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Optimum Packing of Hollow Spheres in Buoyancy Materials

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ABSTRACT

The concept of close packing uniform-size buoyant glass spheres provides an effective means of attaining maximum buoyancy and strength in foam-type flotation materials. The use of this densest known form of packing results in a possible 74% volume fraction of spheres as compared to the 60 to 65% presently attained in syntactic foams. In addition, the volume fraction can be increased to approximately 90% by infiltration with smaller spheres, such as a microsphere mix. This technique could result in a buoyancy system with a density of 30 lb/cu ft or less.

Close packing of uniform-size spheres can be achieved practically by pouring them into inverted regular square and triangular pyramids, which automatically arrange the spheres into a close-packed array. A plate of close-packed material can then be fabricated by using the well-known geometric fact that regular square and triangular pyramids perfectly pack a plate equal in thickness to the height of the square pyramid. The use of plane-faced pyramids results in a lower volume fraction than the 74% theoretically possible. However, the excess voids which cause this loss in packing efficiency can be largely eliminated by contouring the faces of the pyramids to the shape of the spheres.

A variation of the above technique, which also eliminates most of the excess voids, consists in using a very large truncated pyramid with a properly contoured bottom instead of the two pyramids. Pouring spheres into this type of form automatically produces a close-packed plate.

Besides increasing the volume fraction of spheres, a close-packed array should increase the resistance of the spheres to buckling and should provide a homogeneous material with more uniform properties.

The concept of modules in a close-packed array has led to the consideration of the actual role of the resin in a syntactic foam. It would appear feasible to fabricate modules of relatively thick-walled spheres in a close-packed array without a solid resin matrix, a procedure which would provide much greater buoyancy than is attainable with resinous systems. Experimental verification of this concept is obviously needed.

PROBLEM STATUS

This is a final report on one phase of the problem; work on this problem is continuing.

AUTHORIZATION

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NRL Problem F01-14
Project RR 009-03-45-5805
NRL Problem F01-17
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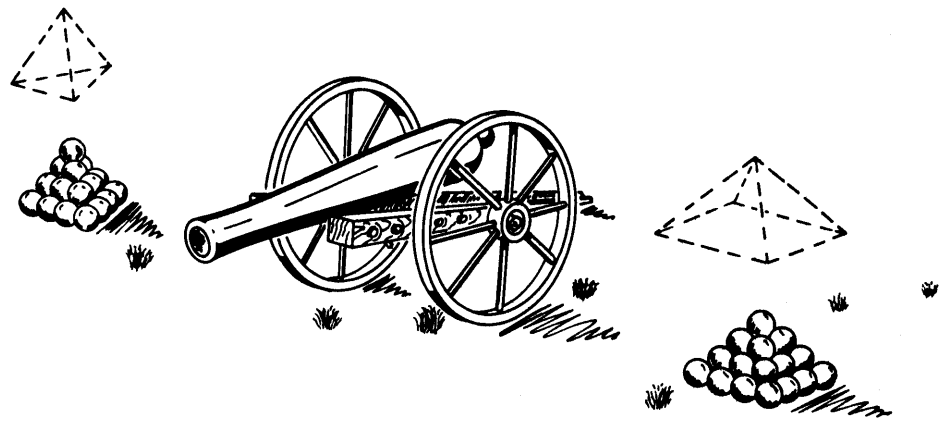

Frontispiece - Historically familiar examples of close-packed spheres

OPTIMUM PACKING OF HOLLOW SPHERES IN BUOYANCY MATERIALS

BACKGROUND AND INTRODUCTION

The Navy's interest in operations and research projects to be carried out deep in the ocean requires the availability of manned vehicles which are not only safe at great pressures from a structural point of view but also satisfy the requirement of positive buoyancy. Since it does not appear possible at present to build the pressure hull from any type of buoyant or even very light material, it has become necessary to develop highly buoyant, pressure-resistant materials to combine with the hull to provide a net positive buoyancy for the vehicle.

An additional need for similar, but not necessarily identical, flotation material occurs in such projects as establishing and maintaining platforms at specified depths in the ocean for long periods of time, possibly years. This type of application has the added requirement of resistance to long-term deterioration.

The syntactic foam now available for such purposes falls far short of meeting the buoyancy and strength targets of the more advanced projects, such as DSSP, now being undertaken.

The need for much stronger and lighter flotation materials for such advanced projects requires the development of the best possible technology, to be followed, if necessary, by the development of materials to implement this technology.

SCOPE OF THIS INVESTIGATION

As part of a consultative support program for DSSP, the Ocean Materials Criteria Branch of NRL has been studying methods of improving the strength, reliability, and buoyancy of flotation materials. This program has primarily taken the form of studies, from the geometrical point of view, of improved methods of packing spherical buoyant material to produce a greater net buoyancy.

This report presents these improved methods of packing, coupled with a brief discussion of the feasibility of nonresinous buoyancy systems.

GEOMETRICAL CONSIDERATIONS

This study of the "geometry" of packing consists of a combined experimental and theoretical analysis of the effect of various geometric parameters on the efficiency of arrangement of large numbers of spheres in a container. Particular attention is given to the effect of these parameters on the strength and density of a foam-type material which uses these spheres as an aggregate.

The primary parameters of interest are (a) the geometrical arrangement of the spheres, (b) the shape and size of the packing container, (c) the absolute and relative sizes of the spheres, and (d) the distance, if any, separating the spheres. It should be

mentioned that the shape of the container is a doubly important parameter, since it not only affects the packing of the spheres within but also determines the usefulness of the resulting module for assembly into larger usable shapes.

The following analysis is designed to provide an estimate of the optimum conditions which are geometrically possible.

CLOSE PACKING OF UNIFORM-DIAMETER SPHERES IN PYRAMIDS

The densest known packing of uniform-size spheres in space is accomplished with a close-packed array. The easiest way to visualize this type of packing is to consider a plane surface of infinite extent covered with a layer of touching spheres in a square array. If a second layer of spheres is then laid down, equilibrium requires that they nest into the first layer and form another square array shifted diagonally with respect to the first array. This process is illustrated in Fig. 1. Successive layering of spheres in this manner produces close packing, and the resulting volume fraction (VF) of spheres is 74%. (Appendix A gives the calculation of this value and also the pertinent geometrical derivations.) A similar process for achieving close packing using layers of spheres in a hexagonal or triangular array is illustrated in Fig. 2.

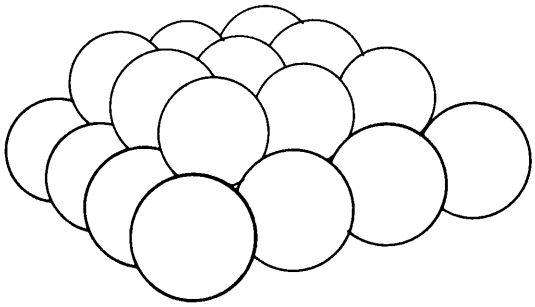

Fig. 1 - Close packing based on square array

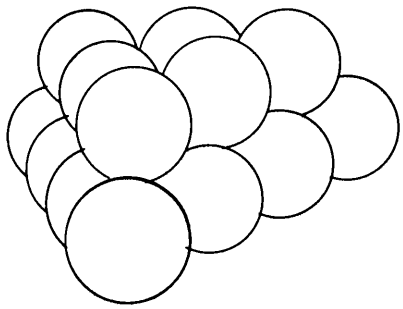

Fig. 2 - Close packing based on hexagonal array

There are two simple, structurally usable geometric shapes which automatically stabilize the close-packed array—the regular triangular pyramid (or tetrahedron) and the regular square pyramid. Illustrations of spheres as they would appear after having been poured into such open pyramidal forms are shown in Fig. 3. The arrays thus created are actually the geometrical equivalents of the historically familiar methods of stacking cannon balls, as is shown in the frontispiece. Although the achievement of close packing in these two shapes is independent of the diameter of the spheres, so that the same forms can be used for packing different sizes, the VF of spheres actually achieved is not. The 74% VF mentioned previously as being characteristic of close packing in general is only the upper limit for the present packing process, since the finite number of layers coupled with the excess voids created at the surfaces and edges of the forms serve to decrease this value. However, for any combination of sphere and pyramid sizes likely to be used, the VF actually achieved is still higher than for the random packing methods presently used for buoyant material. The effective VF and methods of improving it will be discussed in detail in the following sections.

The method of pouring the spheres into the square pyramid so that they fall into the proper positions is relatively noncritical, although it appears that the speed with which

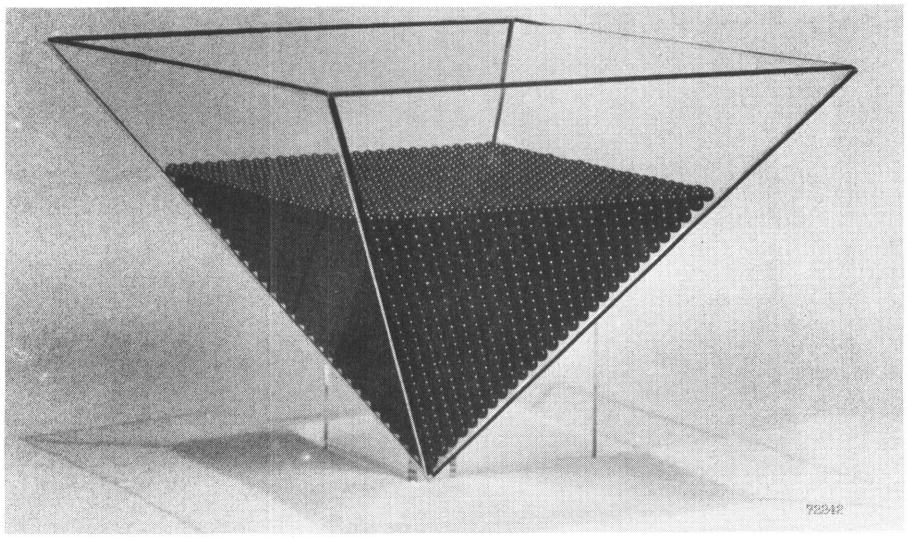

Fig. 3a - Close-packed spheres in square pyramidal form

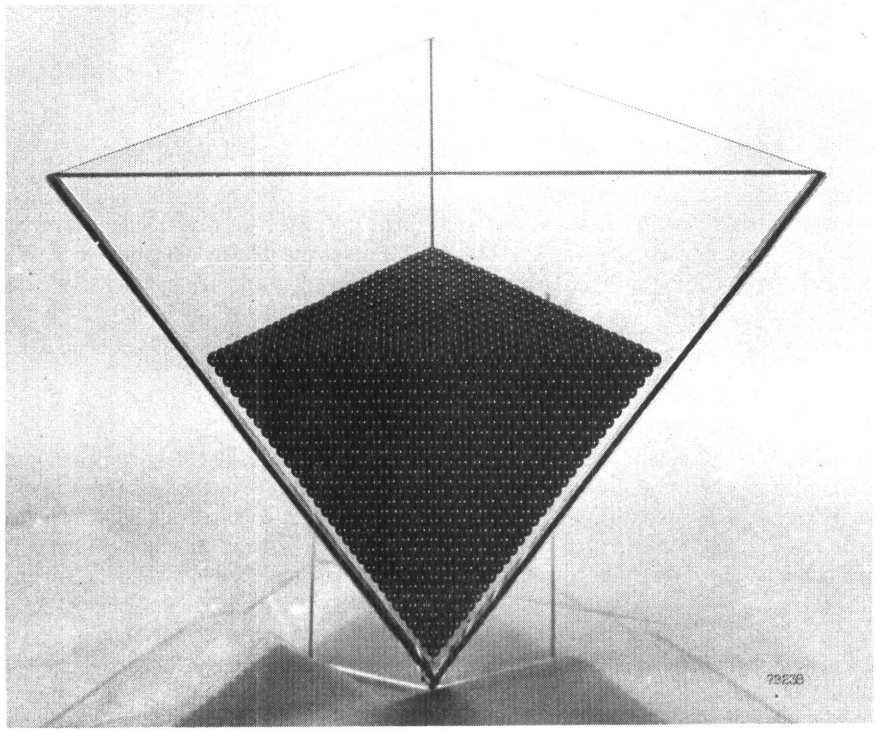

Fig. 3b - Close-packed spheres in hexagonal pyramidal form

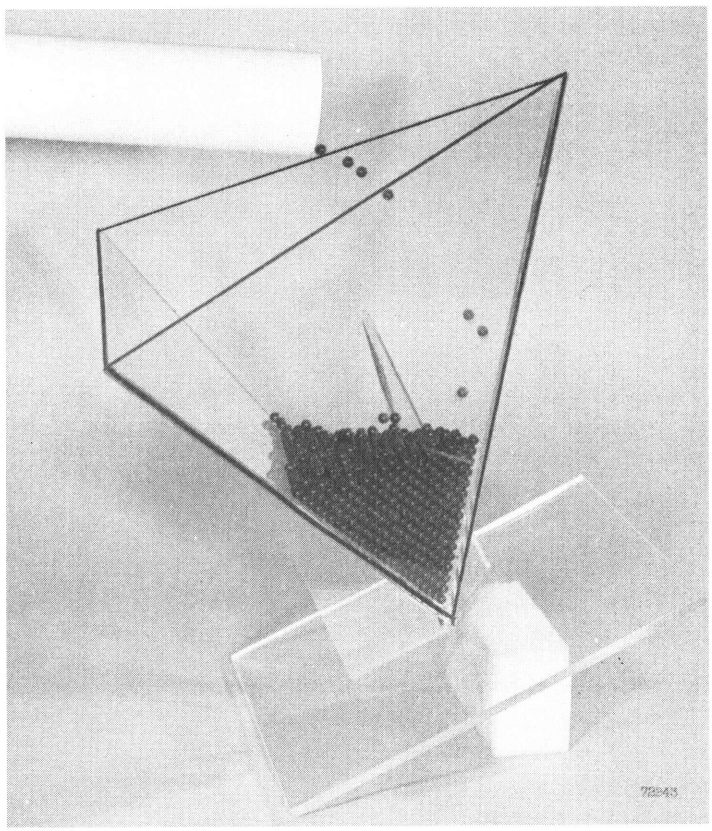

Fig. 4 - Tipping of tetrahedron to facilitate packing

they should be poured depends somewhat on the density of the spheres. However, in the case of the tetrahedron, an excess of equilibrium positions complicates the procedure. A practical way of overcoming this problem is to tip the form as shown in Fig. 4 and pour the spheres into the upper solid angle so that they run down over the top surface and build up from the lower face.

ASSEMBLY OF PYRAMIDS AND EFFECTIVE VOLUME FRACTION

The use of triangular and square pyramids to form a plate equal in thickness to the height of the square pyramid is illustrated in Fig. 5. As mentioned previously, the use of plane-faced pyramids provides a VF somewhat lower than the 74% theoretically possible in close packing. A graphical description of this situation is presented in Fig. 6, in which the actual effective VF for such a plate is plotted against the number of layers of spheres and equivalently against the ratio of pyramid size to sphere size. It is seen from this graph that the effective VF quickly approaches the theoretical limit. A detailed analysis of the VF for the individual shapes will be found in Appendix A.

CONTOURING OF FORMS TO INCREASE EFFECTIVE VOLUME FRACTION

For those applications in which maximum VF is of paramount importance, it is possible to eliminate approximately 75% of the excess voids and thereby approach very closely the 74% VF of close packing by contouring the sides of the pyramidal forms (and

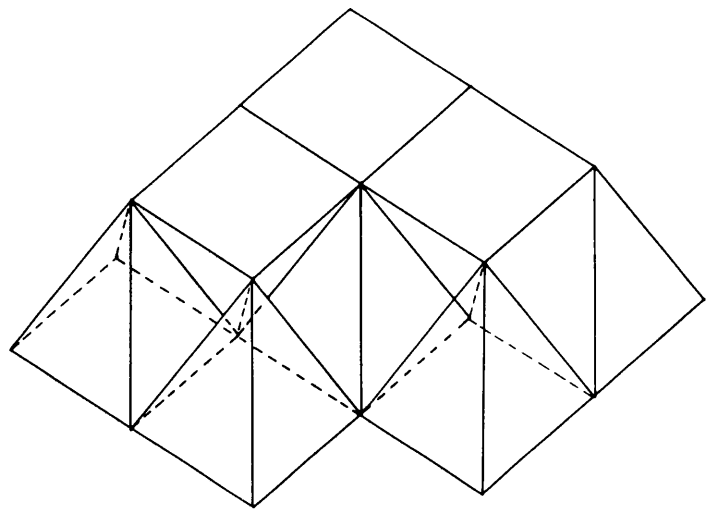

Fig. 5 - Formation of plate from pyramids

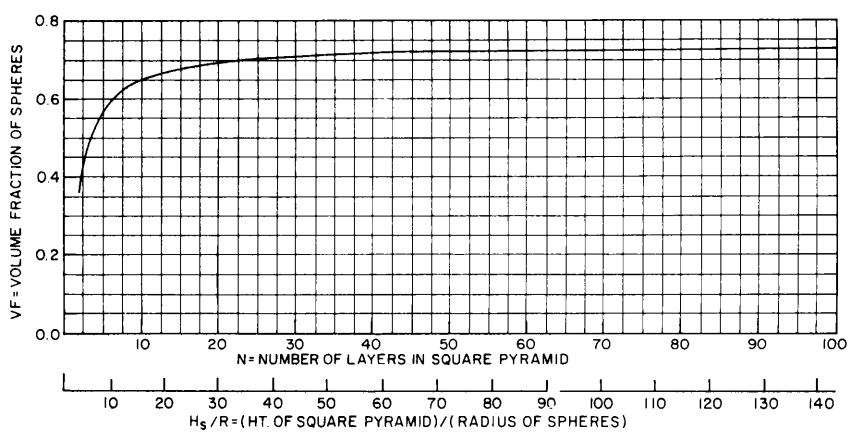

Fig. 6 - Graph of effective volume fraction (VF)

also, necessarily, the top surface of the tetrahedron). There are several ways in which this contouring can be accomplished, but the simplest appears to be a stepping of the sides, as illustrated in Fig. 7, where a sectional view is provided of portions of three pyramids as they would mesh in this case. Of course, such contoured forms would no longer retain their usefulness for other sphere sizes. Although this report is written primarily with the plane-sided forms in mind, the results presented can be easily extended by the reader to the case of contoured forms.

PACKING OF RELATED SIZE-SENSITIVE SHAPES

Besides the two basic pyramids which automatically stabilize close packing of uniform spheres regardless of their size, there are related shapes which will close pack spheres of any particular size. Two such shapes are described on the following page.

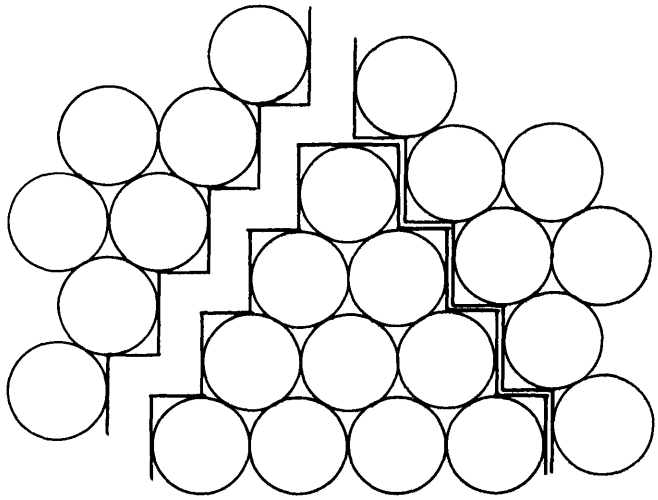

Fig. 7 - Meshing of contoured pyramids

It can be seen from Fig. 5 that when pyramidal forms are arranged to form a plate this plate has inclined edges and is actually the truncated section of a larger pyramid. In the case where large solid plates of arbitrary thickness and extent are the desired end product, they can be quickly close packed by pouring the spheres into a truncated form with a properly contoured bottom, as shown in Fig. 8a. This contouring of the bottom can consist of the simulation of the top surface of one layer of spheres or of any contouring which will cause the first layer of poured spheres to form a square array. The use of this method for packing a plate provides the same increase in effective VF as does the method described in the preceding section of contouring the smaller individual pyramids.

Similarly, proper contouring of the bottom of a rectangular parallelepiped (or box) whose inside dimensions are integral multiples of the sphere diameter also results in a close-packed array, as illustrated in Fig. 8b.

It should be pointed out, however, that such size-sensitive shapes are limited in application to relatively large spheres because of the contouring required and that they also require a considerable amount of care in packing, especially in the case of the rectangular box.

INFILTRATION WITH SMALLER SPHERES

For maximum buoyancy applications a method of increasing the VF is infiltration with smaller secondary spheres having a radius small enough (one-seventh or less of that of the primary spheres) to pass through the interstices of the primary array. Depending on the infiltrating material, the VF can be raised in this way from 74% to approximately 90%. Since the infiltrating material will not be close packed, it is not necessary that this material consist of uniform-size spheres.

STRUCTURAL ADVANTAGES OF CLOSE PACKING

A detailed analysis of the stress field around a glass sphere in a resin-bound array, whether close packed or not, is beyond the scope of this report. However, certain general conclusions are made possible by analogy with more easily analyzed systems.

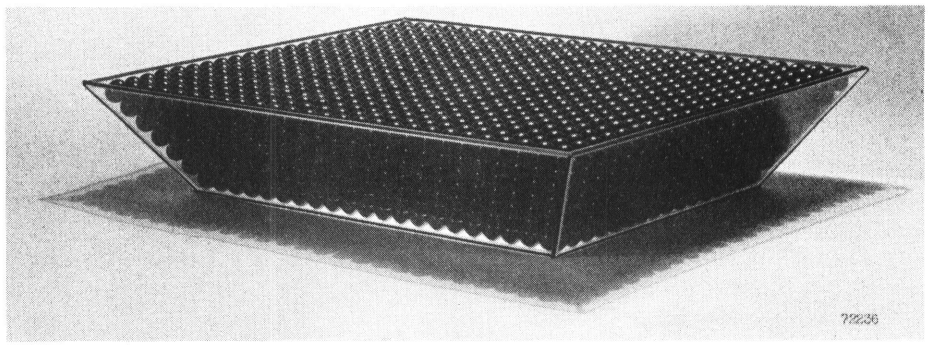

Fig. 8a - Close packing of truncated pyramidal form

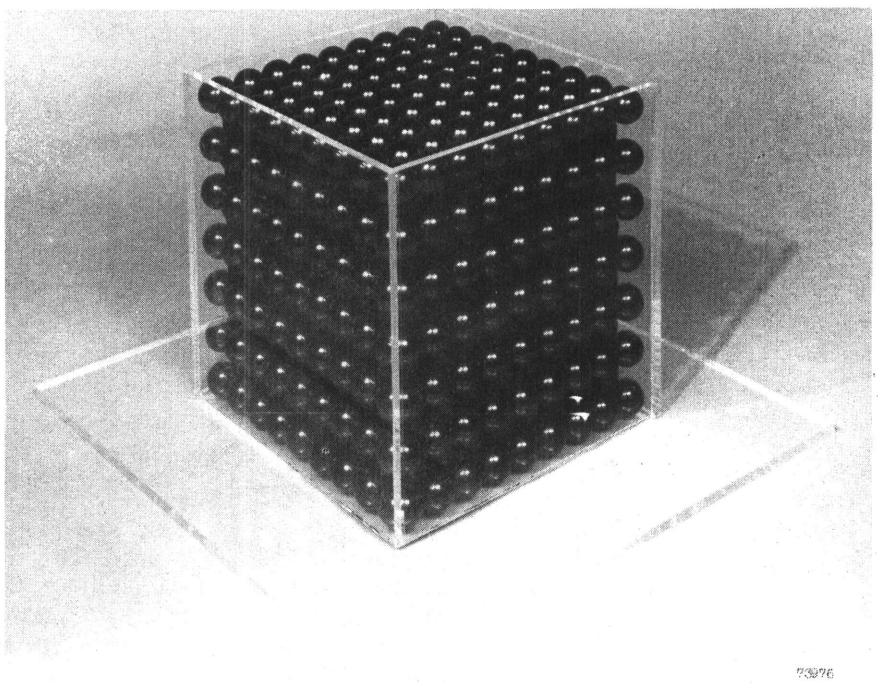

Fig. 8b - Close packing of rectangular box

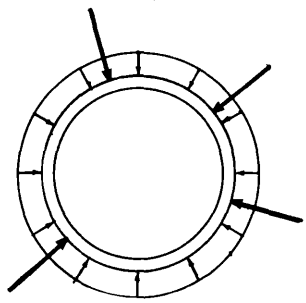

Fig. 9 - Arbitrarily loaded ring

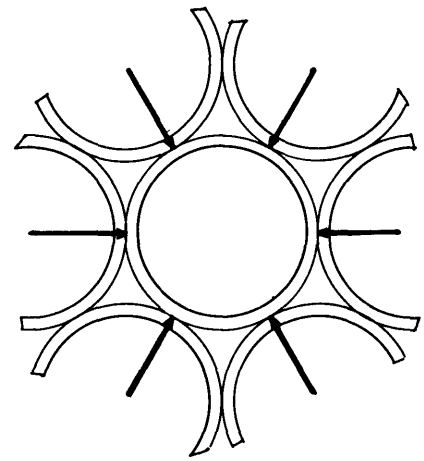

Fig. 10 - Ring loaded by other rings

Consider the circular ring in Fig. 9 loaded with a uniform hydrostatic pressure and with equal arbitrarily spaced concentrated normal loads. An application of any standard procedure for determining the critical buckling load shows that this critical load is raised by spacing the forces equally around the circumference and by increasing their number. This procedure of increasing the strength by increasing the number of loads obviously cannot be continued indefinitely since eventually other failure mechanisms (brittle fracture, etc.) become more critical. However, if the system of concentrated loads is considered to be a first approximation to the contact loadings of surrounding rings, as in Fig. 10, the number of concentrated loads is limited to six (close packing in two dimensions).

If this situation is considered to be the two-dimensional analog of the three-dimensional loading of a sphere in a close-packed array, it leads, more or less heuristically, to the conclusion that close packing of spheres increases their resistance to buckling.

THE EFFECT OF PACKING EFFICIENCY ON BUOYANCY

The effect of increased packing efficiency on the buoyancy of sphere-resin flotation materials depends, of course, on the density of the glass spheres used. Since the 3/4-in. primary spheres now available in limited quantities have a specific gravity (SG) of 0.45 (0.35 projected) and the presently available secondary infiltrating material (microsphere mix) a SG of 0.35 to 0.45, it appears reasonable to look at the buoyancy achieved at the two extremes. A VF of 73%, the value achievable with 3/4-in. spheres in a 12-in.-thick plate, is used for the primary close-packed array and a VF of 60% for the infiltrating material. The resin is assumed to have a SG of 1.2.

Table 1 illustrates that it is not possible to achieve high-buoyancy targets (such as that of DSSP) without infiltration of the stronger primary array with the weaker microsphere mix. Use of 0.45-SG primary spheres and 0.35-SG microsphere mix does produce a buoyancy which approaches desired goals. However, the best tabulated combination would probably be that of the 0.35-SG primary spheres and 0.45-SG (generally stronger than the 0.35 SG) microsphere mix. It appears that in general the greatest overall advantages would be gained by decreasing the density of the primary spheres and possibly increasing the strength of the infiltrating material. The buoyancy values computed above are based on the material properties at sea level. The effect of hydrostatic pressure

Table 1
Densities and Buoyancies for
Infiltrated and Uninfiltrated Material

Primary SG	Secondary SG	Density (lb/cu ft)	Buoyancy (lb/cu ft)
0.45	uninfiltrated	40.7	21.7
0.35	uninfiltrated	35.8	26.6
0.45	0.45	33.2	29.2
0.45	0.35	32.1	30.3
0.35	0.45	28.3	34.1
0.35	0.35	27.2	35.2

will be to increase the buoyancy slightly, since the bulk modulus of a sphere-resin material of this type is greater than that of water. Calculations of the magnitude of this increase require actual experimentally determined values of the bulk modulus for the materials to be used.

SPACING OF SPHERES

Up to this point it has been assumed that the spheres in the primary array touch each other and that the entire increase in VF due to close packing is available to increase the buoyancy of the resulting material. However, this touching of the spheres may present some difficulties, with respect to both contact stresses and sympathetic implosion, especially for the larger spheres. It may, therefore, prove beneficial to use part of the gain in packing efficiency to provide a spacing between the close-packed spheres by providing each sphere with a coating or thin shell of resin prior to packing. The advisability of this procedure can be established only by testing suitable prototype models.

SOME OBSERVATIONS ON THE FEASIBILITY OF NONRESINOUS BUOYANCY SYSTEMS

Although the previous sections on packing have been concerned primarily with optimum geometrical arrangement of spheres, the material density and buoyancy calculations were based on the assumption that the arrays of spheres would be infused with resin. This resin would then harden to form a buoyant "solid material" with bulk mechanical properties similar to those of presently used syntactic foams. This type of solid material, although undeniably easy to handle and form into larger and smaller usable shapes, is not without its disadvantages in buoyancy and strength.

Buoyancy: The adverse effect on the buoyancy of using any binding material heavier than water is obvious. In fact, the use of just a primary uninfiltrated array of close-packed spheres with a matrix, solid or fluid, having the same density as water would produce a buoyancy approaching the goals of such advanced projects as DSSP; a matrix with a SG of 0.8 (achievable with some oils) would result in a buoyancy which easily exceeds these goals.

Static Strength: The fact that resin curing produces a prestressing of the spheres is well known and easily visualized. However, the influence of the resin on the stress field around the spheres during hydrostatic loading of the composite material is not as immediately evident. In spite of the opinion held in some quarters that the presence of

the resin serves to decrease the effective pressure on the spheres, the calculations of Appendix B show that for the type of sphere and resin now used, or likely to be used in the future, the resin actually provides little, if any, reinforcement for the spheres. In fact for the heavier stronger spheres (SG $>$ 0.39), the effect of the resin is to increase the effective pressure on them. This excess pressure could become critical in the case of the thicker-walled primary spheres for which the mode of failure might very well be brittle fracture rather than buckling. This situation needs further theoretical and experimental investigation.

If these were the only characteristics of importance, the solid resin matrix could be replaced by water or some lighter fluid. The difficulty with this idea is that there is evidence that the resin matrix helps to inhibit the propagation of sympathetic implosion, especially for larger spheres, and the resulting possible catastrophic failure of the material. Therefore, if the resin is to be removed in the interest of improving the density and static strength, some means must be found to control or prevent the propagation of sympathetic implosions through the surrounding medium. The most likely methods of providing this control appear to be:

1. Development of buoyant spheres, from either stronger glasses and ceramics or light metals, which have a large enough margin in their static strengths to withstand the dynamic effect of the shock wave.

2. Compartmentizing of spheres, preferably using pyramidal shapes for close packing, in such a way as to prevent the propagation of the shock wave from one compartment to the next, thereby preventing the catastrophic failure of the whole buoyancy system.

3. The use of a viscous fluid or a nonsolid viscoelastic substance (possibly gelatinous and preferably lighter than water) to damp out the shock wave. This approach would probably also require compartmentizing for ease of handling and fabrication.

It seems possible that an application of one or more of these methods might lead to the goal of a satisfactory strong and buoyant material using only a close-packed, uninfiltrated array of spheres imbedded in a light homogeneous medium.

RELATED WORK OF OTHERS

Investigation of pertinent literature sources has not uncovered any evidence of prior studies involving either the achievement or the technological use of three-dimensional, close-packed arrays of uniform-size spheres. Those studies which have been made on the packing of nonrandom-size spheres have been concerned with the experimental determination of VF for containers, such as cylinders and prisms, which, due to improper shape, size, or use, cannot induce anything but random orientation. (An example of such a situation is illustrated in Fig. 11. Here the same form as that used for close packing in Fig. 8b induces random packing because of the absence of a contoured bottom.) The most comprehensive study of this type appears to be that of McGeary.*

Prior investigations of packing directly related to flotation materials have been primarily limited to experimental studies of ways to increase the VF of a random microsphere mix by such techniques as vibration during pouring. However, a significant exception to this procedure exists in the work now being done at the Naval Applied Science Laboratory (NASL), as described by Stechler and Poneros.† At first glance, the NASL approach of using modules which, when assembled, produce layers of large (3-in.) spheres

*R. K. McGeary, J. Am. Ceram. Soc. 44:513 (1961).
†B. Stechler and G. Poneros, Ocean Eng. 1(No. 1):17 (1968).

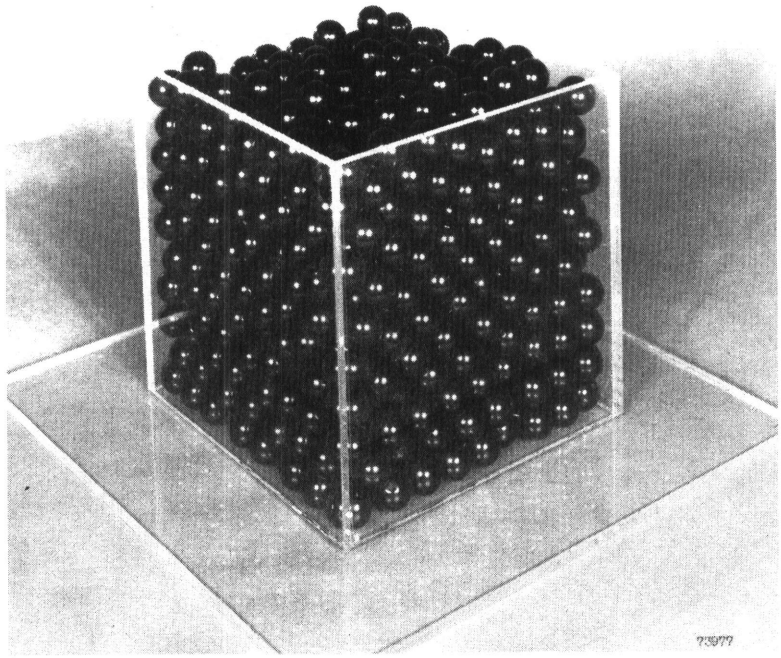

Fig. 11 - Random packing of rectangular box

arranged in a plane hexagonal array appears similar to the NRL approach described in this report. However, since the NRL approach uses a three-dimensional, close-packed array, which can be achieved by simply pouring the spheres into the proper forms, and the NASL approach uses close packing in only two dimensions and requires the placing of the spheres into position individually, it becomes evident that the two approaches are actually different and distinct.

ACKNOWLEDGMENTS

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Appendix A

GEOMETRICAL PROPERTIES OF ARRAYS OF SPHERES CLOSE PACKED IN PYRAMIDS*

Since the geometrical expressions of interest for the two pyramids, although different, are similarly derived, a process of parallel development is used, with quantities and expressions labeled with the subscripts S for the square pyramid and T for the tetrahedron. The geometric parameters of interest are the pyramid height (h), the sphere radius (r), and the number of layers of spheres containable in a pyramid (N). Only two of these parameters may be chosen independently, with the third being defined by

$$\begin{aligned} h_S &= r\left[(N-1)\sqrt{2}+\sqrt{3}+1\right] \\ h_T &= r\left[\frac{2\sqrt{2}}{3}(N-1)+4\right] . \end{aligned} \tag{A1}$$

The volumes of the pyramids are given by

$$\begin{aligned} V_S &= \frac{2}{3}h_S^3 \\ &= \frac{2}{3}r^3\left[(N-1)\sqrt{2}+\sqrt{3}+1\right]^3 \\ V_T &= \frac{\sqrt{3}}{8}h_T^3 \\ &= \frac{\sqrt{3}}{8}r^3\left[(N-1)\frac{2\sqrt{2}}{3}+4\right]^3 . \end{aligned} \tag{A2}$$

The volume of spheres containable in these pyramids is

$$\begin{aligned} (V_{SP})_S &= \frac{4}{3}\pi r^3 \sum_1^N n^2 = \frac{4}{3}\pi r^3 \frac{N(N+1)(2N+1)}{6} \\ (V_{SP})_T &= \frac{4}{3}\pi r^3 \sum_1^N \frac{n(n+1)}{2} = \frac{2}{9}\pi r^3 N(N+1)(N+2) . \end{aligned} \tag{A3}$$

Therefore, the volume fractions are given by

$$\begin{aligned} (VF)_S &= \frac{(V_{SP})_S}{V_S} = \frac{\pi}{3}\frac{N(N+1)(2N+1)}{\left[(N-1)\sqrt{2}+\sqrt{3}+1\right]^3} \\ (VF)_T &= \frac{(V_{SP})_T}{V_T} = \frac{16\pi}{9\sqrt{3}}\frac{N(N+1)(N+2)}{\left[(N-1)\frac{2\sqrt{2}}{\sqrt{3}}+4\right]^3} . \end{aligned} \tag{A4}$$

If we let $N \to \infty$, we find $(VF)_S = (VF)_T = 0.741$, which is the well-known value for close packing.

To find the effective overall VF for a plate formed from these two pyramids, it is instructive to look at Fig. 5. Figure 5 illustrates that h_T is determined by a choice of h_S

*In the following discussion, those expressions which follow directly from an examination of geometrical figures are presented without proof.

and also that the number of square and triangular pyramids is equal. Using the easily derivable relation $h_T = 2/\sqrt{3}\, h_S$, we find that $V_S = 2V_T$ and that therefore, the effective VF of a plate as in Fig. 5 is

$$VF = \frac{2}{3}(VF)_S + \frac{1}{3}(VF)_T$$

$$= \frac{2\pi}{9}\,\frac{N(N+1)(2N+1)}{\left[(N-1)\sqrt{2}+\sqrt{3}+1\right]^3} + \frac{16\pi}{27\sqrt{3}}\,\frac{N(N+1)(N+2)}{\left[(N-1)\dfrac{2\sqrt{2}}{\sqrt{3}}+4\right]^3}\,. \tag{A5}$$

A final quantity of interest is the percentage of pyramid surface area which is interior to the plate, since this provides a measure of the improvement in VF to be obtained by contouring all triangular surfaces as suggested in the text. Since, as is mentioned previously, there is an equal number of square and triangular pyramids, a representative area consists of eight interior equilateral triangles, each with area A_T, and one exterior square with area A_S. The percentage of interior area is then

$$\frac{8A_T}{8A_T + A_S} = \frac{8 \times 0.433\ell^2}{(8 \times 0.433 + 1)\ell^2} = 0.775\,. \tag{A6}$$

Therefore, by contouring the triangular sides of the pyramidal forms, the effective VF can be raised to the value VF′ given by

$$VF' = VF + 0.775(0.741 - VF)\,. \tag{A7}$$

Appendix B

THE PRESSURE ON A HOLLOW SPHERE IN A UNIFORM STRESS FIELD

The qualitative effect of a solid inclusion on the otherwise uniform hydrostatic stress field in an elastic medium can be determined by comparing the effective bulk modulus of the inclusion with that of the surrounding medium. If the inclusion has the larger bulk modulus, the pressure around it is raised; if the inclusion has the smaller modulus, the local pressure is lowered.

In the case of an inclusion consisting of a hollow spherical shell the effective bulk modulus K_s can be easily evaluated by using the definition

$$K_s = P/\left(\frac{\Delta V}{V}\right)$$

where

P = hydrostatic pressure

V = volume of sphere

ΔV = change in volume of sphere due to P.

Letting

$$V = 4/3\pi R^3$$

$$\Delta V = 4\pi R^2 \Delta R$$

$$\Delta R = \frac{P}{2E_s} \; (R^2/t) \; (1 - \nu_s)$$

where

R = radius of sphere (D = diameter)

t = thickness of sphere

E_s, ν_s = elastic constants of sphere material (in this case, glass)

we obtain

$$K_s = \frac{4}{3}\left(\frac{t}{D}\right) \frac{E_s}{1 - \nu_s} \; .$$

For the surrounding homogeneous medium (in this case, resin), the bulk modulus is defined in terms of its corresponding elastic constants as

$$K_R = E_R/3(1 - 2\nu_R) \; .$$

Therefore,

$$K_s/K_R = 4(E_s/E_R) \left[(1 - 2\nu_R)/(1 - \nu_s)\right] (t/D) \; . \tag{B1}$$

If we choose $E_s = 10 \times 10^6$, $\nu_s = 0.22$, $E_R = 0.4 \times 10^6$, and $\nu_R = 0.35$ as being representative of glass and resin, we find

$$K_s/K_R = 38.5(t/D) \ . \tag{B2}$$

Therefore, if P_s is the actual pressure acting on the sphere, the relations resulting are

$$\begin{aligned} &\text{for } D/t > 38.5, \ P_s < P \\ &\text{for } D/t < 38.5, \ P_s > P \ . \end{aligned} \tag{B3}$$

For a thin-walled sphere, the relation between the geometry and density of the sphere is

$$(SG)_s = 6/(t/D)\ (SG)_g \tag{B4}$$

where

$(SG)_s$ = specific gravity of the sphere

$(SG)_g$ = specific gravity of the component glass.

If we choose $(SG)_g = 2.5$ as being representative of glass, then

$$D/t = 15/(SG)_s \ . \tag{B5}$$

Substituting Eq. B5 in Eq. B2 we find that

$$\begin{aligned} &\text{if } (SG)_s < 0.39, \ P_s < P \\ &\text{if } (SG)_s > 0.39, \ P_s > P \ . \end{aligned} \tag{B6}$$

Since a value of 0.39 lies approximately midway in the range of SG values for presently available and projected buoyant glass spheres, the conclusion to be drawn is that the resin reinforces the lighter, weaker spheres but produces an excess loading on the heavier, stronger ones.

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13. ABSTRACT

The concept of close packing uniform-size buoyant glass spheres provides an effective means of attaining maximum buoyancy and strength in foam-type flotation materials. The use of this densest known form of packing results in a possible 74% volume fraction of spheres as compared to the 60 to 65% presently attained in syntactic foams. In addition, the volume fraction can be increased to approximately 90% by infiltration with smaller spheres, such as a microsphere mix. This technique could result in a buoyancy system with a density of 30 lb/cu ft or less.

Close packing of uniform-size spheres can be achieved practically by pouring them into inverted regular square and triangular pyramids, which automatically arrange the spheres into a close-packed array. A plate of close-packed material can then be fabricated by using the well-known geometric fact that regular square and triangular pyramids perfectly pack a plate equal in thickness to the height of the square pyramid. The use of plane-faced pyramids results in a lower volume fraction than the 74% theoretically possible. However, the excess voids which cause this loss in packing efficiency can be largely eliminated by contouring the faces of the pyramids to the shape of the spheres.

A variation of the above technique, which also eliminates most of the excess voids, consists in using a very large truncated pyramid with a properly contoured bottom instead of the two pyramids. Pouring spheres into this type of form automatically produces a close-packed plate.

— Continues

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Besides increasing the volume fraction of spheres, a close-packed array should increase the resistance of the spheres to buckling and should provide a homogeneous material with more uniform properties.

The concept of modules in a close-packed array has led to the consideration of the actual role of the resin in a syntactic foam. It would appear feasible to fabricate modules of relatively thick-walled spheres in a close-packed array without a solid resin matrix, a procedure which would provide much greater buoyancy than is attainable with resinous systems. Experimental verification of this concept is obviously needed.