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The Static Configuration of a Tri-Moored, Subsurface, Buoy-Cable Array Acted on by Current-Induced Forces

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ABSTRACT

A rapid and accurate method is developed for determining the static configuration of and tensions in a buoy-cable array that consists of a subsurface float anchored to the bottom by three cable legs. The array is acted on by weight and buoyancy forces and by current-induced forces that depend on the position and orientation of the cables in the water. A lumped parameter representation of the structure is assumed, and the analysis is performed by using The Method of Imaginary Reactions to determine the equilibrium configuration of the array under constant applied forces and the method of successive approximations to iterate on the position-dependent current forces. Expressions are derived for the hydrodynamic forces, and both normal and tangential drags are included. The analysis allows for the possibility that a variety of discrete-element measuring and flotation devices are attached to the cable legs. Although a lumped parameter representation is used, the analytic method is independent of the number of stations employed, depending only on the six unknown components of the reactions at two of the anchors. Consequently, arbitrary fineness can be used in modeling the structure without increasing the complexity of the problem. A computer program for implementing the analysis is included, and a sample calculation is given to demonstrate the use of the program and to show the rapid convergence of the analytic method.

PROBLEM STATUS

This is an interim report; work is continuing on other phases of the problem.

AUTHORIZATION

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THE STATIC CONFIGURATION OF A TRI-MOORED, SUBSURFACE, BUOY-CABLE ARRAY ACTED ON BY CURRENT-INDUCED FORCES

SYMBOLS

The symbols used in this report are defined as they appear in the context. The most important ones are listed here for reference.

(a_n, b_n, c_n)	the coordinates of the nth cable anchor
$A_{k,m,n}$	the effective cross-sectional area of the (k,m,n)th elemental device
$B_{m,n}$	the extensional rigidity of the (m,n)th cable segment
$C^{c;x}_{m,n}, C^{c;y}_{m,n}, C^{c;z}_{m,n}$	the drag constants of the (m,n)th cable segment
$C^{e;x}_{k,m,n}, C^{e;y}_{k,m,n}$	the drag constants of the (k,m,n)th elemental device
$C^{D}_{k,m,n}$	the coefficient of drag of the (k,m,n)th elemental device
$C^{N}_{m,n}$	the coefficient of drag of the (m,n)th cable segment when this segment is normal to the stream
$C^{P}_{m,n}$	the coefficient of drag of the (m,n)th cable segment when this segment is parallel to the stream
$COMPD$	a cutoff value that defines the acceptable completion of the successive approximation iteration
$COMPE$	a cutoff value that defines the acceptable completion of the Imaginary Reaction iteration
$d_{m,n}$	the diameter of the (m,n)th cable segment
E	a positive definite error function
$f^{\eta}_{m,n}$	the component of drag force per unit length in the $\eta_{m,n}$ direction
$f^{\tau}_{m,n}$	the component of drag force per unit length in the $\tau_{m,n}$ direction
$(F^{x}_{m,n}, F^{y}_{m,n}, F^{z}_{m,n})$	the components of the external force acting at the (m,n)th cable station
$(F^{x}_{M(n),n}, F^{y}_{M(n),n}, F^{z}_{M(n),n})$	the components of the imaginary reactions applied to the $M(n)$th stations of cables 2 and 3

$(h^{c;x}_{m,n},\ h^{c;y}_{m,n},\ h^{c;z}_{m,n})$	the components of the hydrodynamic force per unit length acting on the (m,n)th cable segment
$(h^{e;x}_{k,m,n},\ h^{e;y}_{k,m,n},\ h^{e;z}_{k,m,n})$	the components of the hydrodynamic force acting on the (k,m,n)th elemental device
$(H^{c;x}_{m,n},\ H^{c;y}_{m,n},\ H^{c;z}_{m,n})$	the components of the lumped drag force at the (m,n)th cable station due to the distributed hydrodynamic forces along the cables
$(H^{e;x}_{m,n},\ H^{e;y}_{m,n})$	the components of the lumped drag force at the (m,n)th cable station due to the hydrodynamic forces on the elemental devices
$(\mathbf{i},\ \mathbf{j},\ \mathbf{k})$	unit vectors in the (x,y,z) directions, respectively
$(k,\ m,\ n)$	the index of the kth elemental device on the mth segment of the nth cable
$\tilde{k}(m,n)$	the total number of elemental devices attached to the (m,n)th cable half-segment adjoining the $(m-1,n)$th cable station
$K(m,n)$	the total number of elemental devices attached to the (m,n)th cable segment
$L_{m,n}$	the stressed length of the (m,n)th cable segment
$\bar{L}_{m,n}$	the unstressed length of the (m,n)th cable segment
(m,n)	the index of the mth station or segment on the nth cable
$M(n)$	the total number of stations or segments on the nth cable
$r^{D}_{m,n}$	the ratio of drag coefficients for the (m,n)th cable segment $= C^{P}_{m,n}/C^{N}_{m,n}$
$(R^{x}_{m,n},\ R^{y}_{m,n},\ R^{z}_{m,n})$	the components of the resultant force in the (m,n)th cable segment
$s_{k,m,n}$	the stressed distance of the (k,m,n)th elemental device from the $(m-1,n)$th cable station
$\bar{s}_{k,m,n}$	the unstressed distance of the (k,m,n)th elemental device from the $(m-1,n)$th station
$T_{m,n}$	the tension in the (m,n)th cable segment
$\mathbf{V}$	the current vector
$V(z)$	the current magnitude at a height z above the bottom

$\tilde{V}(m,n:\Xi_1,\Xi_2)$ the integral of $V^2(z)$ along the (m,n)th cable segment from Ξ_1 to Ξ_2 equal to

$$\int_{\Xi_1}^{\Xi_2} V^2[z(m,n:\xi)]\ d\xi$$

$w^c_{m,n}$ the weight (or buoyancy) per unit length in water of the (m,n)th cable segment

$w^e_{k,m,n}$ the weight (or buoyancy) in water of the (k,m,n)th elemental device

$W_{m,n}$ the lumped weight (or buoyancy) force at the (m,n)th cable station

(x,y,z) fixed Cartesian coordinates

$(x_{m,n}, y_{m,n}, z_{m,n})$ the coordinates of the (m,n)th cable station

$z(m,n:\xi)$ the parametric representation of z along the (m,n)th cable segment $= z_{m-1,n} + \gamma_{m,n}\xi$

$(\alpha_{m,n}, \beta_{m,n}, \gamma_{m,n})$ the direction cosines of the (m,n)th cable segment

δ a positive convergence factor having the dimensions of force

$\Lambda_{m,n}$ the sine of the angle between the (m,n)th cable segment and the stream

$(\Lambda F^x_{M(n),n}, \Lambda F^y_{M(n),n}, \Lambda F^z_{M(n),n})$ the components of the additive forces applied to the $M(n)$th stations of cables 2 and 3

$\mu^c_{m,n}$ the hydrodynamic constant of the (m,n)th cable segment $= \rho\ C^N_{m,n}\ d_{m,n}/2$

$\mu^e_{k,m,n}$ the hydrodynamic constant of the (k,m,n)th elemental device $= \rho\ C^D_{k,m,n}\ A_{k,m,n}/2$

ξ a parameter defining distance along the (m,n)th cable segment

ρ the density of the surrounding fluid

$(\boldsymbol{\tau}_{m,n}, \boldsymbol{\pi}_{m,n}, \boldsymbol{\eta}_{m,n})$ respectively, unit vectors along the (m,n)th cable segment, normal to both the (m,n)th cable segment and the stream, and normal to the (m,n)th cable segment but in the plane that includes this segment and the stream

ϕ the angular direction of the current with respect to the x axis

$(\ )'$ the prime symbol denotes trial iteration values

INTRODUCTION

This report presents a method for determining the static configuration of a buoy-cable array, which consists of a subsurface float anchored to the bottom by three cable legs. The array is acted on by weight and buoyancy forces and by current-induced forces which are functions of both the orientation and the depth of the cables in the water. The analysis allows for the possibility that a variety of discrete-element measuring and flotation devices are attached to the legs of the structure.

A lumped parameter representation of the system is used, and the bending stiffness of the cables is ignored. The analysis is performed using The Method of Imaginary Reactions (1) to determine the equilibrium configuration of the array and the method of successive approximations (2) to iterate on the position-dependent current forces. Convergence to the equilibrium shape of the structure is extremely rapid, as is indicated by several numerical examples.

Certain assumptions are made in this report. The two most significant are that

1. The current, though depth dependent in magnitude, is unidirectional, and

2. The drag force component which acts in the direction normal to both the stream and the cable is zero.

It is important to note that neither of these assumptions is necessary because of the method of analysis. The first is made because most design currents are given as depth dependent and unidirectional. The second is necessary because of the lack of experimental information available for determining an analytic expression for the side component of drag on stranded cables.

An array included in the class of structures considered in this report and of great interest to both oceanographers and the Navy is Sea Spider. This structure consists of a tri-moored subsurface float with neutrally buoyant mooring legs. These legs are made neutrally buoyant by positioning flotation devices along their lengths. The method currently in use for determining the equilibrium configuration of Sea Spider was developed by Sniffin and Savage (3). Their basic assumption was that, "If the variation of current velocity with depth is moderate, the use of the root mean square velocity as a uniform velocity results in a good approximation." Also, Sniffin and Savage neglected the tangential component of drag on the cables.

In Appendix C of this report, a particular Sea Spider array is analyzed, and the results are compared with results obtained by using the Sniffin and Savage assumption. The essential conclusion of this example is that *the use of the root-mean-square velocity as a uniform velocity does not result in a good approximation to the equilibrium shape of the array.*

To implement the equations developed in this report, a program was written for the NRL CDC-3800 computer. It appears as Appendix B. The program was written for a velocity profile consisting of a series of straight-line segments, as discussed in Appendix A. However, if it is desired to use another type of velocity profile, only one subroutine of the program need be changed. Card decks are available on request from the authors.

GUIDELINES FOR MODELING THE ARRAY

The structure under consideration, Fig. 1, consists of a subsurface buoy anchored to the bottom by three cables. These cables are referred to by the number n, where

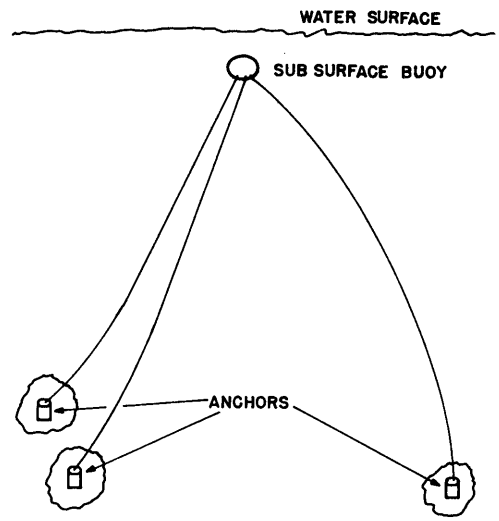

Fig. 1 - The tri-moored, subsurface, buoy-cable array

n = 1, 2, or 3. To describe the array by a lumped parameter model (Fig. 2), each cable is represented by $M(n) + 1$ stations at which the external forces and anchor reactions acting on the array are lumped. These stations are referred to by the subscript pair (m, n), where $m = 0, 1, 2, \ldots, M(n)$. Note that station $(0,1)$ represents the anchor point of cable 1, and that stations $(M(2),2)$ and $(M(3),3)$ represent the anchor points of cables 2 and 3, respectively. The stations $(M(1),1)$, $(0,2)$, and $(0,3)$ all refer to the branch point (subsurface buoy) of the array. The notation $M(n)$ is used to indicate that each cable in the array can be described by a different number of stations.

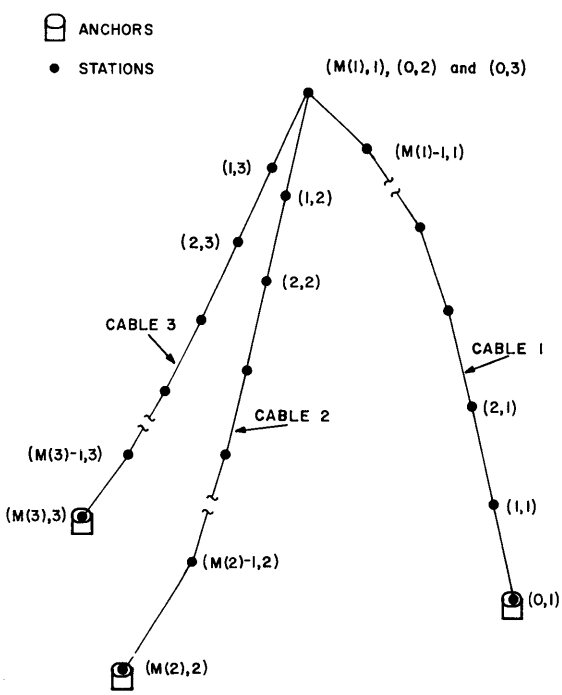

Fig. 2 - Lumped parameter representation of the structure

These stations divide each cable leg into $M(n)$ segments, to which the dimensional and the strength properties of the cable are assigned. The segment between the $(m-1)$ and mth stations on the nth cable is denoted by the subscript pair (m, n).

Since the external forces acting on the structure have been lumped at the cable stations, each cable segment between stations becomes a straight line. Consequently, in order to obtain a successful lumped parameter representation, a few guidelines for modeling the array are necessary. These are summarized as follows:

1. Each point of discontinuity in a physical property of a cable (such as a change in cross-sectional area or a change in weight) is represented by a station. Consequently, each cable segment in the array has constant physical properties.

2. As many additional stations are used as are necessary to obtain a satisfactory approximation to the continuous equilibrium shape of the array. Thus, for example, a section of cable which is expected to have a small radius of curvature should be represented by more stations than a section which is expected to have a large radius of curvature.

With regard to this second point, it is important to note that the method of analysis developed in this report does not depend on the number of stations used to describe the array (see Ref. 1); thus, more stations do not grossly affect the amount of work necessary to determine the equilibrium shape of the structure. It is suggested that each cable be divided into at least ten segments, with the unstressed length of each segment being no greater than one-tenth the unstressed length of the entire cable.

Theoretically, an ultrafine representation of the structure could be used. However, such representations should be applied carefully, due to the errors introduced by computer roundoff and truncation.

DESCRIPTION AND NOTATION OF THE TRI-MOORED, BUOY-CABLE ARRAY

To describe the configuration of the array in space, a right-handed (x, y, z) Cartesian coordinate system is used. The z axis is defined to be parallel to the direction of gravity, and $z = 0$ is taken at the location of the deepest anchor; z is considered increasing toward the surface.

Referred to this system (Fig. 3), the location of the nth cable anchor is given by the coordinate triplet

$$(a_n, b_n, c_n) \ .$$

In the terminology of Imaginary Reactions, the anchor associated with $n = 1$ is called the primary anchor and the anchors associated with $n = 2$ and 3 are called the secondary (redundant) anchors.

As previously stated, each cable in the array is represented by $M(n) + 1$ stations. The location of the mth station on the nth cable is denoted by

$$(x_{m,n}, y_{m,n}, z_{m,n}) \ ,$$

where $m = 0, 1, 2, \ldots, M(n)$. The direction of counting m is from the primary anchor to the branch point of the array along cable 1, and from the branch point of the array to the

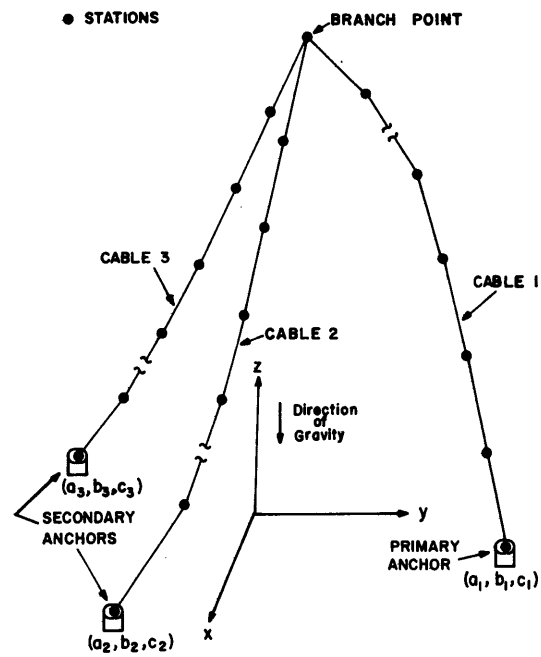

Fig. 3 - Coordinate system and anchor terminology

secondary anchors along cables 2 and 3, respectively. Since station $(0,1)$ represents the primary anchor of the array, the relation

$$(x_{0,1}, y_{0,1}, z_{0,1}) = (a_1, b_1, c_1) \tag{1}$$

must hold. Further, since stations $(M(1),1)$, $(0,2)$, and $(0,3)$ all represent the subsurface buoy, the branch point constraints are obtained as

$$x_{0,2} = x_{0,3} = x_{M(1),1} , \tag{2a}$$

$$y_{0,2} = y_{0,3} = y_{M(1),1} , \tag{2b}$$

and

$$z_{0,2} = z_{0,3} = z_{M(1),1} . \tag{2c}$$

In The Method of Imaginary Reactions, the secondary anchors are released and forces representing the anchor reactions are applied to the free ends of the released cables. For an arbitrary set of "guessed" reactions, the locations of the free ends do not coincide with the true anchor points. However, when the correct reactions are obtained, the equilibrium relations

$$(x_{M(2),2}, y_{M(2),2}, z_{M(2),2}) = (a_2, b_2, c_2) \tag{3a}$$

and

$$(x_{M(3),3}, y_{M(3),3}, z_{M(3),3}) = (a_3, b_3, c_3) \tag{3b}$$

must be satisfied.

The external force acting at the (m,n)th station is defined by its components along the x, y, and z axes as

$$(F^x_{m,n}, F^y_{m,n}, F^z_{m,n}) ,$$

where $m = 1, 2, \ldots, M(n)$. By convention, the force acting at the branch point is indexed as $(M(1),1)$ rather than as $(0,2)$ or $(0,3)$. The particular forces

$$(F^x_{M(n),n}, F^y_{M(n),n}, F^z_{M(n),n})$$

for $n = 2$ and 3 represent the imaginary reactions which are applied to the $M(n)$ ends of these cables, respectively, when the secondary anchors are released.

THE EQUILIBRIUM CONFIGURATION BY USE OF THE METHOD OF IMAGINARY REACTIONS

For the present, assume that the external forces applied to the array, including the imaginary reactions, are constants. The components of the resultant force,

$$(R^x_{m,n}, R^y_{m,n}, R^z_{m,n}) ,$$

acting in the (m,n)th cable segment are then given in terms of the applied external forces through the expressions

for $m = M(n)$ and $n = 2, 3$,

$$R^x_{M(n),n} = F^x_{M(n),n} ,$$

$$R^y_{M(n),n} = F^y_{M(n),n} ,$$

and

$$R^z_{M(n),n} = F^z_{M(n),n} ; \tag{4a}$$

for $m = M(1)$ and $n = 1$,

$$R^x_{M(1),1} = F^x_{M(1),1} + R^x_{1,2} + R^x_{1,3} ,$$

$$R^y_{M(1),1} = F^y_{M(1),1} + R^y_{1,2} + R^y_{1,3} ,$$

and

$$R^z_{M(1),1} = F^z_{M(1),1} + R^z_{1,2} + R^z_{1,3} ; \tag{4b}$$

for $m = 1, 2, \ldots, M(n) - 1$ and $n = 1, 2, 3$,

$$R^x_{m,n} = F^x_{m,n} + R^x_{m+1,n} ,$$

$$R^y_{m,n} = F^y_{m,n} + R^y_{m+1,n} ,$$

and

$$R^z_{m,n} = F^z_{m,n} + R^z_{m+1,n} . \tag{4c}$$

Note that starting from the guessed imaginary reactions at the ends of cables 2 and 3, the resultant forces are readily determined by summing the applied external forces toward the primary anchor.

In terms of the resultant forces, the tensions $T_{m,n}$ in the cable array are given by

$$T_{m,n} = \sqrt{(R^x_{m,n})^2 + (R^y_{m,n})^2 + (R^z_{m,n})^2} \ . \tag{5}$$

Assuming that these tensions are of such a magnitude that the cable segments deform elastically, although this assumption is not necessary (see Ref. 1), the stretched segment lengths $L_{m,n}$ become

$$L_{m,n} = \bar{L}_{m,n} \left[1 + \frac{T_{m,n}}{B_{m,n}} \right] , \tag{6}$$

where $\bar{L}_{m,n}$ is the unstressed length of the (m,n)th segment and $B_{m,n}$ is its extensional rigidity.

Finally, the equilibrium configuration of the array is determined from the equations

$$x_{m,n} = \frac{L_{m,n}}{T_{m,n}} R^x_{m,n} + x_{m-1,n} \ , \tag{7a}$$

$$y_{m,n} = \frac{L_{m,n}}{T_{m,n}} R^y_{m,n} + y_{m-1,n} \ , \tag{7b}$$

and

$$z_{m,n} = \frac{L_{m,n}}{T_{m,n}} R^z_{m,n} + z_{m-1,n} \ , \tag{7c}$$

where the definition of the primary anchor, Eq. 1, is used to start the solution, and where the branch point constraints, Eqs. 2a, b, and c, are used to extend the solution past the branch point.

In general, for a guessed set of imaginary reactions, the $M(n)$ ends of cables 2 and 3 will not be at the true anchor positions. As a measure of the distance of these ends from the correct anchor positions, let the positive definite error function E be defined as

$$E = \sum_{n=2}^{3} \left\{ [a_n - x_{M(n),n}]^2 + [b_n - y_{M(n),n}]^2 + [c_n - z_{M(n),n}]^2 \right\} \ . \tag{8}$$

Note that E vanishes uniquely if and only if the secondary anchor equilibrium conditions, Eqs. 3a and 3b, are identically satisfied.

The Method of Imaginary Reactions defines the following iteration procedure for reducing E and, as a consequence, for obtaining the correct reactions and the actual equilibrium configuration of the array.

Let the imaginary reactions applied to the ends of cables 2 and 3 be recalculated as

$$(F^x_{M(n),n})' = F^x_{M(n),n} + \Delta F^x_{M(n),n} \ , \tag{9a}$$

$$(F^y_{M(n),n})' = F^y_{M(n),n} + \Delta F^y_{M(n),n} \ , \tag{9b}$$

and

$$(F^z_{M(n),n})' = F^z_{M(n),n} + \Delta F^z_{M(n),n} \ , \tag{9c}$$

where primes denote the new imaginary reactions, and the additive forces are defined by

$$\Delta F^x_{M(n),n} = \frac{\delta}{\sqrt{E}}[a_n - x_{M(n),n}] \ , \tag{10a}$$

$$\Delta F^y_{M(n),n} = \frac{\delta}{\sqrt{E}}[b_n - y_{M(n),n}] \ , \tag{10b}$$

and

$$\Delta F^z_{M(n),n} = \frac{\delta}{\sqrt{E}}[c_n - z_{M(n),n}] \ . \tag{10c}$$

The quantity δ is a *positive convergence factor,* having the dimensions of force chosen at each iteration so that $E' < E$. E eventually approaches zero arbitrarily closely, and the correct equilibrium configuration, under the acting external forces, is obtained. A simple technique for determining δ is discussed in Ref. 1.

DISCRETE-ELEMENT NOMENCLATURE

Before deriving the external forces that are to be used in Eqs. 4b and 4c for the analysis of a tri-moored, subsurface, buoy-cable array, it is necessary to represent the discrete-element measuring and flotation devices which are attached to the structure.

Those objects which are attached to the (m,n)th cable segment are indexed by (k,m,n), where $k = 1, 2, \ldots, K(m,n)$, counting in the direction of increasing m as shown in Fig. 4. By convention, an elemental device located at the (m,n)th station is considered as the last device on the (m,n)th segment and is indexed by

$$(K(m,n),\ m,n) \ .$$

Using this convention, the subsurface float becomes the last device on the last segment of the first cable and is thus indexed by

$$(K(M(1),1), M(1),1) \ .$$

The unstressed distance of the (k,m,n)th device from the $(m-1,n)$th station is given by

$$\bar{s}_{k,m,n} \ .$$

In the development of the lumped external forces, it is important to distinguish between those objects attached to the half-segment adjoining the $(m-1,n)$th station and those objects attached to the half-segment adjoining the (m,n)th station. To indicate this division, let $\tilde{k}(m,n)$ represent the value of k such that

$$\bar{s}_{k,m,n} \leq \bar{L}_{m,n}/2$$

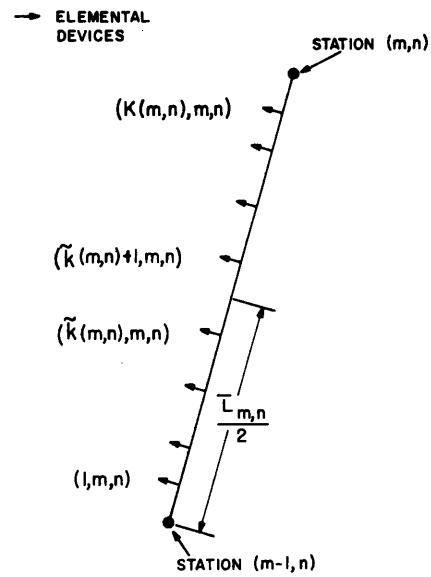

Fig. 4 - Representation of the discrete elemental devices

for $k = 1, 2, \ldots, \tilde{k}(m,n)$, and

$$\bar{s}_{k,m,n} > \bar{L}_{m,n}/2$$

for $k = \tilde{k}(m,n) + 1, \ldots, K(m,n)$.

WEIGHT AND BUOYANCY FORCES

To begin the derivation of the external forces that are applied to the structure, consider the weight and buoyancy forces. These forces are differentiated only by the direction in which they act: weights act toward the bottom (negative z direction), and buoyancies act toward the surface (positive z direction). In this report, it is convenient to have both types of force act in the positive z direction; consequently, weights are considered as having a negative value.

Let the weight (or buoyancy) per unit length in water of the (m,n)th cable segment be given by

$$w^c_{m,n} ,$$

and let the weight (or buoyancy) in water of the (k,m,n)th elemental device be designated as

$$w^e_{k,m,n} .$$

To lump these forces as a weight (or buoyancy) force $W_{m,n}$ acting at the (m,n)th station, a half-segment lumping technique is employed. That is, the distributed and discrete forces acting on the half-segments adjoining the (m,n)th station are integrated and summed, respectively, to give the lumped force at that station.

Using this technique, the lumped weight forces are readily determined by the expressions

for $m = 1, 2, \ldots, M(n) - 1$ and $n = 1, 2, 3$,

$$W_{m,n} = \sum_{k=\tilde{k}(m,n)+1}^{K(m,n)} w^e_{k,m,n} + \sum_{k=1}^{\tilde{k}(m+1,n)} w^e_{k,m+1,n}$$

$$+ \frac{1}{2}\left[w^c_{m,n}\,\bar{L}_{m,n} + w^c_{m+1,n}\,\bar{L}_{m+1,n}\right], \tag{11a}$$

and for $m = M(1)$, $n = 1$,

$$W_{M(1),1} = \sum_{k=\tilde{k}(M(1),1)+1}^{K(M(1),1)} w^e_{k,M(1),1}$$

$$+ \sum_{k=1}^{\tilde{k}(1,2)} w^e_{k,1,2} + \sum_{k=1}^{\tilde{k}(1,3)} w^e_{k,1,3}$$

$$+ \frac{1}{2}\left[w^c_{M(1),1}\,\bar{L}_{M(1),1} + w^c_{1,2}\,\bar{L}_{1,2} + w^c_{1,3}\,\bar{L}_{1,3}\right] \tag{11b}$$

Note that the weight forces defined by Eqs. 11a and 11b are independent of the location of the structure in the water. This is because the density of the water has been assumed to be constant.

HYDRODYNAMIC COORDINATES

To complete the formulation of the problem, the hydrodynamic forces that are acting on the array must be calculated. For this purpose, it is convenient to introduce a new set of natural hydrodynamic coordinates, defined with reference to the directions of the cable and the current.

It is assumed that the current, though depth dependent in magnitude, is unidirectional and normal to the direction of gravity. Thus, the stream possesses no z component. If the angular direction with respect to the x axis is denoted by ϕ, the current in general can be written as

$$\mathbf{V} = V(z)\,[\mathbf{i}\cos\phi + \mathbf{j}\sin\phi]\ ,$$

where $V(z)$ is the magnitude of the current profile at a height z above the deepest anchor ($z = 0$), and where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x and y directions, respectively. The unit vector in the z direction is designated by $\mathbf{k}$.

The natural hydrodynamic coordinate system is then formed in the following manner:

1. Let $\boldsymbol{r}_{m,n}$ be the unit tangent to the (m,n)th cable segment, considered positive in the direction of increasing m;

2. Then

$$\boldsymbol{\pi}_{m,n} = \frac{\mathbf{V} \times \boldsymbol{r}_{m,n}}{|\boldsymbol{\pi}_{m,n}|}$$

is a unit vector normal to both the (m,n)th cable segment and the current; and,

3. $\eta_{m,n} = \tau_{m,n} \times \pi_{m,n}$ is a unit vector normal to the cable and lying in the plane that includes the (m,n)th cable segment and the stream.

The hydrodynamic reference frame is expressed in terms of the basic x,y,z reference frame through the relations

$$\tau_{m,n} = \alpha_{m,n}\,\mathbf{i} + \beta_{m,n}\,\mathbf{j} + \gamma_{m,n}\,\mathbf{k}\,, \tag{12a}$$

$$\pi_{m,n} = \frac{1}{\Delta_{m,n}}\Big\{\big[\gamma_{m,n}\sin\phi\big]\,\mathbf{i} - \big[\gamma_{m,n}\cos\phi\big]\,\mathbf{j} + \big[\beta_{m,n}\cos\phi - \alpha_{m,n}\sin\phi\big]\,\mathbf{k}\Big\}\,, \tag{12b}$$

and

$$\eta_{m,n} = \frac{1}{\Delta_{m,n}}\Big\{\big[(\gamma^2_{m,n}+\beta^2_{m,n})\cos\phi - \alpha_{m,n}\beta_{m,n}\sin\phi\big]\,\mathbf{i} + \big[(\gamma^2_{m,n}+\alpha^2_{m,n})\sin\phi - \alpha_{m,n}\beta_{m,n}\cos\phi\big]\,\mathbf{j} - \big[\gamma_{m,n}\beta_{m,n}\sin\phi + \gamma_{m,n}\alpha_{m,n}\cos\phi\big]\,\mathbf{k}\Big\}\,. \tag{12c}$$

In these relations, $\alpha_{m,n}$, $\beta_{m,n}$, and $\gamma_{m,n}$ are the direction cosines of the (m,n)th cable segment and are defined by

$$\alpha_{m,n} = \frac{x_{m,n} - x_{m-1,n}}{L_{m,n}}\,, \tag{13a}$$

$$\beta_{m,n} = \frac{y_{m,n} - y_{m-1,n}}{L_{m,n}}\,, \tag{13b}$$

and

$$\gamma_{m,n} = \frac{z_{m,n} - z_{m-1,n}}{L_{m,n}}\,. \tag{13c}$$

The quantity $\Delta_{m,n}$ is the sine of the angle between the (m,n)th cable segment and the stream and is given by

$$\Delta_{m,n} = \sqrt{\gamma^2_{m,n} + (\beta_{m,n}\cos\phi - \alpha_{m,n}\sin\phi)^2}\,. \tag{14}$$

Note that $\Delta_{m,n}$ is always taken as positive.

For later reference, the current projections on the hydrodynamic axes are readily calculated from Eqs. 12a, b, and c as

$$\mathbf{V}\cdot\tau_{m,n} = V(z)\,(\alpha_{m,n}\cos\phi + \beta_{m,n}\sin\phi)\,, \tag{15a}$$

$$\mathbf{V}\cdot\pi_{m,n} = 0\,, \tag{15b}$$

and

$$\mathbf{V} \cdot \eta_{m,n} = V(z)\, \lambda_{m,n} \ . \tag{15c}$$

In these expressions, of course, z must be expressed as a function of position along the cable.

THE DISTRIBUTED HYDRODYNAMIC FORCES

The problems inherent in defining the hydrodynamic forces, especially on stranded cables, are well known. Little rigorous experimental work has been done in measuring these forces, the best known results being those described by Pode (4).

If the hydrodynamic forces per unit length are resolved in the τ, π, and η coordinate system, the following expressions for the three components are usually considered as representative and are used in this report.

1. Side force in π direction

Though this force exists for stranded cables (4), no relevant experiments have been performed to determine its functional form. However, the magnitude of this component is small compared to the other two components, and, consequently, the side force is neglected by setting it equal to zero.

2. Normal drag force in η direction

It is well established (4,5) that the normal drag force per unit length which acts in the η direction has a magnitude given by

$$f^{\eta}_{m,n} = \frac{\rho}{2}\, C^{N}_{m,n}\, d_{m,n}\, (\mathbf{V} \cdot \eta_{m,n})^2 \ , \tag{16}$$

where

ρ = the density of the surrounding fluid,

$C^{N}_{m,n}$ = the coefficient of drag (assumed constant) of the (m,n)th cable segment when this segment is normal to the stream, and

$d_{m,n}$ = the diameter of the (m,n)th cable segment.

3. Tangential drag force in τ direction

A variety of assumptions have been made concerning the form of the tangential drag force per unit length. These range from neglecting this component as zero to setting it equal to a constant. The form chosen for the development of this report is the expression given by

$$f^{\tau}_{m,n} = \frac{\rho}{2}\, C^{P}_{m,n}\, d_{m,n}\, V(z)\, [\mathbf{V} \cdot \tau_{m,n}] \ , \tag{17}$$

where

$C^{P}_{m,n}$ = the coefficient of drag (assumed constant) of the (m,n)th cable segment when this segment is parallel to the stream.

From these considerations of the drag components, the hydrodynamic force per unit length which is acting on the (m,n)th cable segment can be written as

$$\mathbf{f}_{m,n} = f^{\eta}_{m,n}\, \eta_{m,n} + f^{\tau}_{m,n}\, \tau_{m,n} ,$$

where, using Eqs. 15, $f^{\eta}_{m,n}$ and $f^{\tau}_{m,n}$ (defined by Eqs. 16 and 17), respectively, are given by

$$f^{\eta}_{m,n} = \mu^{c}_{m,n}\, \Lambda^{2}_{m,n}\, V^{2}(z) \tag{18a}$$

and

$$f^{\tau}_{m,n} = \mu^{c}_{m,n}\, r^{D}_{m,n}\, [\alpha_{m,n} \cos\phi + \beta_{m,n} \sin\phi]\, V^{2}(z) . \tag{18b}$$

The quantity $\mu^{c}_{m,n}$ is called the hydrodynamic constant of the (m,n)th cable segment, and $r^{D}_{m,n}$ is the ratio of the drag coefficients for this segment. These quantities are defined by the expressions

$$\mu^{c}_{m,n} = \frac{\rho}{2}\, C^{N}_{m,n}\, d_{m,n} \tag{19a}$$

and

$$r^{D}_{m,n} = C^{P}_{m,n}/C^{N}_{m,n} . \tag{19b}$$

Taking the projections of $\mathbf{f}_{m,n}$ in the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ directions by making use of Eqs. 12a, b, and c, the components of the hydrodynamic force per unit length acting on the (m,n)th cable segment are found as

$$h^{c;x}_{m,n} = C^{c;x}_{m,n}\, V^{2}(z) , \tag{20a}$$

$$h^{c;y}_{m,n} = C^{c;y}_{m,n}\, V^{2}(z) , \tag{20b}$$

and

$$h^{c;z}_{m,n} = C^{c;z}_{m,n}\, V^{2}(z) , \tag{20c}$$

where the drag constants of the (m,n)th cable segment are defined by

$$\begin{aligned} C^{c;x}_{m,n} = \mu^{c}_{m,n} \big[& \Delta_{m,n} (\gamma^{2}_{m,n} + \beta^{2}_{m,n}) \cos\phi \\ & - \Delta_{m,n}\, \alpha_{m,n}\, \beta_{m,n} \sin\phi \\ & + r^{D}_{m,n} (\alpha_{m,n} \cos\phi + \beta_{m,n} \sin\phi)\, \alpha_{m,n} \big] , \end{aligned} \tag{21a}$$

$$\begin{aligned} C^{c;y}_{m,n} = \mu^{c}_{m,n} \big[& \Delta_{m,n} (\gamma^{2}_{m,n} + \alpha^{2}_{m,n}) \sin\phi \\ & - \Delta_{m,n}\, \alpha_{m,n}\, \beta_{m,n} \cos\phi \\ & + r^{D}_{m,n} (\alpha_{m,n} \cos\phi + \beta_{m,n} \sin\phi)\, \beta_{m,n} \big] , \end{aligned} \tag{21b}$$

and

$$C^{c;z}_{m,n} = \mu^{c}_{m,n}\left[-\Delta_{m,n}(\alpha_{m,n}\cos\phi + \beta_{m,n}\sin\phi)\,\gamma_{m,n}\right.$$

$$\left. + r^{D}_{m,n}(\alpha_{m,n}\cos\phi + \beta_{m,n}\sin\phi)\,\gamma_{m,n}\right] . \tag{21c}$$

To lump the distributed drag forces as a single force which acts at the (m,n)th cable station, the half-segment lumping technique is employed. Using this technique, the lumped hydrodynamic force is given by the equations

for $m = 1, 2, \ldots, M(n) - 1$ and $n = 1, 2, 3$,

$$H^{c;\theta}_{m,n} = C^{c;\theta}_{m,n}\,\tilde{V}(m,n : L_{m,n}/2,\ L_{m,n})$$

$$+ C^{c;\theta}_{m+1,n}\,\tilde{V}(m+1,n : 0,\ L_{m+1,n}/2) , \tag{22a}$$

and for $m = M(1)$ and $n = 1$,

$$H^{c;\theta}_{M(1),1} = C^{c;\theta}_{M(1),1}\,\tilde{V}(M(1),1 : L_{M(1),1}/2,\ L_{M(1),1})$$

$$+ C^{c;\theta}_{1,2}\,\tilde{V}(1,2 : 0,\ L_{1,2}/2)$$

$$+ C^{c;\theta}_{1,3}\,\tilde{V}(1,3 : 0,\ L_{1,3}/2) . \tag{22b}$$

In these expressions, $\theta = x$, y, or z. The function $\tilde{V}(m,n : \Xi_1, \Xi_2)$ represents the integral of $V^2(z)$ along the (m,n)th half-segment and is defined by

$$\tilde{V}(m,n : \Xi_1, \Xi_2) = \int_{\Xi_1}^{\Xi_2} V^2[z(m,n : \xi)]\ d\xi . \tag{23}$$

In this integral, the argument z of $V^2(z)$ is expressed in terms of the integration parameter ξ along the (m,n)th cable segment through the relation

$$z = z(m,n:\xi) = z_{m-1,n} + \gamma_{m,n}\,\xi , \tag{24}$$

where $\gamma_{m,n}$ is defined by Eq. 13c.

THE DISCRETE-ELEMENT HYDRODYNAMIC FORCES

Assuming only that there is no lift associated with the elemental devices, the direction of the drag on these devices is parallel to **V** and of magnitude

$$\frac{\rho}{2}\,C^{D}_{k,m,n}\,A_{k,m,n}\,V^2(z) ,$$

where

$C^{D}_{k,m,n}$ = the coefficient of drag (assumed constant) of the (k,m,n)th elemental device, and

$A_{k,m,n}$ = the effective cross-sectional area of the (k,m,n)th elemental device.

Resolving this force in the x, y, z coordinate system, the hydrodynamic force due to the (k, m, n)th elemental device is found as

$$h^{e;x}_{k,m,n} = C^{e;x}_{k,m,n} V^2(z) , \tag{25a}$$

$$h^{e;y}_{k,m,n} = C^{e;y}_{k,m,n} V^2(z) , \tag{25b}$$

and

$$h^{e;z}_{k,m,n} = 0 . \tag{25c}$$

Here, the drag constants of the (k, m, n)th elemental device are given by

$$C^{e;x}_{k,m,n} = \mu^{e}_{k,m,n} \cos \phi \tag{26a}$$

and

$$C^{e;y}_{k,m,n} = \mu^{e}_{k,m,n} \sin \phi , \tag{26b}$$

and $\mu^{e}_{k,m,n}$ is the hydrodynamic constant of the (k, m, n)th device and is defined by

$$\mu^{e}_{k,m,n} = \frac{\rho}{2} C^{D}_{k,m,n} A_{k,m,n} .$$

Applying the half-segment lumping technique, the hydrodynamic force acting at the (m, n)th station and due to the discrete elemental devices is then calculated through the relations

for $m = 1, 2, \ldots, M(n) - 1$ and $n = 1, 2, 3$,

$$H^{e;\theta}_{m,n} = \sum_{k=\tilde{k}(m,n)+1}^{K(m,n)} C^{e;\theta}_{k,m,n} V^2[z(m,n:s_{k,m,n})] + \sum_{k=1}^{\tilde{k}(m+1,n)} C^{e;\theta}_{k,m+1,n} V^2[z(m+1,n:s_{k,m+1,n})] ; \tag{27a}$$

for $m = M(1)$ and $n = 1$,

$$H^{e;\theta}_{M(1),1} = \sum_{k=\tilde{k}(M(1),1)+1}^{K(M(1),1)} C^{e;\theta}_{k,M(1),1} V^2[z(M(1),1:s_{k,M(1),1})] + \sum_{k=1}^{\tilde{k}(1,2)} C^{e;\theta}_{k,1,2} V^2[z(1,2:s_{k,1,2})] + \sum_{k=1}^{\tilde{k}(1,3)} C^{e;\theta}_{k,1,3} V^2[z(1,3:s_{k,1,3})] . \tag{27b}$$

In these expressions, $\theta = x$ or y, and the argument z of $V^2(z)$ is expressed in terms of the position of the (k,m,n)th device along the (m,n)th segment through the relation in Eq. 24. That is,

$$z(m\ n{:}s_{k,m,n}) = z_{m-1,n} + \gamma_{m,n}\ s_{k,m,n}\ .$$

The stressed distance $s_{k,m,n}$ of the (k,m,n)th device from the $(m-1,n)$th station is obtained from the elasticity relation

$$s_{k,m,n} = \bar{s}_{k,m,n}\left[1 + \frac{T_{m,n}}{B_{m,n}}\right]. \tag{28}$$

GENERALIZED METHOD FOR THE EQUILIBRIUM ANALYSIS

The external force

$$(F^x_{m,n},\ F^y_{m,n},\ F^z_{m,n})\ ,$$

which is acting at the (m,n)th cable station (excluding the $(M(2),2)$ and $(M(3),3)$ stations at which the imaginary reactions act) can be summarized as

$$F^x_{m,n} = H^{c;x}_{m,n} + H^{e;x}_{m,n}\ , \tag{29a}$$

$$F^y_{m,n} = H^{c;y}_{m,n} + H^{e;y}_{m,n}\ , \tag{29b}$$

and

$$F^z_{m,n} = W_{m,n} + H^{c;z}_{m,n} \tag{29c}$$

where the lumped weight forces $W_{m,n}$ are defined in Eq. 11, the lumped hydrodynamic forces $H^{c;\theta}_{m,n}$ due to the drag forces on the cable segments are defined in Eq. 22, and the lumped hydrodynamic forces $H^{e;\theta}_{m,n}$ due to the drag forces on the elemental devices are defined in Eq. 27. It is important to note that the weight forces are constants, but that the hydrodynamic forces depend on the position of the structure through both the orientations and the depths of the cable segments and the depths of the elemental devices.

Since The Method of Imaginary Reactions demands that the applied external forces be constants, it is not entirely applicable to the problem at hand. However, by combining Imaginary Reactions with the method of successive approximations (2), the equilibrium configuration for arbitrary current profiles can be generated to any desired degree of accuracy.

In essence, this combined technique consists of making an initial guess as to the values of the hydrodynamic forces and then using these values while finding the equilibrium position of the structure by Imaginary Reactions. Once this position is found, the hydrodynamic forces are recalculated, and the position of the array under these new force values is again found by Imaginary Reactions. The iteration process is continued until the desired degree of accuracy is obtained. A natural measure of accuracy for the successive approximation routine is to compare the equilibrium coordinates of the (m,n)th cable station for two successive iterations. If the coordinates differ by less than a fixed amount, the iteration is considered satisfied; if any coordinate change is greater than this fixed amount, the iteration is continued. To express this analytically, let *COMPD* denote the fixed accuracy value. Then the successive approximation routine is considered satisfied when

$$\left| \theta^{i}_{m,n} - \theta^{i-1}_{m,n} \right| \leqq COMPD \tag{30}$$

for all m and n, where $\theta^{i}_{m,n}$ represents the equilibrium coordinates ($\theta = x$, y, or z) of the (m,n)th cable station obtained from the ith successive approximation iteration. If I represents the value of i such that Eq. 30 is satisfied, and $\theta^{*}_{m,n}$ represents the exact solution to the equilibrium configuration which would be assumed under the current-induced forces

$$\left(\theta^{*}_{m,n} = \lim_{i \to \infty} \theta^{i}_{m,n} \right) ,$$

it may further be shown (2) that

$$\left| \theta^{*}_{m,n} - \theta^{I}_{m,n} \right| \leqq COMPD \ . \tag{31}$$

An important aspect of this analysis is to compare the no-current configuration of the structure with the configuration assumed under a given current profile. Since the weight forces are constants, the no-current configuration is determined by Imaginary Reactions alone, and thus, it also provides a convenient starting position for the method of successive approximations. The generalized method used to analyze the array is indicated in the block diagram, Fig. 5.

COMMENTS ON THE METHOD

In Fig. 5, it is seen that The Method of Imaginary Reactions is used to determine the equilibrium shape of the structure *within each successive approximation to the hydrodynamic forces.* Theoretically, the Imaginary Reaction routine, defined by Eqs. 8, 9, and 10, can be continued until the error function E is identically zero; that is, until the equilibrium configuration (under the constant applied forces) is obtained exactly. This is, of course, impossible to achieve in practice; therefore, it is necessary to specify some cutoff value that defines the acceptable completion of the Imaginary Reaction routine. Let this value be denoted by $COMPE$. Then, the Imaginary Reaction determination of the equilibrium shape is considered satisfied when

$$E \leqq COMPE \ ; \tag{32}$$

that is, when the end coordinates $x_{M(n),n}$, $y_{M(n),n}$, and $z_{M(n),n}$ for $n = 2$ and 3 are all within $\sqrt{COMPE}$ from their true anchor values.

The introduction of $COMPE$ as a cutoff value also introduces an error, again within $\sqrt{COMPE}$ from the actual equilibrium coordinates, into the coordinates calculated for every cable station. Consequently, it is important that the cutoff value $COMPD$ for the successive approximation iteration be chosen outside the value of this inherent error. A safe minimum value for $COMPD$ is

$$COMPD = 10 \sqrt{COMPE} \ . \tag{33}$$

It is worthwhile to note that E may never obtain the value of $COMPE$. This is for one of two possible reasons.

1. The convergence factor δ, Eq. 10, approaches zero as E approaches zero; consequently, for a very small δ no changes will occur in the applied imaginary reactions *as a result of the significant figure limitations of the computer.* In this case, $COMPE$ is too small.

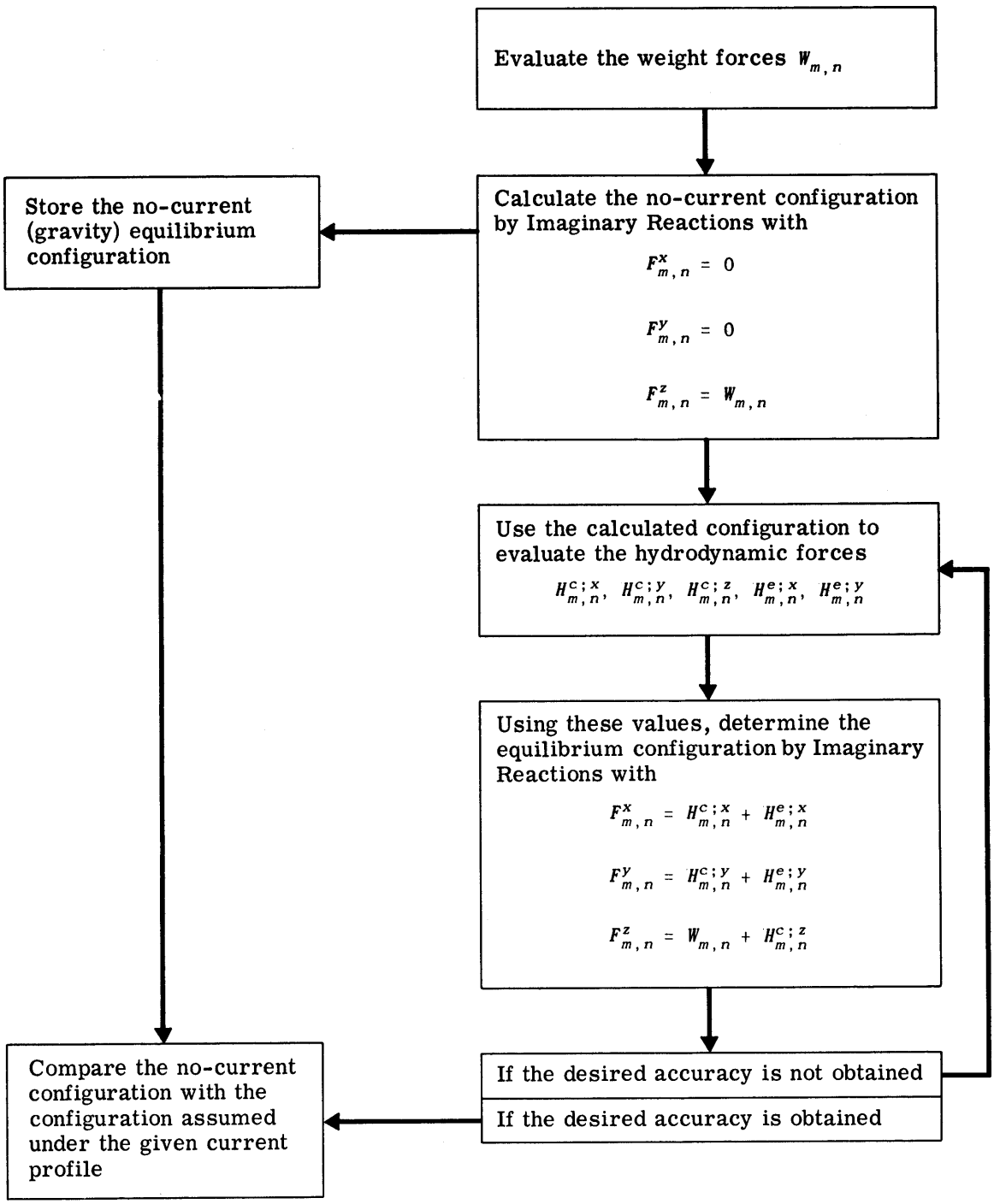

Fig. 5 - Generalized method for the equilibrium analysis

2. A cable segment has gone slack (that is, the segment has zero tension). In this case, the array is statically unstable. This condition is also manifested by no changes occurring in the applied imaginary reactions. (For a fuller discussion of this point see Ref. 1, especially the section Statically Unstable and Divisible Cable Arrays, p. 22).

These sources of error are accounted for in the computer program (Appendix B) and are signaled to the user by the output statement, "PROBLEM NOT COMPLETED, DELTA HAS GOTTEN TOO SMALL TO CHANGE THE IMAGINARY REACTIONS. EITHER ACCURACY REQUIREMENTS ARE TOO SMALL (*COMPE*) OR A CABLE HAS GONE SLACK (CHECK TENSIONS). PRINTOUT IS GIVEN FOR TROUBLE SHOOTING PURPOSES ONLY."

The analysis developed thus far for determining the static configuration of a tri-moored, subsurface, buoy-cable array has been very unrestricted, having been presented for an arbitrary current profile. However, to write a computer program general enough for most analyses, yet simple enough for the user to insert data without revising the entire program, it was decided to limit the velocity profiles to those types which can be represented by a series of straight-line segments. With this limitation, the integral, defined by Eq. 23, that appears in the calculation of the lumped hydrodynamic drag forces, Eqs. 22, can be evaluated exactly. This evaluation is done in Appendix A, and the computer program for implementing the equilibrium analysis is given in Appendix B.

To demonstrate the use of this program and the rapid convergence of the analytic method, a particular Sea Spider array is analyzed in Appendix C. The primary purpose of this example is to test the assumption of Ref. 3 that, "If the variation of current velocity with depth is moderate, the use of the root mean square velocity as a uniform velocity results in a good approximation." Consequently, the structure is analyzed using both an actual current profile and its root mean square value as a constant profile. As the results indicate, the deflections of the array from its equilibrium gravity position obtained using the root mean square profile differ significantly from those obtained using the actual velocity profile. The important conclusion of this example is that *to define a good approximation to the equilibrium shape of the array, the actual velocity profile must be employed.*

SUMMARY AND CONCLUSIONS

This report has presented a method for determining the static configuration of a tri-moored, subsurface, buoy-cable array that is loaded by weight and buoyancy forces and by current-induced drag forces which are dependent on the position of the structure in the water. Both normal and tangential drags on the cables are included in the formulation of the problem, and the possibility that discrete-element measuring and flotation devices are attached to the cables is allowed for in the analysis.

The analysis is performed by using The Method of Imaginary Reactions to determine the equilibrium configuration of the array under constant applied forces and the method of successive approximations to iterate on the position-dependent hydrodynamic forces. Convergence to true equilibrium is extremely rapid.

Although a lumped parameter representation of the system is employed, this should not be a serious drawback. Due to the nature of Imaginary Reactions, arbitrary fineness (within the limits of computer roundoff and truncation error) can be used in representing the array without seriously affecting the rate of convergence of the analysis method.

The method developed in this report should be attractive to both the design engineer and the analyst. A large variety of parametric studies is readily available to the designer, and, for the analyst, the accuracy and rapidity of the calculation are self-recommending.

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Appendix A

THE VELOCITY PROFILE

To develop a program general enough for most analyses yet simple enough for the user to insert data without rewriting large parts of the program, it was decided to limit the velocity profiles to those types which can be represented by a continuous series of straight-line segments, such as shown in Fig. A1. This limitation is really not too severe, since most of the expected design profiles are given or can be well approximated in this manner.

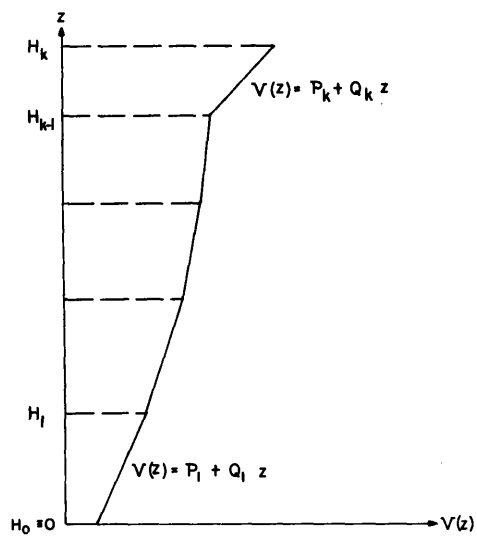

Fig. A1 - A straight-line-segment velocity profile

If k is the index of a particular straight-line segment, then $V(z)$ is defined by

$$V(z) = P_k + Q_k z \tag{A1}$$

for the interval

$$H_{k-1} \leqq z < H_k \ . \tag{A2}$$

In Eq. A2, H_k represents the height above $z = 0$ at which the velocity profile changes slope. By definition of the coordinate system, $H_0 \equiv 0$. The constants P_k and Q_k appearing in Eq. A1 are readily found in terms of H_k and $V(H_k)$ from the expressions

$$Q_k = \frac{V(H_k) - V(H_{k-1})}{H_k - H_{k-1}}$$

and

$$P_k = V(H_{k-1}) - Q_k H_{k-1}$$

With the velocity profile given as a series of straight-line segments, the function $\tilde{V}(m,n:\Xi_1,\Xi_2)$, defined in Eq. 23 and used in Eqs. 22 for the calculation of the lumped hydrodynamic forces, can be evaluated exactly.

Consider first the indefinite integral of $V^2(z)$ along the (m,n)th cable segment. This integral is defined by

$$\int V^2[z(m,n:\xi)]\, d\xi \ , \tag{A3}$$

where the value of z along the cable segment is parametrically expressed in Eq. 24 as

$$z = z(m,n:\xi) = z_{m-1,n} + \gamma_{m,n}\, \xi \ .$$

For values of z (Eq. A2) corresponding to the kth velocity segment (Eq. A1), or, equivalently, for values of ξ delimited by

$$H_{k-1} \leqq z_{m-1,n} + \gamma_{m,n}\, \xi < H_k \ , \tag{A4}$$

the line integral, Eq. A3, is found as

$$\begin{aligned} \tilde{V}_X(m,n:k:\xi) &= \int V^2[z(m,n:\xi)]\, d\xi \\ &= (P_k + Q_k\, z_{m-1,n})^2\, \xi \\ &\quad + (P_k + Q_k\, z_{m-1,n})\, Q_k\, \gamma_{m,n}\, \xi^2 \\ &\quad + \frac{(Q_k\, \gamma_{m,n})^2}{3}\, \xi^3 \ . \end{aligned} \tag{A5}$$

As a result of this calculation, any definite integral obtained from Eq. A3 can be evaluated in terms of the auxiliary indefinite integral $\tilde{V}_X(m,n:k:\xi)$ simply by determining the velocity segments associated with the limits of the integration and the values of ξ associated with the velocity slope discontinuities between these limits.

In particular, the function $\tilde{V}(m,n:\Xi_1,\Xi_2)$ is obtained from Eq. A3 by specifying Ξ_1 as the lower limit and Ξ_2 as the upper limit of the integral. The velocity segments k_1 and k_2 associated, respectively, with the lower and upper limits of integration are determined from the inequalities

$$H_{k_1-1} \leqq z_{m-1,n} + \gamma_{m,n}\, \Xi_1 < H_{k_1} \tag{A6a}$$

and

$$H_{k_2-1} \leqq z_{m-1,n} + \gamma_{m,n}\, \Xi_2 < H_{k_2} \tag{A6b}$$

that are derived from Eq. A4. Further, the values of ξ, ξ_j^*, corresponding to the breakpoints in the current profile between k_1 and k_2, are determined from

$$\xi_j^* = \frac{H_j - z_{m-1,n}}{\gamma_{m,n}} \tag{A6c}$$

for

$$j = k_{\mathrm{MIN}},\ k_{\mathrm{MIN}} + 1,\ \ldots,\ k_{\mathrm{MIN}} - 1 + |k_1 - k_2| \ ,$$

where k_{MIN} is defined as the smaller of k_1 or k_2. The value of $\tilde{V}(m,n:\Xi_1,\Xi_2)$ can then be written in terms of the auxiliary indefinite integral $\tilde{V}_X(m,n:k:\xi)$, Eq. A5, as

$$\tilde{V}(m,n:\Xi_1,\Xi_2) = \tilde{V}_X(m,n:k_2:\Xi_2) - \tilde{V}_X(m,n:k_1:\Xi_1)$$

$$+ \ sgn(k_2 - k_1) \sum_{j=k_{\mathrm{MIN}}}^{k_{\mathrm{MIN}}-1+|k_1-k_2|} [\tilde{V}_X(m,n:j:\xi_j^*) - \tilde{V}_X(m,n:j+1:\xi_j^*)] \ . \tag{A7}$$

This evaluation of $\tilde{V}(m,n:\Xi_1,\Xi_2)$ is used in the computer program to determine the hydrodynamic forces given by Eqs. 22.

Note that, if it is desired to employ another type of velocity profile, it is necessary to change only the subroutine for the calculation of $\tilde{V}(m,n:\Xi_1,\Xi_2)$.

Appendix B

THE COMPUTER PROGRAM DATUBA

DESCRIPTION

The computer program DATUBA (an acronym for Deflection Analysis of a Tri-Moored, Underwater, Buoy-Cable Array) is written in 3600 FORTRAN. This language contains the features of FORTRAN-63 and is a dialect that is compatible with the compilers of Control Data Corporation's (CDC) 3600 and 3800 computers. The program, as listed, should compile on most FORTRAN IV compilers. However, if it does not, the trouble will probably lie with the form of the READ and/or PRINT statements, since INPUT/OUTPUT statements are often compiler dependent. In DATUBA, these statements, all of which occur in the subroutines INPUT, OUTPUT, and VPROFILE, have the form

READ n,L

and

PRINT n,L

where n is the number of the FORMAT statement that controls the transfer of the list of variables L.

Comment cards at the beginning of the main program describe the important nomenclature. The symbols used in the report are duplicated as closely as possible in the program. If the symbol is a letter of the Greek alphabet, its English spelling is used (i.e., ALPHA is used to represent α).

The program, as written, is restricted to 20 segments per cable and to 75 elemental devices per cable segment. These restrictions can be modified simply by changing the dimensions of the COMMON arrays in the main program and in all of the subroutines and functions. It is essential, however, that the M dimension for the arrays $X(M,N)$, $Y(M,N)$, and $Z(M,N)$ exceeds the greatest value of $MMAX(N)$, the number of segments on cable n. This is to insure that the zero subscript notation will be accommodated on the computer.

Approximate memory requirements for the program are 16,000 words for the arrays in COMMON and 2,100 words for the main program and subprograms. If the computer memory is inadequate, then the K and M dimensions for all arrays should be reduced to the largest values of $KMAX(M,N)$ and $MMAX(N)$ that appear in a particular analysis. $KMAX(M,N)$ is equivalent to $K(m,n)$, the number of elemental devices attached to the (m,n)th cable segment.

As indicated by the comment cards in subroutine INPUT, the program, as listed, is not complete and will not run. It is left to the user to provide statements that will read in or compute the listed required items. *Input data cards made necessary by such statements should follow the last input card described in the next section.* It should be noted that $KTILDA(M,N)$, corresponding to $\tilde{k}(m,n)$, the number of discrete objects attached to the (m,n)th half-segment adjoining the $(m-1,n)$th station, is calculated internally and is thus not a necessary input.

INPUT DATA CARDS

Cards 1 through 5 are called from subroutine INPUT.

CARD 1.

COLUMNS	ITEM NAME	FORMAT	DESCRIPTION
1-10	COMPE	Floating Point (FP)	Comparison value for the error function
11-20	COMPD	FP	Comparison value for the successive approximation routine
21-30	STARTPSI	FP	First current angle to be analyzed, in degrees
31-40	DELTAPSI	FP	Change of current angle for each subsequent analysis, in degrees
41-50	ENDPSI	FP	Final current angle, in degrees

CARDS 2 (N = 1), 3 (N = 2), and 4 (N = 3).

COLUMNS	ITEM NAME	FORMAT	DESCRIPTION
1-10	A(N)	FP	x coordinate of nth cable anchor
11-20	B(N)	FP	y coordinate of nth cable anchor
21-30	C(N)	FP	z coordinate of nth cable anchor

CARD 5.

COLUMNS	ITEM NAME	FORMAT	DESCRIPTION
1-10	MMAX(1)	Integer (I)	Number of segments on cable 1
11-20	MMAX(2)	I	Number of segments on cable 2
21-30	MMAX(3)	I	Number of segments on cable 3

CARDS 6 and subsequent.

COLUMNS	ITEM NAME	FORMAT	DESCRIPTION
1-10	H(K)	FP	z coordinate where the velocity profile changes slope. The first card must have $H(0)$ = 0.0, and the last card must have a value of $H(K)$ > 40,000.0 feet.
11-20	V(K)	FP	The magnitude of the current at $z = H(K)$

Following this card, more cards can be called from the subroutine INPUT as are needed to complete the specification of the array.

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      PROGRAM DATUBA
C
C     DEFLECTION ANALYSIS OF A TRI-MOORED UNDERWATER BUOY-CABLE ARRAY
C
C  SOLUTION BY THE METHODS OF IMAGINARY REACTIONS AND SUCCESSIVE APPROXIMATIONS
C
C    NOMENCLATURE
C      N = CABLE INDEX
C      M = STATION INDEX
C      K = DISCRETE ELEMENT INDEX
C
C      CABLE SEGMENT(M,N)
C        LBAR(M,N) = UNSTRESSED LENGTH
C           L(M,N) = STRESSED LENGTH
C          WC(M,N) = WEIGHT/FOOT
C        XTEN(M,N) = EXTENSIONAL RIGIDITY
C          MU(M,N) = DRAG CHARACTERISTIC
C          RD(M,N) = DRAG COEFFICIENT RATIO
C        KMAX(M,N) = NO.OF DISCRETE ELEMENTS IN SEGMENT(M,N)
C      KTILDA(M,N) = NO. OF DISCRETE ELEMENTS IN FIRST HALF OF SEGMENT(M,N)
C           T(M,N) = TENSION
C          (RX(M,N), RY(M,N), RZ(M,N)) = RESULTANT FORCES ON SEGMENT(M,N)
C          (ALPHA(M,N), BETA(M,N), GAMMA(M,N)) = DIRECTION COSINES
C
C      DISCRETE ELEMENT(K,M,N)
C         SBAR(K,M,N) = UNSTRESSED LENGTH FROM STATION(M-1,N) TO ELEMENT(K,M,N)
C           WE(K,M,N) = WEIGHT
C          MUE(K,M,N) = DRAG CHARACTERISTIC
C
C      STATION(M,N)
C          W(M,N) = WEIGHT
C          (X(M,N), Y(M,N), Z(M,N)) = COORDINATES
C          (XB(M,N), YB(M,N), ZB(M,N)) = EQUILIBRIUM POSITION-GRAVITY FORCES
C          (FX(M,N), FY(M,N), FZ(M,N)) = EXTERNAL FORCES
C          (HX(M,N), HY(M,N), HZ(M,N)) = CABLE DRAG FORCES
C          (HXE(M,N), HYE(M,N), HZE(M,N)) = DISCRETE ELEMENT DRAG FORCES
C          MMAX(N) = NO.OF STATIONS ON CABLE(N)
C          (A(N), B(N), C(N)) = ANCHOR COORDINATES
C          HORIZL(M,N) = HORIZL DISPLACEMENT FROM EQUILIBRIUM GRAVITY POSITION
C          HEIGHT(M,N) = VERTICAL DISPLACEMENT FROM EQUILIBRIUM GRAVITY POSITION
C
C      E = ERROR FUNCTION
C      COMPE = COMPARISON VALUE FOR E
C      COMPD = COMPARISON VALUE FOR DISPLACEMENT
C      PSI = ANGLE OF CURRENT
C
      REAL L, LBAR, LT, MU, MUE
      COMMON / C1  / XF, X(21,3), YF, Y(21,3), ZF, Z(21,3)
      COMMON / C2  / FX(20,3), FY(20,3), FZ(20,3)
      COMMON / C3  / W(20,3), WC(20,3), WE(75,20,3)
      COMMON / C4  / MMAX(3), KMAX(20,3), KTILDA(20,3)
      COMMON / C5  / LBAR(20,3),L(20,3),SBAR(75,20,3),T(20,3),LT(20,3)
      COMMON / C6  / A(3), B(3), C(3), E, DELTA, JUMP, LOOPE, LOOPA
      COMMON / C7  / HORIZL(20,3), HEIGHT(20,3)
      COMMON / C8  / CX(20,3), CY(20,3), CZ(20,3)
      COMMON / C9  / AV(25), BV(25), VF, V(25), HF, H(25)
      COMMON / C10 / COMPE, COMPD, PSI, STARTPSI, DELTAPSI, ENDPSI
      COMMON / C11 / XTEN(20,3), MU(20,3), MUE(75,20,3), RD(20,3)
      COMMON / C12 / ALPHA(20,3), BETA(20,3), GAMMA(20,3)
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      COMMON / C13 / RX(20,3), RY(20,3), RZ(20,3)
      COMMON / C14 / X0(20,3), Y0(20,3), Z0(20,3)
      COMMON / C15 / XB(20,3), YB(20,3), ZB(20,3)
      COMMON / C16 / FXP(3), FYP(3), FZP(3), XP(3), YP(3), ZP(3)
      COMMON / C17 / DELTAI
C
C READ AND PRINT INPUT INFORMATION
C
      CALL INPUT
      CALL CABLE
C
C COMPUTE MIDSEGMENT DISCRETE ELEMENT KTILDA(M,N)
C
      DO 2  N = 1, 3
      MX = MMAX(N)
      DO 2  M = 1, MX
      KX = KMAX(M,N)
      DO 1  K = 1, KX
      IF ( SBAR(K,M,N) .GT. LBAR(M,N) / 2. ) GO TO 2
    1 CONTINUE
      K = KX + 1
    2 KTILDA(M,N) = K - 1
C
C ZERO SUBSCRIPT
C
      X(0,1) = A(1)
      Y(0,1) = B(1)
      Z(0,1) = C(1)
C
C INITIAL VALUES
C
      LEAP = 1
      JUMP = 1
      PSI = STARTPSI
      PI = 3.14159265
      LOOPE = 0
      LOOPA = 0
C
C COMPUTE STATION GRAVITY FORCES W(M,N) AND INITIAL FORCES FX,FY,FZ
C
      CALL GFORCES
C
C COMPUTE CABLE FORCES RX,RY,RZ--TENSION T(M,N)--AND STRESSED LENGTH L(M,N)
C
    3 DO 7  NN = 1, 3
      N = 4 - NN
      MX = MMAX(N)
      GO TO ( 5, 4 ) N
    4 RX(MX,N) = FX(MX,N)
      RY(MX,N) = FY(MX,N)
      RZ(MX,N) = FZ(MX,N)
      GO TO 6
    5 RX(MX,1) = FX(MX,1) + RX(1,2) + RX(1,3)
      RY(MX,1) = FY(MX,1) + RY(1,2) + RY(1,3)
      RZ(MX,1) = FZ(MX,1) + RZ(1,2) + RZ(1,3)
    6 T(MX,N) = SQRT ( RX(MX,N)**2 + RY(MX,N)**2 + RZ(MX,N)**2 )
      L(MX,N) = LBAR(MX,N) * ( 1. + T(MX,N) / XTEN(MX,N) )
      LT(MX,N) = L(MX,N) / T(MX,N)
      MX = MX - 1
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      DO 7  MM = 1, MX
      M = MX - MM + 1
      RX(M,N) = FX(M,N) + RX(M+1,N)
      RY(M,N) = FY(M,N) + RY(M+1,N)
      RZ(M,N) = FZ(M,N) + RZ(M+1,N)
      T(M,N) = SQRT ( RX(M,N)**2 + RY(M,N)**2 + RZ(M,N)**2 )
      L(M,N) = LBAR(M,N) * ( 1. + T(M,N) / XTEN(M,N) )
    7 LT(M,N) = L(M,N) / T(M,N)
C
C COMPUTE X,Y,Z COORDINATES OF EACH STATION
C
      DO 10  N = 1, 3
      MX = MMAX(N)
      DO 8  M = 1, MX
      X(M,N) = LT(M,N) * RX(M,N) + X(M-1,N)
      Y(M,N) = LT(M,N) * RY(M,N) + Y(M-1,N)
    8 Z(M,N) = LT(M,N) * RZ(M,N) + Z(M-1,N)
      GO TO ( 9, 10 ) N
    9 DO 10  NN = 2, 3
      X(0,NN) = X(MX,1)
      Y(0,NN) = Y(MX,1)
      Z(0,NN) = Z(MX,1)
   10 CONTINUE
C
C COMPUTE ERROR FUNCTION
C
      LOOPE = LOOPE + 1
      E = 0
      DO 11  N = 2, 3
      M = MMAX(N)
   11 E = E + (A(N)-X(M,N))**2 + (B(N)-Y(M,N))**2 + (C(N)-Z(M,N))**2
      IF ( E .GT. COMPE ) GO TO ( 19, 50, 15 ) LEAP
C
C UPDATED DIRECTION COSINES
C
      DO 300 N = 1, 3
      MX = MMAX(N)
      DO 300 M = 1, MX
      ALPHA(M,N) = (X(M,N)-X(M-1,N)) / L(M,N)
       BETA(M,N) = (Y(M,N)-Y(M-1,N)) / L(M,N)
  300 GAMMA(M,N) = (Z(M,N)-Z(M-1,N)) / L(M,N)
C
C ERROR FUNCTION COMPARISON SATISFIED
C
      GO TO ( 51, 52 ) JUMP
C
C PRINT AND STORE STATIC EQUILIBRIUM POSITION
C
   51 CALL STATPOS
      JUMP = 2
      LOOPE = 0
      DO 53  N = 1, 3
      MX = MMAX(N)
      DO 53  M = 1, MX
      XO(M,N) = X(M,N)
      YO(M,N) = Y(M,N)
      ZO(M,N) = Z(M,N)
      XB(M,N) = X(M,N)
      YB(M,N) = Y(M,N)
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   53 ZB(M,N) = Z(M,N)
      GO TO 61
C
C COMPARE ACCURACY OF COORDINATES
C
   52 DO 55  N = 1, 3
      MX = MVALUE(N)
      DO 55  M = 1, MX
      IF ( ABS (X(M,N)-XO(M,N)) .GT. COMPD .OR.
     1     ABS (Y(M,N)-YO(M,N)) .GT. COMPD .OR.
     2     ABS (Z(M,N)-ZO(M,N)) .GT. COMPD ) GO TO 57
   55 CONTINUE
C
C ACCURACY SATISFIED-PRINT EQUILIBRIUM POSITION FOR HYDRODYNAMIC FORCES
C
      DO 56  N = 1, 3
      MX = MVALUE(N)
      DO 56  M = 1, MX
      HORIZL(M,N) = SQRT ( (X(M,N)-XB(M,N))**2 + (Y(M,N)-YB(M,N))**2 )
   56 HEIGHT(M,N) = Z(M,N) - ZB(M,N)
      CALL DYNAMPOS
      LOOPE = 0
      LOOPA = 0
      GO TO 60
C
C ACCURACY NOT ADEQUATE-REITERATE
C
   57 DO 59  N = 1, 3
      MX = MVALUE(N)
      DO 59  M = 1, MX
      XO(M,N) = X(M,N)
      YO(M,N) = Y(M,N)
   59 ZO(M,N) = Z(M,N)
      GO TO 62
C
C ERROR FUNCTION COMPARISON NOT SATISFIED
C
   50 IF ( E .LT. EP ) GO TO 20
C
C INCREASE IN ERROR FUNCTION
C
      DELTA = DELTA / 2.
C
C COMPUTE IMAGINARY REACTIONS
C
   12 DE = DELTA / SQRT ( EP )
      DO 13  N = 2, 3
      MX = MMAX(N)
      FX(MX,N) = FXP(N) + (A(N)-XP(N)) * DE
      FY(MX,N) = FYP(N) + (B(N)-YP(N)) * DE
   13 FZ(MX,N) = FZP(N) + (C(N)-ZP(N)) * DE
C
C CHECK CHANGES IN IMAGINARY REACTIONS
C
      DO 14  N = 2, 3
      IF ( FX(MX,N) .NE. FXP(N) .OR.
     1     FY(MX,N) .NE. FYP(N) .OR.
     2     FZ(MX,N) .NE. FZP(N) ) GO TO 3
   14 CONTINUE
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```
      LEAP = 3
      GO TO 3
C
C NO CHANGE-TIME TO QUIT
C
   15 CALL EXITT
      GO TO 100
C
C DECREASE IN ERROR FUNCTION
C
   19 LEAP = 2
   20 EP = E
      DO 21  N = 2, 3
      MX = MMAX(N)
      XP(N) = X(MX,N)
      YP(N) = Y(MX,N)
      ZP(N) = Z(MX,N)
      FXP(N) = FX(MX,N)
      FYP(N) = FY(MX,N)
   21 FZP(N) = FZ(MX,N)
      GO TO 12
C
C INCREASE CURRENT ANGLE
C
   60 PSI = PSI + DELTAPSI
      IF ( PSI .GT. ENDPSI+1E-6 ) GO TO 100
C
C ADD HYDRODYNAMIC FORCES TO SYSTEM
C
   61 COSPSI = COS ( PSI * PI / 180. )
      SINPSI = SIN ( PSI * PI / 180. )
   62 DELTA = DELTAI
      LEAP = 1
      LOOPA = LOOPA + 1
C
C DRAG COEFFICIENTS
C
      DO 30  N = 1, 3
      MX = MMAX(N)
      DO 30  M = 1, MX
      CAPDELTA = SQRT ( (BETA(M,N)*COSPSI - ALPHA(M,N)*SINPSI)**2
     1                  + GAMMA(M,N)**2 )
      BUFFER = ALPHA(M,N)*COSPSI + BETA(M,N)*SINPSI
      CX(M,N) = MU(M,N) * ( CAPDELTA *
     1          ( (GAMMA(M,N)**2 + BETA(M,N)**2)*COSPSI
     2          - ALPHA(M,N) * BETA(M,N) * SINPSI )
     3          + RD(M,N) * ALPHA(M,N) * BUFFER )
      CY(M,N) = MU(M,N) * ( CAPDELTA *
     1          ( (GAMMA(M,N)**2 + ALPHA(M,N)**2)*SINPSI
     2          - ALPHA(M,N) * BETA(M,N) * COSPSI )
     3          + RD(M,N) * BETA(M,N) * BUFFER )
   30 CZ(M,N) = MU(M,N) * GAMMA(M,N) * BUFFER * (RD(M,N)-CAPDELTA)
C
C COMPUTE CABLE DRAG FORCES HX,HY,HZ-AND ELEMENT DRAG FORCES HXE,HYE
C
      DO 40  N = 1, 3
      MX = MVALUE(N)
      DO 40  M = 1, MX
      GO TO ( 23, 24 ) MX - M + N
   23 A1 = AREA1(MX,1)
      A2 = AREA2(1,2)
      A3 = AREA2(1,3)
      HX = CX(M,N) * A1 + CX(1,2) * A2 + CX(1,3) * A3
      HY = CY(M,N) * A1 + CY(1,2) * A2 + CY(1,3) * A3
      HZ = CZ(M,N) * A1 + CZ(1,2) * A2 + CZ(1,3) * A3
      DT = DRAG1(MX,1) + DRAG2(1,2) + DRAG2(1,3)
      HXE = DT * COSPSI
      HYE = DT * SINPSI
      GO TO 26
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   24 A1 = AREA1(M,N)
      A2 = AREA2(M+1,N)
      HX = CX(M,N) * A1 + CX(M+1,N) * A2
      HY = CY(M,N) * A1 + CY(M+1,N) * A2
      HZ = CZ(M,N) * A1 + CZ(M+1,N) * A2
      DT = DRAG1(M,N) + DRAG2(M+1,N)
      HXE = DT * COSPSI
      HYE = DT * SINPSI
C
C NEW TOTAL FORCES
C
   26 FX(M,N) = HX + HXE
      FY(M,N) = HY + HYE
      FZ(M,N) = HZ + W(M,N)
   40 CONTINUE
      GO TO 3
  100 END
```

```
      SUBROUTINE INPUT
      REAL L, LBAR, LT, MU, MUE
      COMMON / C3  / W(20,3), WC(20,3), WE(75,20,3)
      COMMON / C4  / MMAX(3), KMAX(20,3), KTILDA(20,3)
      COMMON / C5  / LBAR(20,3),L(20,3),SBAR(75,20,3),T(20,3),LT(20,3)
      COMMON / C6  / A(3), B(3), C(3), E, DELTA, JUMP, LOOPE, LOOPA
      COMMON / C9  / AV(25), BV(25), VF, V(25), HF, H(25)
      COMMON / C10 / COMPE, COMPD, PSI, STARTPSI, DELTAPSI, ENDPSI
      COMMON / C11 / XTEN(20,3), MU(20,3), MUE(75,20,3), RD(20,3)
C
C COMPARISON VALUES AND CURRENT ANGLE REQUIREMENTS
C
      READ 1, COMPE, COMPD, STARTPSI, DELTAPSI, ENDPSI
    1 FORMAT ( 5F10 )
C
C ANCHOR POSITIONS
C
      READ 2, ( A(N), B(N), C(N), N = 1, 3 )
    2 FORMAT ( 3F10 )
C
C NO.OF STATIONS PER CABLE
C
      READ 3, ( MMAX(N), N = 1, 3 )
    3 FORMAT ( 3I10 )
C
C PROVIDE VELOCITY PROFILE IN SUBROUTINE VPROFILE
C
      CALL VPROFILE
C
C USER MUST PROVIDE STATEMENTS TO READ IN OR CALCULATE THE FOLLOWING
C OR INITIALIZE WITH THE USE OF DATA STATEMENTS AFTER COMMON / C11 /
C
C     CABLE PROPERTIES OF SEGMENT(M,N)
C        LBAR(M,N) = UNSTRESSED LENGTH
C          WC(M,N) = WEIGHT/FOOT
C        XTEN(M,N) = EXTENSIONAL RIGIDITY
C          MU(M,N) = WATER DENSITY * NORMAL DRAG COEFF. * CABLE DIAMETER / 2
C          RD(M,N) = PARALLEL DRAG COEFF. / NORMAL DRAG COEFF.
C        KMAX(M,N) = NO.OF DISCRETE ELEMENTS IN SEGMENT
C
C     PROPERTIES OF DISCRETE ELEMENT(K,M,N)
C        SBAR(K,M,N) = UNSTRESSED LENGTH FROM STATION(M-1,N) TO ELEMENT(K,M,N)
C          WE(K,M,N) = WEIGHT
C         MUE(K,M,N) = WATER DENSITY * DRAG COEFF. * X-SECTIONAL AREA / 2
C
C INSERT USER-PROVIDED CALCULATIONS OR READ-IN STATEMENTS FOLLOWING THIS CARD
      END
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      SUBROUTINE GFORCES
      REAL L, LBAR, LT
      COMMON / C2  / FX(20,3), FY(20,3), FZ(20,3,
      COMMON / C3  / W(20,3), WC(20,3), WE(75,20,3)
      COMMON / C4  / MMAX(3), KMAX(20,3), KTILDA(20,3)
      COMMON / C5  / LBAR(20,3),L(20,3),SBAR(75,20,3),T(20,3),LT(20,3)
      COMMON / C6  / A(3), B(3), C(3), E, DELTA, JUMP, LOOPE, LOOPA
      COMMON / C17 / DELTAI
C
C COMPUTE STATION GRAVITY FORCES W(M,N)- AND INITIAL FORCES FX,FY,FZ
C
      WT = 0
      DO 3  N = 1, 3
      MX = MMAX(N) - 1
      DO 3  M = 1, MX
      WX = 0
      FX(M,N) = 0
      FY(M,N) = 0
      KB = KTILDA(M,N) + 1
      KX = KMAX(M,N)
      DO 1  K = KB, KX
    1 WX = WX + WE(K,M,N)
      KX = KTILDA(M+1,N)
      DO 2  K = 1, KX
    2 WX = WX + WE(K,M+1,N)
      W(M,N) = WX + WC(M,N)*LBAR(M,N)/2. + WC(M+1,N)*LBAR(M+1,N)/2.
      FZ(M,N) = W(M,N)
    3 WT = WT + W(M,N)
      WX = 0
      FX(MMAX(1),1) = 0
      FY(MMAX(1),1) = 0
      KB = KTILDA(MMAX(1),1) + 1
      KX = KMAX(MMAX(1),1)
      DO 4  K = KB, KX
    4 WX = WX + WE(K,MMAX(1),1)
      DO 6  N = 2, 3
      KX = KTILDA(1,N)
      FX(MMAX(N),N) = 0
      FY(MMAX(N),N) = 0
      DO 5  K = 1, KX
    5 WX = WX + WE(K,1,N)
    6 WX = WX + WC(1,N) * LBAR(1,N) / 2.
      W(MMAX(1),1) = WX + WC(MMAX(1),1) * LBAR(MMAX(1),1) / 2.
      FZ(MMAX(1),1) = W(MMAX(1),1)
      WT = WT + W(MMAX(1),1)
      DO 7  N = 2, 3
    7 FZ(MMAX(N),N) = -WT / 3.
C
C COMPUTE INITIAL DELTA
C
      SQ2 = SQRT ( 2. )
      DELTAI = ABS ( SQ2 * WT / 3. )
      DELTA = DELTAI
      END




      FUNCTION MVALUE(N)
      COMMON / C4  / MMAX(3), KMAX(20,3), KTILDA(20,3)
      GO TO ( 1, 2 ) N
    1 MVALUE = MMAX(N)
      RETURN
    2 MVALUE = MMAX(N) - 1
      END
```

```
      FUNCTION AREAS ( M, N, K, X1 )
      COMMON / C1  / XF, X(21,3), YF, Y(21,3), ZF, Z(21,3)
      COMMON / C9  / AV(25), BV(25), VF, V(25), HF, H(25)
      COMMON / C12 / ALPHA(20,3), BETA(20,3), GAMMA(20,3)
      AREAS = (AV(K)+BV(K)*Z(M-1,N))**2 * X1
     1        + (AV(K)+BV(K)*Z(M-1,N)) * BV(K) * GAMMA(M,N) * X1**2
     2        + (BV(K)*GAMMA(M,N))**2 * X1**3 / 3.
      END

      FUNCTION LIMIT ( ZT )
      COMMON / C9  / AV(25), BV(25), VF, V(25), HF, H(25)
      DO 1  J = 1, 25
      IF ( ZT .GE. H(J) ) 1, 2
    1 CONTINUE
    2 LIMIT = J
      END

      SUBROUTINE VPROFILE
      COMMON / C9  / AV(25), BV(25), VF, V(25), HF, H(25)
      K = 0
    1 READ 2, H(K), V(K)
    2 FORMAT ( 2F10 )
      IF ( H(K) .GE. 40000. ) GO TO 3
      K = K + 1
      GO TO 1
    3 KX = K
      DO 4  K = 1, KX
      BV(K) = (V(K)-V(K-1)) / (H(K)-H(K-1))
    4 AV(K) = V(K-1) - BV(K) * H(K-1)
      END

      FUNCTION SYGN(J)
      IF ( J ) 1, 2, 3
    1 SYGN = -1.
      RETURN
    2 SYGN = 0
      RETURN
    3 SYGN = 1.
      END

      FUNCTION AREA ( M,N )
      REAL LBAR, L, MU, MUE, LT
      COMMON / C1  / XF, X(21,3), YF, Y(21,3), ZF, Z(21,3)
      COMMON / C4  / MMAX(3), KMAX(20,3), KTILDA(20,3)
      COMMON / C5  / LBAR(20,3),L(20,3),SBAR(75,20,3),T(20,3),LT(20,3)
      COMMON / C9  / AV(25), BV(25), VF, V(25), HF, H(25)
      COMMON / C11 / XTEN(20,3), MU(20,3), MUE(75,20,3), RD(20,3)
      COMMON / C12 / ALPHA(20,3), BETA(20,3), GAMMA(20,3)
C
C LINE INTEGRAL BELOW STATION
C
      ENTRY AREA1
      KL = LIMIT ( Z(M-1,N) + GAMMA(M,N) * L(M,N) / 2. )
      KU = LIMIT ( Z(M,N) )
      TOP = L(M,N)
      BOT = L(M,N) / 2.
    2 KMIN = MIN0 ( KU, KL )
      KTOP = KMIN - 1 + IABS ( KU-KL )
      SUM = 0
      DO 1  K = KMIN, KTOP
      XI = (H(K)-Z(M-1,N)) / GAMMA(M,N)
    1 SUM = SUM + AREAS(M,N,K,XI) - AREAS(M,N,K+1,XI)
      AREA = SYGN( KU-KL ) * SUM + AREAS(M,N,KU,TOP) - AREAS(M,N,KL,BOT)
      RETURN
```

```
C
C LINE INTEGRAL ABOVE STATION
C
      ENTRY AREA2
      KL = LIMIT(Z(M-1,N))
      KU= LIMIT(Z(M-1,N) + GAMMA(M,N) * L(M,N) / 2.)
      TOP = L(M,N) / 2.
      BOT = 0
      GO TO 2
C
C DRAG OF ELEMENTS BELOW STATION
C
      ENTRY DRAG1
      KB = KTILDA(M,N) + 1
      KX = KMAX(M,N)
    5 SUM = 0
      DO 4 K = KB, KX
      ZT = Z(M-1,N) + GAMMA(M,N) * SBAR(K,M,N)*(1.+T(M,N)/XTEN(M,N))
      J = LIMIT( ZT )
    4 SUM = SUM + MUE(K,M,N) * (AV(J) + BV(J) * ZT)**2
      AREA = SUM
      RETURN
C
C DRAG OF ELEMENTS ABOVE STATION
C
      ENTRY DRAG2
      KB = 1
      KX = KTILDA(M,N)
      GO TO 5
      END


      SUBROUTINE OUTPUT
      REAL LBAR, L, MU, MUE, LT
      COMMON / C1  / XF, X(21,3), YF, Y(21,3), ZF, Z(21,3)
      COMMON / C2  / FX(20,3), FY(20,3), FZ(20,3)
      COMMON / C4  / MMAX(3), KMAX(20,3), KTILDA(20,3)
      COMMON / C5  / LBAR(20,3),L(20,3),SBAR(75,20,3),T(20,3),LT(20,3)
      COMMON / C6  / A(3), B(3), C(3), E, DELTA, JUMP, LOOPE, LOOPA
      COMMON / C7  / HORIZL(20,3), HEIGHT(20,3)
      COMMON / C10 / COMPE, COMPD, PSI, STARTPSI, DELTAPSI, ENDPSI
      COMMON / C11 / XTEN(20,3), MU(20,3), MUE(75,20,3), RD(20,3)
C
      ENTRY CABLE
      PRINT 1
    1 FORMAT(1H1,40H PHYSICAL PROPERTIES OF THE CABLE SYSTEM)
      DO 4 N=1,3
      PRINT 2,N,A(N),B(N),C(N)
    2 FORMAT(1H2,//3X,14HCABLE NUMBER =,I1,//10X,15HANCHOR POSITION,/15X
     1       ,2HX=,E16.9,10X,2HY=,E16.9,10X,2HZ=,E16.9)
      MX=MMAX(N)
      DO 4 M=1,MX
      PRINT 3, M, LBAR(M,N), XTEN(M,N)
    3 FORMAT(/5X,16HSEGMENT NUMBER =,I2,/10X,11HLBAR(M,N)= ,E16.9,10X,
     1       11HXTEN(M,N)= ,E16.9)
    4 CONTINUE
      RETURN
C
      ENTRY STATPOS
      PRINT 13
   16 PRINT 5,E,DELTA,LOOPE
    5 FORMAT(  X,41HEQUILIBRIUM POSITION UNDER GRAVITY FORCES,//10X,
     1       2HE=,E16.9,10X, 6HDELTA=,E16.9,10X,19HNO.OF ERROR LOOPS=
     2      ,I5)
    6 DO 10 N=1,3
```

```
      PRINT 7,N
    7 FORMAT(//,3X,13HCABLE NUMBER=,I2)
      MX=MMAX(N)
      DO 10 M=1,MX
      PRINT 8,M,FX(M,N),FY(M,N),FZ(M,N),X(M,N),Y(M,N),Z(M,N),T(M,N),
     1         L(M,N)
    8 FORMAT(/,5X,15HSEGMENT NUMBER=,I2,/
     1    10X,8HFX(M,N)=,E16.9,10X,8HFY(M,N)=,E16.9,10X,8HFZ(M,N)=,E16.9/
     2    10X,8H X(M,N)=,E16.9,10X,8H Y(M,N)=,E16.9,10X,8H Z(M,N)=,E16.9/
     3  10X,8H T(M,N)=,E16.9,10X,8H L(M,N)=,E16.9)
      GO TO(10,9)JUMP
    9 PRINT 11,HORIZL(M,N),HEIGHT(M,N)
   11 FORMAT(15X,12HHORIZL(M,N)=,E16.9,10X,12HHEIGHT(M,N)=,E16.9)
   10 CONTINUE
      RETURN
C
      ENTRY DYNAMPOS
      PRINT 13
   17 PRINT 12,E,DELTA,LOOPE,LOOPA,PSI
   12 FORMAT(  X,40HEQUILIBRIUM POSITION UNDER ACTING FORCES,//
     1        10X,2HE=,E16.9,10X 6HDELTA=,E16.9,/,10X,19HNO.OF ERROR LOOP
     2S=     , I5,10X,21HNO.OF ACCURACY LOOPS=     ,I5,//
     3        14HCURRENT ANGLE=,F8.3)
      GO TO 6
C


      ENTRY EXITT.
      PRINT 13
      PRINT 14
   14 FORMAT(X//X,90HPROBLEM NOT COMPLETED, DELTA HAS GOTTEN TOO SMALL T
     1O CHANGE THE IMAGINARY REACTIONS.
     2/X,100HEITHER ACCURACY REQUIREMENTS ARE TOO SMALL(COMPE) OR A CABL
     3E HAS GONE SLACK (CHECK TENSIONS).
     4/X,55HPRINTOUT IS GIVEN FOR TROUBLE SHOOTING PURPOSES ONLY.      )
      GO TO ( 16, 17 ) JUMP
   13 FORMAT ( 1H1 )
      END
```

SAMPLE OUTPUT

The following printout shows the output format for this program as given by subroutine OUTPUT. The physical properties section of the printout is included as a check on the input. It is followed by the no-current equilibrium configuration of the system, and then by the equilibrium configurations assumed under the various current headings that are considered in the problem.

The user may make changes in subroutine OUTPUT to fit his needs. All variables listed in COMMON statements at the beginning of the main program are in core storage and can be printed if needed.

The particular output given here describes the array considered as an example in Appendix C. The equilibrium configuration under a 0-degree current heading is that obtained for Case B of Appendix C. The printout is shown only for cable 1, since the printouts for cables 2 and 3 have the same format and contain the same information.

```
PHYSICAL PROPERTIES OF THE CABLE SYSTEM

CABLE NUMBER =1

        ANCHOR POSITION
            X=-1.900000000+004          Y= 0.000000000+000          Z= 0.000000000+000

    SEGMENT NUMBER = 1
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 2
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 3
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 4
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 5
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 6
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 7
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 8
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER = 9
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =10
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =11
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =12
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =13
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =14
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =15
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =16
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =17
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

    SEGMENT NUMBER =18
        LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006
```

```
SEGMENT NUMBER =19
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =20
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

CABLE NUMBER =2

     ANCHOR POSITION
          X= 9.500000000+003          Y=-1.645450000+004          Z= 0.000000000+000

SEGMENT NUMBER = 1
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 2
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 3
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 4
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 5
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 6
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 7
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 8
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER = 9
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =10
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =11
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =12
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =13
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =14
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =15
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =16
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006

SEGMENT NUMBER =17
     LBAR(M,N)=  1.341500000+003          XTEN(M,N)=  6.170900000+006
```

```
EQUILIBRIUM POSITION UNDER GRAVITY FORCES

     E= 8.124466426-003          DELTA= 3.598772476-002          NO.OF ERROR LOOPS=    46

 CABLE NUMBER= 1

  SEGMENT NUMBER= 1
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.865007569+004         Y(M,N)= 0.000000000+000         Z(M,N)= 9.501432081+002
       T(M,N)= 9.432919389+003         L(M,N)= 1.343550635+003

  SEGMENT NUMBER= 2
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.710014353+004         Y(M,N)= 0.000000000+000         Z(M,N)= 1.900278537+003
       T(M,N)= 9.432841245+003         L(M,N)= 1.343550618+003

  SEGMENT NUMBER= 3
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.615020350+004         Y(M,N)= 0.000000000+000         Z(M,N)= 2.850405985+003
       T(M,N)= 9.432763102+003         L(M,N)= 1.343550601+003

  SEGMENT NUMBER= 4
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.520025562+004         Y(M,N)= 0.000000000+000         Z(M,N)= 3.800525554+003
       T(M,N)= 9.432684959+003         L(M,N)= 1.343550584+003

  SEGMENT NUMBER= 5
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.425029988+004         Y(M,N)= 0.000000000+000         Z(M,N)= 4.750637242+003
       T(M,N)= 9.432606817+003         L(M,N)= 1.343550567+003

  SEGMENT NUMBER= 6
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.330033629+004         Y(M,N)= 0.000000000+000         Z(M,N)= 5.700741050+003
       T(M,N)= 9.432528676+003         L(M,N)= 1.343550550+003

  SEGMENT NUMBER= 7
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.235036483+004         Y(M,N)= 0.000000000+000         Z(M,N)= 6.650836977+003
       T(M,N)= 9.432450535+003         L(M,N)= 1.343550533+003

  SEGMENT NUMBER= 8
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.140038552+004         Y(M,N)= 0.000000000+000         Z(M,N)= 7.600925023+003
       T(M,N)= 9.432372395+003         L(M,N)= 1.343550516+003

  SEGMENT NUMBER= 9
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-1.045039835+004         Y(M,N)= 0.000000000+000         Z(M,N)= 8.551005189+003
       T(M,N)= 9.432294256+003         L(M,N)= 1.343550499+003

  SEGMENT NUMBER=10
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-9.500403324+003         Y(M,N)= 0.000000000+000         Z(M,N)= 9.501077473+003
       T(M,N)= 9.432216117+003         L(M,N)= 1.343550482+003

  SEGMENT NUMBER=11
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-8.550400438+003         Y(M,N)= 0.000000000+000         Z(M,N)= 1.045114188+004
       T(M,N)= 9.432137978+003         L(M,N)= 1.343550465+003

  SEGMENT NUMBER=12
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-7.600389695+003         Y(M,N)= 0.000000000+000         Z(M,N)= 1.140119840+004
       T(M,N)= 9.432059841+003         L(M,N)= 1.343550448+003

  SEGMENT NUMBER=13
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-6.650371094+003         Y(M,N)= 0.000000000+000         Z(M,N)= 1.235124704+004
       T(M,N)= 9.431981704+003         L(M,N)= 1.343550431+003

  SEGMENT NUMBER=14
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-5.700344634+003         Y(M,N)= 0.000000000+000         Z(M,N)= 1.330128779+004
       T(M,N)= 9.431903568+003         L(M,N)= 1.343550414+003

  SEGMENT NUMBER=15
      FX(M,N)= 0.000000000+000        FY(M,N)= 0.000000000+000        FZ(M,N)= 1.104999930-001
       X(M,N)=-4.750310316+003         Y(M,N)= 0.000000000+000         Z(M,N)= 1.425132067+004
       T(M,N)= 9.431825432+003         L(M,N)= 1.343550397+003
```

```
SEGMENT NUMBER=16
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)=-3.800268140+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.520134566+004
       T(M,N)= 9.431747298+003           L(M,N)= 1.343550380+003

SEGMENT NUMBER=17
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)=-2.850218105+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.615136277+004
       T(M,N)= 9.431669164+003           L(M,N)= 1.343550363+003

SEGMENT NUMBER=18
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)=-1.900160212+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.710137200+004
       T(M,N)= 9.431591030+003           L(M,N)= 1.343550346+003

SEGMENT NUMBER=19
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)=-9.500944605+002           Y(M,N)= 0.000000000+000           Z(M,N)= 1.805137334+004
       T(M,N)= 9.431512897+003           L(M,N)= 1.343550329+003

SEGMENT NUMBER=20
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 2.000616575+004
       X(M,N)=-2.085042000-002           Y(M,N)= 0.000000000+000           Z(M,N)= 1.900136680+004
       T(M,N)= 9.431434765+003           L(M,N)= 1.343550312+003

CABLE NUMBER= 2

SEGMENT NUMBER= 1
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)= 4.750187551+002           Y(M,N)=-8.227867463+002           Z(M,N)= 1.805137378+004
       T(M,N)= 9.431379078+003           L(M,N)= 1.343550300+003

SEGMENT NUMBER= 2
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)= 9.500544314+002           Y(M,N)=-1.645566687+003           Z(M,N)= 1.710137287+004
       T(M,N)= 9.431457211+003           L(M,N)= 1.343550317+003

SEGMENT NUMBER= 3
      FX(M,N)= 0.000000000+000          FY(M,N)= 0.000000000+000          FZ(M,N)= 1.104999930-001
       X(M,N)= 1.425086178+003           Y(M,N)=-2.468339822+003           Z(M,N)= 1.615136407+004

EQUILIBRIUM POSITION UNDER ACTING FORCES

      E= 4.338176168-003          DELTA= 3.598772476-002
      NO.OF ERROR LOOPS=    93          NO.OF ACCURACY LOOPS=    3

CURRENT ANGLE=   0.000

CABLE NUMBER= 1

SEGMENT NUMBER= 1
      FX(M,N)= 2.543530098+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.827260124+001
       X(M,N)=-1.802318953+004           Y(M,N)= 0.000000000+000           Z(M,N)= 9.230087617+002
       T(M,N)= 1.110274405+004           L(M,N)= 1.343913640+003
          HORIZL(M,N)= 2.688616323+001          HEIGHT(M,N)=-2.713444641+001

SEGMENT NUMBER= 2
      FX(M,N)= 2.558872715+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.833164028+001
       X(M,N)=-1.704894148+004           Y(M,N)= 0.000000000+000           Z(M,N)= 1.848719929+003
       T(M,N)= 1.109684900+004           L(M,N)= 1.343912358+003
          HORIZL(M,N)= 5.120204687+001          HEIGHT(M,N)=-5.155860776+001

SEGMENT NUMBER= 3
      FX(M,N)= 2.574358164+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.839035012+001
       X(M,N)=-1.607727849+004           Y(M,N)= 0.000000000+000           Z(M,N)= 2.777142257+003
       T(M,N)= 1.109096911+004           L(M,N)= 1.343911080+003
          HORIZL(M,N)= 7.292501640+001          HEIGHT(M,N)=-7.326372802+001

SEGMENT NUMBER= 4
      FX(M,N)= 2.589986979+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.844871320+001
       X(M,N)=-1.510822339+004           Y(M,N)= 0.000000000+000           Z(M,N)= 3.708284455+003
       T(M,N)= 1.108510450+004           L(M,N)= 1.343909805+003
          HORIZL(M,N)= 9.203222775+001          HEIGHT(M,N)=-9.224109858+001

SEGMENT NUMBER= 5
      FX(M,N)= 2.605759672+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.850671172+001
       X(M,N)=-1.414179926+004           Y(M,N)= 0.000000000+000           Z(M,N)= 4.642155181+003
       T(M,N)= 1.107925529+004           L(M,N)= 1.343908534+003
          HORIZL(M,N)= 1.085006270+002          HEIGHT(M,N)=-1.084820608+002
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SEGMENT NUMBER= 6
     FX(M,N)= 2.621676729+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.856432755+001
      X(M,N)=-1.317822934+004           Y(M,N)= 0.000000000+000           Z(M,N)= 5.578763041+003
      T(M,N)= 1.107342162+004           L(M,N)= 1.343907266+003
          HORIZL(M,N)= 1.223069491+002          HEIGHT(M,N)=-1.219780086+002

SEGMENT NUMBER= 7
     FX(M,N)= 2.637738615+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.862154227+001
      X(M,N)=-1.221693712+004           Y(M,N)= 0.000000000+000           Z(M,N)= 6.518116586+003
      T(M,N)= 1.106760360+004           L(M,N)= 1.343906001+003
          HORIZL(M,N)= 1.334277165+002          HEIGHT(M,N)=-1.327203914+002

SEGMENT NUMBER= 8
     FX(M,N)= 2.653945767+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.867833716+001
      X(M,N)=-1.125854628+004           Y(M,N)= 0.000000000+000           Z(M,N)= 7.460224307+003
      T(M,N)= 1.106180138+004           L(M,N)= 1.343904739+003
          HORIZL(M,N)= 1.418392386+002          HEIGHT(M,N)=-1.407007165+002

SEGMENT NUMBER= 9
     FX(M,N)= 2.670298598+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.873469319+001
      X(M,N)=-1.030288074+004           Y(M,N)= 0.000000000+000           Z(M,N)= 8.405094636+003

      T(M,N)= 1.105601509+004           L(M,N)= 1.343903482+003
          HORIZL(M,N)= 1.475176113+002          HEIGHT(M,N)=-1.459105527+002

SEGMENT NUMBER=10
     FX(M,N)= 2.686797492+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.879059104+001
      X(M,N)=-9.349964609+003           Y(M,N)= 0.000000000+000           Z(M,N)= 9.352735941+003
      T(M,N)= 1.105024487+004           L(M,N)= 1.343902227+003
          HORIZL(M,N)= 1.504387152+002          HEIGHT(M,N)=-1.483415320+002

SEGMENT NUMBER=11
     FX(M,N)= 2.703442809+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.884601109+001
      X(M,N)=-8.399822223+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.030315652+004
      T(M,N)= 1.104449986+004           L(M,N)= 1.343900976+003
          HORIZL(M,N)= 1.505782158+002          HEIGHT(M,N)=-1.479853539+002

SEGMENT NUMBER=12
     FX(M,N)= 2.720234878+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.890093342+001
      X(M,N)=-7.452478133+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.125636461+004
      T(M,N)= 1.103875320+004           L(M,N)= 1.343899729+003
          HORIZL(M,N)= 1.479115620+002          HEIGHT(M,N)=-1.448337879+002

SEGMENT NUMBER=13
     FX(M,N)= 2.737173999+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.895533781+001
      X(M,N)=-6.507957109+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.221236836+004
      T(M,N)= 1.103303204+004           L(M,N)= 1.343898485+003
          HORIZL(M,N)= 1.424139849+002          HEIGHT(M,N)=-1.388786757+002

SEGMENT NUMBER=14
     FX(M,N)= 2.754260443+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.900920373+001
      X(M,N)=-5.566284136+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.317117586+004
      T(M,N)= 1.102732753+004           L(M,N)= 1.343897245+003
          HORIZL(M,N)= 1.340604984+002          HEIGHT(M,N)=-1.301119354+002

SEGMENT NUMBER=15
     FX(M,N)= 2.771494447+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.906251037+001
      X(M,N)=-4.627484419+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.413279510+004
      T(M,N)= 1.102163983+004           L(M,N)= 1.343896009+003
          HORIZL(M,N)= 1.228258973+002          HEIGHT(M,N)=-1.185255647+002

SEGMENT NUMBER=16
     FX(M,N)= 2.788876221+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.911523661+001
      X(M,N)=-3.691583582+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.509723402+004
      T(M,N)= 1.101596909+004           L(M,N)= 1.343894776+003
          HORIZL(M,N)= 1.086847576+002          HEIGHT(M,N)=-1.041116433+002

SEGMENT NUMBER=17
     FX(M,N)= 2.806405938+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.916736104+001
      X(M,N)=-2.758676670+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.606450043+004
      T(M,N)= 1.101031547+004           L(M,N)= 1.343893547+003
          HORIZL(M,N)= 9.161143523+001          HEIGHT(M,N)=-8.686233711+001
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SEGMENT NUMBER=18
     FX(M,N)= 2.824083740+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.921886197+001
      X(M,N)=-1.828580146+003           Y(M,N)= 0.000000000+000           Z(M,N)= 1.703460210+004
      T(M,N)= 1.100467915+004           L(M,N)= 1.343892322+003
         HORIZL(M,N)= 7.158006585+001          HEIGHT(M,N)=-6.676990080+001

SEGMENT NUMBER=19
     FX(M,N)= 2.841909734+001          FY(M,N)= 0.000000000+000          FZ(M,N)=-1.926971739+001
      X(M,N)=-9.015298966+002           Y(M,N)= 0.000000000+000           Z(M,N)= 1.800754666+004
      T(M,N)= 1.099926028+004           L(M,N)= 1.343891100+003
         HORIZL(M,N)= 4.856456399+001          HEIGHT(M,N)=-4.382668161+001

SEGMENT NUMBER=20
     FX(M,N)= 4.632173700+002          FY(M,N)= 0.000000000+000          FZ(M,N)= 2.001019587+004
      X(M,N)= 2.251777269+001           Y(M,N)= 0.000000000+000           Z(M,N)= 1.898334168+004
      T(M,N)= 1.099345904+004           L(M,N)= 1.343889882+003
         HORIZL(M,N)= 2.253862311+001          HEIGHT(M,N)=-1.802512264+001

CABLE NUMBER= 2

SEGMENT NUMBER= 1
     FX(M,N)= 4.895215682+001          FY(M,N)= 1.185663022+001          FZ(M,N)= 1.378709929+001
      X(M,N)= 5.694653929+002           Y(M,N)=-8.037505343+002           Z(M,N)= 1.805622686+004
      T(M,N)= 8.714226786+003           L(M,N)= 1.343394397+003
         HORIZL(M,N)= 9.634596386+001          HEIGHT(M,N)= 4.853085995+000

SEGMENT NUMBER= 2
     FX(M,N)= 4.927635179+001          FY(M,N)= 1.175718962+001          FZ(M,N)= 1.367264254+001
      X(M,N)= 1.109062248+003           Y(M,N)=-1.609621247+003           Z(M,N)= 1.712664939+004
      T(M,N)= 8.711061040+003           L(M,N)= 1.343393709+003
         HORIZL(M,N)= 1.630201226+002          HEIGHT(M,N)= 2.527651930+001

SEGMENT NUMBER= 3
     FX(M,N)= 4.959950413+001          FY(M,N)= 1.165439583+001          FZ(M,N)= 1.355431306+001
      X(M,N)= 1.641250279+003           Y(M,N)=-2.417594227+003           Z(M,N)= 1.619462986+004
      T(M,N)= 8.707939651+003           L(M,N)= 1.343393030+003
         HORIZL(M,N)= 2.220406131+002          HEIGHT(M,N)= 4.326578665+001

SEGMENT NUMBER= 4
     FX(M,N)= 4.992147389+001          FY(M,N)= 1.154822871+001          FZ(M,N)= 1.343208744+001
      X(M,N)= 2.165971622+003           Y(M,N)=-3.227650966+003           Z(M,N)= 1.526018958+004
      T(M,N)= 8.704862920+003           L(M,N)= 1.343392362+003
         HORIZL(M,N)= 2.733255157+002          HEIGHT(M,N)= 5.884218717+001

SEGMENT NUMBER= 5
     FX(M,N)= 5.024211902+001          FY(M,N)= 1.143867018+001          FZ(M,N)= 1.330594465+001
      X(M,N)= 2.683168628+003           Y(M,N)=-4.039772356+003           Z(M,N)= 1.432335055+004
      T(M,N)= 8.701831152+003           L(M,N)= 1.343391702+003
         HORIZL(M,N)= 3.168166012+002          HEIGHT(M,N)= 7.202771425+001

SEGMENT NUMBER= 6
     FX(M,N)= 5.056129550+001          FY(M,N)= 1.132570431+001          FZ(M,N)= 1.317586609+001
      X(M,N)= 3.192783894+003           Y(M,N)=-4.853938683+003           Z(M,N)= 1.338413546+004
      T(M,N)= 8.698844659+003           L(M,N)= 1.343391053+003
         HORIZL(M,N)= 3.524606996+002          HEIGHT(M,N)= 8.284505987+001

SEGMENT NUMBER= 7
     FX(M,N)= 5.087885744+001          FY(M,N)= 1.120931733+001          FZ(M,N)= 1.304183570+001
      X(M,N)= 3.694760283+003           Y(M,N)=-5.670129624+003           Z(M,N)= 1.244256769+004
      T(M,N)= 8.695903754+003           L(M,N)= 1.343390414+003
         HORIZL(M,N)= 3.802064467+002          HEIGHT(M,N)= 9.131761837+001

SEGMENT NUMBER= 8
     FX(M,N)= 5.119465715+001          FY(M,N)= 1.108949776+001          FZ(M,N)= 1.290384002+001
      X(M,N)= 4.189040952+003           Y(M,N)=-6.488324249+003           Z(M,N)= 1.149867136+004
      T(M,N)= 8.693008751+003           L(M,N)= 1.343389785+003
         HORIZL(M,N)= 4.000034061+002          HEIGHT(M,N)= 9.746948957+001

SEGMENT NUMBER= 9
     FX(M,N)= 5.150854533+001          FY(M,N)= 1.096623642+001          FZ(M,N)= 1.276186827+001
      X(M,N)= 4.675569378+003           Y(M,N)=-7.308501010+003           Z(M,N)= 1.055247126+004
      T(M,N)= 8.690159971+003           L(M,N)= 1.343389165+003
```

Appendix C

SAMPLE CALCULATION

PURPOSE

To demonstrate the use of the computer program given in Appendix B and to show the rapid convergence of the analytic method presented in this report, a particular Sea Spider design is analyzed in this appendix. This Sea Spider was previously considered in Ref. 3 as an example for the method of analysis developed there.

The primary purpose of this example is to test the assumption of Ref. 3 that the root-mean-square (rms) velocity can be used instead of the actual velocity profile to predict the motions of the structure. Consequently, the structure is analyzed using both the design velocity profile and its rms value as a constant profile.

To compare the results obtained here with those obtained in Ref. 3, it is necessary to neglect the tangential drags acting on the cables. This is accomplished by setting the ratio of parallel to normal drag coefficients $r^D_{m,n}$ (Eq. 19b) equal to zero for every cable segment. Then, to study the effect of tangential drags on the deflections of the structure, this coefficient is set equal to 0.025 for the entire cable array.

COMPUTER INPUT DATA

The Sea Spider studied in Ref. 3 consists of three identical legs, each 26,830 ft long in the unstressed state. These legs have a diameter of 0.94 in. and a weight in water of -0.313 lb/ft. The extensional rigidity of each cable is given as 6,170,900 lb, and the coefficient of normal drag is assumed to be 1.2 for the entire range of current velocities.

The cables are anchored in the shape of an equilateral triangle in the $z = 0$ base plane with a distance between anchors of 32,909 ft. The x, y, z coordinate system (Fig. C1) is chosen so that the gravity position of the subsurface float will have coordinates $x = 0$ ft and $y = 0$ ft. Note that a 60-degree symmetry exists for the system.

The legs of this Sea Spider are made neutrally buoyant through the use of 10-in.-diameter glass balls that have a net buoyancy of 12.0 lb. These balls are spaced every 38.3 ft along the legs and have an assumed drag coefficient of 1.0. The subsurface float has a net buoyancy of 20,000 lb and a hydrodynamic constant given by 35.1 lb-sec^2/ft^2. The system is designed so that the no-current height above bottom of this float will be 19,000 ft.

The design velocity profile is given as 0.0 knots at the bottom ($z = 0$ ft), increasing linearly to 0.3 knots at $z = 17{,}500$ ft, and then increasing linearly to 2.0 knots at 19,000 ft. Above 19,000 ft, the current remains constant at 2.0 knots. Based on the no-current height of the subsurface float (19,000 ft), the appropriate rms velocity to be used as a constant velocity profile is calculated as 0.39 knots.

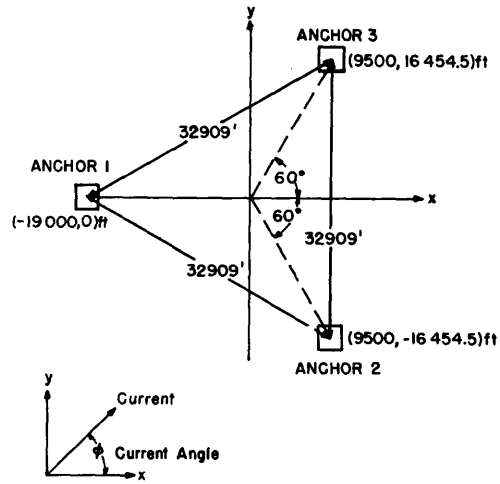

Fig. C1 - View of the anchor plane

To cast this system in a form proper for the analysis developed in this report, each cable is represented by 20 equally spaced stations. The unstressed lengths of the cable segments are, consequently, 1341.5 ft.

The data given above is tabulated and summarized in Tables C1 through C4.

Table C1
The Design Velocity Profile

Current Breakpoint, k	H_k (ft)	$V(H_k)$ (ft/sec)
0	0.0	0.0
1	17500.0	0.50634
2	19000.0	3.3756
3	∞	3.3756

Note: Based on a no-current height of the subsurface float of 19,000 ft, the appropriate rms velocity is 0.658 ft/sec.

Table C2
The Sea Spider Array

Cable Number, n	Cable Length (ft)	Number of Stations, $M(n)$	Anchor Position (ft)		
			a_n	b_n	c_n
1	26830.0	20	-19000.0	0.0	0.0
2	26830.0	20	9500.0	-16454.5	0.0
3	26830.0	20	9500.0	16454.5	0.0

Table C3
Lumped Parameter Representation of the Sea Spider Array

Segment Number, m	$\bar{L}_{m,n}$ (ft)	$B_{m,n}$ (lb)	$w^c_{m,n}$ (lb/ft)	$d_{m,n}$ (in.)	$C^N_{m,n}$	$\mu^c_{m,n}$* (lb-sec^2/ft^3)
CABLE 1						
1 →20	1341.5	6,170,900	-0.313	0.94	1.2	0.092
CABLE 2						
1 →20	1341.5	6,170,900	-0.313	0.94	1.2	0.092
CABLE 3						
1 →20	1341.5	6,170,900	-0.313	0.94	1.2	0.092

*Based on a density of sea water ρ = 1.94 lb-sec^2/ft^4.

Table C4
Properties of the Discrete Elemental Devices

Segment Number, m	$K(m,n)$	Element Index, k	$\bar{s}_{k,m,n}$ (ft)	$w^e_{k,m,n}$ (lb)	$C^D_{k,m,n}$	$A_{k,m,n}$ (ft^2)	$\mu^e_{k,m,n}$* (lb-sec^2/ft^2)
CABLE 1							
1 → 19	35	1 → 35	$19.65 + 38.3(k-1)$	12.0	1.0	$\pi(5/12)^2$	0.529
20	36	1 → 35	$19.65 + 38.3(k-1)$	12.0	1.0	$\pi(5/12)^2$	0.529
		36	1341.5	20,000.0	—	—	35.100†
CABLES 2 and 3							
1 → 20	35	1 → 35	$19.65 + 38.3(k-1)$	12.0	1.0	$\pi(5/12)^2$	0.529

*Based on a density of sea water ρ = 1.94 lb-sec^2/ft^4.
†When the rms velocity of 0.658 ft/sec is used, an equivalent μ of 924.0 is assumed for the subsurface float to insure that the drag force on this object is the same as when the actual profile is used. That is $\mu_{equivalent}$ $(0.658)^2 = (35.1)(3.3756)^2$.

RESULTS

Three possible combinations of current profiles and tangential drags were studied. They are denoted by Cases B, C, and D and are defined by

Case B — the rms velocity profile ($V(z)$ = 0.39 knots) with no tangential drags ($r^D_{m,n}$ = 0.0),

Case C — the actual design velocity profile ($V(z)$ as given in Table C1) with no tangential drags ($r^D_{m,n}$ = 0.0), and

Case D — the actual design velocity profile with tangential drags included in the analysis ($r^D_{m,n}$ = 0.025).

For each case, the equilibrium shapes of the array were calculated for current angles between 0 and 60 degrees at every 5-degree interval.

The acceptable completion of the Imaginary Reaction iteration was defined by setting

$$COMPE = 0.01 \text{ ft}^2$$

and the acceptable completion of the successive approximation routine by setting

$$COMPD = 1.0 \text{ ft.}$$

Consequently, the calculated equilibrium coordinates of the cable stations were all within ±1 ft of their true values, and the end coordinates of cables 2 and 3 were within ±0.1 ft of the actual anchor locations.

In each case studied, the total run time, including compiling, for the fourteen sets of results (no-current equilibrium configuration plus the equilibrium configurations for thirteen current headings), was approximately 2 minutes on the NRL CDC-3800 computer.

Of particular interest for comparison purposes are the current-induced deflections of the subsurface float from its no-current position, which is given by

$$x = 0.0 \text{ ft},$$

$$y = 0.0 \text{ ft},$$

and

$$z = 19001.4 \text{ ft.}$$

The horizontal excursion of the buoy from this position is plotted in Fig. C2 as a function of the current angle ϕ, and its downward movement is plotted in Fig. C3 as a function of this same angle. In both figures, four curves are shown. Curves B, C, and D correspond, respectively, to the deflections obtained in Cases B, C, and D. Curve A shows the results obtained in Ref. 3 and corresponds in its current profile and drag force assumptions to Curve B.

DISCUSSION

As is seen by referring to Curves B and C in Figs. C2 and C3, large discrepancies exist between the deflections predicted using the constant-velocity profile and those predicted using the actual-velocity profile. Significant differences are also obtained for the tensions in the cables and the deflections of the entire array. The obvious conclusion of this calculation is that *to obtain a good approximation for the deflection analysis of a tri-moored, buoy-cable array, the actual current profile should be used.*

Although, for this problem, the largest deflections are obtained at a current heading of 0 degrees using the rms profile, this is not true in general. Other examples have shown that the deflections induced by the actual profile can lie entirely above those induced by the rms profile, and that the deflection curves are not necessarily monotonically decreasing as would seem to be indicated in Figs. C2 and C3. Thus, each particular array must be analyzed independently, and no general conclusions can be reached concerning the pattern of the current-induced deflections.

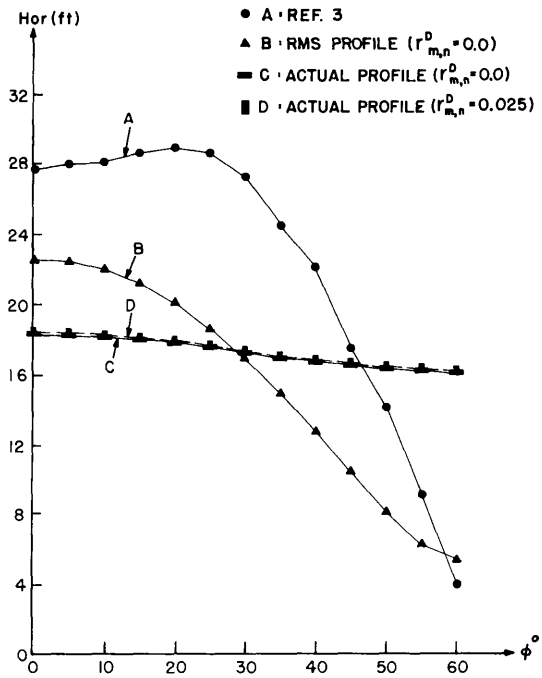

Fig. C2 - Horizontal deflections of the subsurface buoy

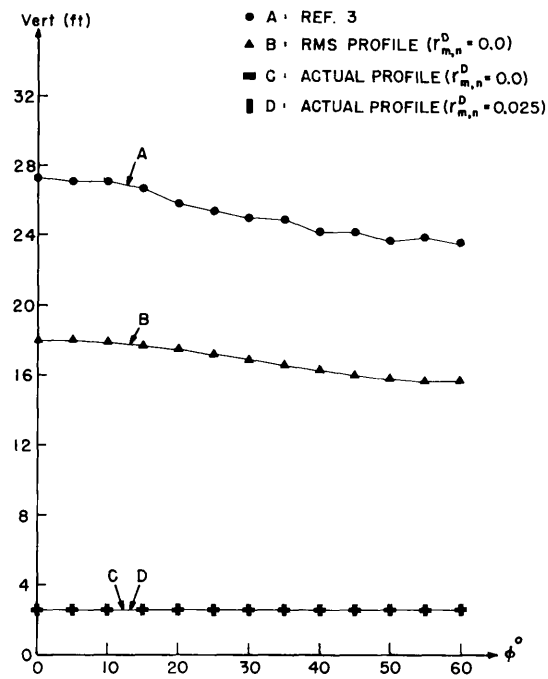

Fig. C3 - Vertical deflections of the subsurface buoy

Since the analytic methods of this report and Ref. 3 are not the same, numerical differences between Curves A and B are to be expected. In essence, Ref. 3 assumes a simplified continuum model of the array by averaging the elemental devices along the cable legs. Consequently, the similarity in shape and the closeness of Curves A and B (a difference in the predicted buoy displacements of less than 10 ft over a typical cable length of approximately 5 mi) tend to validate the lumped parameter representation employed in this report.

The inclusion of tangential drags in the analysis, Curve D, is seen to have little effect on the buoy displacements. However, the inclusion of this drag increases slightly the tensions in the cables that act to resist the current; for this reason, it is recommended that tangential drags be used in future analyses.

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4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

This is an interim report; work is continuing on other phases of the problem.

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13. ABSTRACT

A rapid and accurate method is developed for determining the static configuration of and tensions in a buoy-cable array that consists of a subsurface float anchored to the bottom by three cable legs. The array is acted on by weight and buoyancy forces and by current-induced forces that depend on the position and orientation of the cables in the water. A lumped parameter representation of the structure is assumed, and the analysis is performed by using The Method of Imaginary Reactions to determine the equilibrium configuration of the array under constant applied forces and the method of successive approximations to iterate on the position-dependent current forces. Expressions are derived for the hydrodynamic forces, and both normal and tangential drags are included. The analysis allows for the possibility that a variety of discrete-element measuring and flotation devices are attached to the cable legs. Although a lumped parameter representation is used, the analytic method is independent of the number of stations employed, depending only on the six unknown components of the reactions at two of the anchors. Consequently, arbitrary fineness can be used in modeling the structure without increasing the complexity of the problem. A computer program for implementing the analysis is included, and a sample calculation is given to demonstrate the use of the program and to show the rapid convergence of the analytic method.

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Tri-moored buoys						
Sea Spider						
Structural analysis						
Imaginary reactions						
Drag forces						
Hydrodynamic forces						

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(PAGE 2)

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