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The problem of a two-frequency MTI system in a general case has been formulated. We conclude that (a) the two-frequency MTI system sees an increased clutter-doppler variance as compared to that of a single-frequency system (from σ^2 to $\sigma^2(1 + r^2)$ where r is the ratio of the carrier frequencies in the two frequency channels) and (b) the mean clutter-doppler frequency, at the same time, is reduced accordingly from f_0 to $f_0(1 - r)$.

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CONTENTS

ANALYSIS OF A DUAL-FREQUENCY MTI SYSTEM

INTRODUCTION

It is well known that a radar MTI system performs better at a lower radar frequency. However, other considerations require that a modern radar be operated at a high frequency, usually in the microwave range. This generally presents a difficult problem in the design of an MTI system, particularly if the radar system is required to reject a clutter of wide doppler spread at the same time it is required to detect a high-speed target. It has been suggested that a dual-frequency MTI system may be a solution to this problem. A possible configuration is shown in Fig. 1. The mixing of the two radar returns from two

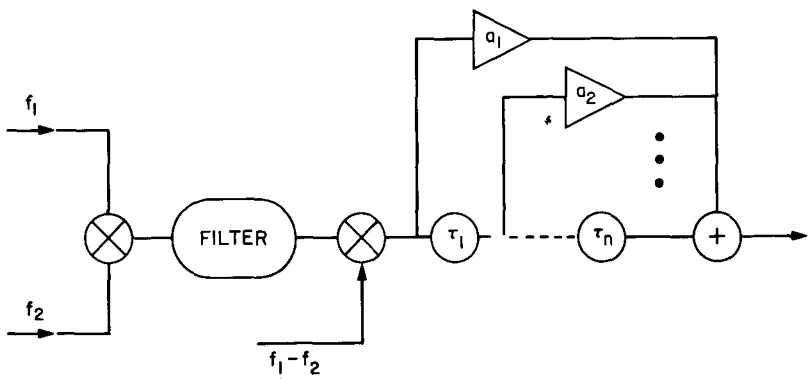

Fig. 1—A dual-frequency MTI system

different frequency channels generates two frequency components, the sum and difference of the doppler frequencies from both frequency channels. The sum frequency component can be filtered out before it is converted to the base band. The difference frequency component, after being shifted to base band, has a net effect of shifting the signal doppler frequency to the difference of the doppler frequencies in the two channels. If the two carrier frequencies are closely spaced, the difference of the doppler frequencies will be small. Thus, this system effectively reduces the target doppler frequency by a large factor, which is equivalent to operating the radar system at a lower frequency. This should improve MTI performance. Unfortunately, such an intuitively simple explanation does not take all the facts into account. The fact is that since the radar return contains both target and clutter, mixing of the two returns generates not only the beat-frequency components of clutter-to-clutter and target-to-target but also the clutter-to-target return from both channels. Because the clutter doppler frequency usually is very low, the frequency component of the beat of the target-to-clutter doppler is high and comparable to that of a single-frequency system. Furthermore, because the spectrum of the clutter returns is Gaussian the resultant clutter doppler spectrum corresponding to the difference

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frequency is also Gaussian. It should be noted, however, that the variance of the clutter doppler after taking the difference frequency is the sum of the clutter doppler variances at each frequency. Thus, as it compres to a single-frequency MTI system, the clutter spectrum of the dual-frequency MTI is wider.

An analysis of the performance of this kind of MTI system has been conducted by Kroszczynski (1,2). Unfortunately, Kroszczynski's analysis is based on the assumption that the clutter return has only a single doppler frequency, which is not true in general. Furthermore, Kroszczynski's analysis includes only two special cases: a 2-pulse and a 3-pulse canceller with binominal weights. In this report, we shall attempt to treat this subject in a more general way. We shall assume first that the clutter spectrum is Gaussian, and we shall treat the MTI filter in a more general sense. Furthermore, we shall find the optimal set of filter weights which will minimize the clutter output. We also will discuss some properties of this type of MTI system and then compare its performance with a conventional single-frequency MTI system.

IMPROVEMENT FACTOR

The dual-frequency MTI system to be analyzed here has a configuration as shown in Fig. 1. The radar returns from both frequency channels contain both clutter and target signals. Let us first assume that the power spectral density function of the clutter returns is Gaussian with mean doppler frequencies f_{10} and f_{20} and variances σ_1 and σ_2. The subscripts 1 and 2 are used here to represent the returns from frequency channels f_1 and f_2. Hence, we have

$$G_1(f_1) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[-\frac{(f_1 - f_{10})^2}{2\sigma_1^2}\right] \tag{1a}$$

$$G_2(f_2) = \frac{1}{\sigma_2\sqrt{2\pi}} \exp\left[-\frac{(f_1 - f_{20})^2}{2\sigma^2}\right]. \tag{1b}$$

Without losing generality, let us assume the power spectrum of the target return to be an impulse function,

$$T_1(f) = \delta(f - f_{1s}) \tag{2a}$$

$$T_2(f) = \delta(f - f_{2s}) \tag{2b}$$

where f_{1s} and f_{2s} represent the doppler frequencies of the target from the two frequency channels. Mixing two signals in the time domain is equivalent to performing convolution in the frequency domain. If we assume that the sum frequency component of the mixed signal has been suppressed, the input spectrum function to the MTI system is then

$$P(f) = C_1 C_2 \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} \exp\left[- \frac{[f - (f_{10} - f_{20})]^2}{2(\sigma_1^2 + \sigma_2^2)} \right]$$

$$+ E_1 C_2 \frac{1}{\sigma_2 \sqrt{2\pi}} \exp \left\{ - \frac{[f - (f_{1s} - f_{20})]^2}{2\sigma_2^2} \right\}$$

$$+ E_2 C_1 \frac{1}{\sigma_1 \sqrt{2\pi}} \exp \left\{ - \frac{[f - (f_{2s} - f_{10})]^2}{2\sigma_1^2} \right\}$$

$$+ E_1 E_2 \delta[f - (f_{1s} - f_{2s})], \tag{3}$$

where C_1, C_2 and E_1, E_2 are the power of the clutter and target returns from both channels.

For a single-frequency case, the power input $P(f)$ contains only two terms, the clutter and target signal terms. Hence

$$P_s(f) = C_1 \frac{1}{\sigma_1 \sqrt{2\pi}} \exp \left[- \frac{(f - f_{10})^2}{2\sigma_1^2} \right] + E_1 \delta(f - f_{1s}),$$

where C_1, C_2 and E_1, E_2 are the power amplitudes of the clutter and target returns from both channels.

The power transfer function of an MTI filter can be represented as

$$|H(f)|^2 = \sum_i \sum_j \mathcal{A}_i \mathcal{A}_j \cos 2\pi f(T_i - T_j) \tag{4}$$

where the $\mathcal{A}_i$'s are the filter weights and T_i is the total delay time of the ith pulse. Therefore, the power output from this filter is

$$W = \int_{-\infty}^{\infty} P(f) |H(f)|^2 df . \tag{5}$$

When Eqs. (3) and (4) are inserted into Eq. (5) and the integration is performed, one finds that

$$W = C_1 C_2 \sum_{i,j} P_{ij} + C_1 E_2 \sum_{i,j} S_{ij}(c_1; e_2) + C_2 E_1 \sum_{i,j} S_{ij}(c_2; e_1)$$

$$+ E_1 E_2 \sum_{i,j} S_{ij}(e_1; e_2) , \tag{6a}$$

where

$$P_{ij} = \mathcal{A}_i \mathcal{A}_j \cos 2\pi(f_{10} - f_{20})T_{ij} \exp[-2\pi^2(\sigma_1{}^2 + \sigma_2{}^2)T_{ij}^2] \tag{6b}$$

$$S_{ij}(c_i; e_2) = \mathcal{A}_i \mathcal{A}_j \cos 2\pi(f_{2s} - f_{10})T_{ij} \exp[-2\pi^2\sigma_1{}^2 T_{ij}^2] \tag{6c}$$

$$S_{ij}(c_2; e_1) = \mathcal{A}_i \mathcal{A}_j \cos 2\pi(f_{1s} - f_{20})T_{ij} \exp[-2\pi^2\sigma_2{}^2 T_{ij}^2] \tag{6d}$$

$$S_{ij}(e_1, e_2) = \mathcal{A}_i \mathcal{A}_j \cos 2\pi(f_{1s} - f_{2s})T_{ij} \tag{6e}$$

$$T_{ij} = T_i - T_j . \tag{6f}$$

This MTI output contains four terms. The first term is the product of the clutter input from both channels, while the last term represents the product of the target signals from both channels. The second and third terms are the cross products of the target signal and the clutter returns from both channels. In the absence of target signal, the second, third and fourth terms are all zero. Thus, the first term in the above expression represents the clutter output. One of the optimal criteria can then be the requirement that this clutter output be minimal. This we shall discuss in the next section.

One feature which should be pointed out is that if the carrier frequences in both channels are identical then this system becomes similar to that of a noncoherent MTI system, in which case the signal output comes from a square law detector. This is equivalent to mixing two identical signals. Hence if f_{1s} is equal to f_{2s}, the target difference frequency signal will have zero doppler, which will be rejected by the MTI filter. Therefore, in a noncoherent MTI system, targets cannot be detected in the absence of clutter. On the other hand, in this dual-frequency system if $f_{1s} \neq f_{2s}$, the target signal will be present even if clutter is absent. More discussion about this effect will be presented later.

Similarly, for a single-frequency case,

$$W_s = C_1 \sum_{i,j} P'_{ij} + E_1 \sum_{i,j} S'_{ij}(e_1) ,$$

where

$$P'_{ij} = \mathcal{A}_i \mathcal{A}_j \cos 2\pi f_{10} T_{ij} \exp(2\pi^2\sigma_1{}^2 T_{ij}{}^2)$$

$$S'_{ij} = \mathcal{A}_i \mathcal{A}_j \cos 2\pi f_{1s} T_{ij} .$$

To simplify the above expression, let us assume that

$$\frac{E_1}{C_1} = \frac{E_2}{C_2} = \frac{E}{C} \tag{7}$$

This assumption says that the power ratio of the target signal to clutter response is invariant with respect to carrier frequency. This assumption is probably valid if the carrier frequencies in both channels are very close. In that case the antenna patterns of both channels will not be significantly different provided that the same antenna is used in both channels.

Let us define the MTI improvement factor (I) as the ratio of two quantities: the output target-to-clutter ratio from the MTI filter and the input target-to-clutter ratio. Hence,

$$I = \frac{\dfrac{S_0}{C_0}}{\dfrac{S_i}{C_i}} \tag{8}$$

where S_i, S_0 are respectively the input and output target-return energy while C_i and C_0 are the input and output clutter energy, respectively. Therefore,

$$\frac{S_i}{C_i} = \frac{E_1 + E_2}{C_1 + C_2} = \frac{E}{C} . \tag{9}$$

In the derivation of this expression, the author has used the fact that the integration of a Gaussian and an impulse function is unity. From Eq. (6), we have

$$\frac{S_0}{C_0} = \frac{C_1 E_2 \sum_{i,j} S_{ij}(c_i; e_2) + C_2 E_1 \sum_{i,j} S_{ij}(c_2; e_1) + E_1 E_2 \sum_{i,j} S_{ij}(e_1; e_2)}{C_1 C_2 \sum_{i,j} P_{ij}} . \tag{10}$$

Therefore, the improvement factor can be represented as

$$I = \frac{\sum_{ij} \left\{ S_{ij}(c_1; e_2) + S_{ij}(c_2; e_1) + \frac{E}{C} S_{ij}(e_1; e_2) \right\}}{\sum_{ij} P_{ij}} . \tag{11}$$

MINIMUM CLUTTER OUTPUT

In the design of an MTI system, the primary goal is to achieve a maximum improvement factor. From the expression of Eq. (11), this is equivalent either to minimizing the denominator or to maximizing the numerator of that expression. Let us discuss the possibility of the minimization of the denominator. This denominator is actually the normalized clutter output from the MTI filter. This is repeated as follows:

$$C_0 = \sum_{i,j} \mathcal{A}_i \mathcal{A}_j \cos 2\pi (f_{10} - f_{20}) T_{ij} \exp \left[-2\pi^2 (\sigma_1{}^2 + \sigma_2{}^2) T_{ij}^2 \right] . \tag{12}$$

The following equations show the relationship of f_{10} and f_{20} to the actual mean clutter velocity and of σ_1 and σ_2 to the velocity standard deviations:

$$f_{10} = \frac{2V_0}{C} f_1 \tag{13a}$$

$$f_{20} = \frac{2V_0}{C} f_2 \tag{13b}$$

$$\sigma_1 = \frac{2\sigma_v}{C} f_1 \tag{13c}$$

$$\sigma_2 = \frac{2\sigma_v}{C} f_2 . \tag{13d}$$

The quantity V_0 includes such effects as constantly moving clouds or raindrops, or, alternatively, the radar platform which moves with respect to stationary clutter. One may consider that the clutter return consists of many individual point targets, each of which has a certain velocity and is independent of other point targets. Thus, one may view this as a random process consisting of a large number of independent random variables. According to the central limit theory of Liapanov, this process has a Gaussian distribution. Thus, V_0 is the mean of this process and σ_v is the variance of this process. Equations (13a), (13b), (13c), and (13d) merely transform this velocity into doppler frequency. From these relationships, if we let

$$\frac{f_1}{f_2} = r, \quad \text{and} \quad f_2 > f_1 \tag{14}$$

then

$$f_{10} = r f_{20} \tag{15a}$$

$$\sigma_1 = r \sigma_2 . \tag{15b}$$

Let us assume that the MTI system has a constant delay time T; then

$$T_{ij} = (i - j) T . \tag{16}$$

The reciprocal of T is the radar pulse repetition frequency (prf). One may normalize f_{20} and σ_2 with respect to this prf;

$$f'_{20} = f_{20} T \tag{17a}$$

$$\sigma'_2 = \sigma_2 T . \tag{17b}$$

Equation (12) can then be written

$$C_0 = \sum_{i,j} \mathfrak{a}_i \, \mathfrak{a}_j \, P_{ij} \tag{18a}$$

$$P_{ij} = \cos\,[2\pi f'_{20}(1 - r)k]\ \exp\,[-2\pi^2 \sigma'^2_2 (1 + r^2) k^2]\ , \tag{18b}$$

where

$$k = i - j.$$

For a single-frequency MTI case, the covariance matrix element P_{ij} is

$$P_{ij} = \cos\,[2\pi f'_{20} k]\ \exp\,[-2\pi^2 T'^2_2 k^2]\ . \tag{18c}$$

This shows that a two-frequency MTI has the following effects:

1. The clutter spectrum variance is increased by a factor of $1 + r^2$.
2. The mean clutter doppler of f_{20} is reduced by a factor of $1 - r$.

Write P_{ij} into a matrix form

$$X = |P_{ij}| \tag{19}$$

and treat the $\mathfrak{a}_i$'s as the components of an N-dimensional vector where N represents the number of pulses in the MTI system. We have

$$C_0 = Q(X) = (\mathfrak{a}, X\mathfrak{a}), \tag{20}$$

where $\mathfrak{a}$ is the row matrix of components $\mathfrak{a}_1, \mathfrak{a}_2 \ldots \mathfrak{a}_N$. The parentheses represent the inner product of vector $\mathfrak{a}$ and its transformed vector $X\mathfrak{a}$. Minimizing C_0 (the clutter output) amounts to finding a vector $\mathfrak{a}$ such that the quadratic form $(\mathfrak{a}, X\mathfrak{a})$ is minimum. To avoid the trivial solution of a null vector, we may add a constraint, such that

$$\sum_i \mathfrak{a}_1^2 = 1\ . \tag{21}$$

Associated with this quadratic form $Q(X)$, there exists a set of characteristics equations

$$\sum_i^N \mathfrak{a}_i P_{ij} - \lambda \mathfrak{a}_j = 0 \qquad j = 1, \ldots N\ , \tag{22}$$

where λ is defined as the characteristic value of $Q(X)$, there is a set of N such characteristic values, $\lambda_1, \lambda_2, \ldots \lambda_N$, all of which satisfy Eq. (22). Substituting these characteristic values into this set of equations, we find a set of vectors $\mathfrak{a}^i$ $(i = 1, 2, \ldots, N)$. Each of these vectors, which are called characteristic vectors, if substituted into the quadratic form $Q(X)$, yields a value of λ_i which is the characteristic value of λ associated with that characteristic vector. These characteristic values can be represented

$$\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_N . \tag{23}$$

The characteristic vectors are also an orthogonal set. Furthermore, due to the constraint of Eq. (21), they are actually orthonormal. Thus, this set of vectors forms a complete set in this N-dimensional space. Any other vector in this space can be formed by a linear combination of this set of orthonormal vectors. Suppose that a vector $\mathcal{A}$ which will yield a minimum value of the quadratic form $Q(X)$ can be represented as

$$\mathcal{A} = \sum_i d_i \mathcal{A}^i . \tag{24}$$

Substituting this into Eq. (20) yields

$$Q(X) = \sum_i d_i^2 \lambda_i .$$

Therefore, the minimum value of $Q(X)$ occurs when

$$\begin{aligned} d_1 &= d_2 = \ldots d_{N-1} = 0 \\ d_N &= 1 \end{aligned} \tag{25}$$

and

$$Q(X) = \lambda_N .$$

Therefore, the minimum clutter output is equal to the minimum characteristic value when the weighting function of the MTI system is set equal to the characteristic vector associated with this minimum characteristic value.

Since the quadratic form $Q(X)$ represents the output clutter power, it is always positive unless all $\mathcal{A}_i$'s are zero. Hence, $Q(X)$ is positive definite. Furthermore, matrix X is real and symmetrical; that is,

$$P_{ij} = P_{ji} \tag{26}$$

and the P_{ij}'s are real. Accordingly, its characteristic values are always positive, that is

$$\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \ldots \geqq \lambda_N > 0 . \tag{27}$$

Since

$$\sum_i \lambda_i = \sum_i P_{ii} \tag{28}$$

and the diagonal elements P_{ii} in matrix X are all unity by Eq. (6b), one may conclude that

$$0 < \lambda_N \leqslant 1 . \tag{29}$$

This says that the denominator of Eq. (11) or the clutter output of the MTI system is always less than unity if the optimal MTI filter weights are used.

AVERAGE TARGET-SIGNAL GAIN

Next, we will examine the effect of the numerator of Eq. (11). This term can be viewed as the target-signal gain. For convenience of discussion, we shall repeat this expression as follows:

$$\begin{aligned} G(f'_{2s}) = \sum_{ij} \mathcal{C}_i\, \mathcal{C}_j \Bigg\{ &\cos \left[2\pi f'_{2s}\left(1 - \frac{f'_{20}}{f'_{2s}} r\right)k\right] \exp\left(-2\pi^2 r^2 \sigma'^2_2 k^2\right) \\ &+ \cos \left[2\pi f'_{2s}\left(r - \frac{f'_{20}}{f'_{2s}}\right) k\right] \exp\left(-2\pi^2 \sigma'^2_2 k^2\right) \\ &+ \frac{E}{C} \cos \left[2\pi f'_{2s}(1 - r)k\right] \Bigg\} \end{aligned} \tag{30a}$$

In deriving this result, the relationships of Eqs. (15), (16), and (17) have been used.

For a single-frequency case, Eq. (30a) becomes

$$G(f'_{2s}) = \frac{E}{C} \sum_{ij} \mathcal{C}_i\, \mathcal{C}_j \cos\left(2\pi f'_{2s} k\right). \tag{30b}$$

For a given set of f_{20} and T_2, this expression is a function of the target doppler frequency f'_{2s}. Since the target doppler frequency is not known, in order to optimize the maximum signal gain, one really has to choose many sets of $\mathcal{C}_i$'s such that each set of $\mathcal{C}_i$'s gives a maximum signal output. Of course, simultaneously, the clutter output should be minimized for a given target doppler. This in general requires a bank of filters, each specifically designed for a particular doppler target.

Our purpose here is to design a single filter which will reject the clutter and yet give a reasonably good signal gain. To this end, we shall optimize the average signal gain by assuming that the target doppler frequency has a uniform distribution. Since the gain function is a periodic function of period f_b, the average gain is then

$$G_a = \frac{1}{f_b} \int_0^{f_b} G(f'_{2s}) df'_{2s}$$

$$= \sum_i \mathfrak{A}_i^2 \cdot \left(2 + \frac{E}{C}\right). \tag{31a}$$

For a single-frequency case this becomes

$$G'_a \doteq \sum_j \mathfrak{A}_j^2 . \tag{31b}$$

By using this relationship and defining the average improvement factor as the ratio of the average target signal gain to the clutter output, we find that

$$I = \frac{\left(2 + \frac{E}{C}\right) \cdot \sum_i \mathfrak{A}_i^2}{(\mathfrak{A}, X\mathfrak{A})}. \tag{32}$$

Factor E/C is the ratio of the power of the target return to the clutter return. Usually it is very small and can be neglected. We have, in that case

$$I \approx \frac{2}{\lambda_N} \tag{33a}$$

where the constraint of Eq. (21) has been used.

For a single-frequency case, this becomes

$$I' \doteq \frac{1}{\lambda'_N}. \tag{33b}$$

However, because the covariance matrix elements of the dual-frequency case and the single-frequency case are different, in general, $\lambda_N \neq \lambda_N{}'$.

This optimal average improvement factor is plotted in Fig. 2. In this plot, it is assumed that mean clutter velocity $f_{20} = 0$. The normalized $(\sigma_2\sqrt{1 + r^2})$ standard deviation for rms clutter is plotted as the abscissa while the average improvement factor is plotted as the ordinate. A family of curves is plotted for different numbers of MTI cancelling pulses N. For each rms clutter variance value at a given N the matrix X is formed and its minimum characteristic value is computed. Thus, these curves represent the optimal improvement factor. Several properties are evident:

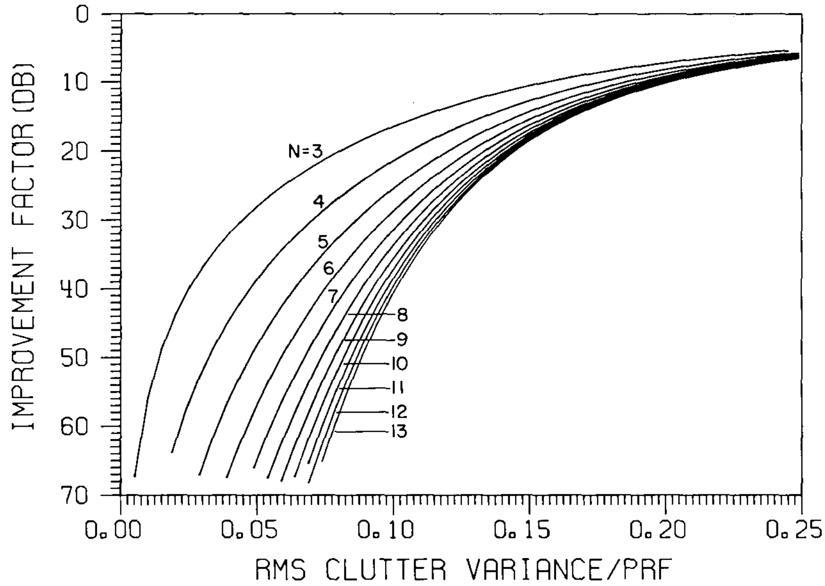

Fig. 2(a)—Average improvement factor vs normalized clutter variance; N = the number of canceling pulses

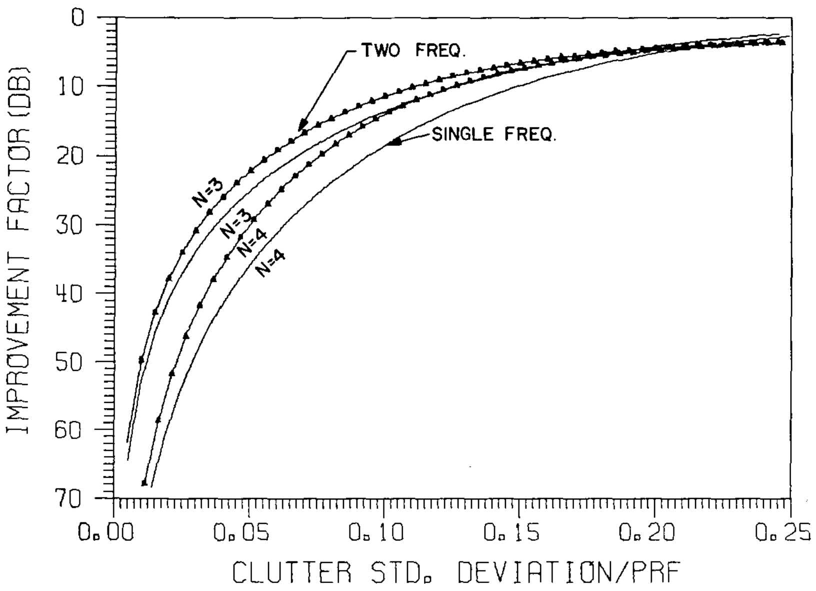

Fig. 2(b)—Comparison of improvement factor of a two-frequency and a single-frequency MTI system r = 1

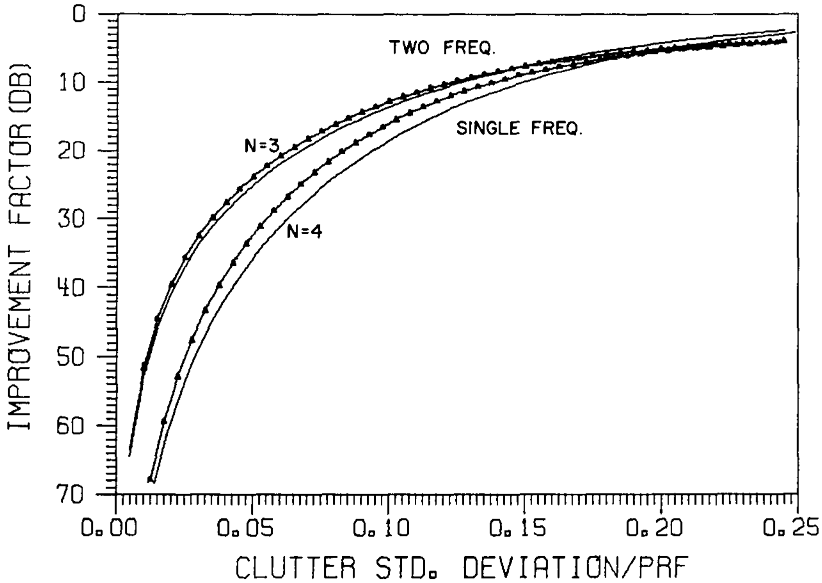

Fig. 2(c)—Comparison of improvement factor of a two-frequency and a single-frequency MTI system (r = 0.8)

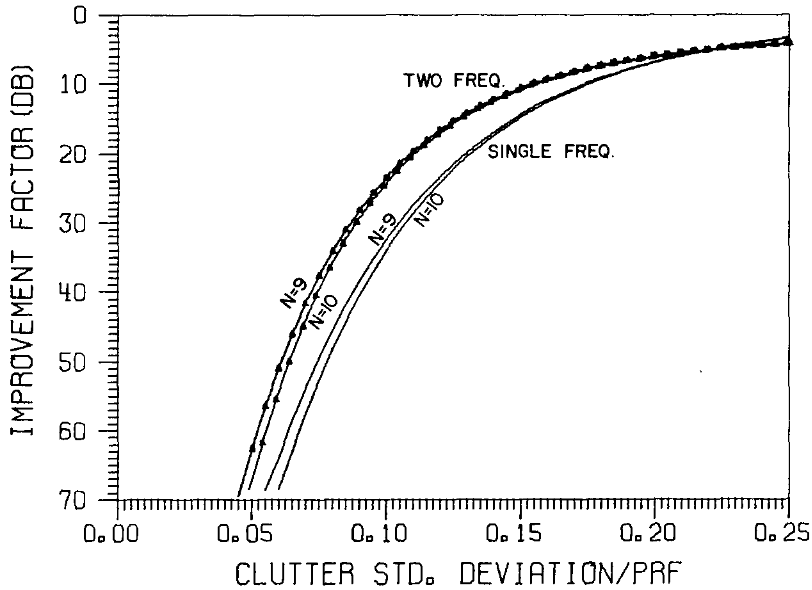

Fig. 2(d)—Comparison of improvement factor a two-frequency and a single-frequency MTI system (r = 0.8)

1. The improvement factor degrades as the rms clutter variance increases.

2. Better improvement factor is achieved if more cancelling pulses are used. However, this improvement reaches a limit as N becomes large ($N = 6$).

3. If rms clutter variance reaches about 0.25 of the radar prf, the improvement factor which can be achieved is only a few decibels and it does not depend on N (the number of cancelling pulses).

For a detailed discussion of this problem, see Ref. 3.

Figures 2b to 2d show a comparison of the improvement factors of two-frequency and single-frequency MTI systems for different frequency ratios. One should notice that the abscissa scale in Fig. 2a is different from that of Figs. 2b, 2c, and 2d. In Fig. 2a, we are using the effective normalized standard deviation for a two-frequency MTI, which is equal to $\sigma_2\sqrt{1 + r^2}$, while in Figs. 2b, 2c, and 2d we are using the normalized standard deviation σ^2.

Figure 2b compares the improvement factor of a 3-pulse ($N = 3$) and that of a 4-pulse ($N = 4$) 2-frequency MTI canceller with a single-frequency MTI canceller. The curves marked with triangles are those of a 2-frequency MTI canceller. In general, for a given clutter spectrum standard deviation (normalized with respect to prf), the improvement factor of a two-frequency MTI system is less than that of a single-frequency system. This is just what one would expect. In this plot the carrier frequencies of the two-frequency MTI are assumed to have a ratio of $r = 1$. These same curves are plotted again in Fig. 2c with r changes from 1 to 0.8. Comparing these two figures, we find that the difference of improvement factor between a two-frequency and a single-frequency MTI is somewhat reduced as the frequency ratio r decreases. However, we also notice that in both plots the difference of improvement factor becomes larger as the number of cancellers increases. This effect is shown on Fig. 2d. In this plot the number of cancellers is assumed to be 9 and 10 with $r = 0.8$. We see that the difference of improvement factor of a two-frequency and a single-frequency MTI system is considerably larger than in the 3- or 4-pulse case. In all these cases, the improvement factor approaches 0 dB as σ' becomes larger.

EFFECT OF NONZERO MEAN-CLUTTER VELOCITY

The clutter output from the MTI system represented in Eq. (18) is repeated as follows:

$$C_0 = \sum_{ij} \mathfrak{A}_i \mathfrak{A}_j P_{ij}$$

$$P_{ij} = \cos\,[2\pi f'_{20}(1 - r)k]\,\exp\,[-2\pi^2 \sigma_2'^2 (1 + r^2)k^2]\,.$$

It can be seen from the above expression that when f'_{20} increases from zero the value of P_{ij} is reduced. This effect is similar to the effect of increasing σ'_2, which was shown in the previous section degrades the average improvement factor, or equivalently increases the clutter output. It is evident that for a given σ_2' this clutter output has a minimum value when $f_{20}' = 0$. If $r = 1$, then no matter what value f_{20}' assumes, the argument of the cosine term remains zero, and the clutter output will not change. On the other hand, if $r = 0$, which corresponds to the case of a single-frequency MTI system, a slight shift of f_{20}' means a larger change of the cosine term as compared to the case of r close to unity. This effect is shown in Fig. 3, where the average improvement factor of a 4-pulse canceller is plotted against the normalized mean clutter-doppler frequency. The filter weights were chosen so that the canceller has an optimal average improvement factor when the mean clutter-doppler frequency is zero. One may notice that when r = 1, the improvement factor remains constant while the normalized mean clutter-doppler frequency varies from zero to unity. When the value of r is reduced the degradation of the improvement factor is evident from this figure.

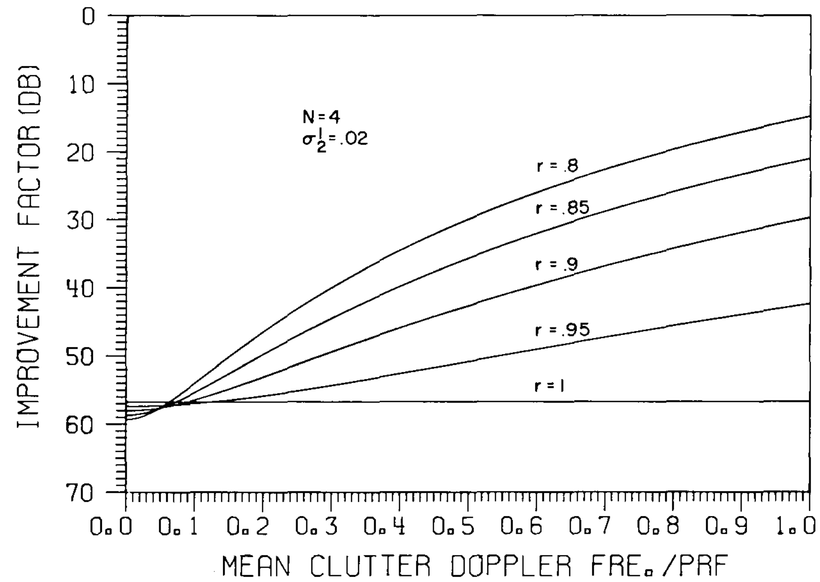

Fig. 3—Effect of mean clutter moving velocity on average improvement factor of a 4-pulse canceller

In Fig. 4, the MTI filter response is plotted as a function of the target doppler frequency. This plot is actually a plot of the target signal gain function which is shown in Eq. (30a). When the mean-clutter velocity is subsequently increased, the null shifts almost to the corresponding frequency point. However, the null becomes shallower as the mean clutter-doppler frequency increases. This demonstrates that the clutter-rejection notch of a dual-frequency MTI system follows the mean clutter velocity and automatically sets the notch at the point where the clutter velocity is located

BLIND SPEED

The target-signal gain function as shown in Eq. (30a) is repeated in the following:

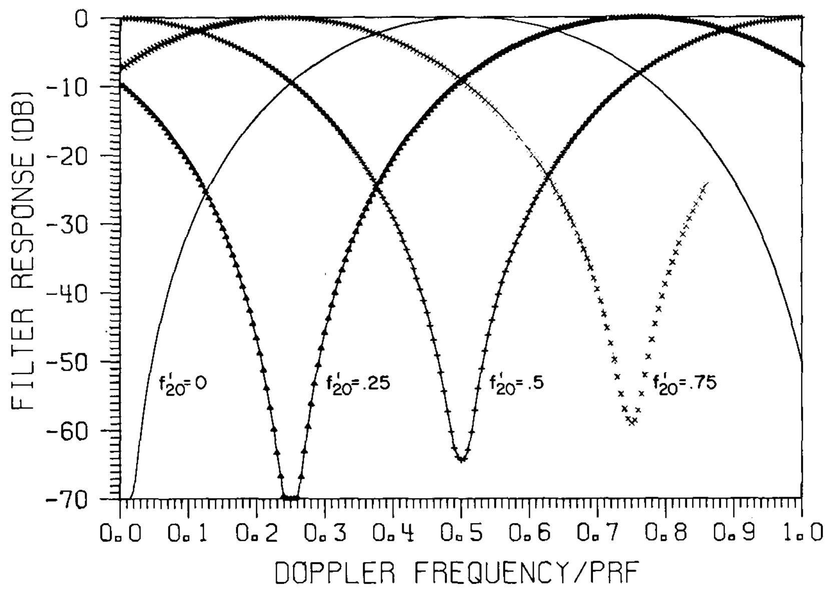

Fig. 4—The MTI filter response of a dual-frequency MTI system; $r = 0.95$; $\sigma_{20} = 0.01$

$$G(f'_{2s}) \approx \sum_{i,j} \mathfrak{a}_i \mathfrak{a}_j \left\{ \cos\,[2\pi f'_{2s}k) \exp\,[\,-\,2\pi^2 r^2 \sigma_2'^2 k^2] + \cos\,[2\pi r f'_{2s}k] \exp\,[\,-\,2\pi^2 \sigma_2'^2 k^2] \right\} \tag{34}$$

In deriving this equation without losing generality, we have assumed that the mean clutter-doppler frequency is zero and that $E << C$, so that the third term in Eq. (30a) can be neglected. It is evident that when

$$f'_{2s} = \ell_1 \tag{35a}$$

$$f'_{2s}r = \ell_2 \tag{35b}$$

where ℓ_1 and ℓ_2 are integers, the function $G(f'_{2s})$ repeats. Thus a dual-frequency MTI system has a property similar to that of a staggered prf MTI system—its blind speed is greatly extended. For example, if $r = 8/9$ the first blind speed will occur when $f'_{2s} = 9$. This is shown in Fig. 5, where the target signal shows a period when $f'_{2s} = 9$. It also shows properties similar to those of a staggered prf MTI system (4). One should notice here that for a single-frequency, constant-prf system this blind speed will occur at $f'_{2s} = 1$.

In this plot no effort has been made to smooth out the passband variations. Conceivably, one may choose a good frequency ratio r, which may yield a better result. However if this were compared with a staggered-prf, single-frequency system, one would expect that it probably would be less efficient in the sense that there is only one

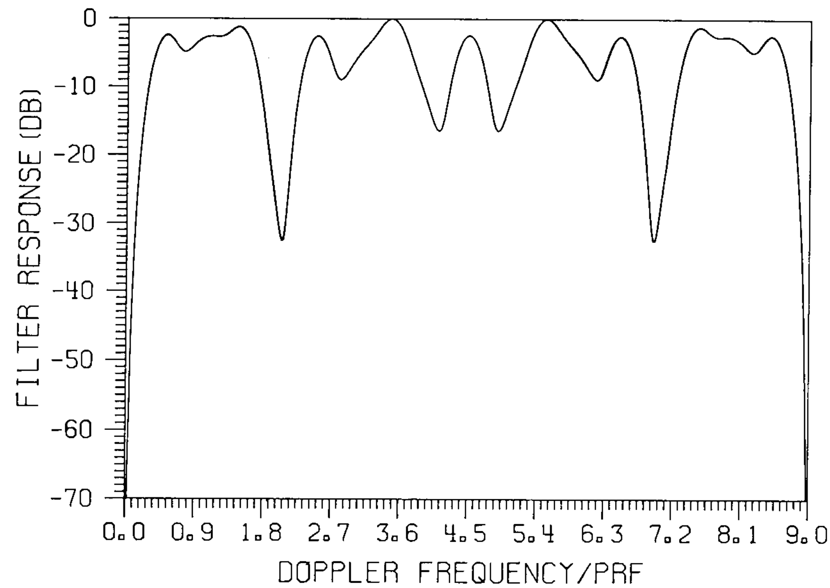

Fig. 5—Target-signal gain function of a dual-frequency MTI system; $N = 4$, $\sigma_{20} = 0.01$, $r = 0.89$

parameter which could be chosen, while in a staggered-prf system with a long train of pulses, far more parameters could be varied. However the exact effect has not been analyzed.

COMPARISON WITH A SINGLE-FREQUENCY MTI SYSTEM

For convenience of comparison, let us write the improvement factor of a single-frequency MTI system as follows:

$$I_s = \frac{\sum_{ij} \mathcal{C}_i \, \mathcal{C}_j \cos(2\pi f_s' k)}{\sum_{i,j} \mathcal{C}_i \, \mathcal{C}_j \cos(2\pi f_0' k) \exp(-2\pi^2 \sigma'^2 k^2)},$$

where f_s' is the normalized doppler frequency of the target and f_0' is the normalized mean clutter velocity, while σ' is the normalized clutter standard deviation. The denominator of this equation represents the clutter output which has a functional form almost identical to that of a dual-frequency system (Eq. (18)). There are three differences, however:

1. The clutter-doppler standard deviation σ' in a single-frequency MTI system is changed to $\sigma'\sqrt{1 + r^2}$ in a dual-frequency system.

2. The mean clutter-doppler frequency of a dual-frequency MTI system is changed from f_{20} in a single-frequency MTI to f_{20} $(1 - r)$; therefore, the value of f_{20} has very little effect on the clutter output, if the value of r is close to unity and f_{20} $(1 - r) << 1/T$.

3. One may also notice that the target doppler frequency is also reduced by a factor of $1 - r$. In case clutter is absent, one cannot use the clutter and target and cross-correlation terms. Therefore under this correlation, some low-velocity targets that can be seen in a conventional single-frequency MTI system may look like a stationary target to a two-frequency MTI system and be rejected, especially if r is chosen to be close to 1.

4. Finally, from the discussion of blind speed, we see that a dual-frequency MTI system eases the blind-speed problem somewhat.

CONCLUSION

In this report we have formulated the problem of a dual-frequency MTI system in a general case. We concluded that in comparison with a single-frequency MTI, the dual-frequency MTI system sees

1. A wider clutter-doppler spectrum spread with an increase of variance by a factor of $\sqrt{1 + r^2}$, where r is the ratio of the two carrier frequencies.

2. A mean clutter and target doppler frequency reduced by a factor of $1 - r$.

The performance of an MTI system is limited by the clutter spectra spread. Hence the effect of increasing doppler variance is very undesirable. The effect of reducing the mean clutter velocity may have some use. However, one should also realize that this same effect applies to the target doppler frequency. Therefore, in case the clutter is absent, some targets may not be detected.

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