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# Circuit Implementation of Spearman Rho Rank Detector for Scanning Radars

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| 20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)<br>A simple algorithm and circuit is devised for computing the Spearman rho test statistic used in detecting targets by scanning radars. The circuit requires $(n-1)$ comparators, $(n+1)$ adders, $n$ multipliers, $(n-1)$ one-bit shift registers of average length $(n-1)/2$ and a shift register of length $n$ where $n$ is the number of radar sweeps incorporated in the test. | | |

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# CIRCUIT IMPLEMENTATION OF SPEARMAN RHO RANK DETECTOR FOR SCANNING RADARS

## INTRODUCTION

The Spearman rho rank detector used in scanning search radars operates as follows: After the $k$th pulse transmission, the set of observations $[X(k)]$ over the last $n$ pulse transmission are stored at a given range cell.* The rank information is normally specified by the vector of ranks $[R(k)] = [r_1(k), r_2(k), \ldots r_n(k)]$, where $r_i(k)$ is the rank of the observation $x_i(k)$. For example, if $[X(k)] = [3,5,9,1]$, $[R(k)] = (2,3,4,1)$. The Spearman rho test computes the test statistic

$$T(k) = \sum_{i=1}^{n} (n-i)\, r_i(k) \tag{1}$$

at each range cell after each new pulse transmission $k$. The objective of this paper is to describe a simple circuit for computing the vector of ranks $[R(k)]$.

## DEFINITIONS

The vector of observations $[X(k)]$ contains $n$ elements for all values of $k = 1, 2, 3, \ldots$ and is defined by

$$[X(k)] = [x_1(k), x_2(k), \ldots x_n(k)] \; .$$

The vectors $[X(k)]$ and $[X(k+1)]$ are related through their elements by

$$x_i(k) = x_{i+1}(k+1) \; . \tag{2}$$

For example, consider a long sequence of positive numbers,

$$\ldots\ldots, 33, 56, 22, 67, 82, 15, 11;$$

then

$$\begin{aligned}
[X(1)] &= (67, 82, 15, 11)\\
[X(2)] &= (22, 67, 82, 15)\\
[X(3)] &= (56, 22, 67, 82)\\
[X(4)] &= (33, 56, 22, 67)\\
&\text{etc.}
\end{aligned}$$

Note: Manuscript submitted November 21, 1973.

*V.G. Hansen, "Detection Performance of Some Nonparametric Rank Tests and an Application to Radar," IEEE Trans. IT-16 (No. 3), 309-318 (May 1970).

The rank information is specified by a vector of ranks $[R(k)] = [r_1(k), r_2(k), \ldots r_n(k)]$ where $r_i(k)$ is the rank of the observation $x_i(k)$ defined by

$$r_i(k) = \sum_{j=1}^{n} u_{ji}(k) + 1 \tag{3}$$

where

$$u_{ji}(k) = \begin{cases} 1 & \text{if} \quad x_i(k) \geqslant x_j(k) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad x_i(k) < x_j(k) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad i = j \end{cases} \tag{4}$$

for $j = 1, 2, \ldots, n$. The auxiliary variable $v_{ji}(k)$ is defined as

$$v_{ji}(k) = \begin{cases} 1 & \text{if} \quad x_j(k) \geqslant x_i(k) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad x_j(k) < x_i(k) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad i = j\,. \end{cases} \tag{5}$$

The variables $u_{ji}(k)$ and $v_{ji}(k)$ are outputs of comparators. The vectors are defined by

$$[V(k)_i] = [v_{1i}(k),\ v_{2i}(k),\ \ldots,\ v_{ni}(k)]$$

$$[D(k)_i] = [u_{1i}(k-n),\ u_{ni}(k-1),\ u_{n-1,i}(k-2),\ u_{n-2,i}(k-3),\ \ldots\ u_{2i}(k-n+1)]$$

$$[R^T(k-1)] = [r_1(k-1),\ r_2(k-1),\ \ldots\ r_{n-1}(k-1)]\,.$$

A simple algorithm for computing $[R(k)]$ from $[X(k)]$ is next developed.

*Algorithm*—Two lemmas are first proved which are used in the main theorem.

*Lemma 1.* The elements of $u_{ji}(k)$ are related by

$$u_{j+1,i}(k) = u_{j,i-1}(k-1)\,.$$

*Proof*: By Eq. (4)

$$u_{j,i-1}(k-1) = \begin{cases} 1 & \text{if} \quad x_{i-1}(k-1) \geqslant x_j(k-1) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad x_{i-1}(k-1) < x_j(k-1) \quad \text{and} \quad i \neq j \\ 0 & \qquad i = j \end{cases}$$

for $j = 1, 2, \ldots, n$. We substitute Eq. (2) into the above definition

$$u_{j,i-1}(k-1) = \begin{cases} 1 & \text{if} \quad x_i(k) \geqslant x_{j+1}(k) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad x_i(k) < x_{j+1}(k) \quad \text{and} \quad i \neq j \\ 0 & \qquad\qquad\qquad\qquad\qquad\quad\; i = j \end{cases}$$

for $j = 1, 2, \ldots, n$. By Eq. (4) the above expression becomes

$$u_{j,i-1}(k-1) = u_{j+1,i}(k) .$$

This completes the proof.

*Lemma 2.* The comparators $v_{j1}(k)$ and $u_{1i}(k)$ outputs are equal.

*Proof*: By Eq. (4)

$$u_{1i}(k) = \begin{cases} 1 & \text{if} \quad x_i(k) \geqslant x_1(k) \quad \text{and} \quad i \neq j \\ 0 & \text{if} \quad x_i(k) < x_1(k) \quad \text{and} \quad i \neq j \\ 0 & \qquad\qquad\qquad\qquad\quad\; i = j . \end{cases}$$

But this is just the definition of $v_{j1}(k)$; therefore $u_{1i}(k) = v_{j1}(k)$. This completes the proof.

*Theorem.* The vector of ranks at the $k$th step can be computed from the vector of ranks at the $(k-1)$ step, and the results of comparisons between only the first element $x_1(\cdot)$ and all other elements $x_j(\cdot)$ by

$$[R(k)] = [r_1(k), [R^T(k-1)]] + [V(k)_1] - [D(k)_1] .$$

*Proof*: The first elements in the vector are related by

$$r_1(k) = r_1(k) + v_{11}(k) - u_{11}(k-n) .$$

By definition $v_{11}(k) = u_{11}(k-n) = 0$. The general elements in the vector are related by

$$r_i(k) = r_{i-1}(k-1) + v_{j1}(k) - u_{n-j+1,1}(k-j) . \tag{6}$$

By Lemma 2, $v_{j1}(k) = u_{1i}(k)$ and by recursively applying Lemma 1 to $u_{n-j+1,1}(k-j)$, we find that

$$u_{n-j+1,1}(k-j) = u_{n+1,i}(k)$$

The first term in Eq. (6) is defined by Eq. (3),

$$r_{i-1}(k-1) = \sum_{j=1}^{n} u_{j,i-1}(k-1) + 1 .$$

Applying Lemma 1, we find that

$$r_{i-1}(k-1) = \sum_{j=1}^{n} u_{j+1,i}(k) + 1 .$$

Defining the index $m = j + 1$, we find that

$$r_{i-1}(k-1) = \sum_{m=2}^{n+1} u_{m,i}(k) + 1 .$$

Combining the three terms in Eq. (6), we obtain

$$r_{i-1}(k-1) + v_{j1}(k) - u_{n-j+1,1}(k-j) = \sum_{m=2}^{n+1} u_{m,i}(k) + 1 + u_{1i}(k) - u_{n+1,i}(k)$$

$$= \sum_{m=1}^{n} u_{m,i}(k) + 1 ,$$

which by definition is $r_i(k)$. This completes the proof.

### Example

A circuit for computing the ranks of the vector $[X(k)]$ of length $n = 7$ is shown in Fig. 1. The new observations are shifted into a shift register on each clock cycle $k$ and their resulting ranks appear in the registers at the output of the adders. On the $k$th clock cycle, the timing pulse is delayed so that the vector $[R(k-1)]$ is present in the rank registers so that the vector $[R^T(k-1)]$ is formed. After the delayed timing pulse $[R(k)]$ appears in the rank registers. The terms $v_{11}(k)$ and $u_{11}(k)$ are not used since they are always zero. The circuit is initialized by placing all zeros in all storage elements before clocking in a sequence of positive numbers. After at least $n$ or more clock cycles the correct rank of the sequence appearing in the shift register appears in the rank registers. The Spearman rho test is then computed by Eq. (1). The same circuit can be modified to compute the test statistic $T(k)$ for $m$ range cells by making all storage elements in Fig. 1 $m$ times as longer than are shown.

## SUMMARY

An algorithm and circuit was devised for computing the vector of ranks $[R(k)]$. The circuit requires $(n-1)$ comparators, $n$ adders, $2(n-1)$ and gates, $(n-1)$ one-bit shift registers of average length $(n-1)/2$, $n$ registers, and a shift register of length $n$ which contains the radar observations. The Spearman rho test is computed from the vector of ranks $[R(k)]$ with $n$ multipliers and one adder. If one wishes to implement $m$ range cells, the total number of storage locations is multiplied by $m$. The circuit is reasonably simple to implement and is fast enough to use in real time for conventional scanning search radars.

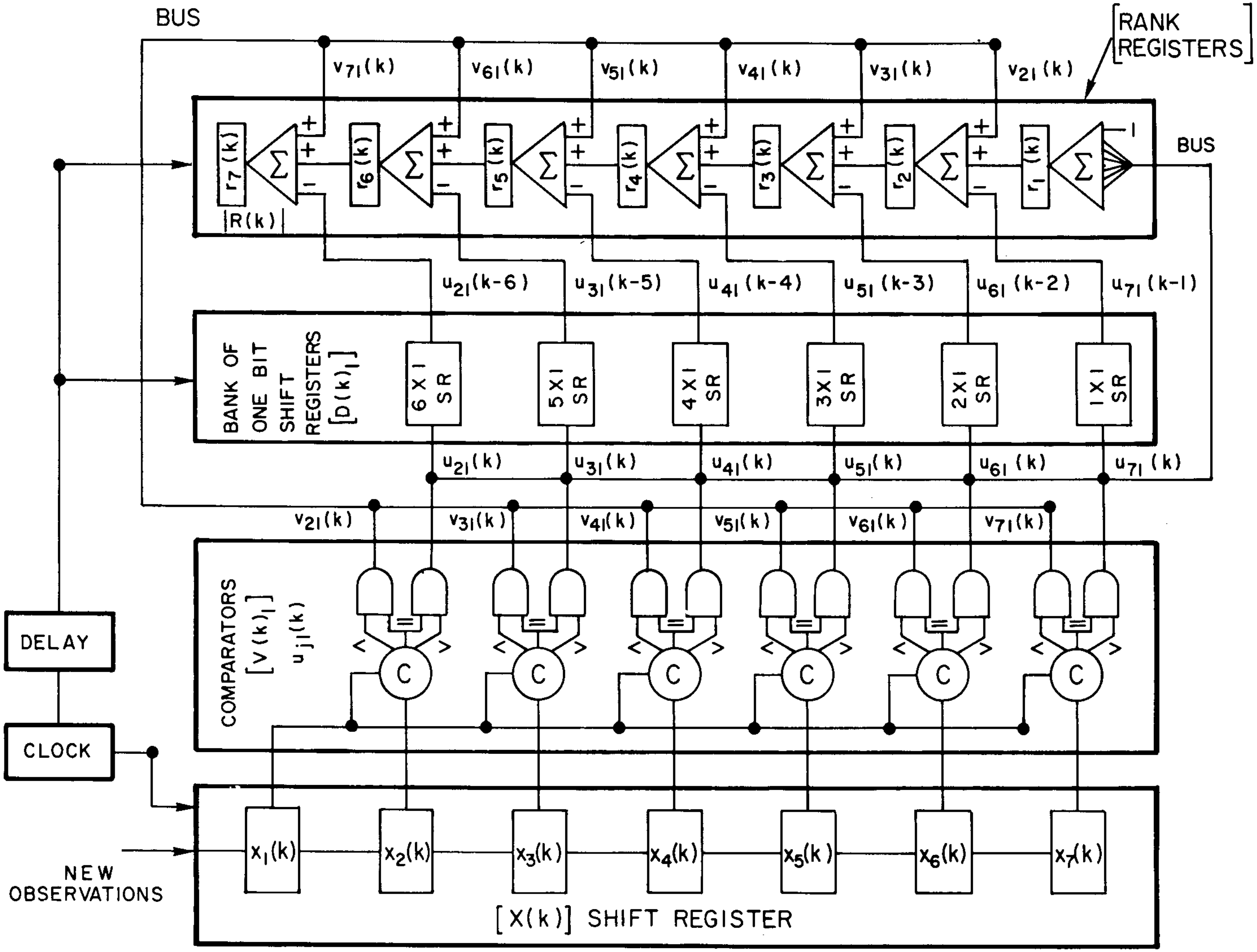


**Fig. 1—Circuit for computing ranks $n = 7$**