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Noncoherent Detection of Nonfluctuating Targets in Contaminated-Normal Clutter

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ABSTRACT

Measurements of sea clutter using high-resolution radar indicate that clutter cross-section returns do *not* follow a Rayleigh distribution. In this report, a contaminated-normal distribution is proposed and found to be a good description of some of the clutter data taken by the high-resolution radar. The report then develops the theory necessary for detecting constant signals in contaminated-normal clutter when N pulses are summed noncoherently. Curves are presented for the threshold values, normalized to the median clutter voltage, versus the probability of false alarm for N = 1, 3, 10, and 30 pulses. Finally, probability of detection curves are presented for the previous N's and for false alarm probabilities ranging from 10^{-2} to 10^{-9}.

PROBLEM STATUS

This is the first of a series of reports dealing with the detection of targets in noise; it is an interim report on one phase of the problem, and work is continuing.

AUTHORIZATION

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NONCOHERENT DETECTION OF NONFLUCTUATING TARGETS IN CONTAMINATED-NORMAL CLUTTER

SUMMARY

Sea clutter measurements made by the Naval Research Laboratory using a high-resolution X-band radar with a 20-nsec pulse show that the clutter cross section is neither Rayleigh distributed (which has always been assumed) nor log-normally distributed (which has recently been suggested). It was conjectured that a contaminated-normal distribution might be a good fit for the NRL data.

In the first part of the report, the contaminated-normal clutter model is presented and its noncoherent distribution is found. A very good fit of some NRL data for a smooth sea is found by using a contaminated fraction of 0.25 and a ratio of standard deviations of 2.25, and a fair fit for a moderate sea is found by using the values 0.25 and 3.5 respectively.

In the second part, the density function of signal in clutter is derived. This density can be written only in terms of an integral which appears to have no known closed-form solution.

In the third part, the probabilities of false alarm and detection are developed for the noncoherent sum of N pulses by use of the characteristic function. This requires two applications of the Fourier transform. Both calculations were performed numerically using the fast Fourier transform.

In the last part, curves are presented for the threshold values, normalized to the median clutter voltage, versus the probability of false alarm for N = 1, 3, 10, and 30 pulses. Finally, probability of detection curves are presented for the previous N's and for false alarm probabilities ranging from 10^{-2} to 10^{-9}.

INTRODUCTION

High-resolution radar returns from the sea show that the log-normal probability distribution (1) gives a better description of the sea than the usually assumed Rayleigh distribution. Data reported by NRL (2) for a patch area of about 120 sq yd is plotted in Fig. 1 on probability paper with the clutter return in dB. While the log-normal distribution is a better fit than the Rayleigh, the tail of the log-normal is much too long. Since a contaminated-normal distribution (3) has a longer tail than the Rayleigh but a shorter tail than the log-normal, the contaminated-normal distribution will be investigated as a possible model for sea clutter.

CONTAMINATED-NORMAL CLUTTER MODEL

The contaminated-normal density is given by

$$p_{\gamma,K}(x) = \frac{(1-\gamma)}{\sqrt{2\pi\sigma^2}} \exp(-x^2/2\sigma^2) + \frac{\gamma}{\sqrt{2\pi K^2\sigma^2}} \exp(-x^2/2K^2\sigma^2) , \tag{1}$$

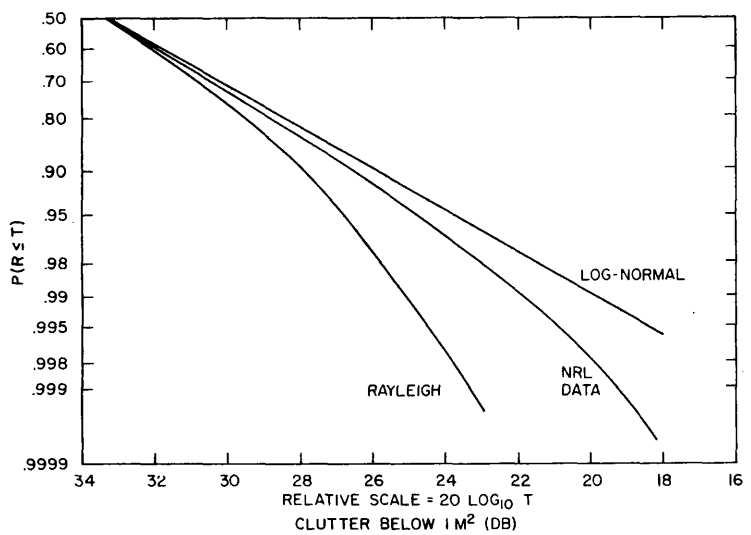


Fig. 1 - Comparison of log-normal and Rayleigh distributions with NRL data of a smooth sea

where γ is the contamination fraction, K is the ratio of the standard deviations of the gaussian densities, and σ is the standard deviation of uncontaminated distribution, i.e., $\gamma = 0$. (Note that $p_{\gamma,K}(x)$ is the sum of gaussian densities, not of gaussian variables; hence $p_{\gamma,K}(x)$ is not a gaussian density.)

Since the clutter data in Fig. 1 are obtained as the output of a noncoherent linear (envelope) detector, it is necessary to calculate the output of an envelope detector when the inphase and quadrature components of clutter are independent and have contaminated-normal distribution. The joint density of the independent clutter components, x and y respectively, is

$$\begin{aligned} p_{\gamma,K}(x,y) &= p_{\gamma,K}(x)p_{\gamma,K}(y) \\ &= \frac{1}{2\pi\sigma^2}\left[(1-\gamma)\exp(-x^2/2\sigma^2) + \frac{\gamma}{K}\exp(-x^2/2K^2\sigma^2)\right] \\ &\quad \times \left[(1-\gamma)\exp(-y^2/2\sigma^2) + \frac{\gamma}{K}\exp(-y^2/2K^2\sigma^2)\right]. \end{aligned} \tag{2}$$

Let

$$x = R\cos\theta \tag{3}$$

and

$$y = R\sin\theta \tag{4}$$

so that $R = \sqrt{x^2 + y^2}$ is the output of an envelope detector. Now, the joint density of R and θ is related to the joint density of x and y by the Jacobian transformation

$$p_{\gamma,K}(R,\theta) = \begin{bmatrix} \frac{\partial x}{\partial R} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial R} & \frac{\partial y}{\partial \theta} \end{bmatrix} p_{\gamma,K}(x,y)$$

$$= R p_{\gamma,K}(R\cos\theta,\ R\sin\theta). \tag{5}$$

Substituting Eqs. (2), (3), and (4) into Eq. (5) yields

$$p_{\gamma,K}(R,\theta) = \frac{R}{2\pi\sigma^2}\left\{(1-\gamma)^2\exp(-R^2/2\sigma^2) + \frac{\gamma^2}{K^2}\exp(-R^2/2K^2\sigma^2)\right.$$

$$+ \frac{\gamma(1-\gamma)}{K}\exp\left[-R^2\left(\sin^2\theta + \frac{\cos^2\theta}{K^2}\right)\Big/2\sigma^2\right]$$

$$\left. + \frac{\gamma(1-\gamma)}{K}\exp\left[-R^2\left(\cos^2\theta + \frac{\sin^2\theta}{K^2}\right)\Big/2\sigma^2\right]\right\}. \tag{6}$$

The probability density of R is obtained from Eq. (6) by

$$p_{\gamma,K}(R) = \int_0^{2\pi} p_{\gamma,K}(R,\theta)\,d\theta, \tag{7}$$

and the probability distribution of R is

$$P_{\gamma,K}(R \leq T) = \int_0^T p_{\gamma,K}(R)\,dR\ . \tag{8}$$

Unfortunately, the integral in Eq. (7) can be evaluated only numerically or in terms of Bessel functions. Either of these two solutions makes the evaluation of Eq. (8) rather difficult. However, the order of integration can be inverted and the procedure simplified as follows:

$$P_{\gamma,K}(R \leq T) = \int_0^T \int_0^{2\pi} p_{\gamma,K}(R,\theta)\,d\theta\,dR$$

$$= \int_0^{2\pi} \int_0^T p_{\gamma,K}(R,\theta)\,dR\,d\theta$$

$$= 1 - (1-\gamma)^2\exp(-T^2/2\sigma^2) - \gamma^2\exp(-T^2/2K^2\sigma^2)$$

$$- \frac{4\gamma(1-\gamma)}{K\pi}\int_0^{\pi/2} \frac{\exp[-T^2(\sin^2\theta + \cos^2\theta/K^2)/2\sigma^2]}{\sin^2\theta + \cos^2\theta/K^2}\,d\theta \tag{9}$$

Next, several values of γ and K were chosen, the integral in Eq. (9) was evaluated numerically, and the calculated distributions were compared to the NRL data (with σ being determined by requiring that the 50% points of the contaminated-normal distribution and NRL data occur at the same clutter value). A typical example is shown in Fig. 2. As indicated by Fig. 2, γ needs to be increased, since the probability of the center points (32 to 24 dB) is too low, and K needs to be reduced, since the probability of the tail points (22 to 16 dB) is too high. If γ is increased to 0.25 and K reduced to 2.25, a very good similarity is obtained. As shown in Fig. 3 the same curve can be drawn through the data points and the calculated points for the contaminated-normal distribution.

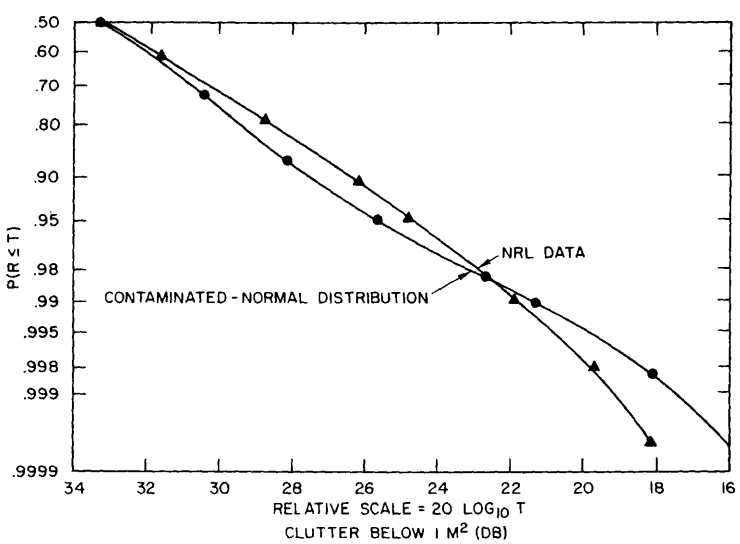


Fig. 2 - Comparison of NRL data and a contaminated-normal distribution with $\gamma = 0.05$ and $K = 3$

Though a very good fit of a smooth sea can be obtained with a contaminated-normal distribution, the question remains as to whether the contaminated-normal distribution can describe other sea state conditions by variations of γ and K. To answer this question the previous procedure was repeated for a moderate sea state, and the result is shown in Fig. 4. As can be seen, a fair fit is obtained; however, the fit is not as good as was obtained for the smooth sea. This is because the tail of the moderate sea is quite long and a good fit cannot be obtained over the entire range by using the sum of only two gaussians. If the sum of three gaussians were used, a much better fit would be obtained.

An obvious mechanism, which in effect produces a sum of gaussian distributions over a time interval, is a nonstationary gaussian process whose nonstationarity is caused by a time varying variance. The validity of this conjecture should be investigated as soon as possible.

The calculations for the rest of this report will be carried out only for the $\gamma = 0.25$ and $K = 2.25$ case, since the calculations are quite time consuming.

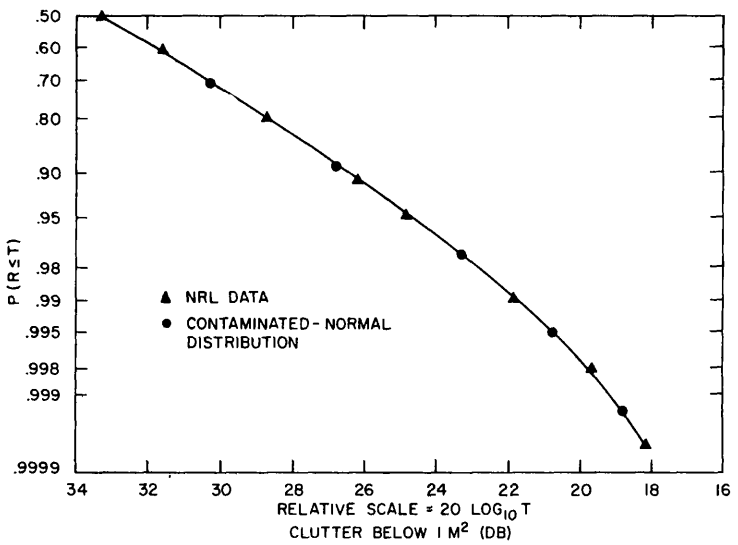


Fig. 3 - Fit of NRL data of a smooth sea by a contaminated-normal distribution with $\gamma = 0.25$ and $K = 2.25$

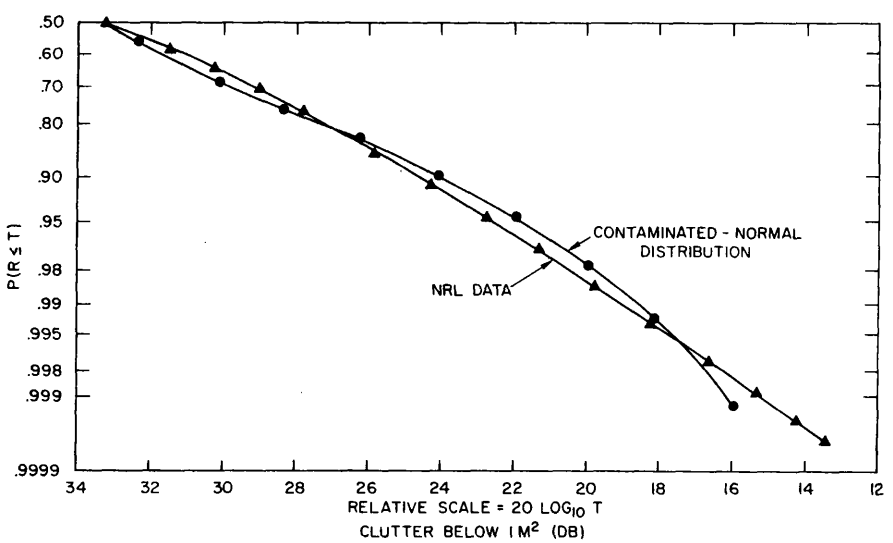


Fig. 4 - Fit of NRL data of a moderate sea by a contaminated-normal distribution with $\gamma = 0.25$ and $K = 3.5$

DENSITY FUNCTION OF SIGNAL IN CLUTTER

Let it be assumed that the signal of interest is of constant amplitude A. Since the detection is noncoherent, the signal can be added into one component. Hence, the densities of the inphase and quadrature components are

$$p_{\gamma,K,A}(x) = \frac{(1-\gamma)}{\sqrt{2\pi\sigma^2}} \exp\left[-(x-A)^2/2\sigma^2\right] + \frac{\gamma}{\sqrt{2\pi K^2\sigma^2}} \exp\left[-(x-A)^2/2K^2\sigma^2\right] \tag{10}$$

and

$$p_{\gamma,K}(y) = \frac{(1-\gamma)}{\sqrt{2\pi\sigma^2}} \exp(-y^2/2\sigma^2) + \frac{\gamma}{\sqrt{2\pi K^2\sigma^2}} \exp(-y^2/2K^2\sigma^2) . \tag{11}$$

The joint density of the envelope R and angle θ can be found by first noting that the joint density of x and y is simply the product of Eqs. (10) and (11). Then, by using the Jacobian transform in Eq. (5) and again using Eqs. (3) and (4), one obtains

$$\begin{aligned} p_{\gamma,K,A}(R,\theta) = \frac{R}{2\pi\sigma^2} \{ & (1-\gamma)^2 \exp[-(R^2 - 2RA\cos\theta + A^2)/2\sigma^2] \\ & + \frac{\gamma^2}{K^2} \exp[-(R^2 - 2RA\cos\theta + A^2)/2K^2\sigma^2] \\ & + \frac{\gamma(1-\gamma)}{K} \exp[-(R^2\cos^2\theta - 2RA\cos\theta + A^2 + K^2R^2\sin^2\theta)/2K^2\sigma^2] \\ & + \frac{\gamma(1-\gamma)}{K} \exp[-(R^2\sin^2\theta - 2RAK^2\cos\theta + A^2K^2 + K^2R^2\cos^2\theta)/2K^2\sigma^2]\} \end{aligned} \tag{12}$$

for the joint density of R and θ and

$$p_{\gamma,K,A}(R) = \int_0^{2\pi} p_{\gamma,K,A}(R,\theta)\, d\theta \tag{13}$$

for the density of R.

PROBABILITIES OF FALSE ALARM AND DETECTION

Let it be assumed that N pulses are to be integrated (summed). If the N pulses are independent, the easiest method of determining the probability density for the sum of N envelope detected pulses is by use of the characteristic function. Using the density of the envelope of signal plus clutter given in Eq. (13), the characteristic function is

$$\phi(w) \equiv E\{e^{iwR}\} = \int_0^{\infty} p_{\gamma,K,A}(R) \exp(iwR)\, dR, \tag{14}$$

which is the Fourier transform of $p_{\gamma,K,A}(R)$. The characteristic function for the sum of N independent pulses is

$$\phi_N(w) = [\phi(w)]^N . \tag{15}$$

The inverse Fourier transform yields the probability density for the sum of N pulses:

$$p_N(R) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_N(w) \exp(-iwR)\, dw. \tag{16}$$

The probability of false alarm is simply

$$P_{FA} = \int_T^{\infty} P_N(R)\, dR, \tag{17}$$

with $A = 0$, and the probability of detection is

$$P_D = \int_T^{\infty} P_N(R)\, dR , \tag{18}$$

with $A \neq 0$. In calculating these probabilities, Simpson's rule was used for Eqs. (13), (17), and (18), and the fast Fourier transform was used for Eqs. (14) and (16).

PROBABILITY OF DETECTION RESULTS

Threshold values for N = 1, 3, 10, and 30 were determined numerically by the previous procedure for false alarm probabilities from 10^{-2} to 10^{-9} for γ = 0.25 and K = 2.25. The threshold is expressed in 1.39 σ units, where 1.39 σ is the numerically calculated median of the contaminated-normal distribution with γ = 0.25 and K = 2.25. These curves are plotted in Fig. 5.

Using the threshold values from Fig. 5, the probability of detection versus the signal-to-noise ratio per pulse is shown in Fig. 6 for N = 1, 3, 10, and 30. If the power required to obtain a $P_{FA} = 10^{-8}$ and P_D = 0.9 is compared between the log-normal distribution with σ = 6 dB and the contaminated-normal distribution with γ = 0.25 and K = 2.25, the contaminated-normal distribution requires 12, 10, 7, and 5 dB less power for N = 1, 3, 10, and 30 pulses respectively. This tremendous decrease in power is caused by the very long tail of the log-normal distribution. On the other

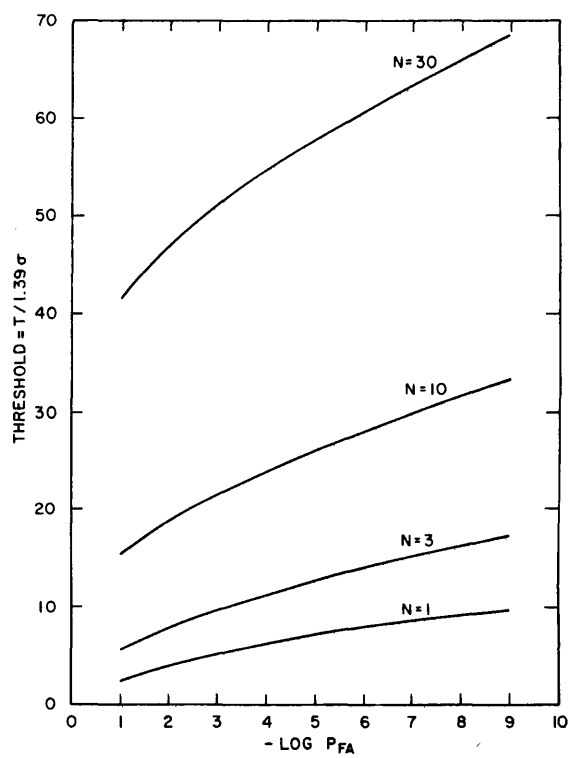
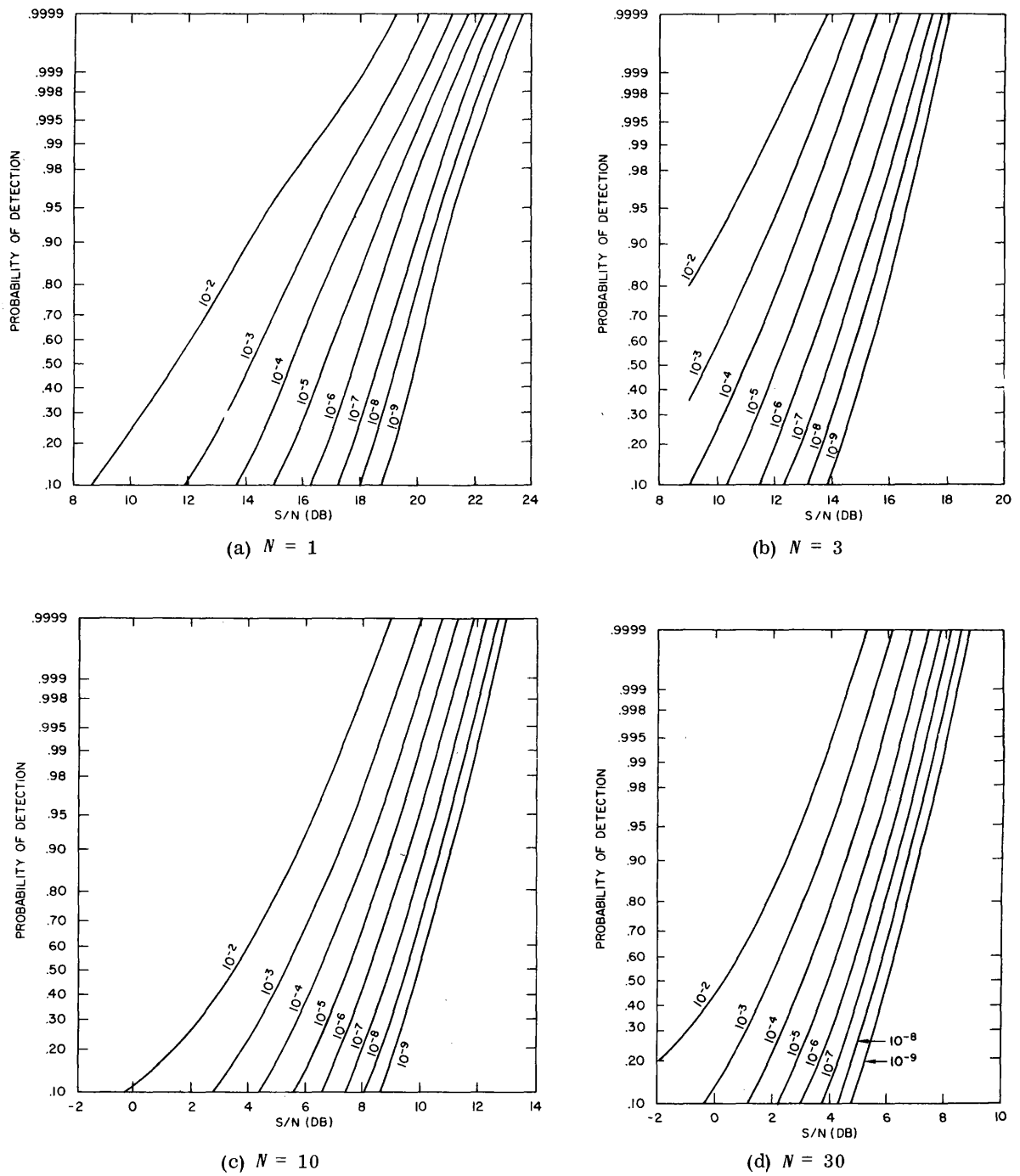


Fig. 5 - Threshold values for a contaminated-normal distribution with γ = 0.25 and K = 2.25



(a) $N = 1$

(b) $N = 3$

(c) $N = 10$

(d) $N = 30$

Fig. 6 - Probability of detection versus S/N for $P_{FA} = 10^{-2}$ to 10^{-9}

hand, if this same contaminated-normal distribution is compared with the Rayleigh distribution, the contaminated-normal requires 4.1, 3.5, 2.4, and 1.4 dB more power for N = 1, 3, 10, and 30 pulses respectively.

CONCLUSIONS AND RECOMMENDATIONS

Because of the mechanism that produces a contaminated-normal distribution, it is possible that sea clutter can be described by a gaussian variable that has a time varying variance. This conjecture should be tested.

Secondly, since both the contaminated-normal and log-normal distributions have long tails, detectors which are not influenced by long tails (such as the median and trimmed mean detectors) should be investigated to see if they provide an improvement over the simple sum detector.

ACKNOWLEDGMENTS

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13. ABSTRACT

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