NRL Report 6784

Apparent Local Jump in the Momentum Transfer Rate at the Front of Flare-Fresh Solar Plasma in the Interplanetary Space

THOMAS S. LEE

Department of Electrical Engineering
University of Minnesota

and

W. W. BALWANZ

Plasma Section
Applied Physics Branch
Applications Research Division

December 31, 1968

NAVAL RESEARCH LABORATORY
Washington, D.C.

This document has been approved for public release and sale; its distribution is unlimited.

CONTENTS

Abstract	ii
Problem Status	ii
Authorization	ii
INTRODUCTION	1
ANALYSIS	3
LOCALIZED INERTIAL EFFECTS AT THE DRIVING INTERFACE	9
DISCUSSIONS	10
ACKNOWLEDGMENT	14
REFERENCES	14

ABSTRACT

This report is limited to calling to attention a feature, not scrutinized before in the literature, of the ideal interplanetary blastwave according to the Parker type of formal theory. It is shown analytically that, because of inertial effects, a gradient singularity of the radial momentum transfer rate arises in association with the local coronal accumulation in front of the rear propelling contact surface postulated in the theory. Also, it is shown that such a local accumulation is a general result for power-law explosive waves extruding against an ambient hydrogen solar wind having a strictly inverse-square radial decay in density. The usual numerical schemes are rendered ineffective for the determination of the expected singular local wave behavior. To circumvent the difficulty, a combined numerical-integral technique has been developed, and in this work it has been applied on six specific model waves, including both the decelerating and the nondecelerating, which have energies increasing with time as t^K with $K = 28, 4, 1, 1/2, 1/4$, and $1/8$. Also, a local solution corresponding to the limiting approximation of a constant material velocity gradient has been analytically constructed. The importance of the local singular behavior is quantitatively appraised, and certain interplanetary implications are given cursory inferences. It is considered that the new results should have relevance to the quantitative determination of the progressive peak transverse magnetic field during an interplanetary storm and to the theoretical determination of the macroscopic stability of the postulated contact surface.

PROBLEM STATUS

This is an interim report on a continuing problem.

AUTHORIZATION

NRL Problem R07-18
Project RR 002-10-45-5063

Manuscript submitted August 1, 1968.

APPARENT LOCAL JUMP IN THE MOMENTUM TRANSFER RATE AT THE FRONT OF FLARE-FRESH SOLAR PLASMA IN THE INTERPLANETARY SPACE

INTRODUCTION

Geomagnetic storms, rapid modulations of primary cosmic radiation of galactic, and occasionally solar, origins, and other related transient geophysical disturbances belong to a family of phenomena now believed to be either direct or indirect manifestations of the transit of hydrogen plasma clouds in the inner interplanetary space. Of these events, those with more significant magnitude for observation are further believed to be associated with the infrequent but large surface activities on the sun. Current deductive thinkings on the subject, though still largely qualitative, all evolve from a central premise. According to this premise separate considerations are given to a normal quiescent state of the interplanetary medium and to a disturbed state of the same medium following a strong solar activity of the nonrecurrent type.

The quiescent medium consists of an extended solar corona in more or less steady outward streaming. Such a "solar wind," as called by Parker (1,2), is basically collisionless, and its fluidlike bulk behavior is generally attributed to the special coupling role played by the generally weak yet irregularity-infested magnetic field transported by convection through the interplanetary region. By and large, inferences from indirect observations (e.g., by Biermann (3) and Winckler and Anderson (4)) and, more recently, direct space probe measurements (e.g., by Gringauz et al. (5), Neugebauer and Snyder (6), Bonetti et al. (7), Coleman et al. (8), and Ness et al. (9)) have confirmed most of these theoretical expectations.

The understanding of the disturbed state of the interplanetary medium is less complete. There, the observations have been mostly of an indirect kind (e.g., Akasofu (10) and Cole (11)). The few pertinent deep-space direct detections have been largely incidental (8, 10-18). On the other hand, the few available theoretical attempts at storm modeling up to now have not progressed far beyond stages of elementary long-hand sketches as observed by Chapman (19). Currently favored assessments, e.g., by Cole (11), Akasofu (10), Parker (20), and McCracken (21), of the disturbed state of the interplanetary space have origins in the early Chapman-Ferraro (22) picture of discrete expulsion of a flare plasma cloud from the sun. The now nontrivial recognition that the solar environment is normally not empty complicates all aspects of the situation. Gold (23) was the first to suggest that the fresh flare plasma in motion must be preceded by a shock wave of a collisionless nature. He envisioned a configuration for the driving plasma cloud in which an assumed "magnetic bottle" is the dominant feature. Parker (20,24) has since proposed a blastwave model in which the chromospheric temperature enhancement is preferred as the sole cause for the subsequent violent dynamic motion. According to this view, flux lines permanently anchored on the sun and stretched out into the interplanetary region are essentially passive at all times. With modifications, actually, these two models do not differ as greatly as might appear at first sight as noted by Davis (25). We point out here that this is especially true of the region bounded between the leading shock front and the rear plasma interface. By definition this region is filled by material belonging to the original solar wind which is now swept up by the outrushing flare plasma. Its global dynamics can be expected to be insensitive to the specific assumptions made on the interior magnetic structure of the driving plasma. In Fig. 1 the region occupied by the fresh flare plasma is shaded, and the

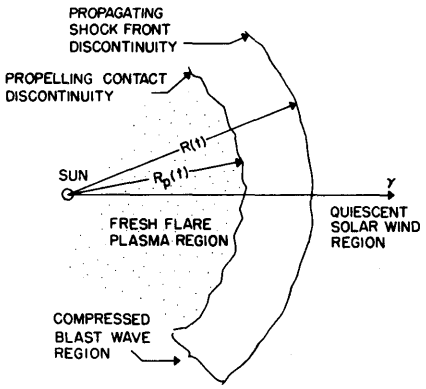

Fig. 1 - Sketch of a representative solar wind compression produced by an enhanced fresh solar plasma propelled into the interplanetary space following the onset of a flare

compressed wind plasma in question together with its leading shock front is appropriately indicated in front for identification. That such a sketch can well depict an average storm configuration draws support from Chapman's conclusion (19,26), based on geophysical observations, that the usual angular extent of the expected wave away from the sun can be substantial, because even flares on the limbs often lead to significant geomagnetic storms.

Parker's work (20,24) represents the only serious attempt thus far at offering a self-consistent analytical formulation for the purely hydrodynamical aspects of the driven solar plasma. Essentially, the solution of an idealized problem is contemplated by invoking the similarity principles of the progressive wave for which Courant and Friedrichs' transformation hierarchy (27) is used. Specifically, Parker considers a class of relevant power-law blastwaves moving into an ambient solar corona which follows an inverse square law in the decay of plasma density with distance, in keeping with his own adiabatic solar wind model. The resultant theory constitutes an extension to an earlier work by Rogers (28) which treats the problem of instantaneous energy releases in an inhomogeneous medium. It applies in an asymptotic sense to conditions correponding to the undiminished pushing by the fresh flare plasma over a substantial portion of the distance between the sun and the earth. As is known from optical (H_α line) observations, e.g., by DeJager (29, 30), the active lifetime of a flare ranges from a fraction of an hour up to as long as eight hours. We point out that this order of lifetime should be regarded merely as a lower bound to the actual lifetime since coronal forcing may continue well into the later non-emissive phase of the phenomenon. Based on mechanical effectiveness, then, the actual lifetime could, reasonably, be important compared with the known sun-earth transit time of a large flare cloud, which is on the order of a day or two (31,32). When the circumstances are approximately as described, investigations paralleling Parker's intention are warranted.

In this report it is our limited objective to call to attention a feature of the explosive wave which appears to have been neglected by Parker and has since gone undetected in the literature. Since the blastwave model has been of no small quantitative interest, in view of the Pioneer V magnetic field data (20), a detailed examination is in order. We recognize the idealized nature of the formulation but take the view that it is imperative to precisely establish and scrutinize the expected essential characteristics of the blastwave under this formulation before either a comparison of these characteristics with available observational facts or a realistic examination of individual assumptions in the theory can be meaningfully made. In the following section the analysis shows the existence of a singularity of local gradient in the momentum transfer rate (i.e., thermal pressure in hydrodynamics) at the expected plasma contact surface in coincidence with the known density singularity. Here, an integral technique based on the energy conservation principle will be developed for yielding proper results. A subsequent section establishes the physical argument that the noted singularity is due exclusively to local

inertial effects. The final discussion appraises the new result for implications in the light of the current knowledge on the interplanetary dynamics.

ANALYSIS

The Parker-type blastwave formulation has the following simplifying idealizations:

1. The only significant motion is in the outward radial direction, and a spherical symmetry is assumed.

2. The ambient corona is of the idealized solar-wind-model type, for which the density distribution $\rho_0(r)$ varies in inverse proportion to the square of the radial distance r in the far field.

3. The blastwave is so strong that both the solar-gravitational energy and the ambient internal energy are negligible.

4. The wave expansion is rapid enough for the kinetic energy in the ambient solar wind to be ignored.

Based on these assumptions, the physical situation portrayed in the previous section can be governed by the conservation equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \qquad \text{(momentum)}, \tag{1a}$$

$$\frac{\partial \rho}{\partial t} + r^{-2}\frac{\partial}{\partial r}(r^2\rho u) = 0 \qquad \text{(continuity)}, \tag{1b}$$

and

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} = \frac{5}{3}\frac{p}{\rho}\frac{d\rho}{dt} \qquad \text{(entropy)} \tag{1c}$$

in terms of the radial velocity $u(r,t)$, the density $\rho(r,t)$, and the pressure $p(r,t)$, which are appropriate for a lossless hydrogen corona for which the adiabatic index is taken to be 5/3.

Instead of choosing Courant and Friedrichs' transformation, it is convenient, for reasons of explicitness, to specialize Lee's similarity transformation (33) in a form pertinent to the assumptions enumerated above.* Specifically, we introduce the normalized quantities for the pressure, the density, and the velocity:

$$f(\eta) = 5p/3\,\rho_o'(R)\,V^2, \tag{2a}$$

$$\psi(\eta) = \rho/\rho_{o(R)}, \tag{2b}$$

and

$$\phi(\eta) = u/V, \tag{2c}$$

*The form of the transformation used here corresponds to an adaptation of the basic transformation (33) with a unity ambient unsteadiness parameter $k = 1$, a strong shock condition of $M^{-2} = 0$, and an ambient density index $\beta = -2$.

where $R(t)$ and $V = dR/dt$ denote respectively the instantaneous position and the instantaneous velocity of the outgoing shock front, and $\eta = r/R$ is a scaled radius. Equations (1a) to (1c) thus reduce to

$$f' = (5/3)\ \psi\phi\,[N + (\eta - \phi)\ (\ln \phi)'], \tag{3a}$$

and

$$(\phi - \eta)\ (\ln \psi)' + \phi' = 2 - (2\phi/\eta), \tag{3b}$$

and

$$(\phi - \eta)\ [\ln(f/\psi^{5/3})]' = 2N - (4/3) \tag{3c}$$

respectively. In these expressions, $N \equiv -d \ln V/d \ln R$ is a front propagation parameter. At the $r = R$ front, the boundary conditions for strong shock waves may be stated as

$$f(1) = 5/4, \tag{4a}$$

$$\psi(1) = 4, \tag{4b}$$

and

$$\phi(1) = 3/4. \tag{4c}$$

Referring to Fig. 1, we can calculate the total energy contained in the wave per unit solid angle subtended at the sun as

$$E = \int_{R_p}^{R} \left(\frac{3}{2}\,p + \frac{1}{2}\,\rho u^2\right) r^2\, dr$$

$$= \rho_o(R)\ R^3 V^2\ L, \tag{5}$$

where a definite integral

$$L \equiv \int_{\eta_p}^{1} \left(\frac{9}{10} f + \frac{1}{2}\,\psi\phi^2\right) \eta^2\, d\eta \tag{6}$$

has been introduced. In Eq. (5), $R_p(t)$ gives the instantaneous location of the contact surface, at the rear of the blastwave, where the condition $(dr/dt)_\eta = u$ is expected to hold. This condition is equivalent to

$$\phi(\eta) - \eta = 0, \tag{7}$$

which arises in the solution of Eqs. (3) and (4). The root of Eq. (7) $\eta_p = R_p/R$ represents the normalized location of the contact surface.

Physically we are mainly interested in the class of power-law waves which emanate from the sun beginning at some fixed time, say $t = 0$. For this class of flows, the range of relevance for N is $-1 < N < 1/2$. The lower bound $N = -1$ is a singular state for which the power-law propagation ceases to apply. Instead, the propagation becomes exponential in time, a situation of no immediate relevance to the problem in question. On the other hand, it may be shown that the upper bound $N = 1/2$ implies the idealized problem of instantaneous point energy release in an inhomogeneous medium for which Rogers (28) has provided an exact solution. We shall regard the entire indicated range of N as theoretically both possible and acceptable. Since it may be shown that $V \propto R^{-N}$, we can write the desired power-law propagation trajectory of the front as

$$R(t) = R_0\ t^{1/(N+1)},$$

where R_0 is a constant. The position and velocity of the contact surface are $V_p = \eta_p V$ and $R_p = \eta_p R$ respectively. When N is negative, material near this surface undergoes local

acceleration continuously. When N is positive, however, deceleration ensues instead. The particular value of $N = 0$ corresponds to a constant-velocity local motion, a feature superficially reminiscent of that of an expanding-sphere problem early solved by Taylor (34), except that the latter considers a wave of invariant but arbitrary strength entering a homogeneous medium.

This report deals with solutions in which an unbounded density at the contact surface is an essential property. From the boundary condition on ϕ given by Eq. (4), we may conclude that the quantity $\eta - \phi$ remains positive throughout the entire blastwave domain of $\eta_p < \eta < 1$. In the limit $\eta \to \eta_p$, then, Eqs. (3c) and (7) combine to show that it is inevitable for the local gradient in the specific entropy to become infinitely positive. With justification to be gathered shortly from the self-consistency of the total theory, we take it that this singular inhomogeneity in specific entropy is accounted for by the local density behavior alone, rather than by the local variation of the thermal pressure. For a blastwave from finite coronal driving, the velocity distribution is expected to be regular throughout its entire domain. Near the contact surface at η_p, it is possible to expand the quantity $\eta - \phi$ in a Taylor's series in powers of $\Delta\eta \equiv \eta - \eta_p$. For the immediate neighborhood, in particular, it is sufficient to keep only up to the linear term. Hence, we may write

$$\eta - \phi = A\Delta\eta, \tag{8}$$

where $A(N)$ is a constant for a specified value of N. This being the fundamental level of approximation, a corresponding local limiting solution will now be constructed. Following the stipulation on the local variation of specific entropy just alluded to above, we have the limiting form of Eq. (3c) which is locally valid, namely,

$$(\phi - \eta)(\ln\psi)' = 2(2 - 3N)/5. \tag{9}$$

Substitution of Eq. (9) into Eq. (3b) yields $A(N) = 3(3 - 2N)/5$. It can now be surmised from Eq. (9) that the local divergence of the density distribution follows

$$\psi(\eta) = S(N)\Delta\eta^{2(3N-2)/3(3-2N)}, \tag{10}$$

where $S(N)$ for a specific N value is a constant. This equation explicitly demonstrates that ψ approaches positive infinity as $\Delta\eta$ approaches zero. The momentum equation, Eq. (3a), may also be shown now to assume its limiting form

$$f' = \frac{5}{3}\eta_p N\psi. \tag{11}$$

For positive values of N, f' approaches $+\infty$ locally. For negative values of N, on the other hand, it approaches $-\infty$ locally instead. Only for the neutral value of $N = 0$ is f' locally well-behaved. The discussion of the physical interpretation will be deferred until the following section. Here, it is necessary only to note that the existence of this singular behavior of the local gradient in the momentum transfer rate is an inescapable feature of a general idealized blastwave. To complete the solution, Eq. (11) is integrated to give

$$f(\eta) - f(\eta_p) = (3 - 2N)\,\eta_p N S(N)\,\Delta\eta^{5/3(3-2N)}. \tag{12}$$

At this point, it is feasible to show incidentally that

$$\lim_{\Delta\eta \to 0} \left[3(\ln f)'/5(\ln\psi)'\right] = \lim_{\Delta\eta \to 0} \Delta\eta^{5/3(3-2N)} = 0.$$

This result validates the consistency stipulation on the local variation of the specific entropy made earlier by justifying the replacement of $\left[\ln(f/\psi^{5/3})\right]'$ by $-5(\ln\psi)'/3$, which led Eq. (3c) to Eq. (9).

The set of Eqs. (8), (10), and (12) constitutes the required limiting solution. The external solution for the blastwave field in the large can be found through the numerical integration of Eqs. (3), starting from the shock front at $\eta = 1$ and proceeding toward the expected contact surface in the rear. Presumably the quantities $f(\eta_p)$ and $S(N)$ are to be determined by matching the local limiting solution with the external numerical solution. In practice, however, we recognize the failure of such a procedure because of the existence of the noted local singularity in f'. The difficulty particularly affects the determination of $f(\eta_p)$. This is because a Taylor's series for f does not exist locally, except when N vanishes identically. In Fig. 2, local variations in pressure are qualitatively illustrated for three typical yet distinct cases where the local accelerations are respectively negative, zero, and positive. The local limiting solution in each case is sketched to show the tendency of merging into the external solution away from the contact surface. It is clear by inspection that straightforward numerical methods, like the one apparently (and equivalently) used by Parker (20,24), cannot correctly give local results. In principle, the dependent and independent variables in Eqs. (3) could be interchanged in position through a suitable transformation to facilitate local numerical integrations. However, the added complexity involved would render such a scheme impracticable. Fortunately, the value of $f(\eta_p)$ can be independently obtained from a gross statement of energy conservation; namely, the rate of energy gain being received by the blast plasma is exactly equal to the rate of work being done by the propelling coronal plasma. In other words,

$$\frac{dE}{dt} = p(R_p, t)u(R_p, t)R_p^2(t).$$

By utilizing Eqs. (2) and (5), this energy balance relation becomes

$$f(\eta_p) = 5L(1 - 2N)/3\eta_p^3. \tag{13}$$

In Eq. (13), the integral L is regularly convergent. This behavior may be seen by first noting that

$$\int_{\eta_p}^{1} \psi\eta^2 \, d\eta = 1,$$

a relation of material conservation inherent in Eq. (16), which will be derived later. Recognizing this, we may calculate L from Eq. (6) according to

$$L = \frac{1}{2}\eta_p^2 + \int_{\eta_p}^{1} \left[\frac{9}{10} f + \frac{1}{2}(\phi^2 - \eta_p^2)\psi\right]\eta^2 \, d\eta. \tag{14}$$

By virtue of the limiting solution of Eqs. (8), (10), and (12), it is straightforward to show that the integrand of the second term in Eq. (14) is bounded at $\eta = \eta_p$. Furthermore, the same integrand remains finite throughout the closed interval $(\eta_p, 1)$. Thus L, and consequently $f(\eta_p)$, can be accurately determined despite the existence of the local combined $\psi - f'$ singularity.

If, on the other hand, an extrapolation scheme were directly applied and combined with the usual numerical method, a different situation would arise. Suppose that f is calculated up to the edge of the immediate neighborhood of η_p and then extended smoothly by direct extrapolation to η_p. Let the resulting value of f at η_p be referred to as the "apparent normalized driving pressure" and represented by $f_a(\eta_p)$. In Fig. 2, this quantity is shown to differ from $f(\eta_p)$ in general. It is understood that the precise value of $f_a(\eta_p)$ depends on the width of the neighborhood subjectively chosen for extrapolation. Nevertheless, the concept serves to exhibit the degree of discrepancy between the genuine boundary driving pressure according to the idealized formal theory and what is likely to yield from a usual numerical-extrapolating method.

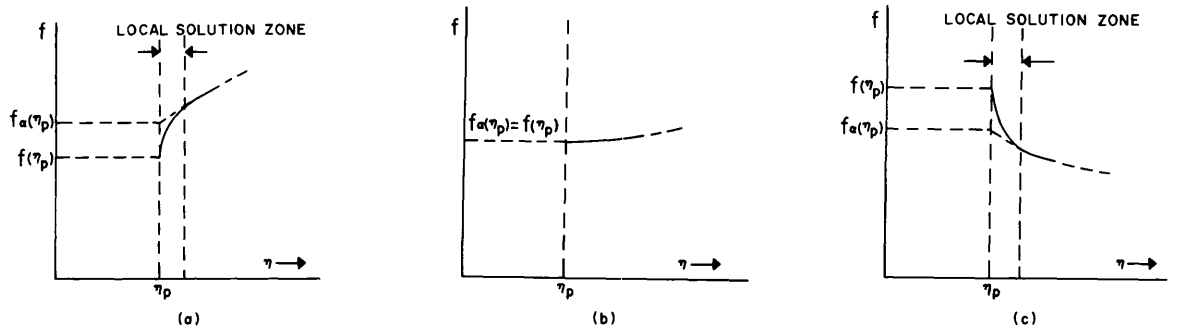

Fig. 2 - Expected local transition of the pressure distribution near the rear contact surface at η_p on which the enhanced corona is pushing. The three sketches here are respectively representative of the cases of (a) decelerating wave ($0 < N < ½$), (b) a constant-velocity wave ($N = 0$), and (c) an accelerating wave ($-1 < N < 0$). In each case the curve outside the local solution zone merges into the blastwave solution in the large.

A selected group of blastwaves, corresponding to $N = -9/10$, $-1/2$, 0, $1/5$, $1/3$, and $7/17$, have been obtained by numerical integration. The last four are nonaccelerating waves. Their conditions are separately equivalent to those on which Parker's four intended solutions (corresponding to $\lambda =$ 1, 6/5, 4/3, and 24/17 in Ref. 20) are based. The normalized densities, velocities, and pressures are plotted in Figs. 3, 4, and 5, successively, for all six obtained blastwaves. Table 1 lists the essential results and gives in addition the calculated values of the noted pressure discrepancy in percentages. The last item is sketched as a function of N in Fig. 6. The values of $S(N)$ yield from the direct matching of numerical calculations with Eq. (10). Again we emphasize that the results associated with $f_a(\eta_p)$ are subject to the choice of neighborhood width for extrapolation and are enlisted mainly for purposes of disucssion.

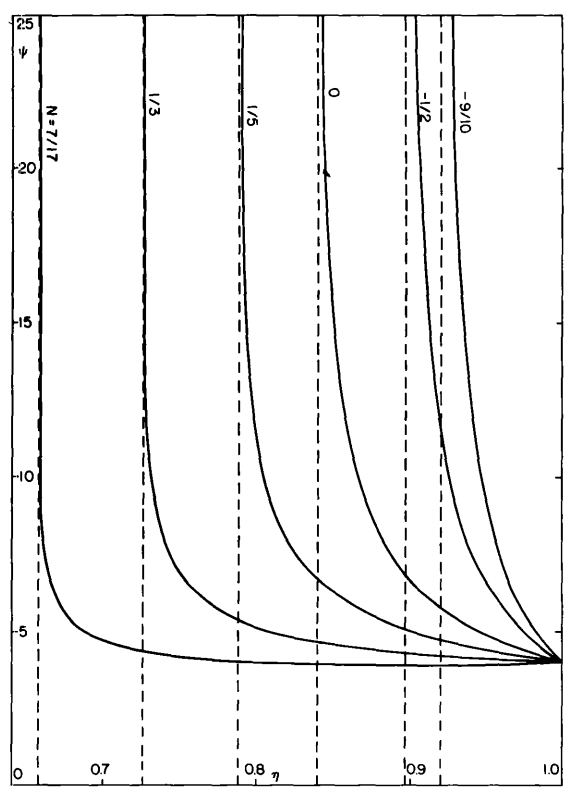

Fig. 3 - Normalized density profile $\psi(\eta)$ through the strong spherical blastwave for $N = -9/10$, $-1/2$, 0, $1/5$, $1/3$, and $7/17$, according to assumptions of an inverse-square ambient density decay and of a hydrogen-ion gas. The vertical asymptote in each case represents the position of the rear contact surface where propulsion by the enhanced corona is exerted.

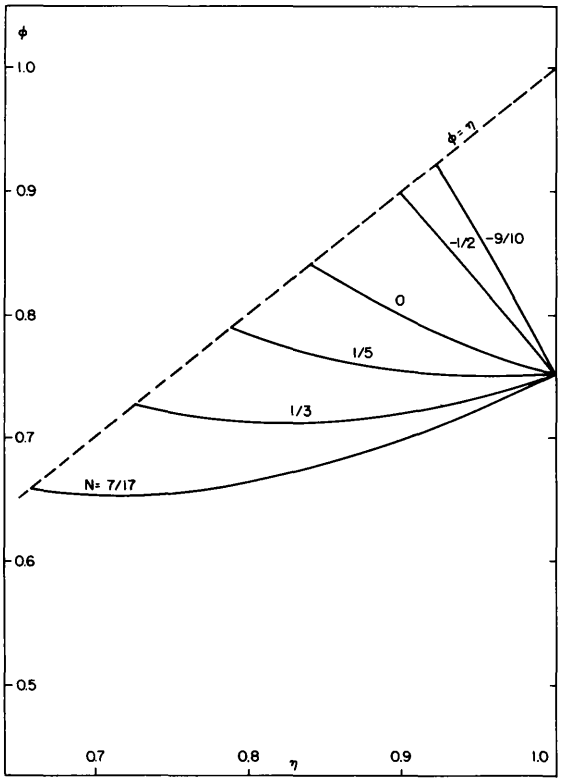

Fig. 4 - Normalized profile of material velocity distribution $\phi(\eta)$ through the strong spherical blastwave for N = -9/10, - 1/2, 0, 1/5, 1/3, and 7/17, according to assumptions of an inverse-square ambient density decay and of a hydrogen ion gas

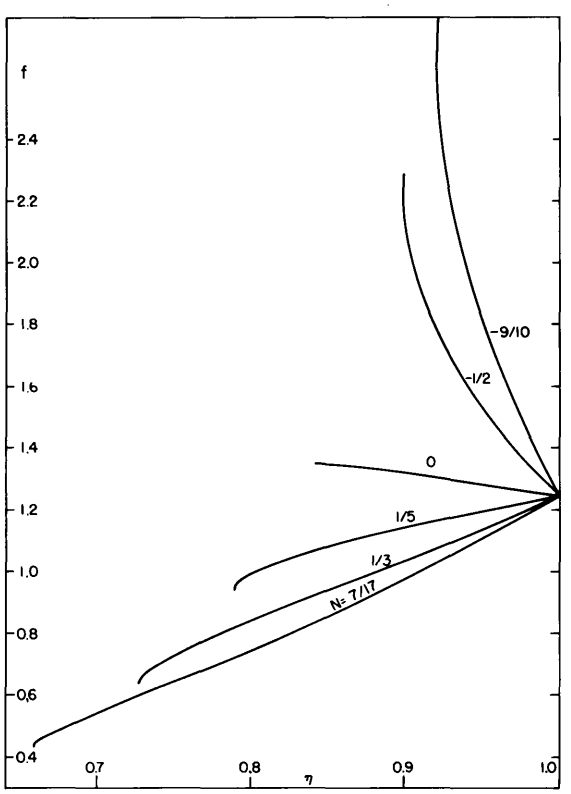

Fig. 5 - Normalized profile of pressure distribution $f(\eta)$ through the strong spherical blastwave for N = -9/10, -1/2, 0, 1/5, 1/3, and 7/17, according to assumptions of an inverse-square ambient density decay and of a hydrogen-ion gas

Table 1
Parameters Associated with the Blastwave Driving Contact Surface for N = -9/10, -1/2, 0, 1/5, 1/3, and 7/17

	N	η_p	L	$S(N)$	$f(\eta_p)$	$f_a(\eta_p)$	$\frac{f_a(\eta_p) - f(\eta_p)}{f(\eta_p)}$	$\frac{m}{m_t}$
Accelerating waves	-9/10	0.9212	0.4999	1.07	3.014	2.815	-0.0650	0.122
	-1/2	0.8987	0.4956	1.50	2.279	2.172	-0.0470	0.115
Constant-velocity wave	0	0.8402	0.4802	2.26	1.349	1.349	0	0
Decelerating waves	1/5	0.7892	0.4645	2.22	0.946	0.971	0.0265	0.0591
	1/3	0.7265	0.4443	2.19	0.645	0.665	0.0310	0.0262
	7/17	0.6584	0.4253	2.14	0.438	0.448	0.0251	0.0096

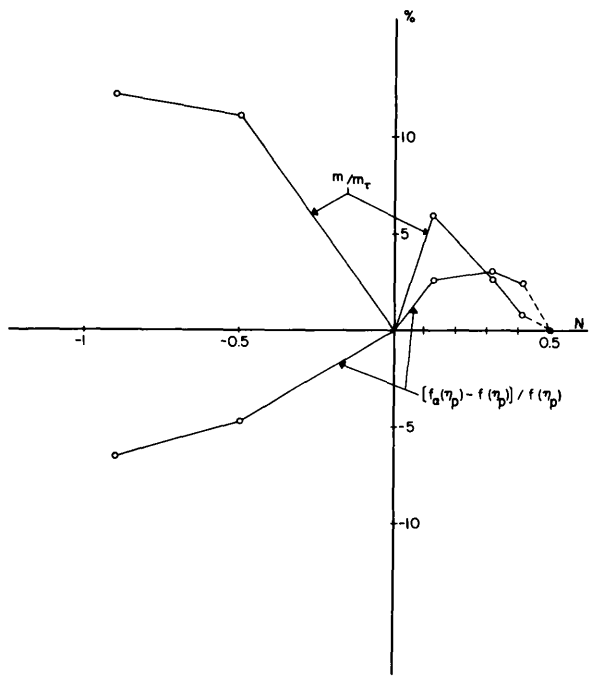

Fig. 6 - Plot of $\left[f_a(\eta_p) - f(\eta_p)\right]/f(\eta_p)$, the percentage discrepancy between values of the boundary momentum transfer rate separately obtained by a straightforward numerical-extrapolating method and by the combined numerical-integral method, and plot of m/m_t, the fractional blastwave mass in percentiles attributable to the effective material accumulation at the driving interface. The discrete values calculated for N = -9/10, -1/2, 0, 1/5, 1/3, and 7/17 are denoted by small circles, which, for the purpose of representation, are joined by straight line segments. Both quantities are expected to be zero at N = 0.5, which corresponds to a constant-energy wave without sustained rear propulsion.

LOCALIZED INERTIAL EFFECTS AT THE DRIVING INTERFACE

In the last section, we have seen that because of the mass accumulation effect there is always locally an unbounded spatial gradient in the thermal pressure at the interface. The difference between $f(\eta_p)$ and $f_a(\eta_p)$ resembles a jump in pressure over an exceedingly narrow layer next to the interface. This behavior is accounted for entirely by Newtonian-inertial consideration.

Consider in Fig. 7 a thin slab of concentrated material with a finite mass surface density in motion. The pressures exerted from the front and from the rear differ by an amount equal to the time rate of linear momentum change per unit slab area. When the differential pressure and the finite acceleration are both known, the surface mass density is uniquely fixed. An application of this analogy to the blastwave problem of interest calls for the useful introduction of the concept of an effective mass sheet in front of the plasma interface. Let m be the mass contained over an area of this sheet subtending a unit solid angle at the sun. Then it is readily shown through Eqs. (2) and (8) that this inertial principle determines m as

$$m(R) = 0.6\left[f_a(\eta_p) - f(\eta_p)\right]\rho_o(R)R^3\eta_p/N, \tag{15}$$

a quantity which grows with propagation. On the other hand, let us denote by m_t the total coronal mass in the blastwave domain subtending the same unit solid angle at the sun. Then, by the simple mass conservation argument, m_t must be equal to the amount of the solar wind mass which in the absence of the explosion would occupy the entire interplanetary space inside $R(t)$ and within the unit solid angle, i.e.,

$$m_t(R) = \int_{R_p(t)}^{R(t)} \rho(r,t)\, r^2 dr = \int_0^{R(t)} \rho_o(r)\, r^2 dr = \rho_o(R)R^3, \tag{16}$$

where during the evaluation of the integral the inverse-square decay property of $\rho_o(r)$ has been used. As shown, m_t also grows with propagation. Now, by taking the ratio of m to m_t from Eqs. (14) and (15), we obtain the fractional blastwave mass attributable to the effective material accumulation at the interface

$$m/m_t = 0.6\left[f_a(\eta_p) - f(\eta_p)\right]\eta_p/N, \tag{17}$$

which, as expected, is time-invariant. Table 1 includes a listing of this quantity for the calculated blastwave solutions. Also, to show the variation of relative importance, Fig. 6 includes a plot of m/m_t as a function of the front propagation parameter N.

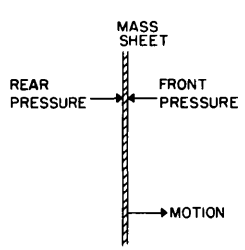

Fig. 7 - Newtonian-inertial effect governing the pressure differential across a moving concentrated mass sheet

To sum up, we state that an infinite mass density induces a local infinite pressure gradient whenever the local acceleration is nonzero. Finite pressure jumps arise from inertial effects due to finite local mass concentrations.

DISCUSSIONS

The physical understanding of flare observations is still in an incomplete stage. Blastwaves pushed out by the flare plasma in uninterrupted acceleration over some fractions of the sun-earth distance cannot be ruled out on theoretical grounds. For this reason, two representative solutions ($N = -9/10$ and $-1/2$) of such waves have been included in this work. On the other hand, four ($N = 0$, $1/5$, $1/3$, $7/17$) have been calculated from those

waves which are nonaccelerating. Altogether, these six blastwaves correspond to individual energy contents varying with time as t^K with $K = 28, 4/3, 1, 1/2, 1/4$, and $1/8$. We have shown that, since mass accumulation at the driving interface is a general result, the local pressure gradient singularity is always present except in the neutral case of $K = 1$. This conclusion is strictly true under assumptions underlying the theory, of course. We shall now discuss the probable effects which might accompany a relaxation of some of these assumptions.

Geometry enters the problem through the continuity equation only. In case the geometric divergence deviates from the assumed spherical divergence, the immediately affected quantity is the coefficient in the second term on the right-hand side of Eq. (3b). It can be shown that for moderate deviations the basic conclusion concerning the contact surface singularity remains the same, except that the form of the local limiting solution would no longer follow Eqs. (8), (10), and (12). The only notable difference is that now the upper bound of the physical N range mentioned before would shift away as a result, increasing or decreasing from 1/2 as the divergence is respectively less or more than the r^2 dependence of the spherical symmetry.

The unbounded density and pressure gradient singularity at the contact surface is not a necessary feature of a blastwave propagating into a medium of power-law density distribution. For an ambient density decay considerably milder than $\propto 1/r^2$, which follows from the idealized solar wind, the local behavior could be radically different. A general investigation encompassing this important extension will be reported elsewhere.

Parker's model for the solar wind is based on the picture of steady adiabatic expansion for which it is possible to write the wind pressure as $p_o(r) \propto [\rho_o(r)]^{5/3}$. The strength of a blastwave is measured by the hydrodynamic shock Mach number $M = [3\rho_o(R)V^2(R)/5p_o(R)]^{1/2}$. It is straightforward to show that with the idealized solar wind as the ambient medium we have $M^{-2} \propto R^{2N-(4/3)}$, which approaches zero at large R. The strong shock assumption is hence asymptotically valid for far field propagations. Once the wave has enough strength, self-strengthening becomes perpetual during subsequent propagation, in spite of the fact that the velocity may actually be decreasing. The noted M^{-2} variation is not expected to be valid in the near field where the wave is still weak. We note, however, that the $\rho_o(r) \propto 1/r^2$ decay is strictly correct only for the far field solar wind in any case.

Following similar lines of thought, we reiterate the idealized nature of two conditions inherent in the blastwave theory: (a) the assumed singularity of the r^{-2}-law solar wind at the sun and (b) the infinite-velocity power-law propagation near the sun (for the $N > 0$ case only). According to the theory, the concentrated solar wind mass at the center expands at $t = 0$ through the infinitely rapid passage of a shock wave. However, this shock wave has just been shown to be weak near the sun. (Strictly speaking, the formal theory is built for strong shocks only.) Thus it is possible to interpret the existence of a contact surface density singularity in the blastwave consistently as follows: The infinitely concentrated solar wind mass at the center undergoes shock heating at $t = 0$. Though rapid, the shock wave is weak and incurs no entropy production in the basically adiabatic solar wind. Therefore the initial expansion is illusory and the inherent density singularity of the idealized solar wind is retained by the blastwave to give the singular behavior of the density at the contact surface. The same conclusion holds true also for the $N < 0$ case, where both the velocity and the strength of the shock wave increase monotonically with distance. A realistic assessment based on this recognition favors the view that the predictions of the basic blastwave theory are essentially and asymptotically correct for far field quantities, but in the wave structure the mass density immediately in front of the flare coronal plow can only attain large but finite magnitudes (Fig. 8), as has been pointed out by Parker (20,24).

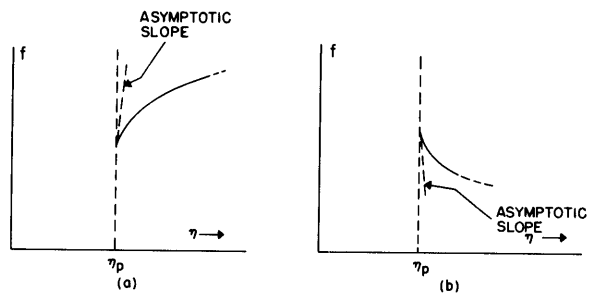

Fig. 8. Effective modifications of the local gradient of momentum transfer rate near the propelling contact surface expected from realistic consideration of finite local mass densities. (a) Case of a decelerating wave (b) Case of an accelerating wave

A more complete picture of the blastwave theory requires incorporating the consideration of the magnetic field. At present a fully magnetohydrodynamic treatment has not been achieved. However, Parker (20) observes that beyond the corotating inner corona of the sun the magnetic field has only a limited bearing on the large-scale interplanetary motions. By implication, a hydrodynamic theory should suffice provided due allowances are made for localized regions where the magnetic energy density cannot be ignored compared with the thermal energy density. Essentially, with appropriate initial distributions given of the permeating magnetic field, a plasma "frozen-in" concept can be invoked to follow the evolution of the magnetic field which arises from a large-scale dynamic motion in the interplanetary space. Parker has used this approach to deal with the kinking of flux lines by blastwaves viewed near the ecliptic plane, assuming that the coronal streaming and the solar rotation combine to yield a spiraling structure for the magnetic field in the quiescent interplanetary region. Near the contact surface of the blastwave, where the transverse magnetic field is expected to be important, the purely hydrodynamic formulation is not satisfactory. The chief value of that theory is, rather, to give an upper bound to the estimate on the peak field attainable, by recognizing that the local magnetic pressure cannot exceed the expected total momentum transfer rate per unit area across the contact surface which is basically inferred from the hydrodynamic theory. The evaluation of the last quantity has been shown in this report to encounter an ambiguity due to local inertial effects. Parker's results on boundary pressure (see Fig. 1 in Ref. 24 or Fig. 8.1 in Ref. 20) are obtained by numerical integrations which by all appearances have relied on local extrapolation. As such, they are considered to be at variance with what the idealized theory should supply. Nevertheless, as can be seen from Fig. 6 and Table 1, the difference between $f_a(\eta_p)$ and $f(\eta_p)$ is not large in practice. This is true at least for waves moving into ambient media having inverse square density decays. Thus, although there exists a difference in the prediction on the peak magnetic field in a typical explosive wave, which is important in quantitatively correlating the associated cosmic-ray Forbush effect, a more detailed certification is not warranted at present in the face of other more serious uncertainties in the interplanetary dynamics.

Occasional space probe detections of flare plasmas have been reported — Pioneer V on the March 31-April 1, 1960, event by Fan et al. (12) and Coleman et al. (8); Mariner II on the Oct. 7, 1962, event by Zelwer et al. (15) and Sonett et al. (13,14); Imp I on Dec. 2, 1963 event by Ness (9); Vela II on the April 17 - 18, 1965, event by Van Allen and Ness (18). On the basis of inferences from data and with the exception of the March 31 - April 1, 1960, event, these storms appear to either possess shock waves of only finite strength or have prestorm velocities of the solar wind too great to be ignored. They are not readily convenient for a comparison with the blastwave theory. Obayashi (35) has commented on the multiflare observations during the days preceding March 30, 1960, injecting complexities

to the interpretation of the general Pioneer V recordings. The magnetometer on Pioneer V gathered data indicating a possible peak storm transverse magnetic field of the order of 50 gammas. As pointed out by Parker (20), this is a reasonable value to be expected from the blastwave theory if it is attributed only to the local plasma compression at the propelling contact surface.

The recognition of the existence of large local density and pressure gradient has significance in connection with the observability of plasma interfaces in the interplanetary space. Here, two related aspects need be considered:

1. Since the coronal ions are basically collision-free, the ability of the interface to maintain identity in interplanetary transit can legitimately be questioned. According to the Parker concept in its original form, the blastwave contact surface connects the separate coronas in such a way that the local penetrating flux lines are sharply skewed. Colburn and Sonett (36) have speculated that contact surfaces could not be expected to be especially hindrant to diffusion, since plasma can migrate across the discontinuity, and that the lifetime determined by field-free transport parameters would be too short for the discontinuity to exist at earth orbit. We point out that the basic Parker-model concept is insufficient in that the dynamics of the driving corona itself, especially the part adjacent to the contact surface, has not been treated. Had the theory been combined with the consideration of a recompression layer (37 - 39), for instance, the noted magnetic skewing at the contact surface might not be effected and the associated strong local transverse magnetic field could be effective against diffusion, even without the aid of the inherent turbulent magnetic irregularities and/or the potential local magnetic dissipation where sufficient resistivity is present. The question of interface identity, the distinctness of which has been linked to a mechanism for geomagnetic storm sudden commencements (36), remains open and is best left to evidences from further observations.

2. Two macroscopic instabilities are known to exist at a plane interface. One is the Kelvin-Helmholtz instability, which applies at a tangential discontinuity across which the tangential component of the velocity jumps. It is known that the discontinuity is unstable when the two fluids are imcompressible and nonconductive. The situation becomes complicated, however, when the fluids carry a magnetic field and are compressible and conductive (40 - 43). The compressibility tends to augment the instability, whereas the magnetic field, if present, tends to suppress it. By appearance the basic mechanism could conceivably have relevance, if we considered portions of the blastwave near the angular fringes where nonradial velocity components due to loss of spherical symmetry are expected to be important. The other instability is the Rayleigh-Taylor instability. It occurs when a plasma is accelerated by either another plasma or a pure transverse magnetic field (e.g., Thompson (44)). Inertial effects presumably take the place of gravitational effects in the static problem originally considered by Taylor. We point out that the formal theories on these two instabilities, having been based on rather simple assumptions made on the two ambient fluids which combine to support the contact surface between them, do not readily apply to the blastwave contact surface where the local external fluid environment is one of large variation in both the mass density and the momentum transfer rate. Again we emphasize that a self-consistent companion theory for the dynamics of the driving plasma adjacent to the contact surface does not exist. As a result, the prescription of conditions ambient to the contact surface can at best be partially complete. The question of stability must be considered unsettled at present. Nevertheless, more complete stability theories to be developed in the future specifically for the study of the blastwave interface must include, as an indispensable consideration, the local behavior of the driven coronal plasma inferred and discussed in this work.

We conclude this report by stating that the singular character of the local inertia-induced gradient of the momentum transfer rate at the contact surface is a necessary consideration not only for estimating the transient peak magnetic field in the disturbed

interplanetary space but also for isolating the role played by such discontinuities in generating complicated solar-terrestrial relations.

ACKNOWLEDGMENTS

Individual thanks are due Dr. P.J. Kellogg, Dr. J.R. Winckler, and Dr. C.J. Waddington for generously supplying helpful information.

This work was supported, in part, by NASA Grant ONGR-005-063 to the Space Science Center of the University of Minnesota.

REFERENCES

1. Parker, E.N., Astrophys. J. 128:664 (1958)
2. Parker, E.N., Astrophys. J. 132:821 (1960)
3. Biermann, L., Zeit. fur Astrophys. 29:274 (1951)
4. Winckler, J.R., and Anderson, K.A., Phys. Rev. 108:148 (1957)
5. Gringauz, K.I., Bezrukikh, V.V., Ozerov, V.D., and Rybchinskii, R.E., Soviet Physics-Doklady 5:361 (1960)
6. Neugebauer, M., and Snyder, C.W., Science 138:1095 (1962)
7. Bonetti, A., Bridge, H.S., Lazarus, A.J., Rossi, B., and Scherb, F., J. Geophys. Res. 68:4017 (1963)
8. Coleman, P.J., Jr., Davis, L., and Sonett, C.P., Phys. Rev. Letters 5:43 (1960)
9. Ness, N.F., Scearce, C.S., and Seek, J.B., J. Geophys. Res. 69:3531 (1964)
10. Akasofu, S.I., Space Sci. Rev. 6:21 (1966)
11. Cole, K., Space Sci. Rev. 5:699 (1966)
12. Fan, C.Y., Meyer, P., and Simpson, J.A., Phys. Rev. Letters 5:269 (1960)
13. Sonett, C.P., Colburn, D., Davis, L., Jr., Smith, E.J., and Coleman, P.J., Jr., Phys. Rev. Letters 13:153 (1964)
14. Sonett, C.P., Colburn, D.S., and Briggs, B.R., "The Solar Wind" ed. by R.J. Mackin, Jr., and M. Neugebauer, New York:Pergamon Press, p. 165, 1966.
15. Zelwer, R., Fernando, P.C.B., and Ward, S.H., J. Geophys. Res. 72:3471 (1967)
16. Ness, N., "The Solar Wind," ed. by R.J. Mackin, Jr., and M. Neugebauer, New York: Pergamon Press, p. 83, 1966
17. Gosling, J.T., Asbridge, J.R., Bame, S.J., Hundhausen, A.J., and Strong, I.B., J. Geophys. Res. 72:1813 (1967)
18. Van Allen, J.A., and Ness, N.F., J. Geophys. Res. 72:935 (1967)

19. Chapman, S., "Foreword" in "The Solar Wind," ed. by R.J. Mackin, Jr. and M. Neugebauer, New York:Pergamon Press, 1966

20. Parker, E.N., "Interplanetary Dynamical Processes," New York:Interscience, 1963

21. McCracken, K.G., J. Geophys. Res. 67:423-58 (1962)

22. Chapman, S., and Ferraro, V.C.A., Terr. Mag. 38:79-96 (1933)

23. Gold, T., "Gas Dynamics of Cosmic Clouds," ed. by J.M. Burgers and H.O. VandeHulst, New York:Interscience, p. 103, 1955

24. Parker, E.N., Astrophys. J. 133:1014 (1961)

25. Davis, L., Jr., "Space Science," ed. by D.P. LeGalley, New York:John Wiley, p. 486, 1963

26. Chapman, S., Proc. Plasma Space Science Symposium, June 1963 ed. by C.C. Chang and S.S. Huang New York:Gordon and Breach, p. 263, 1965

27. Courant, R., and Friedrichs, K.O., "Supersonic Flow and Shock Waves," New York: Interscience, 1948

28. Rogers, M.H., Astrophys. J. 125:478 (1957)

29. De Jager, C., "Handbuch der Physik," ed. by S. Flügge, 52:80, (1959)

30. De Jager, C., "Introduction to Solar Terrestrial Relations" ed. by J. Ortner and H. Maseland, Dordrecht, Holland, D. Reidel Pub. Co., p. 78, 1965

31. Caroubalos, C., Doctoral Thesis, Université de Paris, 1964

32. Sandström, A.E., "Cosmic Ray Physics," New York:Interscience, 1965

33. Lee, T.S., Phys. Fluids 8:1266 (1965)

34. Taylor, G.I., Proc. Roy. Soc. (London) A186:273 (1946)

35. Obayashi, T., J. Geophys. Res. 67:2039 (1962)

36. Colburn, D., and Sonett, C., Space Sci. Rev. 5:439 (1966)

37. Sonett, C.P., and Colburn, D.S., Planetary Space Sci. 13:675 (1965)

38. Sturrock, P.A., and Spreiter, J.R., J. Geophys. Res. 70:5345 (1965)

39. Simon, M., and Axford, W., Cornell-Sydney University Astronomy Center, Report 38, 1966

40. Sen, A.K., Phys. Fluids 6:1154 (1963)

41. Fejer, J.A., Phys. Fluids 7:499 (1964)

42. Talwar, S.P., J. Geophys. Res. 69:2707 (1964)

43. Talwar, S.P., Phys. Fluids 8:1295 (1965)

44. Thompson, W.B., "An Introduction to Plasma Physics" Reading, Mass.:Addison-Wesley, 1962

This page intentionally left blank.

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

APPARENT LOCAL JUMP IN THE MOMENTUM TRANSFER RATE AT THE FRONT OF FLARE-FRESH SOLAR PLASMA IN THE INTERPLANETARY SPACE

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

This is an interim report on a continuing problem.

5. AUTHOR(S) *(First name, middle initial, last name)*

T.S. Lee and W.W. Balwanz

6. REPORT DATE December 31, 1968	7a. TOTAL NO. OF PAGES 20	7b. NO. OF REFS 44
8a. CONTRACT OR GRANT NO. NRL Problem R07-18 b. PROJECT NO. RR 002-10-45-5063 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 6784 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES This work was supported, in part, by NASA Grant ONGR-005-063 to the Space Science Center of the University of Minnesota.	12. SPONSORING MILITARY ACTIVITY Department of the Navy (Office of Naval Research), Washington D.C. 20360

13. ABSTRACT

This report is limited to calling to attention a feature, not scrutinized before in the literature, of the ideal interplanetary blastwave according to the Parker type of formal theory. It is shown analytically that, because of inertial effects, a gradient singularity of the radial momentum transfer rate arises in association with the local coronal accumulation in front of the rear propelling contact surface postulated in the theory. Also, it is shown that such a local accumulation is a general result for power-law explosive waves extruding against an ambient hydrogen solar wind having a strictly inverse-square radial decay in density. The usual numerical schemes are rendered ineffective for the determination of the expected singular local wave behavior. To circumvent the difficulty, a combined numerical-integral technique has been developed, and in this work it has been applied on six specific model waves, including both the decelerating and the nondecelerating, which have energies increasing with time as t^K with K = 28, 4, 1, 1/2, 1/4, and 1/8. Also, a local solution corresponding to the limiting approximation of a constant material velocity gradient has been analytically constructed. The importance of the local singular behavior is quantitatively appraised, and certain interplanetary implications are given cursory inferences. It is considered that the new results should have relevance to the quantitative determination of the progressive peak transverse magnetic field during an interplanetary storm and to the theoretical determination of the macroscopic stability of the postulated contact surface.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Security Classification

Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Interplanetary space						
Solar corona						
Solar flares						
Solar plasma						
Solar wind						
Shock waves						
Blastwave						
Magnetic fields						
Geomagnetic storms						
Plasma interface						
Dynamics						
Hydrodynamic theory						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

Security Classification