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Median Detector for Noncoherent Distributions

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ABSTRACT

Sea-clutter measurements using high-resolution radar indicate that clutter cross-section returns do not follow a Rayleigh distribution. The log-normal and contaminated-normal distributions have been suggested, and detection curves for these distributions have been published. In this report, the median detector is proposed as a detector against these clutter distributions, and the theory is developed which is necessary for detecting constant signals in log-normal and contaminated-normal clutter when one has N independent samples. Curves are obtained for the threshold values and probabilities of detection for false-alarm probabilities ranging from 10^{-2} to 10^{-9}. If one compares the mean and median detectors, the conclusion is reached that the median is better for the log-normal distribution and that the mean is slightly better for the contaminated-normal distribution for small sample sizes. Using the concept of asymptotic relative efficiency, the median is shown to be better than the mean for large sample sizes for both distributions.

PROBLEM STATUS

This is an interim report on one phase of the problem; work on the problem is continuing.

AUTHORIZATION

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MEDIAN DETECTOR FOR NONCOHERENT DISTRIBUTIONS

SUMMARY

Sea-clutter measurements made by NRL using a high-resolution x-band radar show that clutter cross section is not Rayleigh distributed. The log-normal and contaminated-normal distributions have been suggested, and detection curves have been generated for these distributions. Because of the long tails of these distributions, it was conjectured that the median might be a more effective detector than the sampled mean.

First, the distribution of ordered statistics is derived, and the ordered distributions are presented for the log-normal and contaminated-normal distributions.

Second, curves are presented for the threshold values versus the probability of false alarm for the median detector, for both distributions and for $n = 3$ and 30 pulses. Then, probability-of-detection curves are presented for the previous values of n and for false-alarm probabilities ranging from 10^{-2} to 10^{-9}.

Third, the median is compared to the sampled mean. The median is clearly better for the log-normal distribution. However, the mean is slightly better for the contaminated-normal distribution when the sample size is small.

Next, a possible implementation of the median detector is shown. It is only slightly more complicated than the sampled mean. Then, the concept of asymptotic relative efficiency (ARE) is introduced. The ARE is a comparison between the asymptotic behavior of two detectors.

Finally, the ARE's of the mean and median are calculated. To obtain the same probabilities of detection and false alarm, the mean requires asymptotically almost twice as many samples as the median for the contaminated-normal distribution and over 100 times as many for the log-normal distribution

INTRODUCTION

Sea-clutter measurements made by NRL (1,2) using a high-resolution x-band radar with various pulse widths show that the clutter cross section is not Rayleigh distributed. Consequently, distributions with longer tails have been studied. Miner (3), Ballard (4), and George (5) have considered the log-normal distribution, and Trunk (6) has considered the contaminated-normal distribution. George has found that when one is integrating three pulses, to obtain a detection probability of 0.9 and a false-alarm probability of 10^{-8}, the log-normal distribution requires a signal-to-noise (S/N) ratio 14 dB higher than the S/N ratio for the Rayleigh distribution. Trunk has found that the contaminated-normal distribution requires a S/N ratio 4 dB higher than the Rayleigh distribution. What causes the higher S/N ratios is the longer tails of these distributions. Thus, a better detector could possibly be one which ignores the tails of these distributions. Such a detector is the median detector.

DISTRIBUTIONS OF ORDER STATISTICS

Since the median detector has no special appeal in noncoherent detection (as it does in coherent detection when dealing with symmetrical distributions), other order statistics will also be considered.

Let $X_1, X_2, \ldots, X_n$ be independent samples taken from the probability density $f(X)$, and let $F(X)$ be the distribution function of $f(X)$. Let the order statistics be

$$X_{r_1} \leq X_{r_2} \leq X_{r_3} \leq \ldots \leq X_{r_n} , \tag{1}$$

where X_{r_k} is called the kth-order statistic, is denoted by y_k^n, and is the value of the kth smallest observation in the sample of size n from $F(X)$. The density of y_k^n can be found by the following reasoning: The probability of having $k-1$ samples less than the value X is $F(X)^{k-1}$; the probability of having $n-k$ samples greater than X is $(1-F(X))^{n-k}$; the probability of having one sample at X is $f(X)$; and the number of ways of dividing n samples into three groups of one, $(k-1)$, and $(n-k)$ samples each is

$$\frac{n!}{(n-k)!\ 1!\ (k-1)!} . \tag{2}$$

Consequently, the density of y_k^n is

$$p_{n,k}(x) = \frac{d}{dx} P\left\{y_k^n < x\right\} = \frac{n!}{(n-k)!\ (k-1)!} F(x)^{k-1}[1-F(x)]^{n-k} f(x) . \tag{3}$$

In this report, the log-normal distribution, where

$$f(x) = \int_0^{2\pi} \frac{x \exp\left[-\frac{2}{\sigma^2}\left(\ln\sqrt{x^2 - 2xA\cos\theta + A^2}\right)^2\right] d\theta}{\pi\sqrt{2\pi\sigma^2}\,(x^2 - 2xA\cos\theta + A^2)} \tag{4}$$

and

$$\mathrm{S/N} = 20 \log A \tag{5}$$

and the noncoherent contaminated-normal distribution, where

$$\begin{aligned} f(x) = \frac{x}{2\pi\sigma^2}\int_0^{2\pi} & (1-\gamma)^2 \exp\left[-(x^2 - 2xA\cos\theta + A^2)/2\sigma^2\right] \\ & + \frac{\gamma^2}{K^2}\exp\left[-(x^2 - 2xA\cos\theta + A^2)/2K^2\sigma^2\right] \\ & + \frac{\gamma(1-\gamma)}{K}\exp\left[-(x^2\cos^2\theta - 2xA\cos\theta + A^2 + K^2x^2\sin^2\theta)/2K^2\sigma^2\right] \\ & + \frac{\gamma(1-\gamma)}{K}\exp\left[-(x^2\sin^2\theta - 2xAK^2\cos\theta + A^2K^2 + K^2X^2\cos^2\theta)/2K^2\sigma^2\right] d\theta \end{aligned} \tag{6}$$

and

$$S/N = 20 \log (A/X_{med}) \tag{7}$$

will be investigated.

DETECTION RESULTS

When using a rank detector, the probability of false alarm is

$$P_{FA} = \int_T^{\infty} p_{n,k}(x)\, dx, \quad \text{with } A = 0 . \tag{8}$$

and the probability of detection is

$$P_D = \int_T^{\infty} p_{n,k}(x)\, dx , \quad \text{with } A \neq 0 . \tag{9}$$

Equations (8) and (9) cannot be solved in a closed-form solution. Rather the integrals have to be performed numerically. In fact, for the contaminated-normal distribution a triple numerical integration is required for Eqs. (8) and (9).

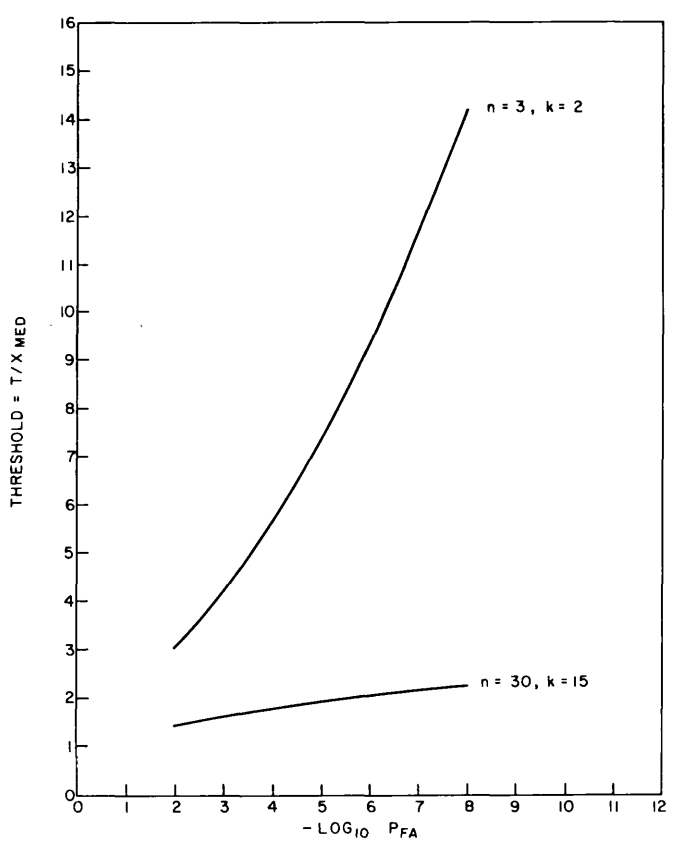


Fig. 1 - Thresholds for the median detector for the log-normal distribution (σ = 6 dB)

Threshold values for the log-normal distribution (σ = 6 dB) were obtained for case 1, where n = 3 and k = 2, and case 2, where n = 30 and k = 15, by substituting Eq. (4) into Eq. (8) and performing the necessary numerical integrations. Curves summarizing these results are plotted in Fig. 1. Using Eq. (9) and Fig. 1, the probability of detection versus the S/N ratio per pulse was calculated and is shown in Figs. 2 and 3. The same calculations were performed for the contaminated-normal distribution (γ = 0.25 and K = 2.25). The threshold values are plotted in Fig. 4, and the probability-of-detection curves are shown in Figs. 5-6.

DISCUSSION OF RESULTS

To compare the noncoherent integrator for the log-normal distribution (5) with the median detector (Figs. 2 and 3), Table 1 was generated. Referring to Table 1, it is obvious that the median detector is better than the integrator. This is because the long tail of the log-normal distribution requires an exceedingly high threshold for the integrator as opposed to the lower threshold for the median. That is, for the integrator the threshold equals 57, and for the median the threshold equals 14 when n is 3 and P_{FA} is 10^{-8}.

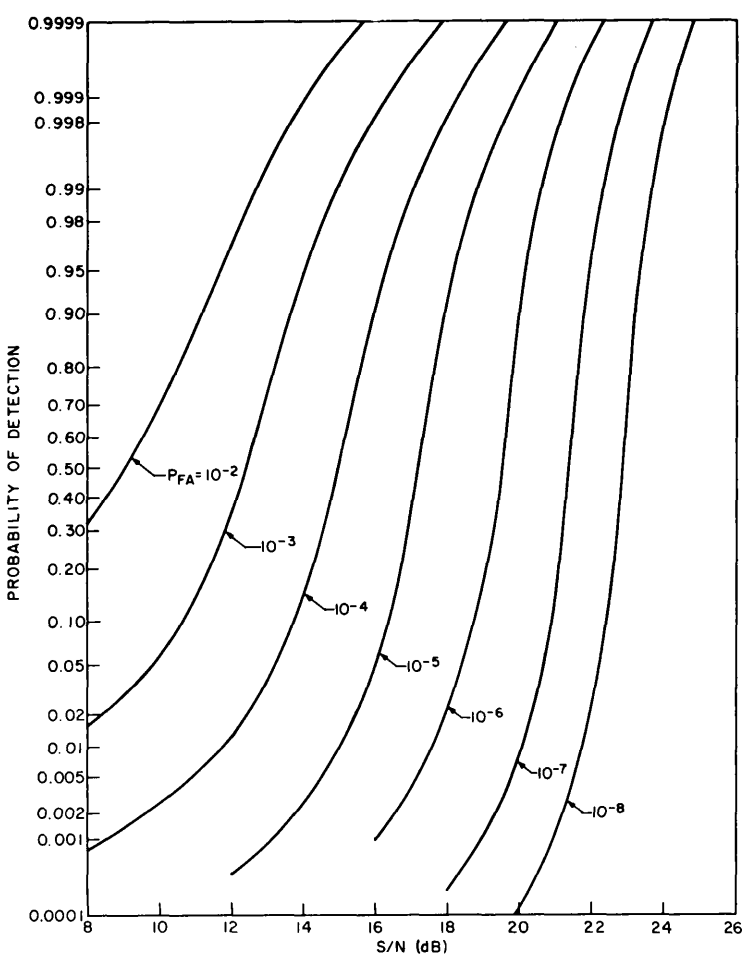


Fig. 2 - Probability of detection for the log-normal distribution (σ = 6 dB) for n = 3, and k = 2

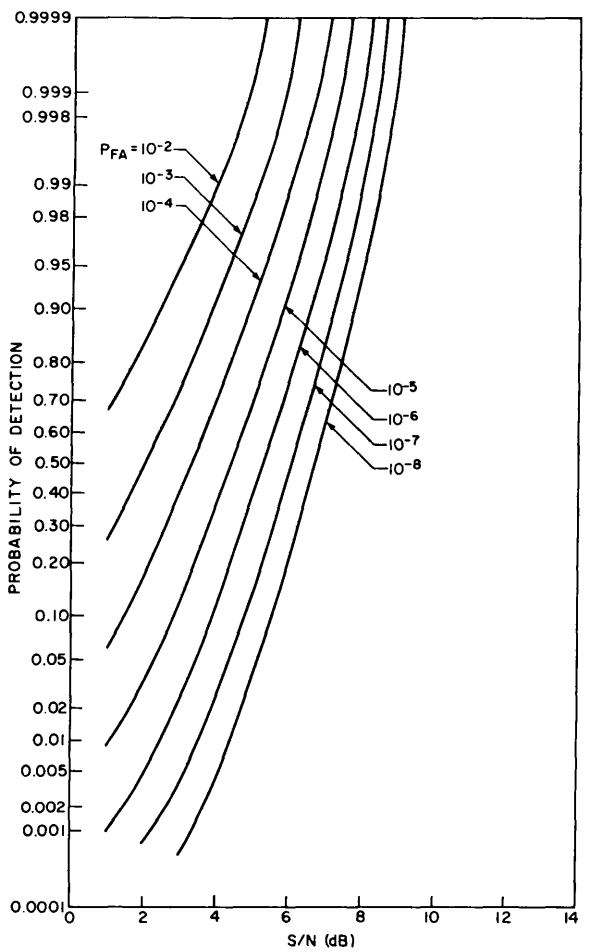


Fig. 3 - Probability of detection for the log-normal distribution (σ = 6 dB) for n = 30, and k = 15

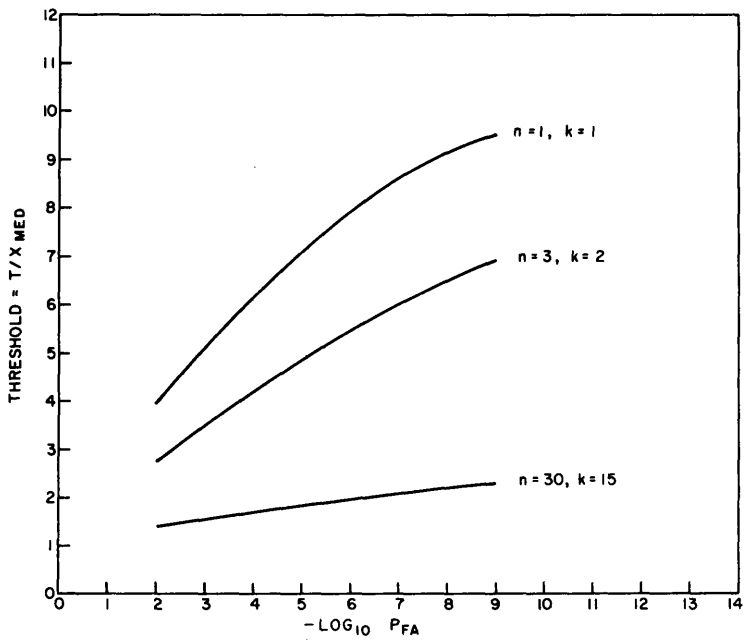


Fig. 4 - Thresholds for the median detector for the contaminated-normal distribution (γ = 0.25 and K = 2.25)

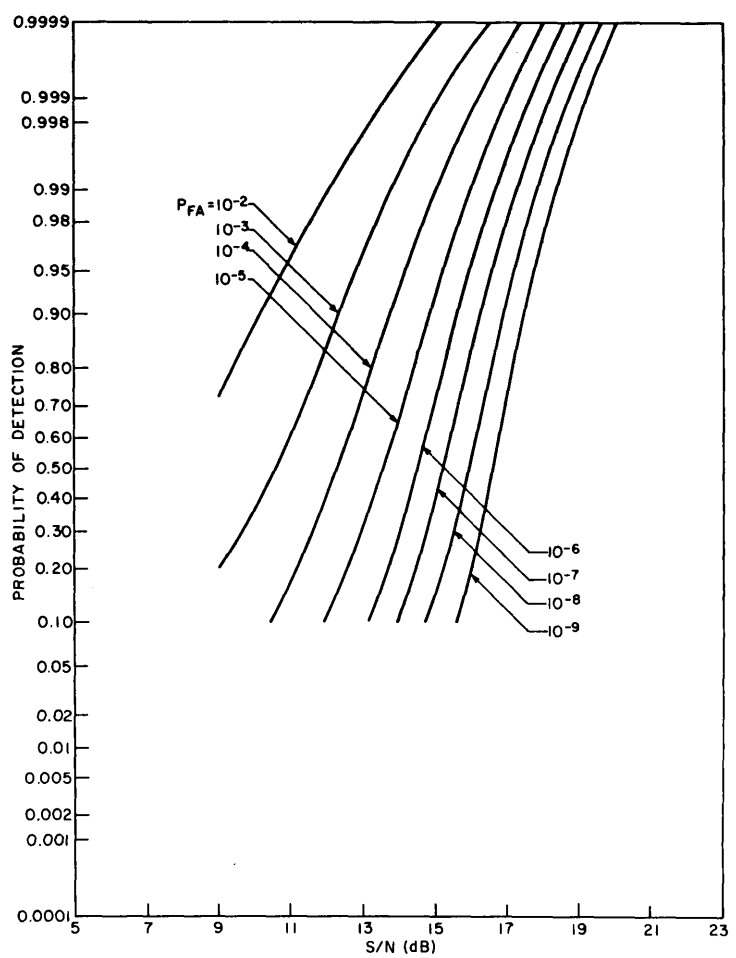


Fig. 5 - Probability of detection for the contaminated-normal distribution (γ = 0.25 and K = 2.25) for n = 3 and k = 2

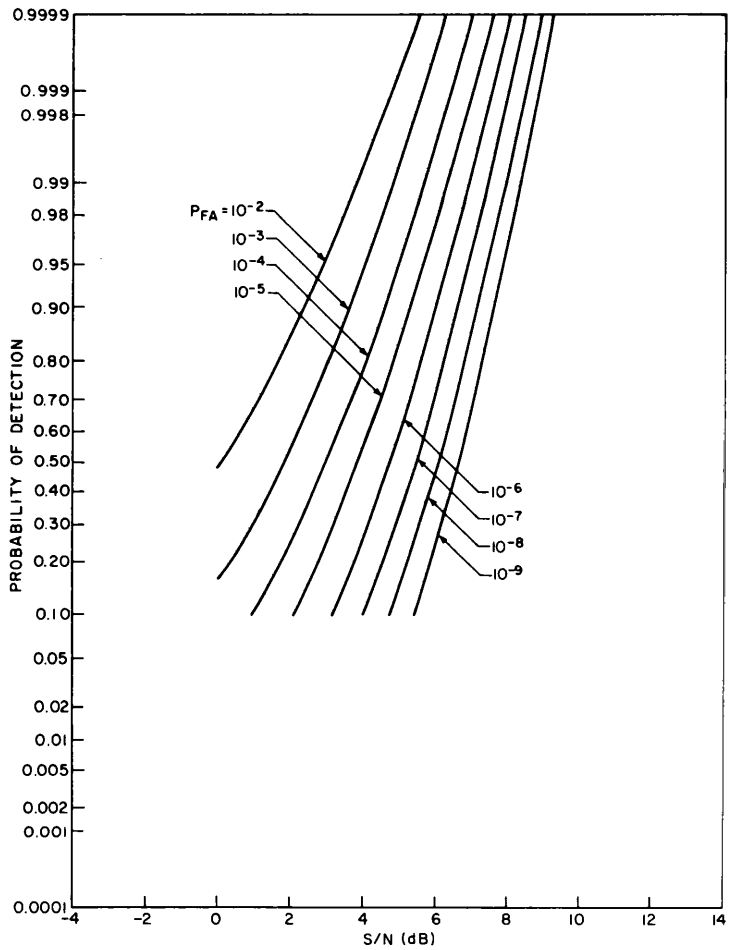


Fig. 6 - Probability of detection for the contaminated-normal distribution (γ = 0.25 and K = 2.25) for n = 30 and k = 15

To compare the noncoherent integrator for the contaminated-normal distribution (6) with the median detector (Figs. 5 and 6), Table 2 was generated. It appears obvious that for a small number of samples the integrator is slightly better than the median. However, as number of pulses increases, the difference decreases. One may speculate that for a very large number of pulses, the median may even be better than the integrator.

Table 1
Comparison of the Integrator and Median Detectors for the Log-Normal Distribution

n	P_{FA}	P_D	S/N Ratio for the Integrator (dB)	S/N Ratio for the Median Detector (dB)
3	10^{-2}	0.9	11.4	11.2
3	10^{-8}	0.9	25.8	23.2
30	10^{-2}	0.9	3.9	2.4
30	10^{-8}	0.9	11.4	7.7

Table 2
Comparison of the Integrator and Median Detectors for the Contaminated-Normal Distribution

n	P_{FA}	P_D	S/N Ratio for the Integrator (dB)	S/N Ratio for the Median Detector (dB)
3	10^{-2}	0.9	9.9	10.2
3	10^{-9}	0.9	16.3	17.5
30	10^{-2}	0.9	2.4	2.4
30	10^{-9}	0.9	7.0	7.6

Up until now, the only ordered statistic used has been the median. However, since noncoherent distributions are unsymmetrical about their median value, there is no intuitive reason to consider only the median. Therefore, another ordered statistic will be considered. For no other reason except that because of the long tails something smaller than the median should be chosen, the 33% smallest sample will be used, i.e., when n is 3, k is 1 and when n is 30, k is 10. The threshold curves are plotted in Fig. 7, and the detection curves are plotted in Figs. 8 and 9. Referring to Figs. 8 and 9, the following observations can be made:

1. Whether the 33rd percentile value or the 50th percentile value (median) is better is a function of P_{FA} and P_D. For the values of P_{FA} and P_D shown, there is only a 1-dB difference between the two detectors.

2. For a fixed P_D, as P_{FA} becomes smaller, the 33rd percentile becomes the better detector.

3. For a fixed P_{FA}, as P_D becomes larger, the 50th percentile becomes the better detector.

IMPLEMENTATION OF THE kth-ORDERED SAMPLE DETECTOR

If the value of the kth-ordered sample were required, the implementation of the detector would be very difficult because of the necessity of ordering the samples. Fortunately, all that is required is a knowledge of the fact of whether or not the kth-ordered sample is larger than the threshold T; this is equivalent to knowing the number of samples larger than T. The exact implementation of this device, which uses an n-stage shift register, is shown in Fig. 10a. An approximate implementation, which uses a feedback integrator instead of a shift register, is shown in Fig. 10b. Either of these implementations is much simpler (equipment wise) than the device needed to order the samples.

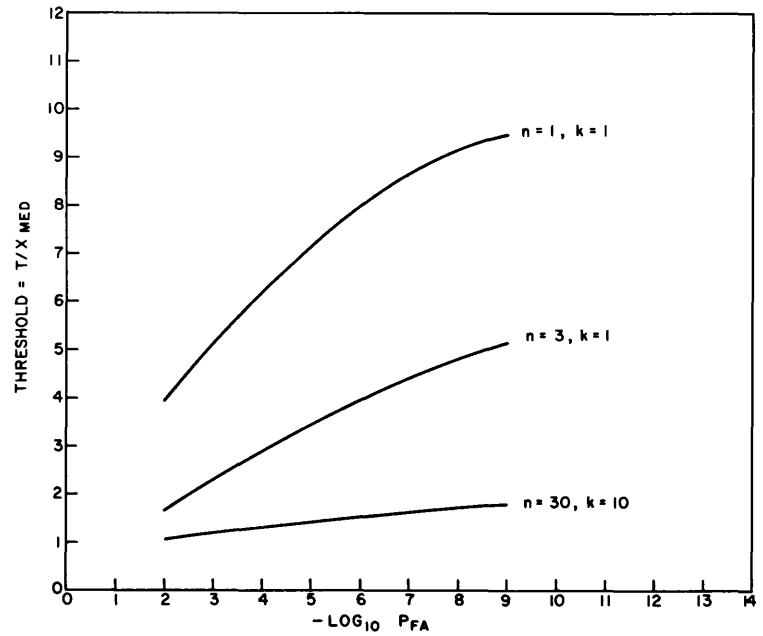


Fig. 7 - Thresholds for rank detectors for the contaminated-normal distribution (γ = 0.25 and K = 2.25)

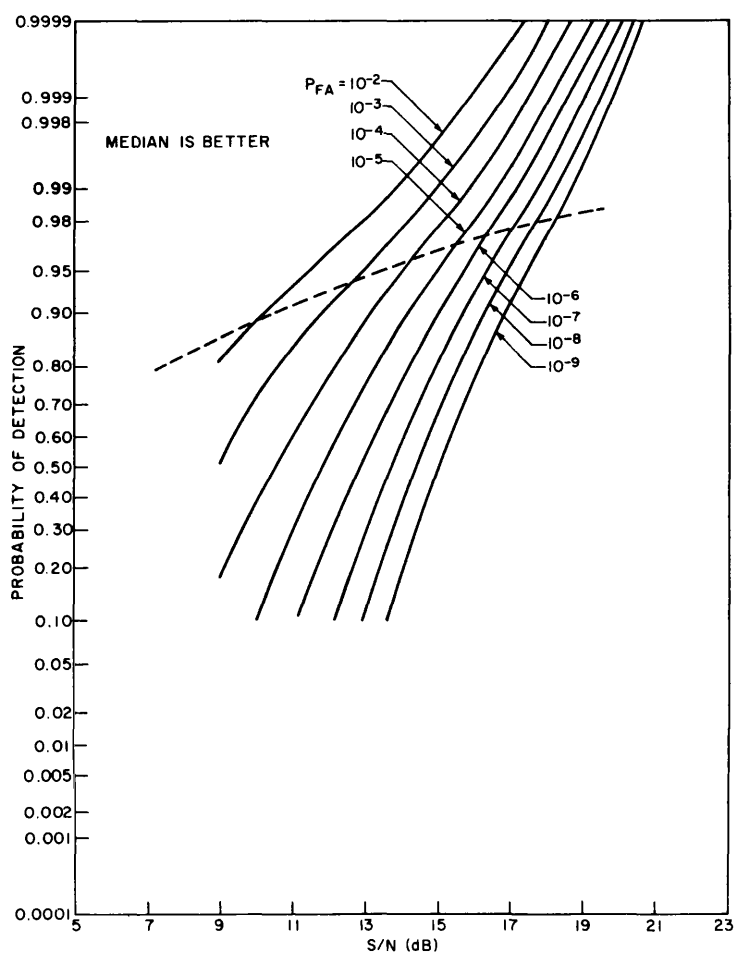


Fig. 8 - Probability of detection for the contaminated-normal distribution (γ = 0.25 and K = 2.25) for n = 3 and k = 1

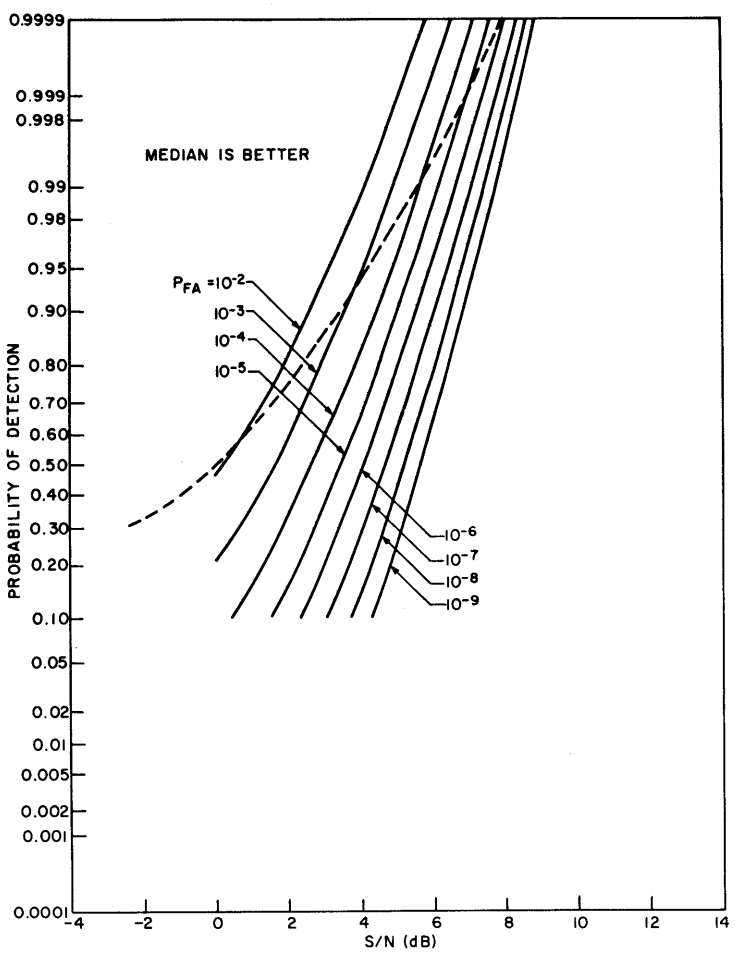
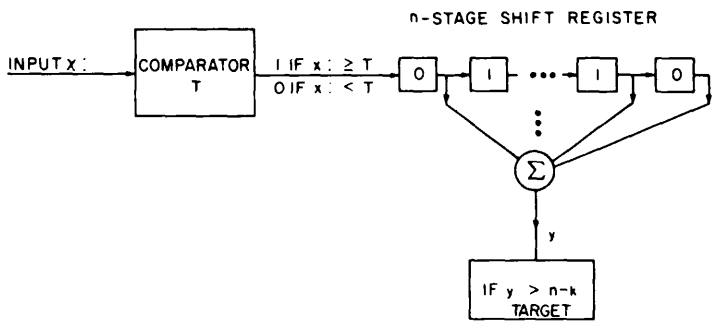
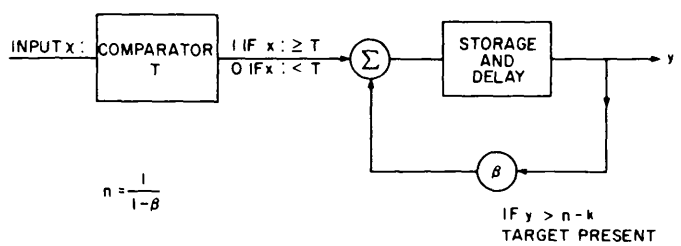


Fig. 9 - Probability of detection for the contaminated-normal distribution (γ = 0.25 and K = 2.25) for n = 30 and k = 10



(a) Exact implementation



(b) Approximate implementation

Fig. 10 - Implementation of an ordered-sample detector

In fact, the approximate implementation is only slightly more complicated than the feedback integrator used to perform the noncoherent integration.

ASYMPTOTIC RELATIVE EFFICIENCY AND EFFICACY

A common criterion that is used to compare detectors is Pitman's asymptotic relative efficiency (ARE) (7). This criterion is quite appropriate when the signals are very weak. Specifically, it is the ratio of the number of samples required to maintain a P_{FA} and P_D for the first detector to the number required for the second detector as the S/N ratio approaches zero:

$$\mathrm{ARE}(d_1, d_2) = \lim_{S/N \to 0} \frac{N_1(P_{FA}, P_D, S/N)}{N_2(P_{FA}, P_D, S/N)} , \tag{10}$$

where N_i is the minimum number of observations required for detector d_i. The simplest way of calculating the ARE is by employing the concept of efficacies, which was also introduced by Pitman. To define an efficacy and to give the conditions when it can be used, Noether's extension of a theorem by Pitman will be summarized (8): Let us assume we have a binary hypothesis testing problem

$$H_0: \theta = \theta_0$$

and

$$H_1: \theta > \theta_0 .$$

Furthermore, we will assume that the alternative $\theta = \theta_n$ changes with the sample size n in such a way that as $n \to \infty$, $\theta_n = \theta_0$. Let the test be based on the statistic

$$T_n = T(x_1, \ldots, x_n) ,$$

and let $E\{T_n\} = \psi_n(\theta)$ and $\mathrm{Var}\{T_n\} = \sigma_n^2(\theta)$. Now, if conditions A, B, C, and D are true,

A. $\psi_n'(\theta_0) = \ldots = \psi_n^{m-1}(\theta_0) = 0 , \quad \psi_n^m(\theta_0) > 0 ,$

B. $\lim_{n\to\infty} n^{-m\delta} \psi_n^m(\theta_0)/\sigma_n(\theta_0) = C > 0$ for some $\delta > 0$,

C. $\lim_{n\to\infty} \psi_n^m(\theta_n)/\psi_n^m(\theta_0) = 1, \ \lim_{n\to\infty} \sigma_n(\theta_n)/\sigma_n(\theta_0) = 1,$

D. The distribution of $[T_n - \psi_n(\theta_n)]/\sigma(\theta_n)$ tends to be normally distributed with mean 0 and variance 1, under both H_0 and H_1,

the efficacy of the ith test is

$$E_i = \left[\frac{\psi_{in}^m(\theta_0)}{\sigma_{in}(\theta_0)}\right]^{1/m\delta} \tag{11}$$

and the ARE is

$$\mathrm{ARE}(d_1, d_2) = \lim_{\substack{S \\ N_1 N_2 \to \infty}} \frac{N_1(\cdot)}{N_2(\cdot)} = \frac{E_2}{E_1} . \tag{12}$$

To demonstrate the utility of efficacies, let's determine the ARE's for the integrator (sum), median, and sign detectors for the coherent contaminated-normal distribution. The coherent contaminated-normal density is

$$p(x,\theta) = (1-\gamma)\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\theta)^2/2\sigma^2} + \frac{\gamma}{\sqrt{2\pi K^2\sigma^2}} e^{-(x-\theta)^2/2K^2\sigma^2} . \tag{13}$$

<u>Efficacy of the integrator (sum)</u>: The integrator is an unbiased estimator. Hence,

$$\psi(\theta) = n\theta ,$$

$$\psi'(\theta) = n , \tag{14}$$

and

$$\psi'(0) = n .$$

The variance is

$$\sigma_n^2(0) = n[(1-\gamma)\sigma^2 + \gamma K^2\sigma^2] ; \tag{15}$$

consequently, since $m = 1$ and $\delta = 1/2$, the efficacy is

$$E_1 = \frac{n}{\sigma^2[(1-\gamma) + \gamma K^2]} \quad . \tag{16}$$

Efficacy of the median: The median is also an unbiased estimator. Thus,

$$\psi(\theta) = \theta \ ,$$

$$\psi'(\theta) = 1 \ , \tag{17}$$

and

$$\psi'(0) = 1 \ .$$

Using Ref. 9, the variance of the median $\tilde{x}$ is

$$\mathrm{Var}(\tilde{x}) = [4p^2(0)]^{-1}/n \ , \tag{18}$$

which for the contaminated-normal distribution is

$$\sigma^2(0) = \frac{\pi\sigma^2}{2n[(1-\gamma) + \gamma/K]^2} \quad . \tag{19}$$

Consequently, the efficacy is

$$E_2 = \frac{2n[(1-\gamma) + \gamma/K]^2}{\pi\sigma^2} \quad . \tag{20}$$

Efficacy of the sign test: The mean of the sign test is

$$\psi(\theta) = np$$

and

$$\psi'(\theta) = n\frac{\partial p}{\partial \theta} \ , \tag{21}$$

where p is the probability that the sample is greater than 0; i.e.,

$$p = \int_0^\infty \left[\frac{(1-\gamma)}{\sqrt{2\pi\sigma^2}} e^{-(x-\theta)^2/2\sigma} + \frac{\gamma}{\sqrt{2\pi K^2\sigma^2}} e^{-(x-\theta)^2/2K^2\sigma^2}\right] dx \quad . \tag{22}$$

Taking the derivative inside the integral gives

$$\frac{\partial p}{\partial \theta} = \int_0^\infty \left[\frac{(x-\theta)(1-\gamma)}{\sigma^2\sqrt{2\pi\sigma^2}} e^{-(x-\theta)^2/2\sigma^2} + \frac{(x-\theta)\gamma}{K^2\sigma^2\sqrt{2\pi K^2\sigma^2}} e^{-(x-\theta)^2/2K^2\sigma^2}\right] dx \ , \tag{23}$$

which reduces to

$$\frac{\partial p}{\partial \theta} = \frac{[(1-\gamma) + \gamma/K]}{\sqrt{2\pi\sigma^2}} \quad . \tag{24}$$

Substituting Eq. (24) into Eq. (21) yields

$$\psi'(\theta) = \frac{n[(1-\gamma) + \gamma/K]}{\sqrt{2\pi\sigma^2}} \quad . \tag{25}$$

Since the variance of the sign test is

$$\sigma^2(\theta_0) = np(1-p) = \frac{n}{4} \quad , \tag{26}$$

the efficacy is

$$E_3 = \frac{2n[(1-\gamma) + \gamma/K]^2}{\pi\sigma^2} \quad . \tag{27}$$

Comparing Eqs. (20) and (27), it is obvious that they are equal, which was to be expected since the way the median is implemented showed that it was equivalent to the sign test. The ARE for the sampled mean and median is

$$\text{ARE} = \frac{E_2}{E_1} = \frac{2}{\pi}\,[(1-\gamma) + \gamma/K]^2\,[(1-\gamma) + \gamma K^2] \quad , \tag{28}$$

which equals $2/\pi$ if $\gamma = 0$ (gaussian noise) and equals 0.95 if $\gamma = 0.25$ and $K = 2.25$. This implies that for the small signal case the sampled mean requires only $2/\pi$ as many samples as the median to obtain identical probabilities of false alarm and detection when the noise is gaussian and 95% as many samples when the noise is a contaminated-normal.

ARE'S FOR NONCOHERENT CONTAMINATED-NORMAL AND LOG-NORMAL DISTRIBUTIONS

When one is trying to calculate the ARE's for noncoherent distributions, the difficulty encountered is that the first derivative evaluated at zero signal strength is zero. Thus, the second derivative needs to be evaluated ($m = 2$ and $\delta = 1/4$), and many times the integral of the derivatives needs to be evaluated numerically. Unfortunately the contaminated-normal and log-normal distributions are not exceptions. (Since the calculations are straightforward, only the results will be presented.) The efficacies for the contaminated-normal distribution are

$$\begin{aligned} E_i = \Bigg\{ \int_0^{T_i} F_i(R) \Bigg[& \frac{(1-\gamma)^2}{\sigma^4}\,(-1 + R^2/2\sigma^2)\,\exp(-R^2/2\sigma^2) \\ & + \frac{\gamma^2}{K^4\sigma^4}\,(-1 + R^2/2K^2\sigma^2)\,\exp(-R^2/2K^2\sigma^2) \\ & + \frac{2\gamma(1-\gamma)}{K\pi\sigma^4} \int_0^{\pi/2} \Big((-1 + R^2\cos^2\theta/\sigma^2)\,\exp\{-[R^2 + (K^2-1)R^2\cos^2\theta]/2K^2\sigma^2\} \\ & + \frac{(-1 + R^2\cos^2\theta/K^2\sigma^2)}{K}\,\exp\{-[R^2K^2 - (K^2-1)R^2\cos^2\theta]/2K^2\sigma^2\}\Big)\Bigg] d\theta\; dR/\sigma_i \Bigg\}^2 \quad , \end{aligned} \tag{29}$$

where for the efficacy of the sampled mean

$$F_1(R) = nR^2,$$

$$T_1 = \infty,$$

and

$$\sigma_1 = 1.11037\,\sigma\sqrt{n}, \qquad \text{for } \gamma = 0.25 \text{ and } K = 2.25,$$

and for the efficacy of the median

$$F_2(R) = R,$$

$$T_2 = 1.39149\,\sigma, \qquad \text{for } \gamma = 0.25 \text{ and } K = 2.25,$$

and

$$\sigma_2 = 0.5\sqrt{n}.$$

The efficacies of the log-normal distribution are

$$E_i = \left(\int_0^{T_i} F_i(R)\left\{\frac{4(\sigma^2-1)/\sigma^2R^3 + 16[\sigma^2\log R + (\log R)^2]/\sigma^4R^3}{\sqrt{2\pi\sigma^2}}\right\}\exp\left[-2(\log R)^2/\sigma^2\right]dR/\sigma_i\right)^2,$$

where for the efficacy of the sampled mean

$$F_1(R) = nR,$$

$$T_1 = \infty,$$

and

$$\sigma_1 = \left[ne^{\sigma^2}(e^{\sigma^2}-1)\right]^{1/2},$$

and for the efficacy of the median

$$F_2(R) = 1,$$

$$T_2 = 1,$$

and

$$\sigma_2 = 0.5/\sqrt{n}.$$

The integrals were calculated numerically, and the results are presented in Table 3. From Table 3, it can be inferred that for the contaminated-normal distribution and for very small S/N ratios, the sampled mean requires almost twice as many samples as the median to obtain the same results. This is in contrast to the small sample results (Table 2), where the sampled mean was actually a more effective detector. Thus, as one obtains more and more samples, the median becomes the more effective detector.

For the log-normal distribution (σ = 6 dB), the sampled-mean detector requires over 100 times as many samples as the median, because of the enormous tail of the log-normal distribution. This is another example of why the log-normal distribution gives exaggerated answers for certain questions.

Table 3
Efficacies and ARE's

Distribution	Efficacy Sampled Mean (n)	Efficacy Median (n)	ARE
Contaminated-normal (γ = 0.25 and K = 2.25)	0.0137	0.0238	0.576
Log-normal (σ = 3)	0.286	5.334	5.0535
Log-normal (σ = 6)	0.0103	1.333	0.00778

CONCLUSIONS

There are several things that recommend the median as a detector. First, it can be implemented just as easily as the sampled mean. Second, for the contaminated-normal distribution (note: γ = 0.25 and K = 2.25 is a good fit for a smooth sea, Ref. 6), the median is just as good as the mean for a sample size of 30 and is about twice as good in the limit ($n \to \infty$). Also, for rougher sea conditions sea clutter will have longer tails; then, the median will perform better than the sampled mean. However, before one can say that the median is a better detector than the mean, the effects of fluctuating targets and correlated noise on both of these detectors has to be investigated.

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13. ABSTRACT

Sea-clutter measurements using high-resolution radar indicate that clutter cross-section returns do not follow a Rayleigh distribution. The log-normal and contaminated-normal distributions have been suggested, and detection curves for these distributions have been published. In this report, the median detector is proposed as a detector against these clutter distributions, and the theory is developed which is necessary for detecting constant signals in log-normal and contaminated-normal clutter when one has N independent samples. Curves are obtained for the threshold values and probabilities of detection for false-alarm probabilities ranging from 10^{-2} to 10^{-9}. If one compares the mean and median detectors, the conclusion is reached that the median is better for the log-normal distribution and that the mean is slightly better for the contaminated-normal distribution for small sample sizes. Using the concept of asymptotic relative efficiency, the median is shown to be better than the mean for large sample sizes for both distributions.

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