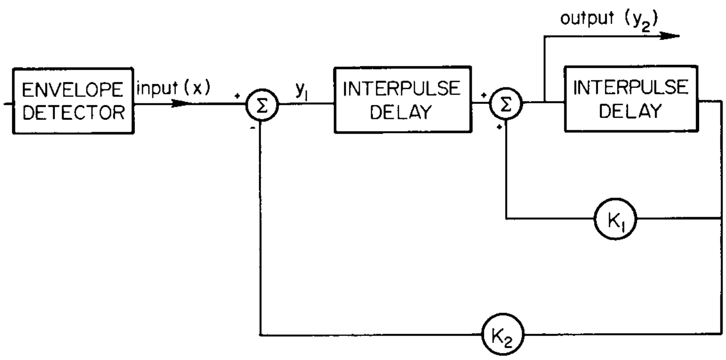

Fig. 2—Two-pole filter

Two inputs, which are used later in this report, are considered. The first is white Gaussian noise whose variance is σ^2. Consequently, the output of the envelope detector is Rayleigh distributed and its variance $Q(i)$ is $(2 - \pi/2)\sigma^2$, independent of i. The other input is due to the scanning of the beam over a point target and is proportional to the fourth power of the antenna (voltage) gain pattern for small signals; i.e.,

$$G^4(i) = \begin{cases} \dfrac{\sin^4(i\alpha\Delta\theta - \pi)}{(i\alpha\Delta\theta - \pi)^4} & 0 \leqslant i \leqslant \dfrac{2\pi}{\alpha\Delta\theta} \\ 0 & \text{else} \end{cases}$$

where

$a = 1.3916/\beta$
2β = 3-dB beamwidth,
$\Delta\theta$ = angular increment of the scanning radar and equals $2\pi(\tau/T)$,

T being the scan time of the radar and τ being the sampling time which is the reciprocal of the pulse repetition rate. The number of pulses N in the 3-dB beamwidth is $2\beta/\Delta\theta$.

The output of the filter when excited with noise only is calculated first. The covariances of the state variables are [12]*

$$\mathbf{P}(i) = \Theta\mathbf{P}(i - 1)\Theta^T + \Gamma Q(i)\Gamma^T \tag{3}$$

where

$$\mathbf{P}(i) = \begin{bmatrix} p_{11}(i) & p_{12}(i) \\ p_{21}(i) & p_{22}(i) \end{bmatrix}$$

$$p_{\ell j}(i) = E(y_\ell - \overline{y}_\ell)(y_j - \overline{y}_j).$$

Substituting Eq. (1) into Eq. (3) and setting $p_{\ell j}(i) = p_{\ell j}(i - 1)$, one obtains the steady state solution for the covariances. The output variance of the filter is

$$\sigma_Q^2 \triangleq p_{22}(i) = \frac{\left(2 - \dfrac{\pi}{2}\right)\sigma^2}{\left[1 - k_1^2 - k_2^2 + \dfrac{2k_1^2 k_2}{(1 + k_2)}\right]}, \tag{4}$$

independent of i.

The filter output at time i can be written as

$$y_2(i) = \sum_{j=o}^{\infty} h(j)x(i - j), \tag{5}$$

where $h(j)$ is the impulse response of the system.

The impulse response of the filter is now obtained. The filter's transfer function in terms of the z-transform is

$$H(z) = \frac{y_2(z)}{x(z)} = \frac{z^{-1}}{(1 - k_1 z^{-1} + k_2 z^{-2})}. \tag{6}$$

If the roots are complex, Eq. (6) can be written as

$$H(z) = \frac{z^{-1}}{1 - 2e^{-\xi\omega_0\tau}\cos\omega_d\tau z^{-1} + e^{-2\xi\omega_0\tau}z^{-2}}, \tag{7}$$

*Although the input noise source is assumed to be white Gaussian in obtaining Eq. (3) in the reference cited, the Gaussian assumption is not required.

where

$$\xi = \frac{\ell n\left(\frac{1}{\sqrt{k_2}}\right)}{\left\{\left[\ell n\left(\frac{1}{\sqrt{k_2}}\right)\right]^2 + \left[\cos^{-1}\frac{k_1}{2\sqrt{k_2}}\right]^2\right\}^{1/2}} \tag{8}$$

$$\omega_d \tau = \cos^{-1}\frac{k_1}{2\sqrt{k_2}} \tag{9}$$

$$\omega_0 = \frac{\omega_d}{(1 - \xi^2)^{1/2}}. \tag{10}$$

The impulse response of the filter is then given as

$$h(i) = Be^{-\xi\omega_0\tau i} \sin \omega_d i\tau \tag{11}$$

where

$$B = \frac{e^{\xi\omega_0\tau}}{\sin \omega_d \tau}. \tag{12}$$

The response of the system to the $G^4(i)$ waveform is obtained by convolving Eq. (2) with Eq. (11).

PARAMETER OPTIMIZATION

The optimum filter coefficients are found by using the Hooke and Jeeves (13) direct search technique to maximize the output $(S/N)_0$ defined by

$$\left(\frac{S}{N}\right)_0 = \frac{\left[\sum_{j=o}^{\infty} h(j)x(i^* - j)\right]^2}{\sigma_Q^{\,2}}$$

where i^* is the sample at which the filter obtains its maximum value and $x(i) = G^4(i)$. An optimum weighting function is shown in Fig. 3 and the optimum k_1 and k_2 and resulting $(S/N)_0$ for various N's are in Table 1. The S/N denoted by $(S/N)_{opt}$ in Table 1 refers to the S/N obtained by letting $h(j) = G^4(j)$. Comparing $(S/N)_0$ and $(S/N)_{opt}$, the optimum two-pole filter requires a 0.15-dB increase in S/N in order to provide the same detection capability as the optimum detector; i.e., that detector which has $h(j) = G^4(j)$.

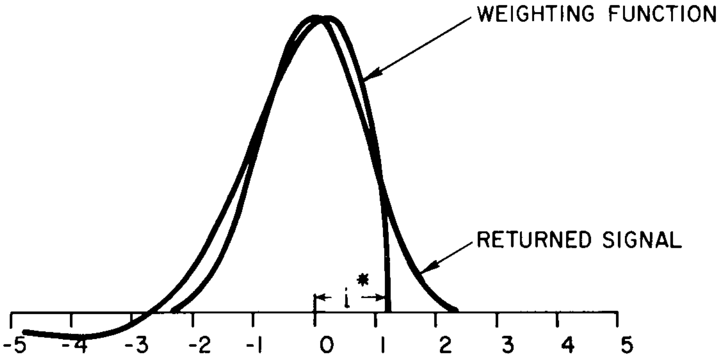

Fig. 3—Optimum weighting function of the two-pole filter

Table 1.
Optimum Filter Coefficients as a Function of the Number of Pulses on Target N.

N	k_1	k_2	$(S/N)_0$ (dB)	$(S/N)_{opt}$ (dB)
5	1.263821	0.490718	4.78	4.99
10	1.629371	0.697855	7.83	8.00
15	1.753619	0.786028	9.60	9.76
20	1.815702	0.834514	10.86	11.01
25	1.853011	0.865273	11.83	11.98
30	1.877655	0.886277	12.62	12.77
35	1.895277	0.901669	13.29	13.44
40	1.908494	0.913418	13.87	14.02
45	1.918714	0.922626	14.38	14.53
50	1.9265951	0.9297813	14.84	14.99
55	1.9334151	0.93605575	15.25	15.40
60	1.9342834	0.9367502	15.63	15.78
65	1.937274	0.9393953	15.98	16.12
70	1.9458011	0.9475199	16.30	16.45
75	1.95203	0.953459	16.60	16.75
80	1.947554	0.948961	16.88	17.03
85	1.9551646	0.95633073	17.15	17.29
90	1.960444	0.9614337	17.39	17.54
95	1.962102	0.9630165	17.63	17.77
100	1.9628717	0.96370641	17.85	18.00

Next, an approximate expression for the coefficients k_1 and k_2 was found in terms of the number of pulses N within the beamwidth. By substituting the coefficients in Table 1 into Eqs. (8) and (9), we found that $\xi = 0.63$ and $N\omega_d\tau = 2.2$ for all values of N. Consequently, substituting into Eqs. (8) and (9), and solving for k_1 and k_2, we obtain

$$k_1 = 2e^{-\xi\omega_d\tau/N\sqrt{1-\xi^2}} \cos(\omega_d\tau/N) \tag{14}$$

$$k_2 = e^{-2\xi\omega_d\tau/N\sqrt{1-\xi^2}} . \tag{15}$$

Using k_1 and k_2 determined from Eqs. (14) and (15), we found that the S/N was the same as in Table 1.

PROBABILITY OF FALSE ALARM

The probability of false alarm P_{fa} is computed by obtaining $p(y_2 > T_y)$ where T_y is the threshold. Because the probability density of y_2 is untractable (y_2 being an infinite sum of weighted Rayleigh variables), the P_{fa} for a given T_y will be found by simulation. Specifically, the importance sampling technique described in the appendix will be used. The results of the simulation, Figs. 4 and 5, show how the normalized threshold T_y/σ behaves as a function of the number of pulses N on target for various P_{fa}'s.

PROBABILITY OF DETECTION

Using the threshold settings given in Figs. 4 and 5, the probability of detection P_D is also obtained by simulation. The ith envelope-detected pulse can be represented by

$$x(i) = \sqrt{\left[G^2(i) + n_{i_1}\right]^2 + n_{i_2}^2} , \tag{16}$$

where n_{i_1} and n_{i_2} are independent zero-mean Gaussian random variables with variance σ^2, and $G^2(i)$ is the returned signal from a unit-amplitude point scatterer. The midbeam S/N is $10 \log (1/2\sigma^2)$. The initial value of the two-pole integrator, $y_2(0)$, was set equal to its average noise value and the generation of signal-plus-noise samples using Eq. (16) started at approximately the null of the antenna beam. Specifically, the position of the first pulse was randomized so that the angle between the maximum signal return and the center of the antenna beam was a random variable that was uniformly distributed between $-\Delta\theta/2$ and $+\Delta\theta/2$. The simulation was performed for many combinations. The number of pulses N was set to 5, 10, 15, and 20; and for each N the midbeam S/N was varied in decibel increments. For each of the above combinations, 2000 cases were run; the results are summarized in Fig. 6.

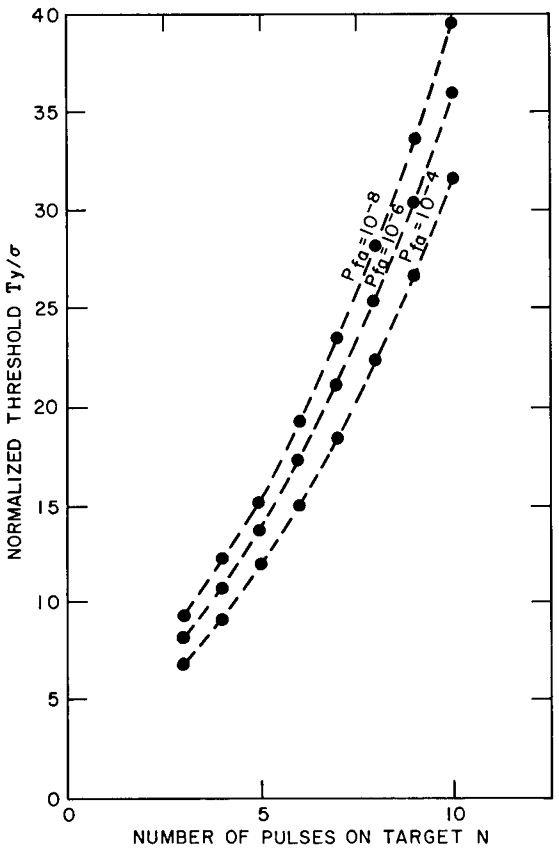

Fig. 4—Normalized threshold as a function of the number of pulses on target for various probabilities of false alarm.

ANGULAR ACCURACY

Two estimates of azimuth position will be considered: an estimate using a threshold-crossing procedure and an estimate using the maximum output of the two-pole filter. Let the first target detection (FTD) be defined as the smallest i such that

$$y_2(i) > T_y, \tag{17}$$

where T_y is the detection threshold for the two-pole filter, and let the last target detection (LTD) be defined as the largest i such that

$$y_2(i) > T_y. \tag{18}$$

The azimuth position of a target can be estimated by using a threshold-crossing procedure defined by

$$\hat{\theta} = \frac{1}{2}(\text{FTD} + \text{LTD})\Delta\theta, \tag{19}$$

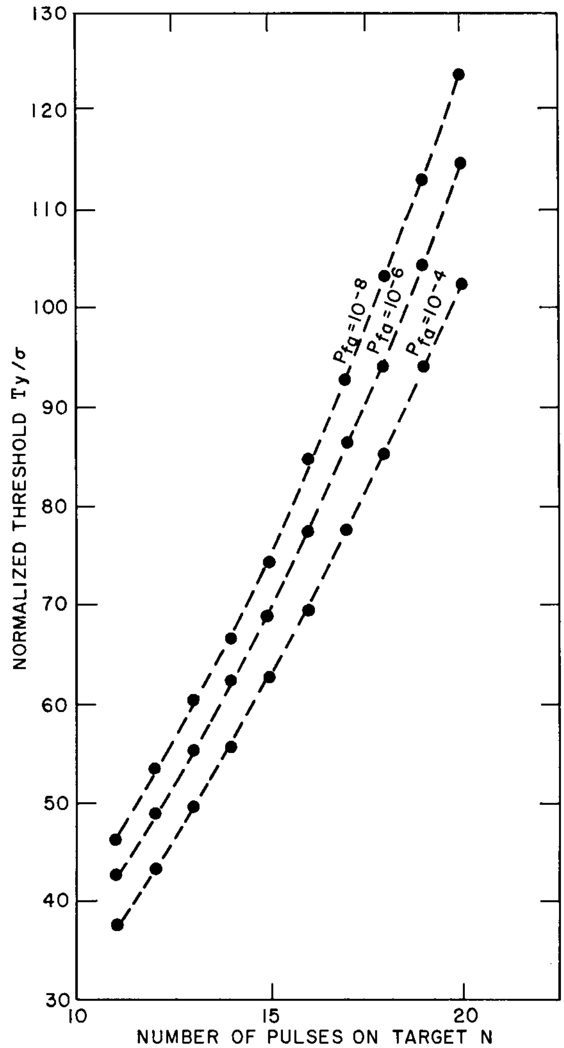

Fig. 5—Normalized threshold as a function of the number of pulses on target for various probabilities of false alarm.

where $\Delta\theta$ is the angular scanning increment between successive pulses. Another possible azimuth estimate is

$$\hat{\theta}_m = \Delta\theta(\mathrm{MP}), \tag{20}$$

where MP is the subscript of the largest integrated output; i.e., $y_2(\mathrm{MP}) \geqslant y_2(i)$ for all i.

The accuracy of the estimators was again determined by a Monte Carlo simulation. The simulation was performed for many combinations. The number of pulses in the beamwidth N was set equal to 15, 20, 30, 40, 60, and 100; and for each N the midbeam S/N was set to 1, 3, 5, 7, 9, and 11 dB. For each of the above combinations, 200 cases were run; the results are summarized in Fig. 7. Comparing these results with those

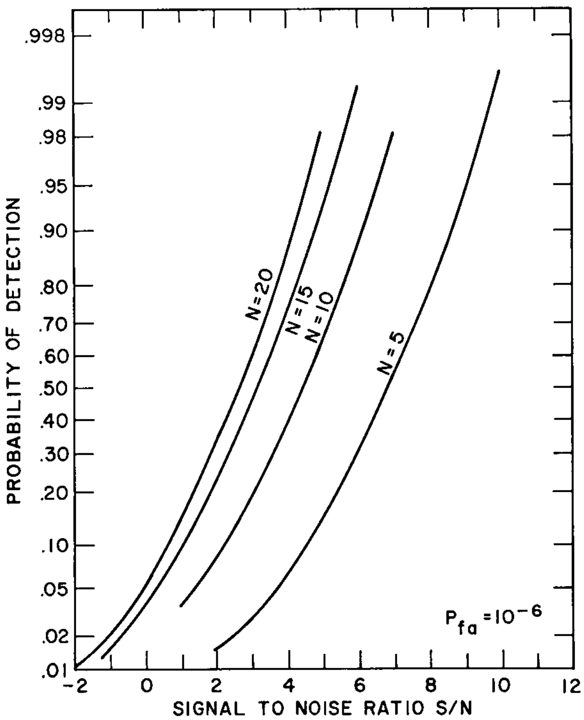

Fig. 6—Probability of detection vs signal-to-noise ratio for various number of pulses on target

given in Fig. 1, one can state that the two-pole filter yields more accurate results than the double delay-loop integrator. Moreover, for the two-pole filter, whereas the bias value for the threshold-crossing procedure varies between 1.00β and 1.1β as a function of the S/N, the bias value for the maximum value estimator is 1.05β independent of the S/N. Consequently, the *maximum value estimator* should be used since the bias does not have to be estimated.

SUMMARY

Previous estimators of azimuth position had deficiencies: The moving window requires the storage of many pulses, and the feedback integrator has a bias which has to be estimated. The two-pole filter has been investigated and yields good detection performance and accurate azimuth estimates. The detection performance of the two-pole filter is only 0.15 dB less efficient than the optimal detector. The SD of the azimuth estimate is only 15% greater than the Cramér-Rao lower bound, and the estimator using the maximum value has a constant bias. Consequently, because of its detection performance, angular accuracy, and simplicity of implementation, the two-pole filter would be an excellent detector to use with scanning radars.

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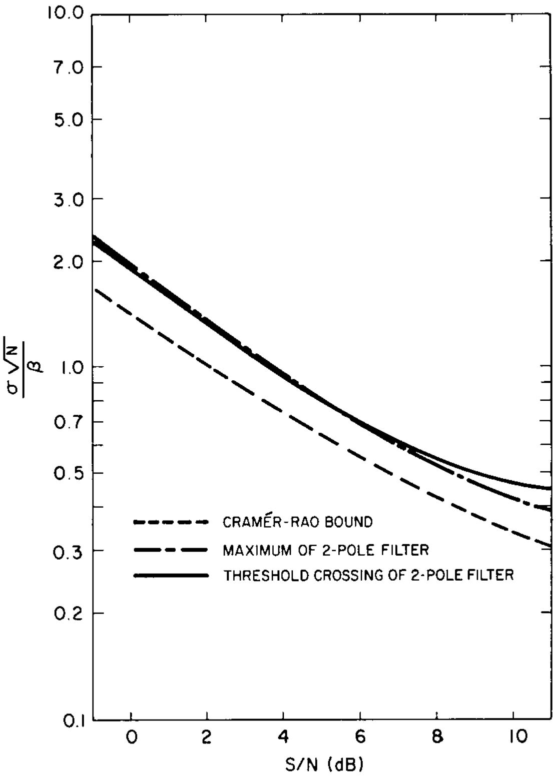

Fig. 7—Comparison of angular estimates with the Cramér-Rao lower bound. σ is the standard deviation of the estimation error and N is the number of pulses within the 3-dB beamwidth which is 2β.

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Appendix
IMPORTANCE SAMPLING

The fundamental principle of the technique of importance sampling* is to modify the probabilities that govern the outcome of the basic experiment of the simulation in such a way that the event of interest (i.e. the false alarm) occurs more frequently. This distortion is then compensated for by weighting each event by the ratio of the probability that this specific event would have occurred if the true probabilities had been used in the simulation to the probability that this same event would occur with the distorted probabilities. Consequently, by proper choice of the distorted probabilities the number of repetitions can be reduced greatly.

To illustrate the method, the simulation involving the two-pole filter is run for $2N$ pulses, the output is noted, and the output serves as the initial value for the next run of $2N$ pulses. However, to obtain more events on the tail of the distribution, the original probabilities are distorted so that more events occur on the tail. This is achieved by making the variance of the Gaussian noise larger. The simulation is run and the probability density of y_2 is estimated by

$$p(y_2 \text{ is in the } n\text{th interval}) = \frac{1}{M} \sum_{j=1}^{M} \delta_j$$

where

$$\delta_j = \begin{cases} \displaystyle\prod_{L=1}^{2N} \frac{P_T(n_{i_1})P_T(n_{i_2})}{P_D(n_{i_1})P_D(n_{i_2})}, & y_2 \text{ lays in the } n\text{th interval} \\ 0 & \text{otherwise.} \end{cases}$$

n_{i_1} and n_{i_2} are the I and Q Gaussian noise components, and $P_T(\cdot)$ and $P_D(\cdot)$ are the true and distorted probability densities, respectively, evaluated at the values of the noise sequence. Because both $P_T(\cdot)$ and $P_D(\cdot)$ are Gaussian,

$$\delta_j = \prod_{L=1}^{2N} \frac{\sigma_D^2}{\sigma^2} \exp\left[-\frac{n_{i_1}^2}{2}\left(\frac{1}{\sigma_D^2} - \frac{1}{\sigma^2}\right) - \frac{n_{i_2}^2}{2}\left(\frac{1}{\sigma_D^2} - \frac{1}{\sigma^2}\right)\right].$$

In generating Figs. 4 and 5, $\sigma = 1$, σ_D varied from 1.25 σ to 1.65 σ for various N's, and $M = 40{,}000$.

*F.S. Hillier and G.J. Lieberman, Introduction to Operations Research, Holden-Day, Inc., San Francisco, 1967, p. 457-459.