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Eliminating Radar Ambiguities by Processing Staggered Pulse Trains or Spectra

H. H. Woerrlein

Search Radar Branch
Radar Division

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NAVAL RESEARCH LABORATORY
Washington, D.C.

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ABSTRACT

A method for reducing or eliminating doppler and/or range ambiguities ordinarily associated with using repetitive waveforms in radar has been developed. The basic idea calls for building two radar systems into one. Two different waveforms are impressed on one carrier, the waveforms being designed in such a fashion that they can be separated and processed in different receiver channels. The combined information of the two processed signals is used to reduce or to eliminate ambiguities. Analog computation has been used to study the feasibility of generating and processing the proposed waveforms.

A new method for measuring small frequency shifts is suggested. A pair of pulses is used; the first and the second pulse will produce ringing at the outputs of a first and a second narrowband filter, respectively, and the relative phase of the ringing oscillations is shown to be linearly related to the difference between the carrier frequency and the filter center frequency.

PROBLEM STATUS

This is an interim report on one phase of the problem; work on other phases continues.

AUTHORIZATION

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ELIMINATING RADAR AMBIGUITIES BY PROCESSING STAGGERED PULSE TRAINS OR SPECTRA

INTRODUCTION

The outstanding and unique advantage of pulse-burst or pulse-doppler waveforms is their capacity of yielding combined range and doppler resolution associated with possibly very small "self-clutter" residues of the ambiguity function; yet the target response may be ambiguous in range and/or in doppler due to numerous spikes of the ambiguity function, the kind and number of ambiguities depending on the range-doppler extent of potential targets and on the particular waveform.

To eliminate ambiguities we suggest that two different modulations be impressed on one carrier, the modulations being such that they can be separated in the receiver. The modulations may be separated in time or in frequency, depending on the particular waveforms chosen. The separated waveforms might be further processed by filters matched to each waveform.

The target information is carried in the amplitude and in the phase of the separated waveforms. We will explain how the proper evaluation of amplitude and phase relationships may enable us to eliminate ambiguities indicated by the ambiguity functions of either of the separated waveforms and by the ambiguity function of the nonseparated waveforms.

In the first part of this report we explain how to eliminate doppler ambiguities if unambiguous range is available. The system discussed in the second part may be called dual to the first system, since the distribution of spectral lines belonging to the second type of waveform and the time distribution of pulses belonging to the first type follow the same rules.

If the pulse width is properly designed, it will be possible to eliminate both range and doppler ambiguities (third part), the modulation being otherwise the same as that described in the second part.

As a by-product of this work we discovered a method to measure accurately the carrier frequency of a pair of pulses. This method is presented in the fourth part.

We hope to introduce some ideas which are believed to be potentially useful and new. Results obtained by analog computation are presented. Many of the details will necessitate and, hopefully, justify further study.

REPORT OUTLINE

The object of this report is to explain how range and/or doppler ambiguities which are ordinarily associated with the use of pulse-burst or pulse-doppler waveforms can be recognized or identified as such. Identification of ambiguities furnishes the key to their potential elimination.

The report is divided into four parts. The first part deals with how one can eliminate the doppler ambiguities of a low-prf pulse doppler radar measurement if unambiguous range is available. The spectrum of a burst consists of lines distributed at intervals equal to the prf; they are broadened as a consequence of the finite duration of the burst. Their amplitudes are determined by the shape of the pulses; and their phases may be written as $\phi = 2\pi(f - f_0)R_1$, where $f - f_0$ is the difference between the frequency f of a line and the frequency of the carrier f_0, and R_1 is the time shift with respect to a reference burst. The time shift R_1 may be the only distinction between the original burst and the reference burst; both bursts are transmitted and received together at the same carrier frequency. Their separation in the time domain is made possible by alternating switching circuitry, if the assumption of unambiguous range measurement is fulfilled. Processing the two separated waveforms makes the phase difference $\phi = 2\pi(f - f_0)R_1$ amenable to measurement, and $\phi = 0$ occurs only at the carrier frequency and not at the location of adjacent prf lines.

The second part presents a high-prf system capable of eliminating range ambiguities whenever the doppler measurement is unambiguous. Like the first system, it actually uses two different pulse bursts which are transmitted and received together, but which can be separated, in this case, in the frequency domain by filtering. A more detailed mathematical description and analysis of the waveforms involved is postponed until the third part. In the second part we try to establish how the high-prf system may be derived from the low-prf system by interchanging time and frequency.

The time shift of the two initially frequency-separated bursts leads to an additional phase factor, and the third part explains how simultaneous range and doppler ambiguities may be eliminated if this time shift is chosen judiciously.

The fourth part explains a technique for accurately measuring the carrier frequency. The method was found quasi-accidentally during analog simulation of the low-prf system. It is based on the idea of processing two pulses, both being modulated on the same unknown carrier. The first pulse would be used to produce ringing of a narrow-band resonant filter, and the second pulse would be used to produce ringing of another similar filter. The phase between the ringing oscillations is linearly related to the difference between the carrier frequency of the pulses and the resonant frequency of the filters. Measuring the phase between the two sinusoids would thus enable determination of the carrier with an error which may be much smaller than the reciprocal of the time between two pulses. The obtainable accuracy will be limited by noise or other types of interference.

PART 1: ELIMINATION OF DOPPLER AMBIGUITIES

We will now show how the familiar ambiguity problem which arises in pulse-doppler radar can be resolved. To understand the fundamental idea, consider Fig. 1. It shows the spectrum of a train of $\cos^2$ pulses modulated on a carrier frequency f_0. This spectrum might, for example, be given in the case of a low-prf pulse-doppler radar. One could understand the ambiguity problem to arise because there are many lines on both sides of the carrier and these lines look very similar to the line at f_0. In fact, the amplitude difference between the line at f_0 and the neighboring lines becomes very small if the duty ratio of the modulation becomes small, and, in practically all cases of duty ratios considered in pulse-doppler radar, this amplitude difference would be too small to permit a distinction between the carrier and one of its neighbors.

The plot of Fig. 1 represents only part of the information contained in the spectrum of the pulse train; namely, it represents just the amplitude information. It is important to realize that phase terms are associated with each of the lines. Certainly these phase terms can be evaluated only if there is a phase reference. Since this phase reference is usually not available in radar systems, the ambiguity is mostly understood in terms of the absolute value of the spectrum (the ambiguity function ignores this phase term). A phase reference may be introduced by the methods described below, thereby making the phase terms amenable to measurement and also introducing a new means of reducing or eliminating ambiguities.

The details are most easily explained and understood if we take advantage of Woodward's notation.* A radar signal might be written as

$$\left[\mathrm{rep}_{R_1+R_2}\, s(t/T)\right] e^{j2\pi f_0 t} \tag{1}$$

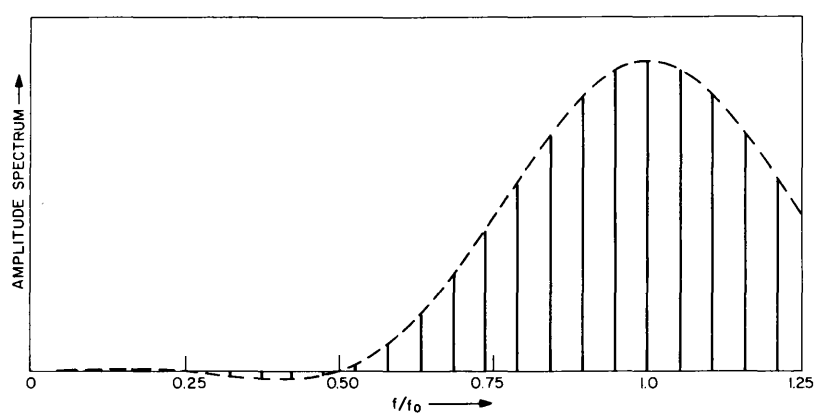

Fig. 1 - Spectrum of a train of $\cos^2$ pulses

*P.M. Woodward, "Waveform Analysis and Noise," Chapter 2 in "Probability and Information Theory with Applications to Radar," Woodward, P.M., 2nd ed., New York: Pergamon, 1963.

The term in brackets signifies the video waveform, where $s(t/T)$ represents the envelope of one individual pulse, while the exponential term stands for the carrier f_0, and $R_1 + R_2$ is equal to the repetition time. The spectrum of this waveform is given by

$$\frac{T}{R_1 + R_2} \left[\mathrm{comb}_{1/(R_1+R_2)}\, S(Tf) \right] * \delta(f - f_0) . \tag{2}$$

This is the spectrum S of one of the pulse envelopes, sampled every $1/(R_1 + R_2)$ frequency units with its center shifted to the carrier frequency f_0.

To follow the above idea we would introduce another pulse train into our concept to create a phase reference, as desired. In the simplest case one could consider transmitting a waveform whose modulation is identical with the modulation of the above-described waveform (except for a constant time shift R_1) and whose carrier is, for coherency purposes, the same as the carrier of the first waveform. This waveform would be described by

$$\left[\mathrm{rep}_{R_1+R_2}\, s\left(\frac{t - R_1}{T}\right) \right] e^{j2\pi f_0 t} . \tag{3}$$

The spectrum associated with this waveform is given by

$$\frac{T}{R_1 + R_2} \left[\mathrm{comb}_{1/(R_1+R_2)}\, S(Tf)\, e^{-j2\pi f R_1} \right] * \delta(f - f_0) . \tag{4}$$

This spectrum is identical with that of Eq. (2), apart from the phase term $\exp(-j2\pi f R_1)$ arising from the shift R_1.

In the upper portion of Fig. 2 schematic plots of the first and second spectra are shown. Each spectrum consists of lines separated at intervals given by the prf, and each spectrum by itself gives rise to the familiar doppler ambiguity problem. Notice the phase difference $2\pi(f - f_0)R_1$ which is depicted in the lower part of Fig. 2. The phase difference between corresponding lines from the first and from the second spectrum is a linear function of the separation of the line under consideration with respect to the line corresponding to the carrier f_0. At f_0 the phase difference is zero, or any assigned value, if the carriers of the first wave and the second wave are phase-shifted. The phase increase associated with stepping from one pair of lines to the next is given by $2\pi R_1/(R_1+R_2)$, since $1/(R_1 + R_2)$ is the fundamental repetition frequency.

We can consider the interval R_1 to be an integer multiple n_1 of a small ΔR and, similarly, R_2 to be an integer multiple n_2 of the same increment ΔR, i.e.:

$$R_1 = n_1 \Delta R; \quad R_2 = n_2 \Delta R . \tag{5}$$

We are free to select R_1 and R_2 or n_1 and n_2, respectively, such that the event "phase difference $\bmod 2\pi$ equals 0" does not coincide with lines close to the carrier f_0. Thus, we are able to identify the carrier frequency in a given spectrum — for example, in the spectrum of a received radar signal.

Before we proceed let us describe the determination of n_1 and n_2. The radar design would be such that we discriminate one line out of m_1 lines. After stepping m_1 lines to the left or to the right of the given carrier, we would find another pair of lines whose phase difference would be indistinguishable from zero. The formulas are

$$m_1 2\pi \frac{R_1}{R_1 + R_2} = m_1 2\pi \frac{n_1}{n_1 + n_2} = m_2 2\pi . \tag{6}$$

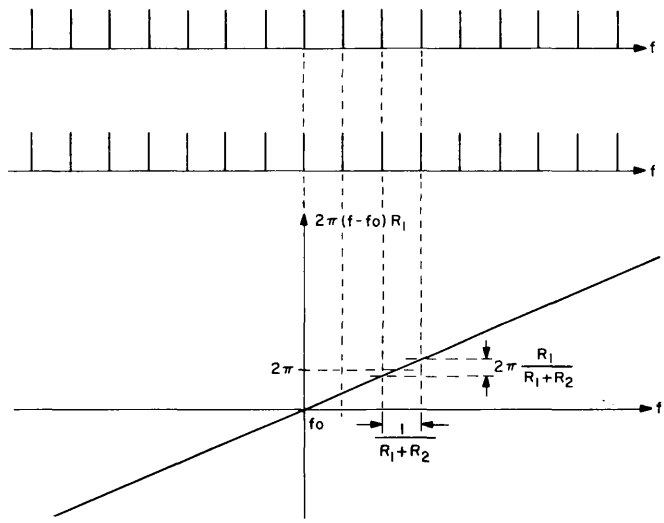

Fig. 2 - Phase relationships associated with a staggered pulse train

The solution of this equation is $m_1 = n_1 + n_2$ and $m_2 = n_1$ if we choose n_1 and n_2 such that neither n_1 nor n_2 have any prime factors in common with $n_1 + n_2$. This seems also to be the rule for the choice of staggering ratios in ordinary MTI radar (for example $n_1 : n_2 = 4:5$, 9:11, 14:15, or 20:21).

An analog computer simulation has been set up to study the feasibility of the measurements involved. Figure 3 depicts the simulated system. We begin by generating a staggered pulse train (staggering ratio $R_1 : R_2 = n_1 : n_2$) with the carrier f_0 adjustable to simulate different doppler effects. In the receiver part of our simulation we split the signal into two channels by a switch; i.e., we make two regular pulse trains with a mutual time shift R_1 out of the originally staggered train. We thereby generate signals defined by Eqs. (1) and (3). The next step is to pass each of the regular pulse trains through a narrow-band filter tuned to the fixed center frequency f_1. At the output of a narrow-band filter we see a signal oscillating with the frequency f_1 whose amplitude is proportional to the module of the Fourier transform at f_1 of that part of the input signal which has elapsed up to the present moment and whose phase contains the phase of the input spectrum at f_1. The oscillations at the filter outputs of the two channels are compared using a phase meter, which completes the instrumentation corresponding to the above mathematics.

Figures 4 through 14 give the results of the simulation for different values of the carrier f_0, thus simulating several different doppler frequencies. In the first line of each diagram is a pulse train whose staggering ratio has been chosen to be 4:5. The carrier itself is plotted on line 2. The signals, after being sorted by a switch — i.e., at the inputs of the narrow-band filters — are plotted in lines 3 and 4. Lines 5 and 6 show the output signals of the filters which are excited by the waveforms of lines 3 and 4, respectively. Finally, line 7 shows the signal at the output of the phase meter. The scale is such that a zero signal in channel 7 (a line at the lower margin of the trace of channel 7) corresponds to a zero phase shift, whereas maximum signal (at the upper margin — channel 7) corresponds to a $\pm 180°$ phase shift; the values in between are given by linear subdivision of the interval. The points at which the various waveforms appear are marked in Fig. 3.

Figure 4 shows the case in which the carrier frequency f_0 and the filter frequency f_1 are identical. Channels 5 and 6 indicate that there is a strong oscillation built up at

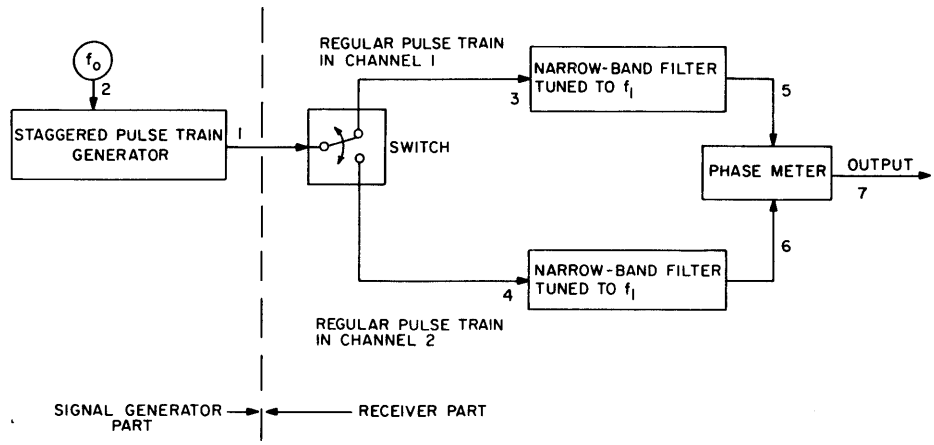

Fig. 3 - Low-prf system simulation

the outputs of the narrow-band filters. This is because the filters are tuned to a line of the incoming spectrum. Channel 7 shows a phase signal which is near zero as soon as oscillations occur at the outputs of both filters and which remains close to zero throughout the run.

Figures 5 through 14 demonstrate how prf lines neighboring the carrier can be distinguished from the carrier by applying the principles described above. These figures should be valuable to readers wishing to verify that the phase shifts are equal to multiples of $2\pi R_1/(R_1 + R_2) = 2\pi 4/9$. The filter tuning is the same for all runs, whereas the carrier f_0 is changed such that certain prf lines fall into the passbands of the filters. Those prf lines to the right of the carrier are (a) the first (Fig. 5), (b) the second (Fig. 6), (c) the third (Fig. 7), (d) the fourth (Fig. 8), and (e) the fifth (Fig. 9). Those prf lines to the left of the carrier are (a) the fifth (Fig. 10), (b) the fourth (Fig. 11), (c) the third (Fig. 12), (d) the second (Fig. 13), and (e) the first (Fig. 14).

In all these cases strong oscillations occur in channels 5 and 6. Observing one of the channels 5 or 6 alone, as is done in conventional low-prf pulse doppler, leads us to conclude that the incoming frequency is equal to f_0 plus or minus an undetermined integer multiple of the prf. Observing the phase signal (channel 7, Figs. 4 through 14) would permit us, in the cases of Figs. 5 through 14, to recognize that the narrow-band filters are tuned to a prf line different from the carrier frequency; furthermore, by evaluating the particular value of the phase difference we could decide which prf-line is causing the oscillation at the filter output.

Figures 4 through 14 are considered to demonstrate the feasibility of the new concept. We had to select a relatively large duty ratio to be able to simulate the system on the analog computer and to get clear recordings. A comprehensive study of the system under the influence of noise can hardly be undertaken on the analog computer; rather, for this purpose, it is necessary to set up a laboratory model.

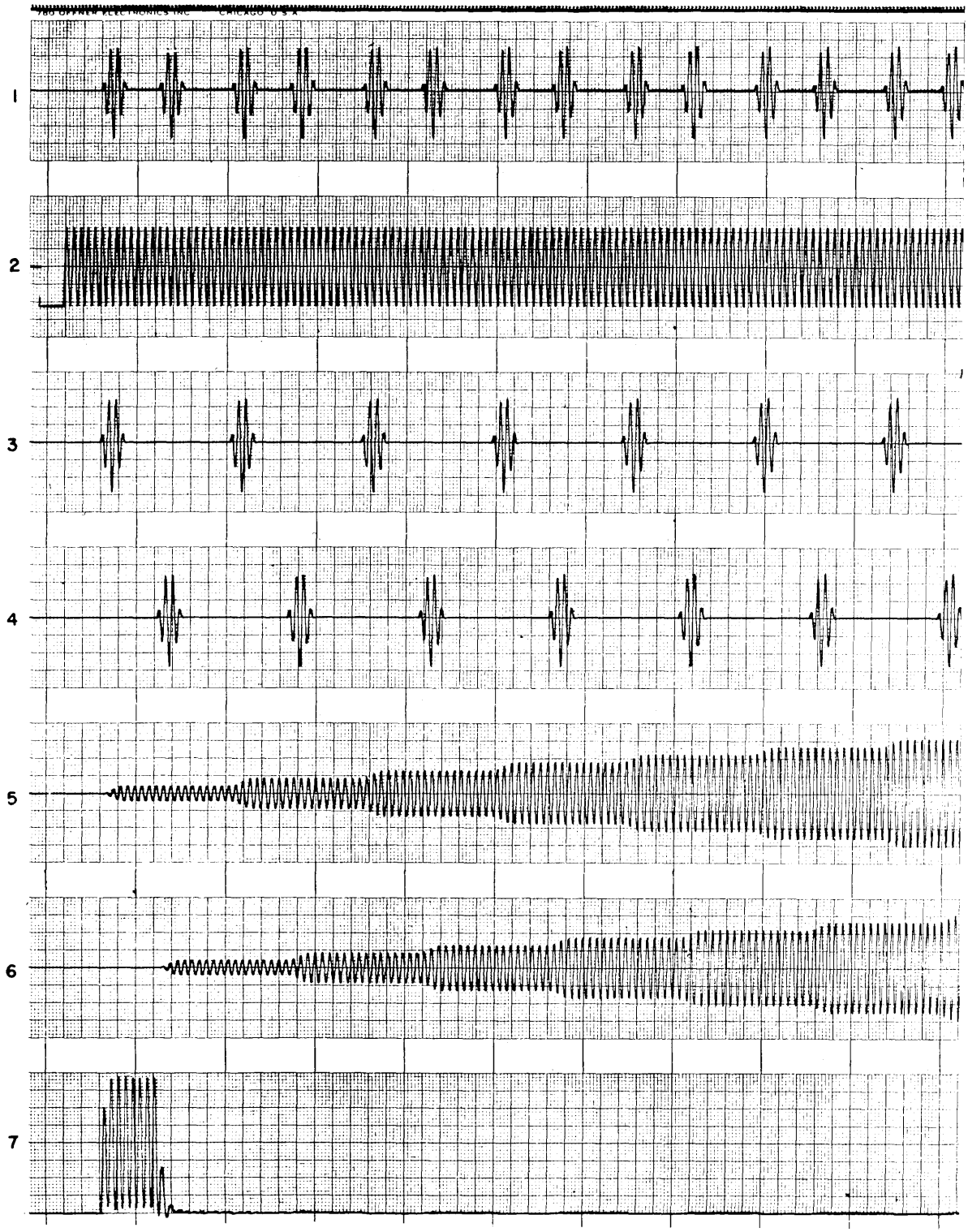

Fig. 4 - Low-prf system waveforms, with no doppler shift and the staggering ratio 4:5 (lines 1 to 7 exhibit waveforms as they appear at points 1 to 7 of Fig. 3)

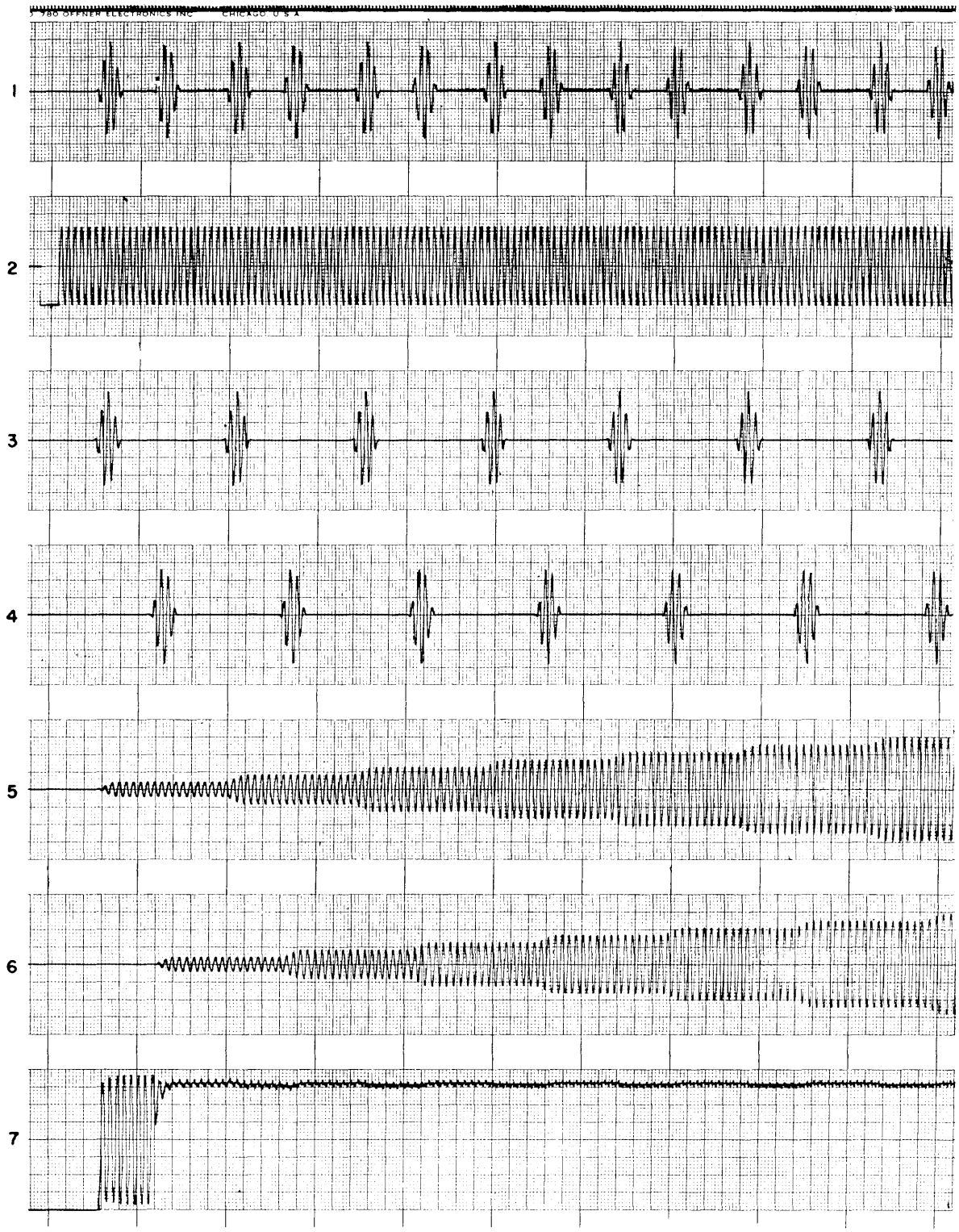

Fig. 5 - Low-prf system waveforms with doppler = prf

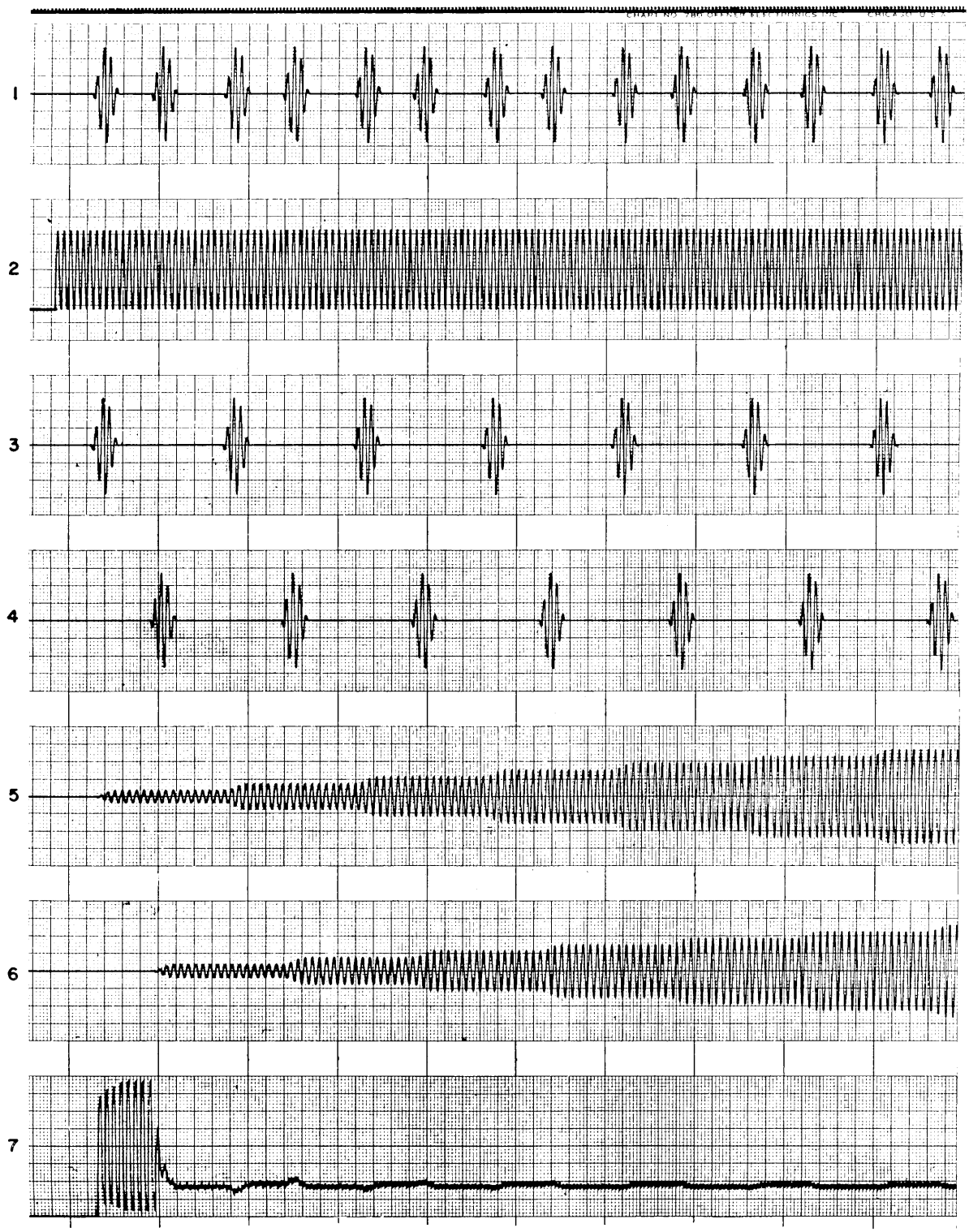

Fig. 6 - Low-prf system waveforms with doppler = 2 prf

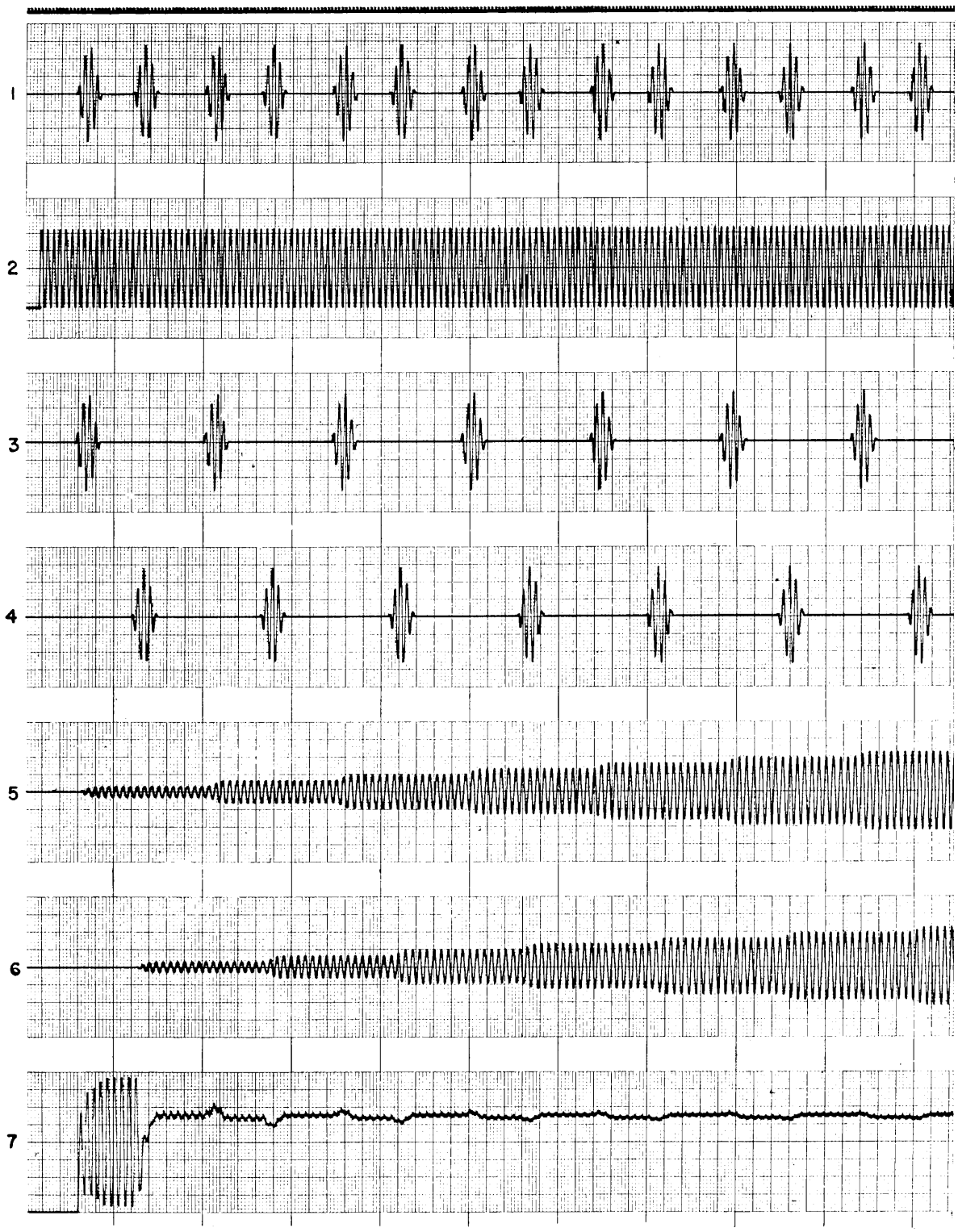

Fig. 7 - Low-prf system waveforms with doppler = 3 prf

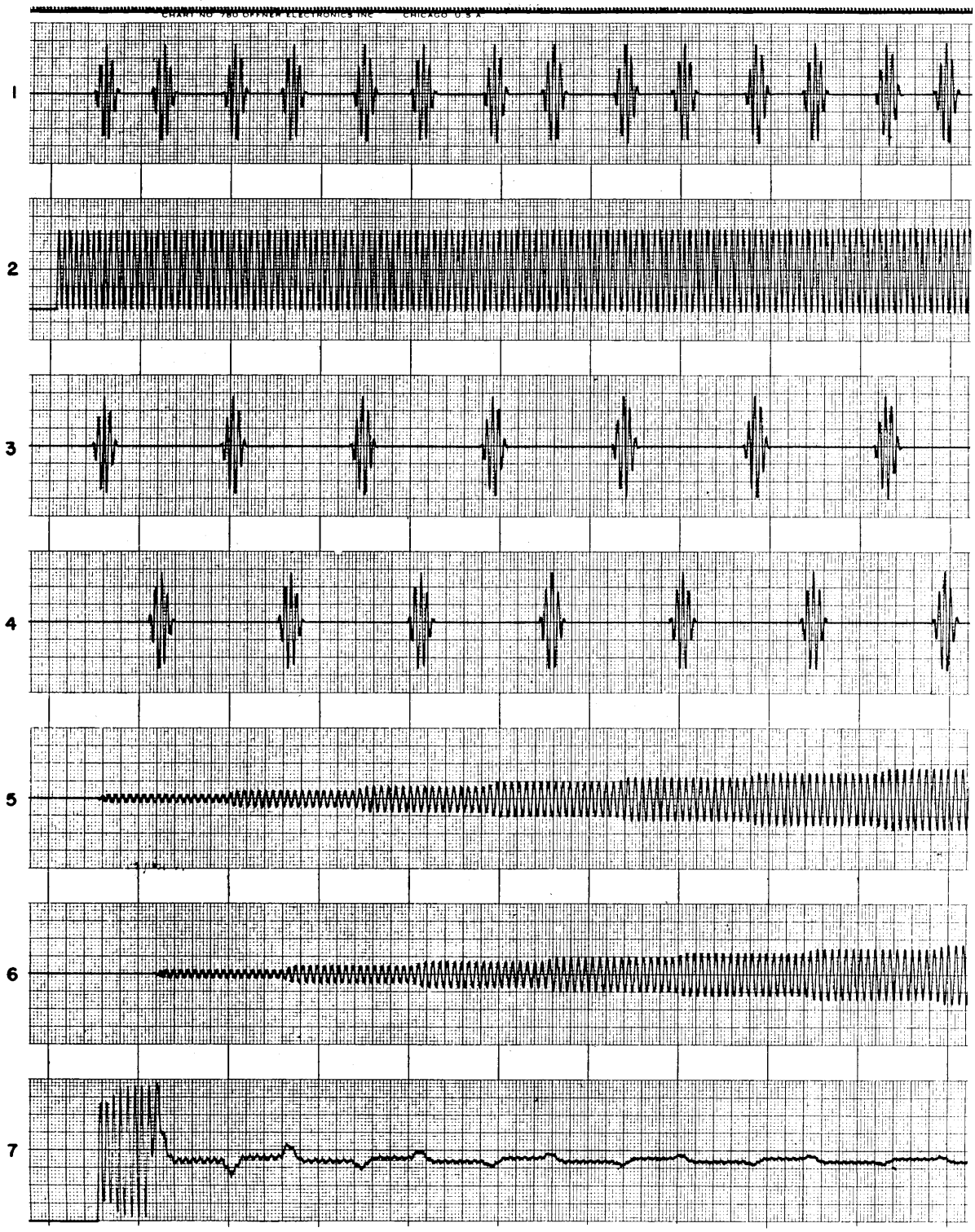

Fig. 8 - Low-prf system waveforms with doppler = 4 prf

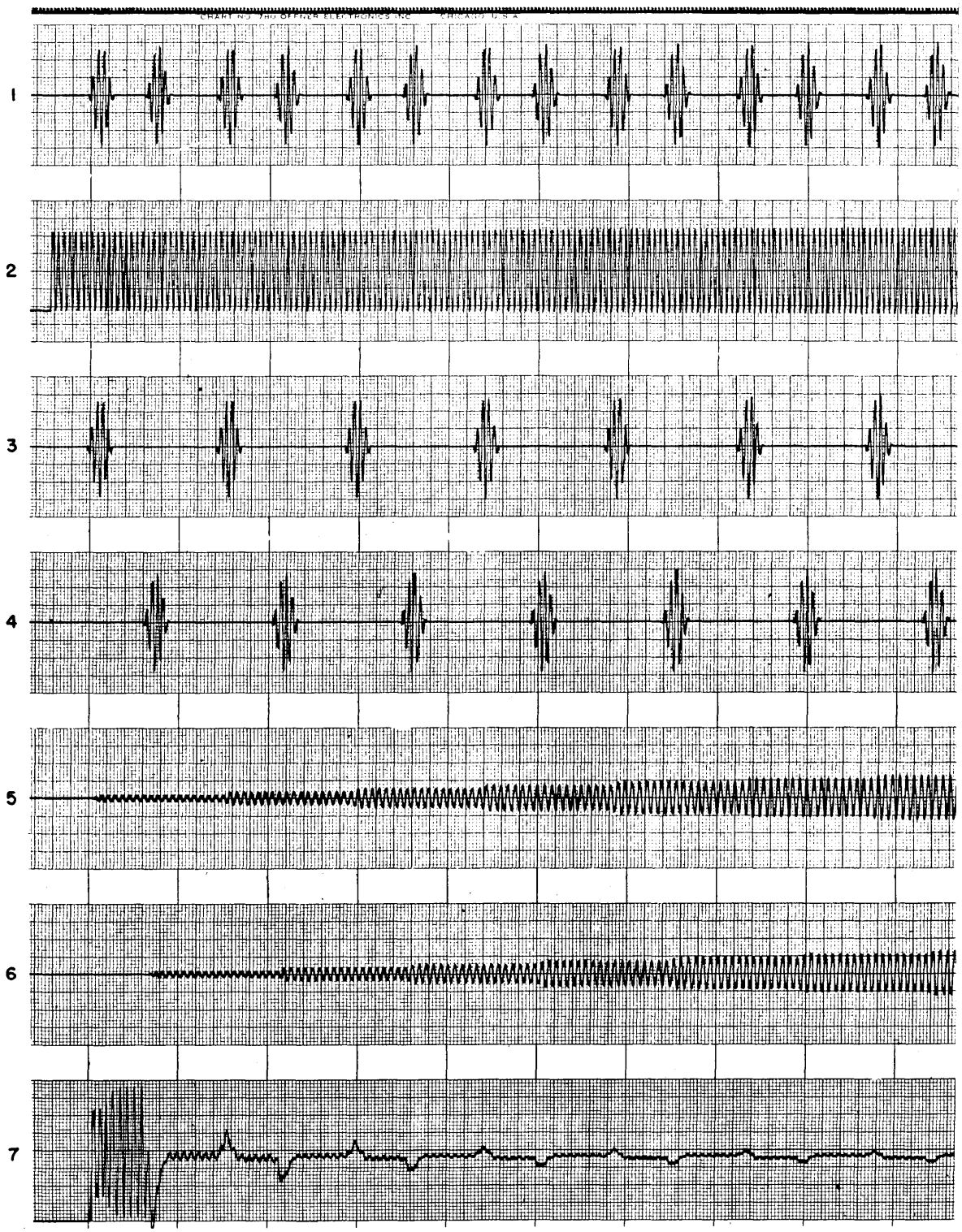

Fig. 9 - Low-prf system waveforms with doppler = 5 prf

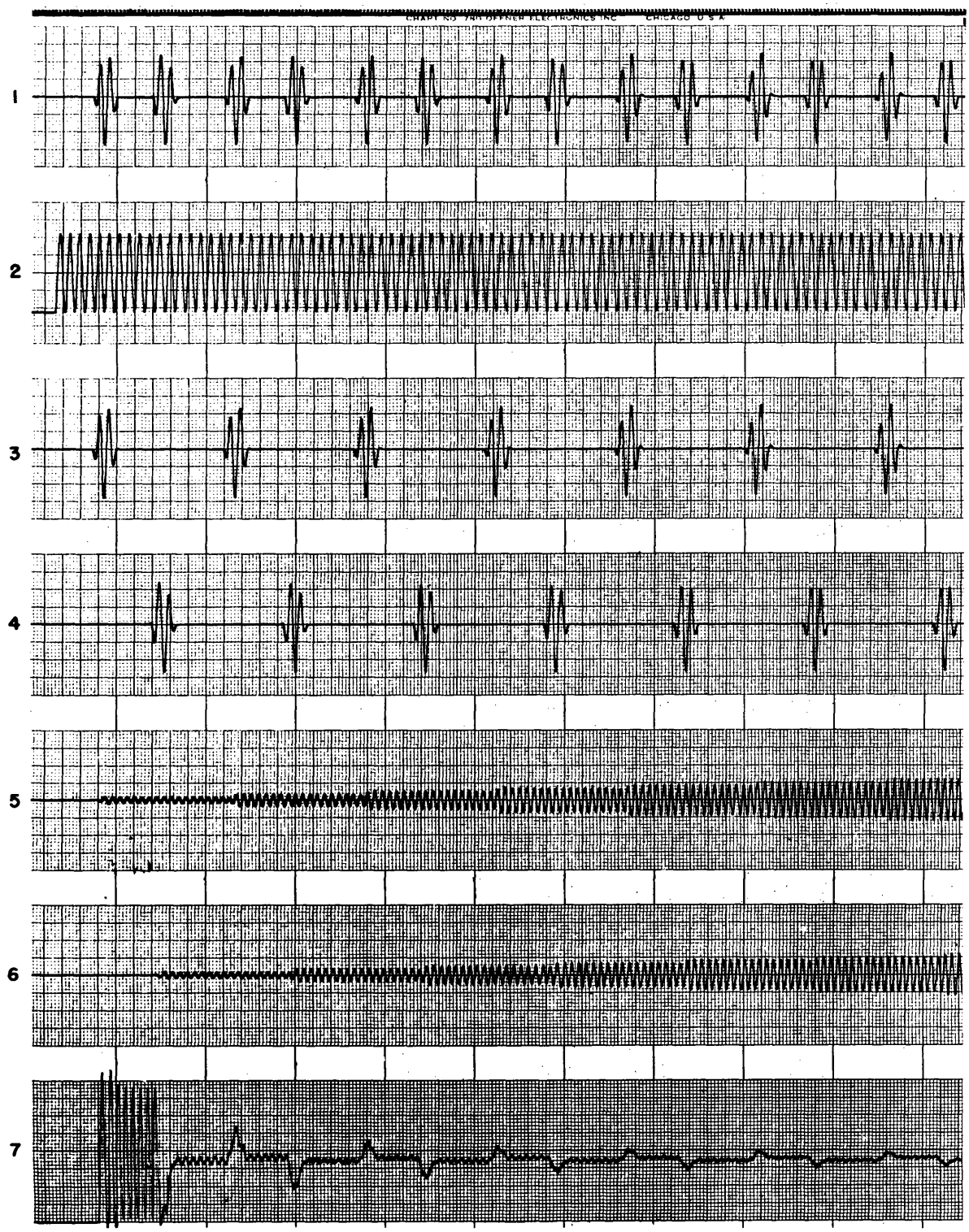

Fig. 10 - Low-prf system waveforms with doppler = -5 prf

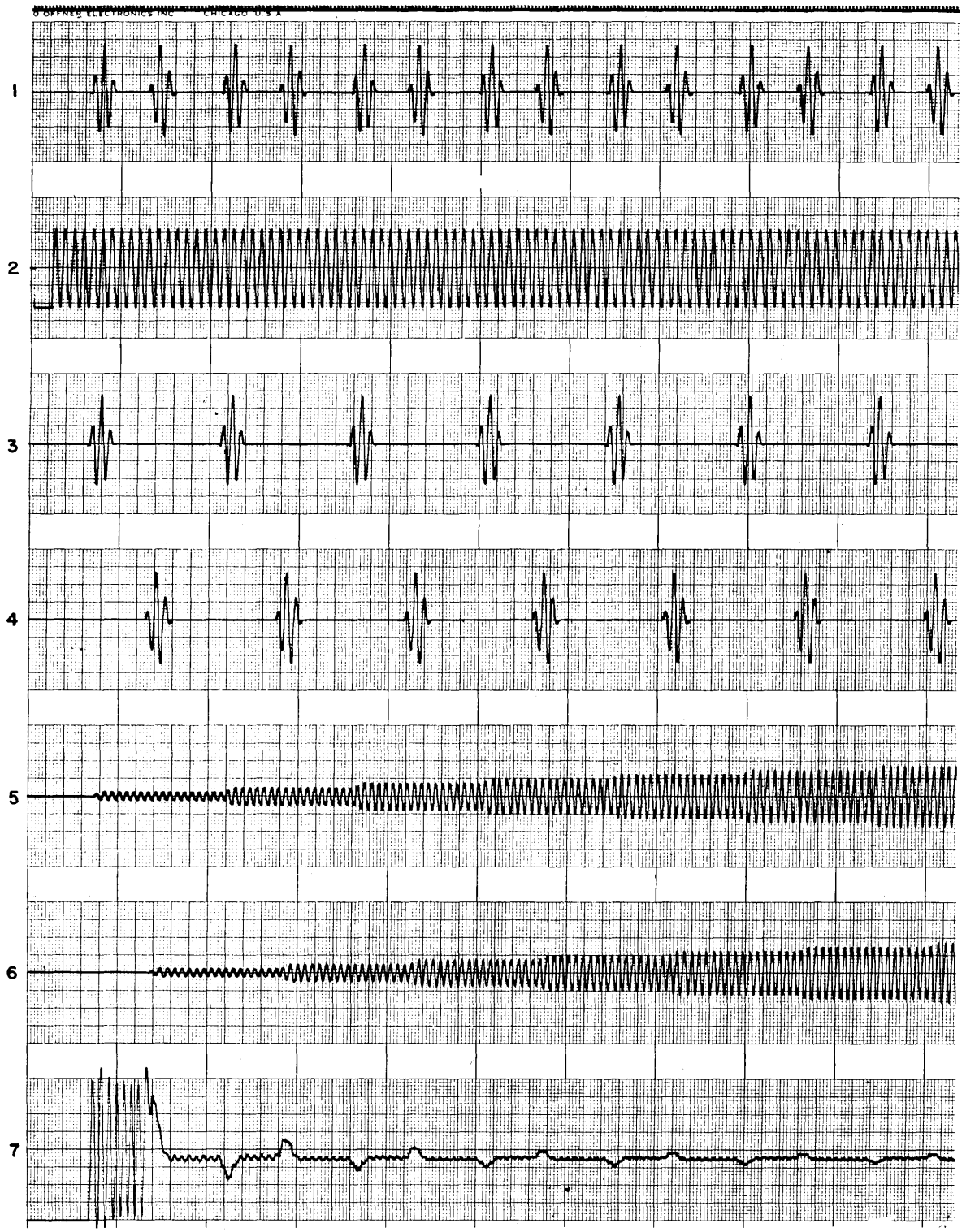

Fig. 11 - Low-prf system waveforms with doppler = -4 prf

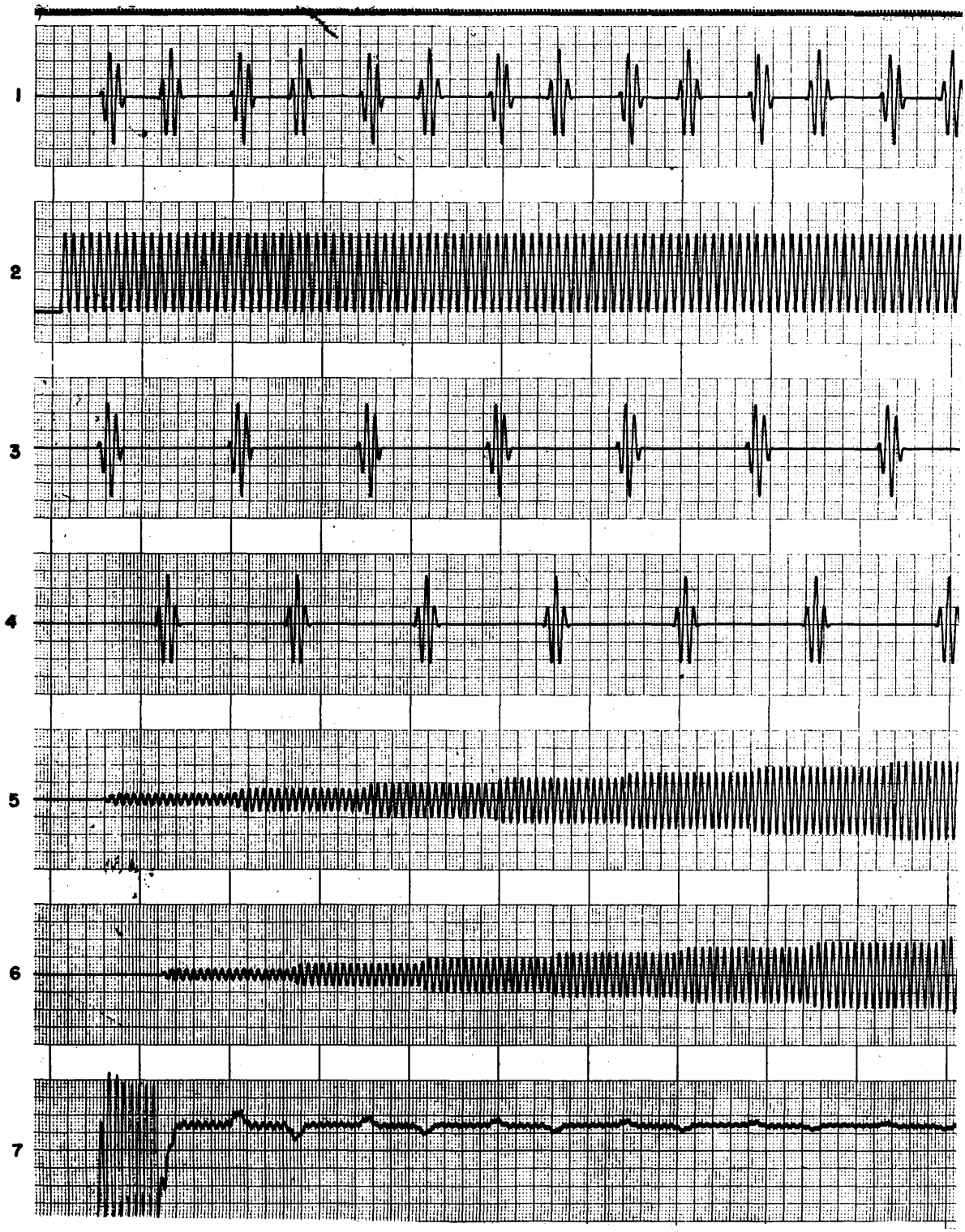

Fig. 12 - Low-prf system waveforms with doppler = -3 prf

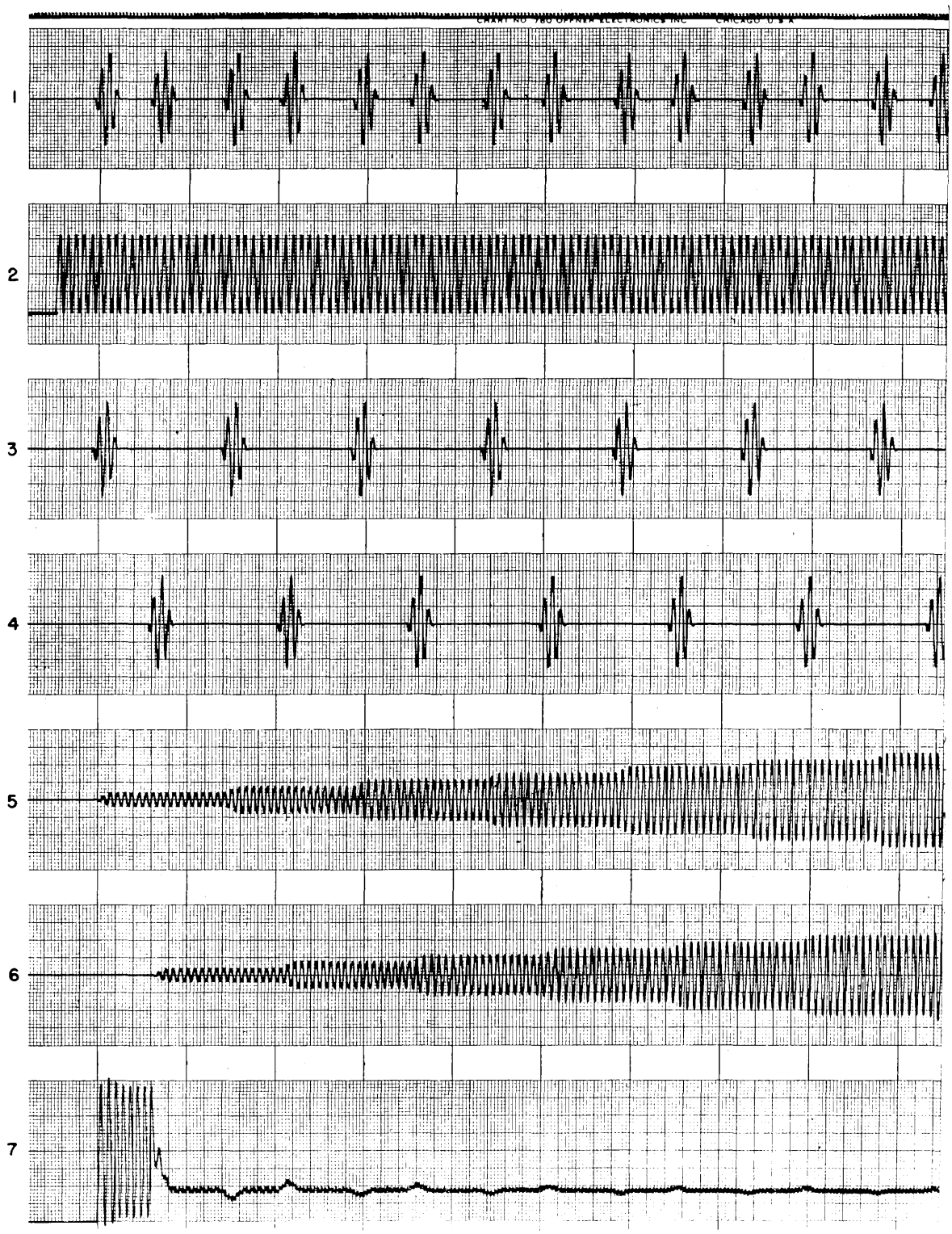

Fig. 13 - Low-prf system waveforms with doppler = -2 prf

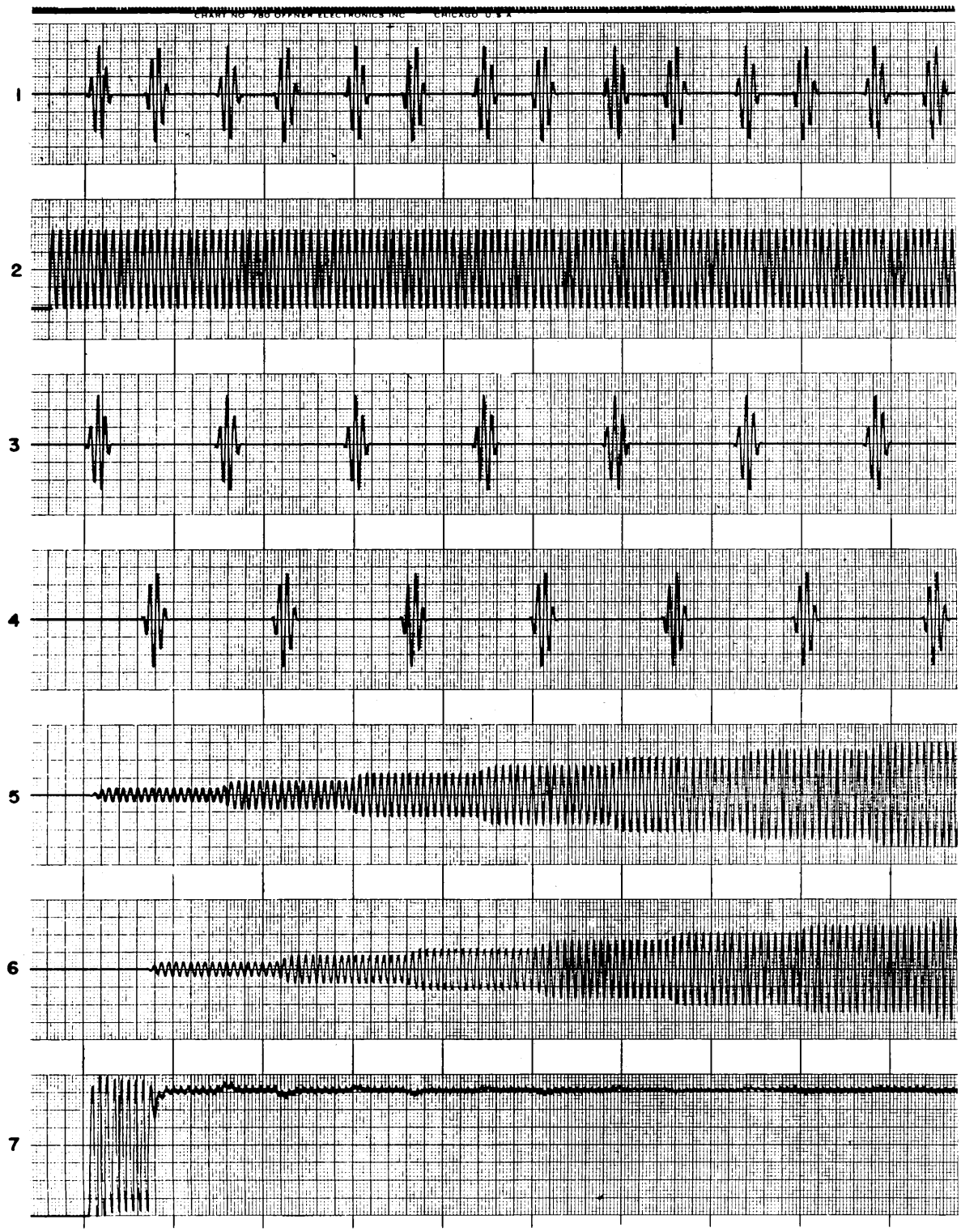

Fig. 14 - Low-prf system waveforms with doppler = -prf

PART 2: ELIMINATION OF RANGE AMBIGUITIES

The system previously described has the capability of eliminating doppler ambiguities if range is measured unambiguously. Unambiguous range is required to permit the switching operation, as indicated in Fig. 3. Our system has this requirement in common with existing staggered-prf-MTI radars, and we consequently call it a low-prf system.

Let us try to find a system related to the system described, in the sense that one can be transformed into the other by interchanging frequency and time coordinates. This "dual" system would be capable of eliminating range ambiguities if there are no doppler ambiguities, and it should consequently be called a high-prf system.

Consider Fig. 15. Using the terminology of the low-prf system we would interpret the markings of this plot as pulses as a function of time, repeating themselves at two different spacings $n_1 \Delta R$ and $n_2 \Delta R$. To arrive at the dual system we must interchange frequency and time. The "pulses" occur then as a function of frequency, and they should consequently be understood as spectral lines arranged at the two different spacings $n_1 \Delta f$ and $n_2 \Delta f$. Let us call the spectrum arising from this interchange of time and frequency a staggered spectrum. If we consider three neighboring lines of this spectrum, then the middle line will not be exactly in the center, but it will be displaced from the center by a small amount. The frequency difference $n_1 \Delta f$ is associated with the period $1/n_1 \Delta f$, and we are in a position to choose the staggering ratio $n_1 : n_2$ such that m_2 times this period will be a multiple m_1 of the pulse repetition time $1/(n_1 + n_2)(1/\Delta f)$, where

$$\frac{m_1}{n_1 + n_2} \frac{1}{\Delta f} = \frac{m_2}{n_1} \frac{1}{\Delta f} . \tag{7}$$

The solution of this equation is $m_1 = n_1 + n_2$ and $m_2 = n_1$, if we select the staggering ratio according to the same rules as in the case of the low-prf system, namely, that n_1 and $n_1 + n_2$ or n_2 and $n_1 + n_2$ should not have any common prime factors.

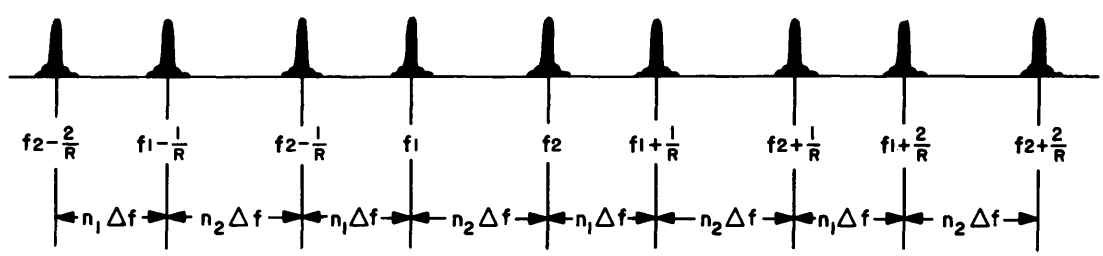

Fig. 15 - Typical staggered distribution

An analog computer program has been set up to demonstrate the feasibility of the concepts involved in this system and to show the associated waveforms (Fig. 16). The simulation contains a signal generator whose carrier frequency can be tuned to permit simulation of different doppler frequencies. In the receiver the signal is split frequency-wise into two different channels; this means that two narrow-band filters are tuned to the frequencies f_1' and f_2', where f_1' and f_2' are identical with the frequencies of two neighboring lines in the transmitted spectrum offset by known doppler. To complete the instrumentation we measure the phase between the two narrow-band filter outputs. This phase difference increases as a linear function of time, since f_1' and f_2' are constant

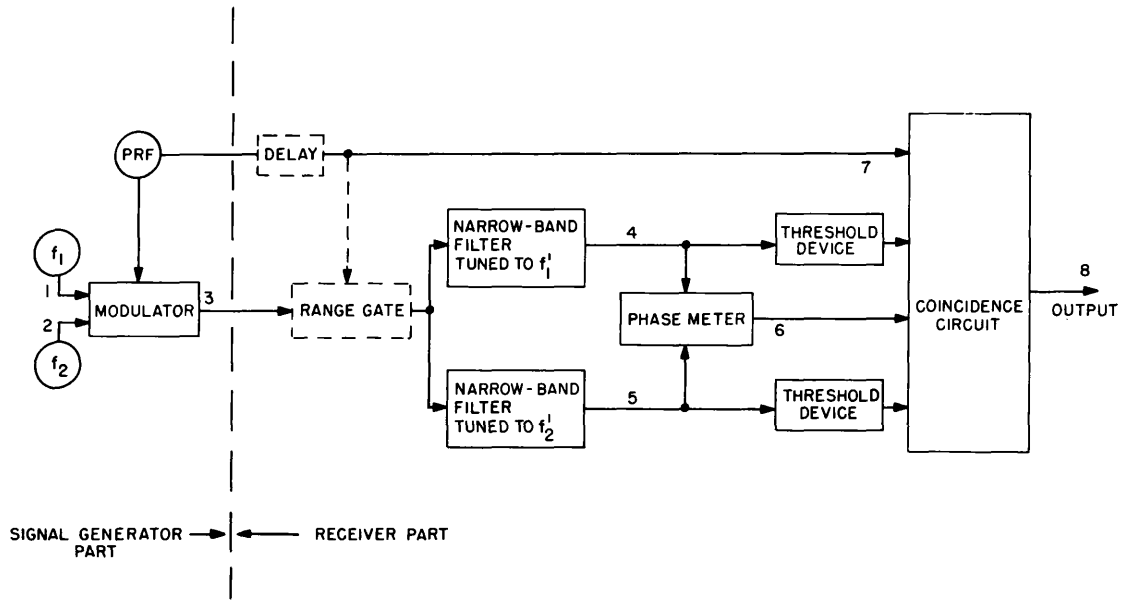

Fig. 16 - High-prf system simulation

frequencies, and it will increase by 2π in a time which is the reciprocal of the difference between f_1' and f_2'. As a consequence of the V-shaped measurement characteristics of the phase meter, the phase meter will generate a periodic signal having the period equal to the reciprocal of $f_1' - f_2'$. The linear function of time $2\pi f_1' t - 2\pi f_2' t$ will therefore be folded over into a repetitive, sawtooth function. To complete the circuitry according to our design principles, we determine coincidence between the prf signal and a signal derived when the phase-meter output is close to zero. We found, during the simulation, that it is useful to include two threshold devices which prohibit the generation of a coincidence signal, unless the oscillations in both channels are larger than a critical value.

Figure 17 describes the performance of this system if the pair of filters is tuned exactly to the doppler of the incoming signal, i.e., if f_1 is equal to f_1' and f_2 is equal to f_2', whereas Figs. 18 through 20 show characteristic waveforms for various values of the doppler.

Let us first describe Fig. 17. In channels 1 and 2 we see two sinusoids with the constant carrier frequencies f_1 and f_2. In channel 3 we see a composite signal understood to be the sum of two pulse trains, one pulse train sliced from the first oscillation, the other pulse train placed adjacent to the first pulse train and sliced from the second constant-frequency oscillation. As a consequence of the choice of frequencies, the spectrum of the signal in channel 3 will appear like a staggered spectrum. Channel 4 is a recording of the output signal at the frequency f_1', similarly, we see in channel 5 the corresponding output signal at frequency f_2'. The amplitudes of both oscillations increase monotonically with time, as implied by the coherent integrator performance of the pair of narrow-band filters. Finally, channel 6 shows the phase-meter output, and channel 7 shows a prf signal, coincidence of both being derived every m_1 times the basic pulse-repetition time. In our case we have chosen the staggering ratio such that six out of seven range ambiguities will be eliminated, as is apparent from inspecting the signal in channel 8, which shows a pulse at each coincidence of the zero phase and prf signals.

In Fig. 18 we see how the system performs if the doppler frequency amounts to a value such that none of the transmitted lines falls into the passband of the filters. At the

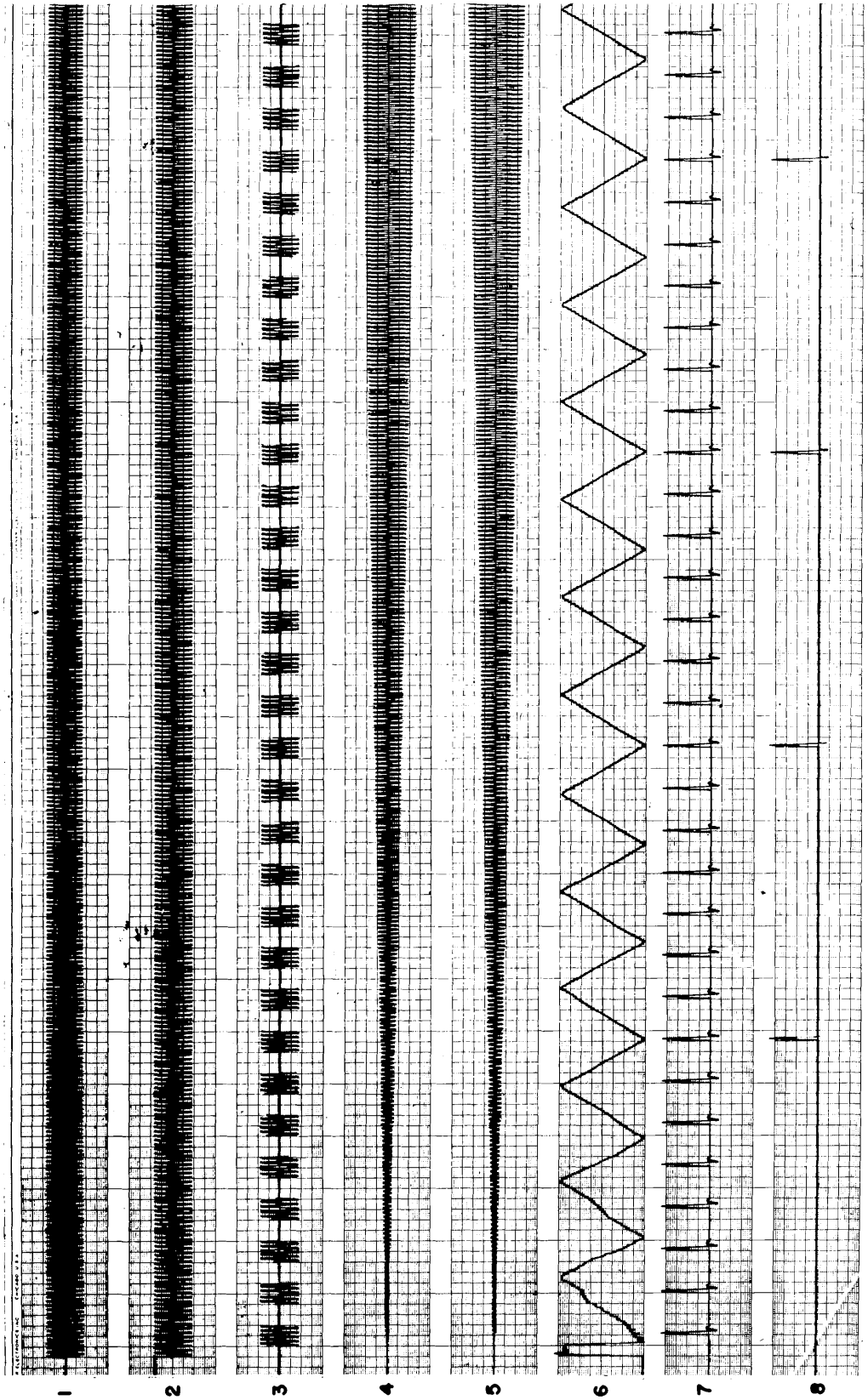

Fig. 17 - High-prf system waveforms with no doppler and the staggering ratio 3:4 (lines 1 to 8 exhibit waveforms as they appear at points 1 to 8 of Fig. 15)

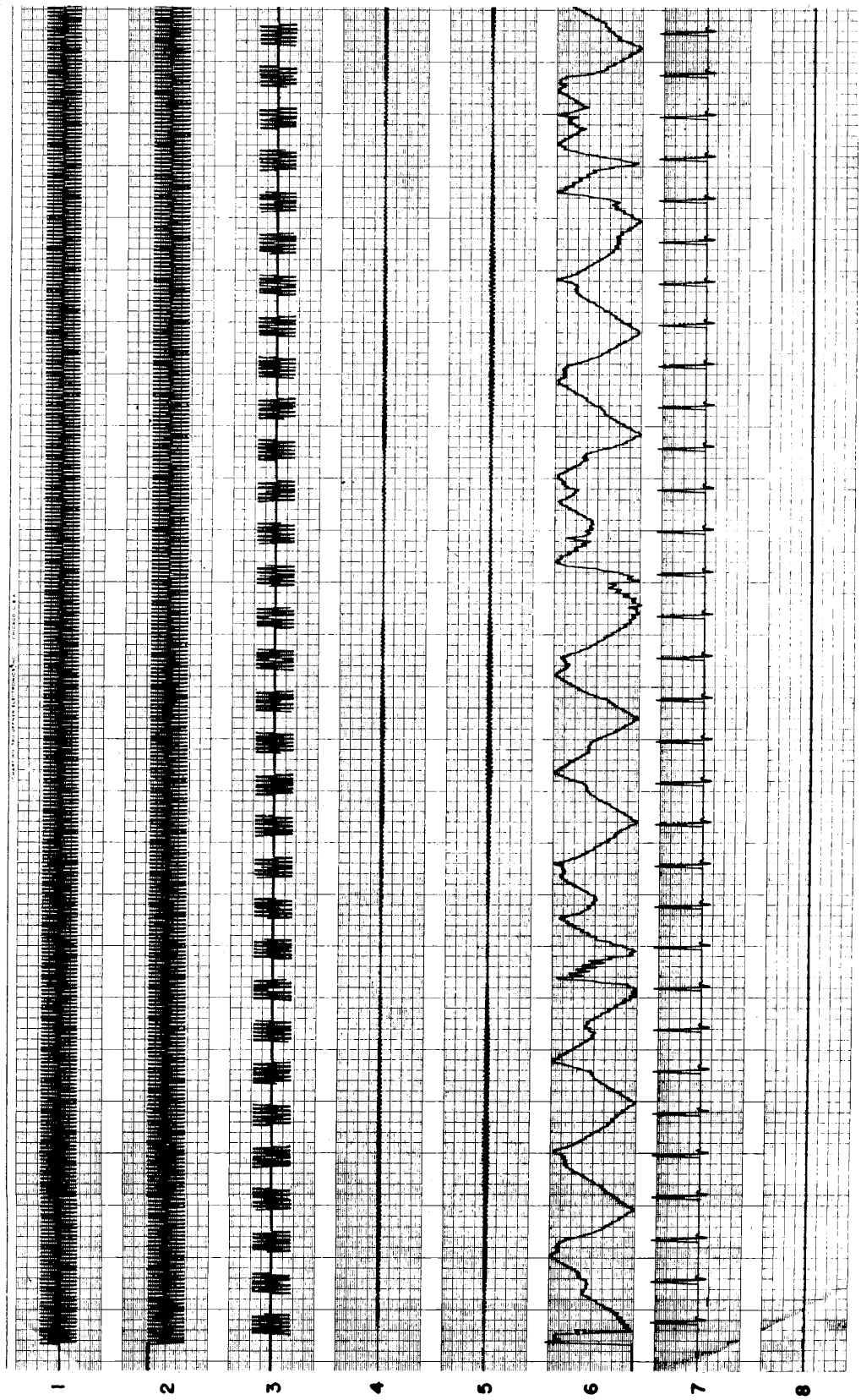

Fig. 18 - High-prf system waveforms with doppler smaller than the spectral line separation

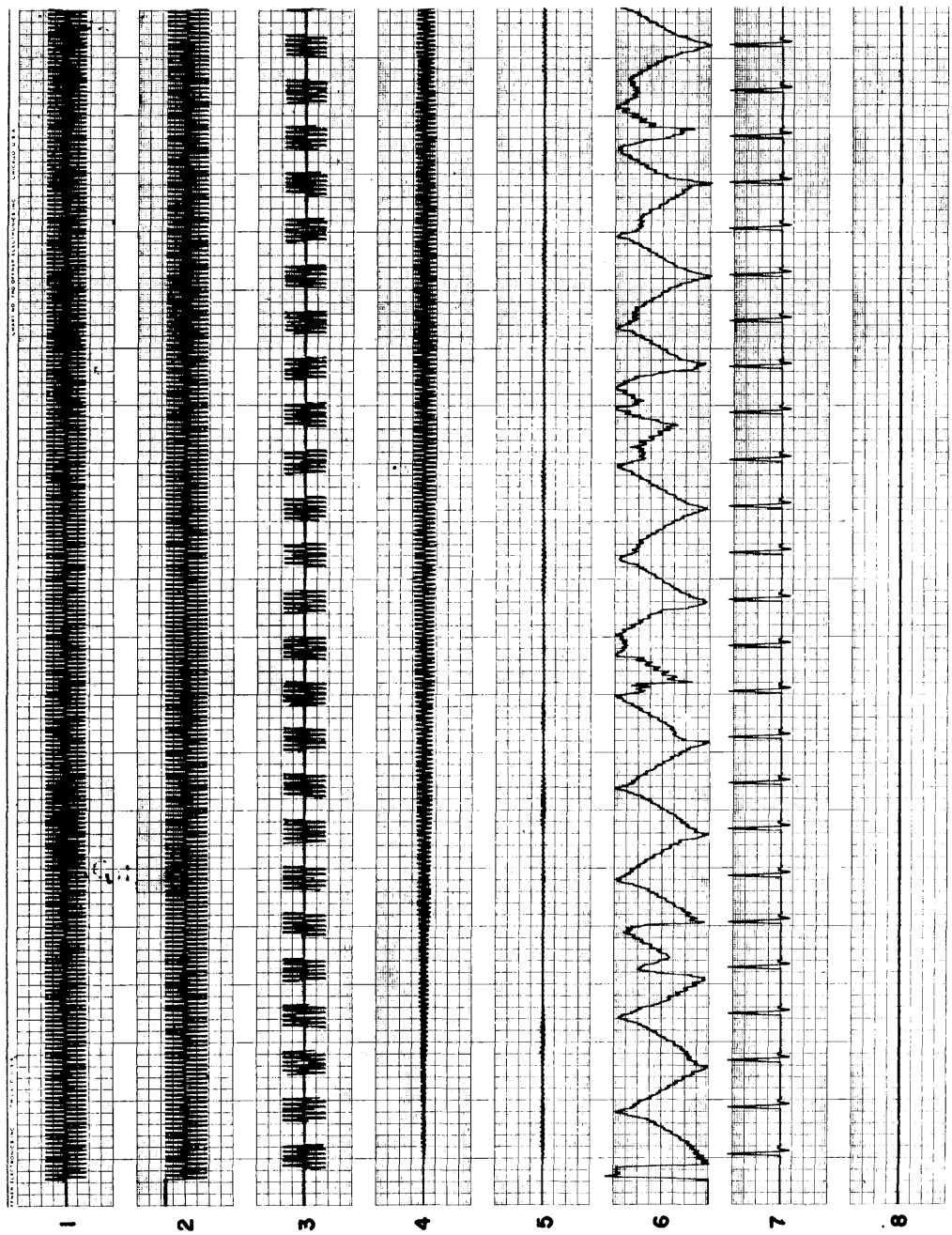

Fig. 19 - High-prf system waveforms with doppler close to the spectral line separation

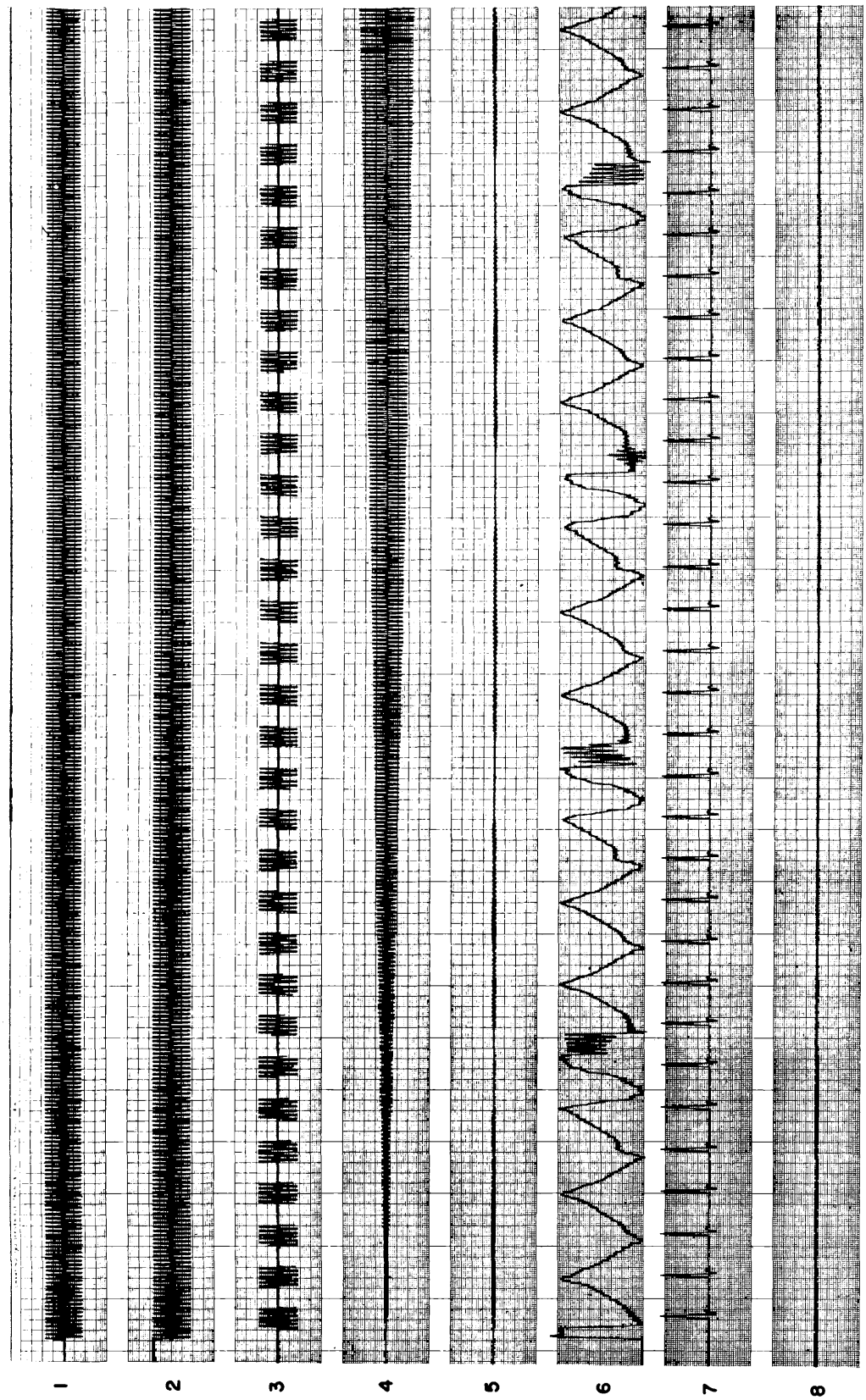

Fig. 20 - High-prf system waveforms with doppler equal to the spectral line separation

output of both filters we obtain small-amplitude oscillations. Therefore, the coincidence signal will be prohibited through the action of the threshold devices. The phase signal in channel 6 is sometimes irregular. This occurs because the phase meter tries to establish the phase with reference to a very-small-amplitude waveform in one of the channels 4 or 5.

Figure 19 shows waveforms for a case in which the frequency shift of the input signal is close to $n_1 \Delta f$, i.e., the smaller of the two line spacings. In this case the filter tuned to f_1' will transfer appreciable energy from a line at a frequency f_2, giving rise to a relatively strong oscillation, as recorded. However, the filter in the opposite channel will be far from being tuned to one of the lines of the input spectrum. Therefore, in this case as in the former, the threshold devices will prohibit generation of a final output signal in channel 8.

Finally, Fig. 20 shows the situation if the frequency shift between the incoming signal and the frequency for which the receiver has been designed is exactly equal to $n_1 \Delta f$. The final signal in channel 8 is suppressed exactly as described previously.

Figure 21 shows the performance of the same system at another staggering ratio; i.e., one of the frequencies f_1 or f_2 has been slightly changed. The recording shows the characteristic waveforms if we desire to eliminate, for example, 11 ambiguous returns out of 12 returns.

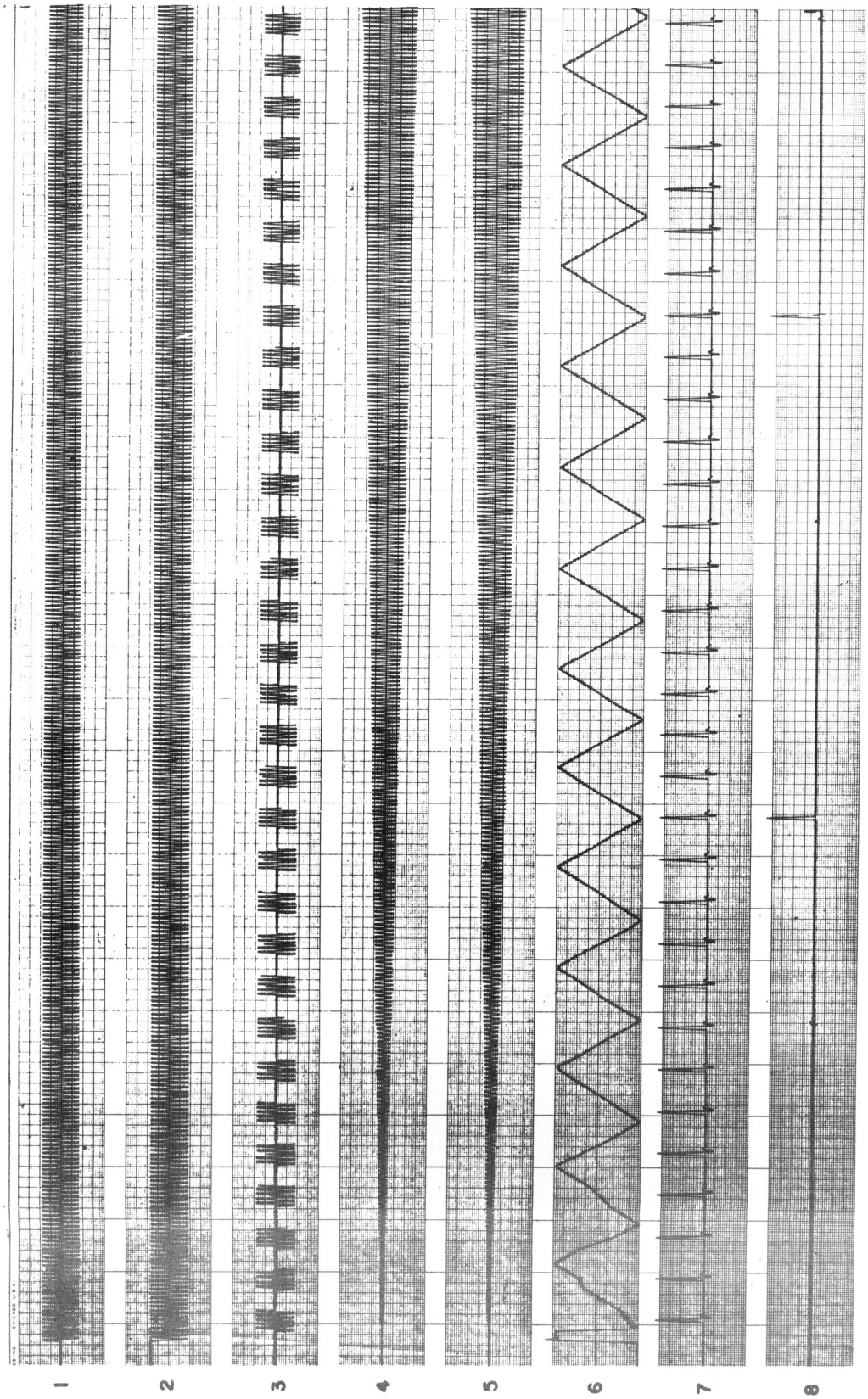

Fig. 21 - High-prf system waveforms with no doppler and the staggering ratio = 5:7

PART 3: ELIMINATION OF BOTH RANGE AND DOPPLER AMBITUITIES

Let us consider the waveforms associated with the high-prf system and the corresponding spectrum in more detail. Using Woodward's notation, we write the signal waveform as

$$\left(\mathrm{rep}_R\ \mathrm{rect}\ \frac{t+T/2}{T}\right)e^{j2\pi f_1 t} + \left(\mathrm{rep}_R\ \mathrm{rect}\ \frac{t-T/2}{T}\right)e^{j2\pi f_2 t} \tag{8}$$

and obtain the spectrum

$$\frac{T}{R}\left(\mathrm{comb}_{1/R}\ \mathrm{sinc}\ Tf\ e^{j2\pi f T/2}\right) * \delta(f-f_1) + \frac{T}{R}\left(\mathrm{comb}_{1/R}\ \mathrm{sinc}\ Tf\ e^{-j2\pi f T/2}\right) * \delta(f-f_2)\ . \tag{9}$$

The spectrum is the sum of two terms arising from the two different transmitted pulse trains, and in both expressions there is a phase-term arising from the finite duration T of the transmitted pulse. Figure 22 shows schematically the spectrum obtained, indicating the value of said phase term.

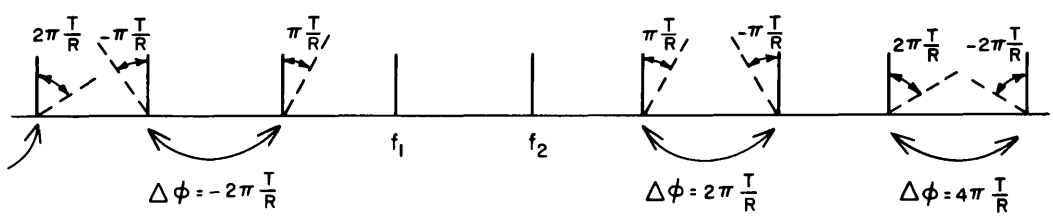

Fig. 22 - Phase relationships in staggered spectrum

We see that this phase term is equal to zero for the first component pulse train if we consider the line corresponding to the carrier f_1, but the phase term amounts to multiples of $\pi T/R$ for the neighboring harmonics. Similarly, the equivalent phase term of the second expression is zero just for the carrier frequency f_2, and it will amount to multiples of $-\pi T/R$ if we move away from f_2 again to one of its harmonics. Thus, we recognize that there are additional phase shifts equal to multiples of $2\pi T/R$ between pairs of harmonics. This means that the sawtooth waveform derived from comparing pairs of harmonics will suffer an additional constant phase or time shift with respect to the function obtained by comparing the carriers themselves. In other words, through proper choice of the pulse width T we should be able to cause the "phase signal equals zero" events never to coincide with the prf signal, if the doppler should amount (possibly against the initial assumption of a high-prf system) to more than the basic prf, $(n_1 + n_2)\Delta f$. Therefore, we can define a medium-prf system which should be capable of reducing or eliminating both range and doppler ambiguities.

This system's performance can be demonstrated on the analog computer by increasing the frequency difference between f_1 and f_1' or between f_2 and f_2', respectively, using the computer program of Fig. 16. The simulation has been performed for the system which should eliminate six of the seven returns, and we see in Fig. 23 that the zero phase signals (channel 6) "squeeze" through the prf pulse sequence (channel 7) in a way that we do not obtain any coincidence signals in channel 8, apart from small transients easily avoided through careful design.

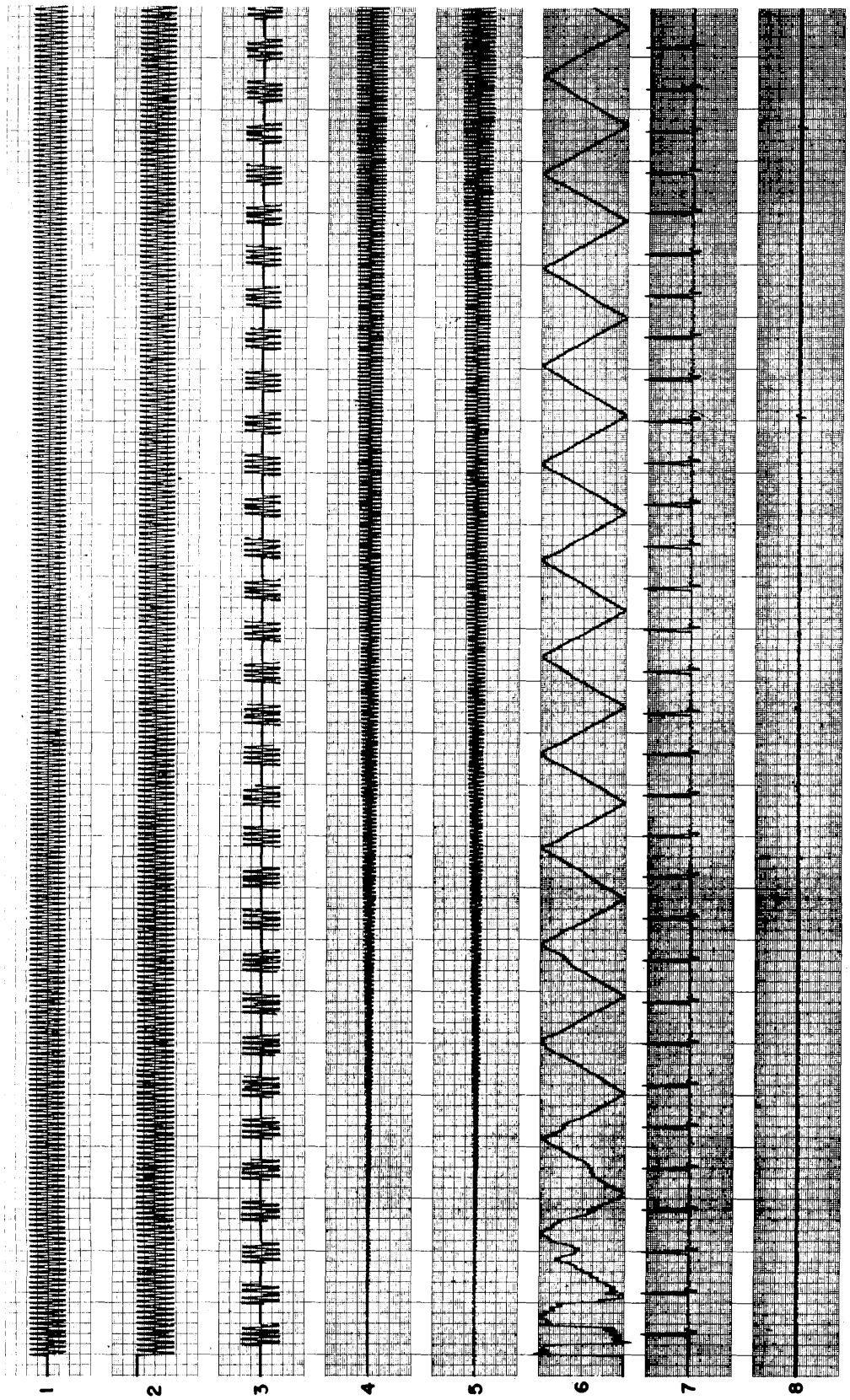

Fig. 23 - High-prf system waveforms with doppler = -2 prf and staggering ratio = 3:4

PART 4: DETERMINATION OF SMALL FREQUENCY DIFFERENCES

As a by-product of the low-prf system simulation, we found a method of precisely determining frequency differences possibly smaller than the reciprocal of the observation time. We now propose a new MTI technique based on this new scheme.

To explain the ideas involved let us consider in more detail the signal output of a narrow-band filter. The filter may be characterized by its impulse response

$$h(t) = \begin{cases} \sin 2\pi f_1 t & (t > 0) \\ 0 & (t < 0) \end{cases} , \tag{9}$$

and the output associated with the input signal $s(t)$ is given by the convolution of $s(t)$ with the impulse response $h(t)$ as

$$\begin{aligned} s(t) * h(t) &= \int_{-\infty}^{t} s(\tau) \sin 2\pi f_1 (t - \tau) \, d\tau \\ &= \left[\int_{-\infty}^{t} s(\tau) \cos 2\pi f_1 \tau \, d\tau \right] \sin 2\pi f_1 t - \left[\int_{-\infty}^{t} s(\tau) \sin 2\pi f_1 \tau \, d\tau \right] \cos 2\pi f_1 t \\ &= |S(f_1, t)| \sin \left[2\pi f_1 t + \varphi(f_1, t) \right] . \end{aligned} \tag{10}$$

From Eq. (10) we see that the narrow-band circuit acts as a computer of the Fourier transform of the incoming signal. The frequency of the oscillation is equal to the filter's center frequency f_1. The amplitude is proportional to the module of the Fourier transform of the past input signal, and the phase term $\varphi(f_1, t)$ is the phase of the input spectrum at f_1. If the input signal consists of an envelope $s(t)$ modulated onto the carrier frequency f_0, then the resulting spectrum is obtained by shifting the spectrum of the envelope to f_0:

$$s(t) \, e^{j 2\pi f_0 t} \rightarrow S(f - f_0) = |S(f - f_0)| \, e^{j\varphi(f - f_0)} . \tag{11}$$

Consequently, the oscillation at the output of a narrow-band filter tuned to f_1 will equal

$$|S(f_1 - f_0)| \sin \left[2\pi f_1 t + \varphi(f_1 - f_0) \right] .$$

If the envelope is shifted by the delay T, the carrier remaining the same as in the example above, then we observe an additional phase term $-2\pi(f - f_0)T$ in the spectrum of $s(t - T) \exp(j 2\pi f_0 t)$. Passing this signal through another narrow-band circuit gives the output signal

$$|S(f_1 - f_0)| \sin \left[2\pi f_1 t + \varphi(f_1 - f_0) - 2\pi(f_1 - f_0) T \right] .$$

The phase difference between the ringing at the two filter outputs is then given by $\phi = 2\pi(f_1 - f_0)T$. This phase difference is determined by the design parameter T, and it is proportional to the frequency mismatch $f_1 - f_0$, which could be understood as a small doppler shift. Since ϕ can be determined with increasing precision if the signal-to-noise ratio increases, we are also able to determine the doppler with precision. The corresponding diagram is identical with the one of Fig. 3. Certainly the frequency resolution of our system cannot be better than the reciprocal of the observation time, and since we never know whether some radar return has been caused by one target at one speed or by several targets with several slightly different speeds within the same range-doppler-resolution cell, it appears doubtful whether it would be useful to push the precision beyond the resolution capabilities as long as we are interested in measuring target speed. However, the principle may be usefully applied if the problem is to determine whether the radar return is due to stationary targets only or to stationary targets and moving targets in the same resolution cell, and this is exactly the task of MTI radar.

Figures 24 through 32 show the performance of the low-prf system as described in the first part, if the frequency shift between the signal and the tuning of the filters is equal to 1/10, 2/10, . . ., 9/10 prf. In all these cases the phase signal of line 7 is not equal to zero, or it does not remain close to zero throughout the runs. Therefore, the system would perform properly as far as distinguishing the incoming carrier from the center frequency of the filters is concerned. To understand the MTI system contained in the simulation, consider the phase signal from the moment the first pulse in line 4 has arrived to the moment the second pulse in line 3 starts to excite the corresponding filter. It is easily verified, by inspection of Figs. 24 through 32, that the phase signal which is constant during the described time interval increases proportional to the frequency difference.

It is hoped that the suggested MTI system would have a noise and clutter performance very close to the theoretical limitations imposed on the possible accuracy of frequency measurement. Whether this is really true or not will be determined by future theoretical and practical study. Certainly, the new MTI system would offer a simple method of changing or adapting the threshold at which radar returns are considered to be caused by moving or by stationary targets. Also, this method might be useful if signal processing in digital computers is being considered. A new feature, certainly, is the use of one pair of resonant circuits per range cell as a memory device.

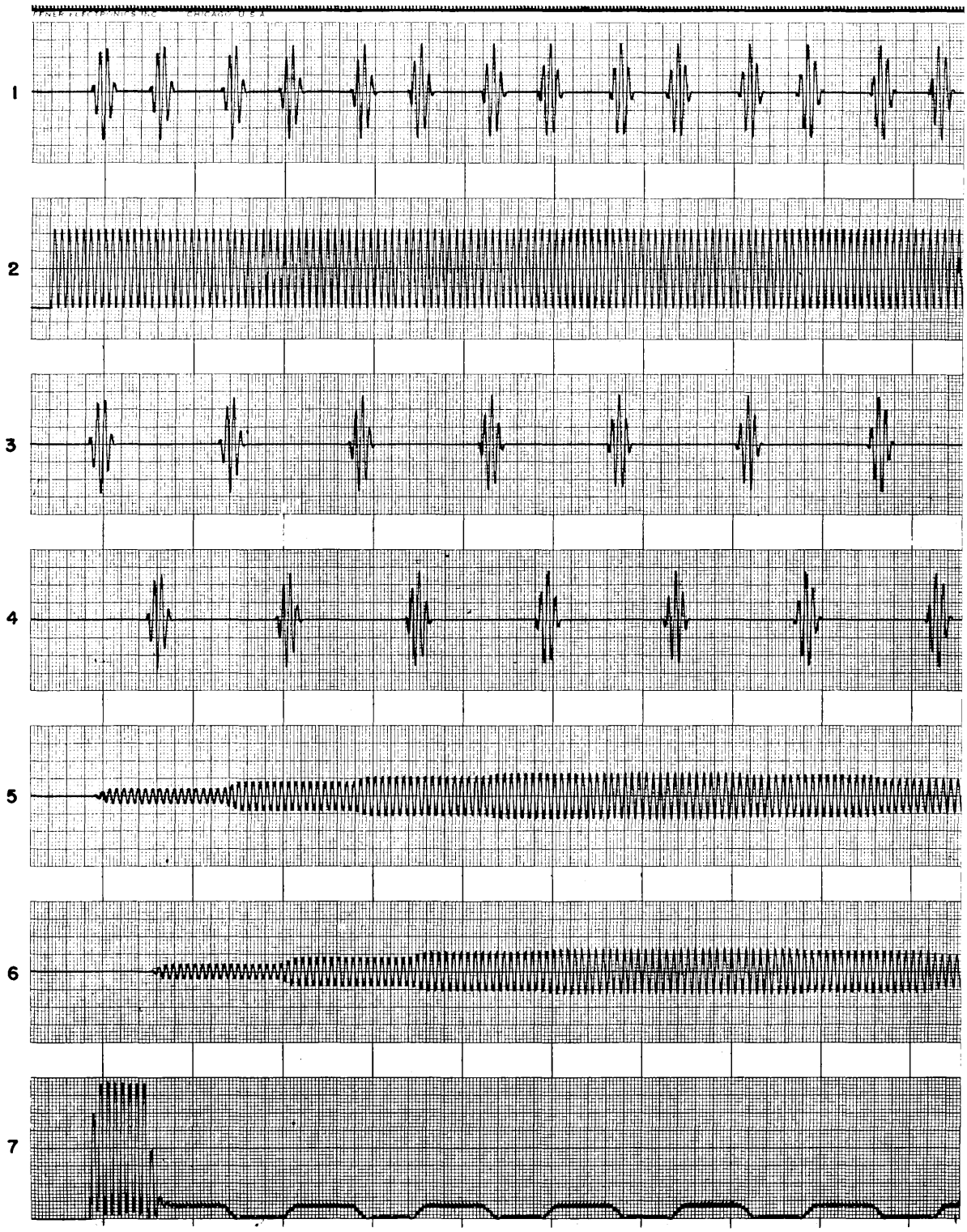

Fig. 24 - Low-prf system waveforms with doppler = 1/10 prf

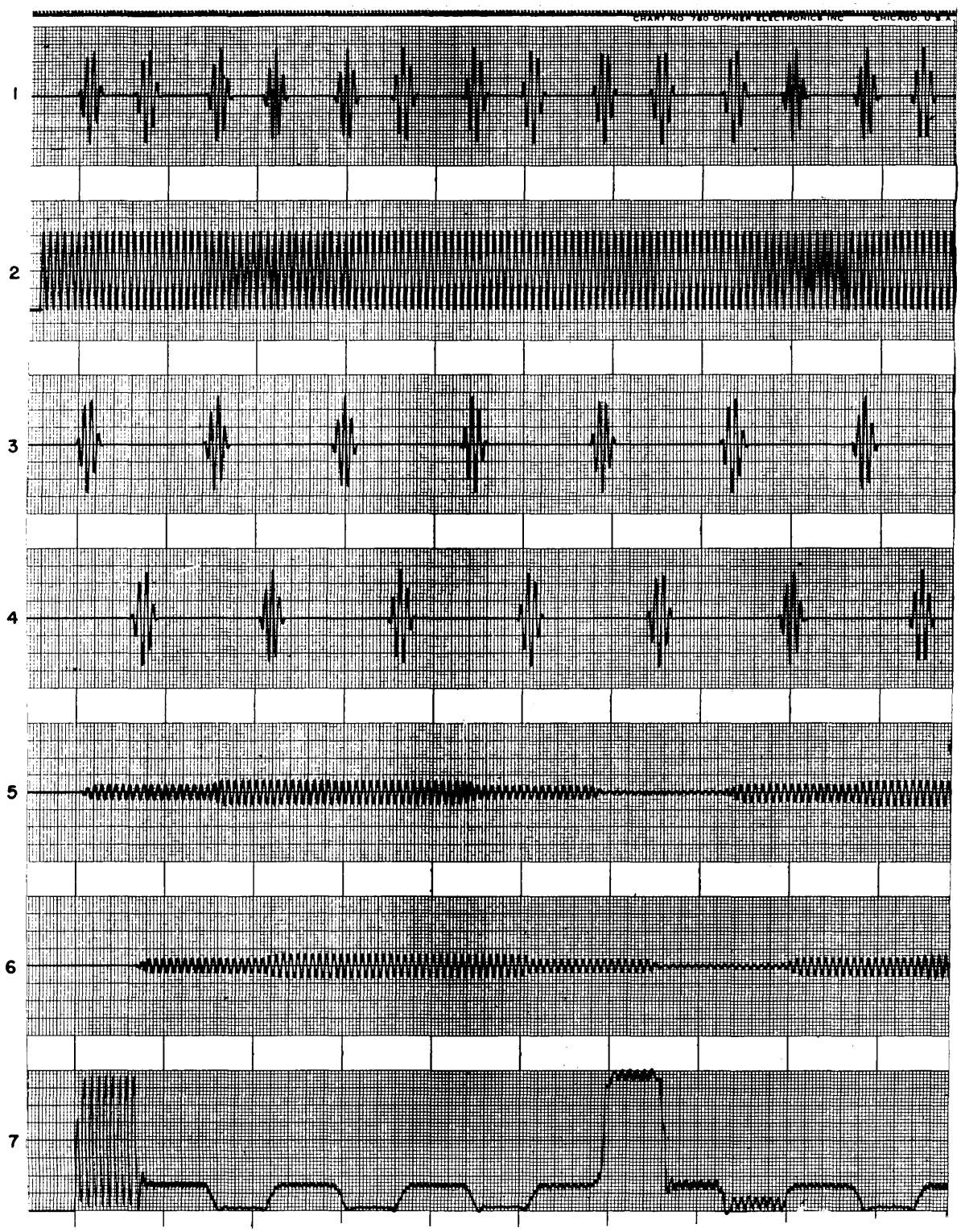

Fig. 25 - Low-prf system waveforms with doppler = 2/10 prf

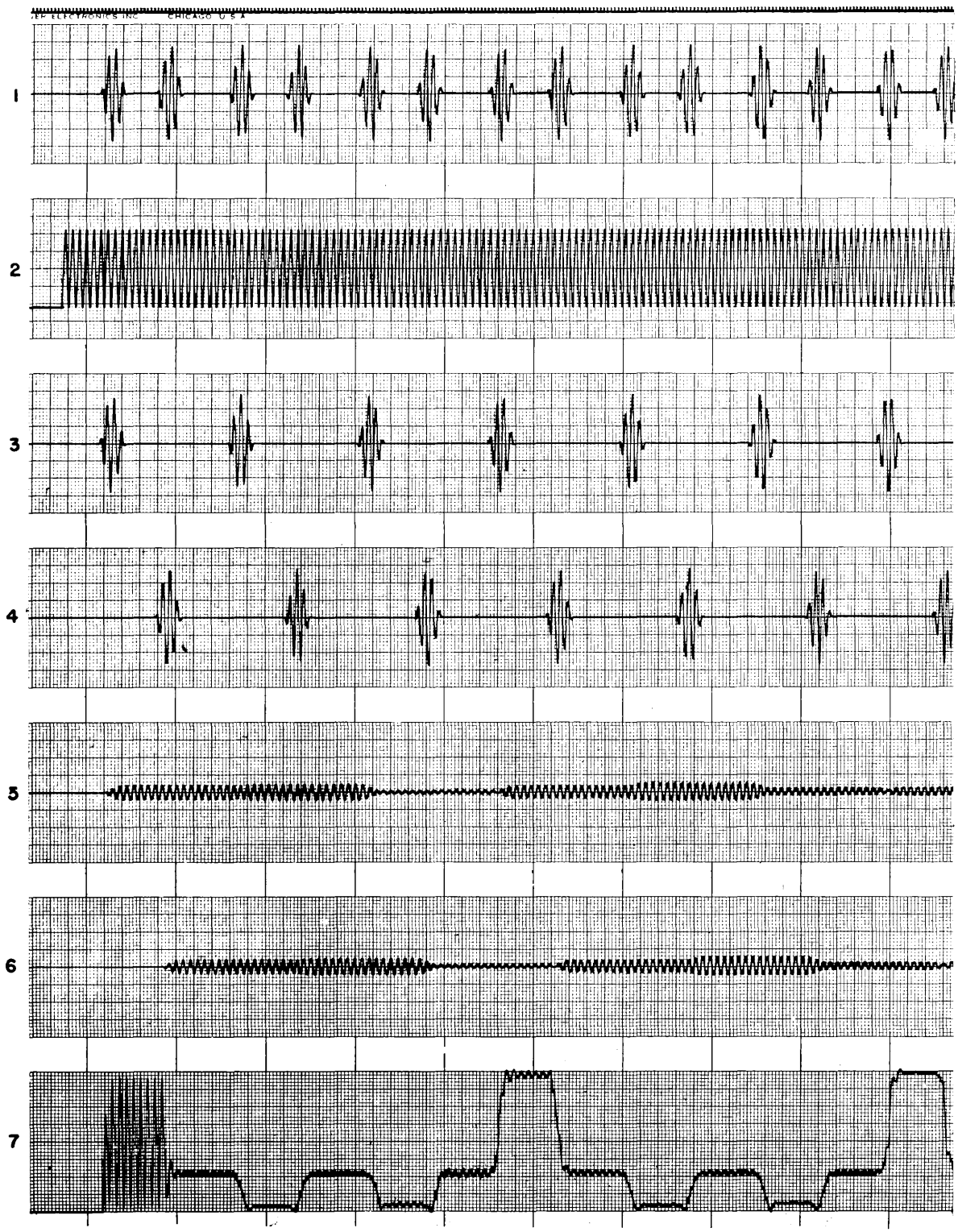

Fig. 26 - Low-prf system waveforms with doppler = 3/10 prf

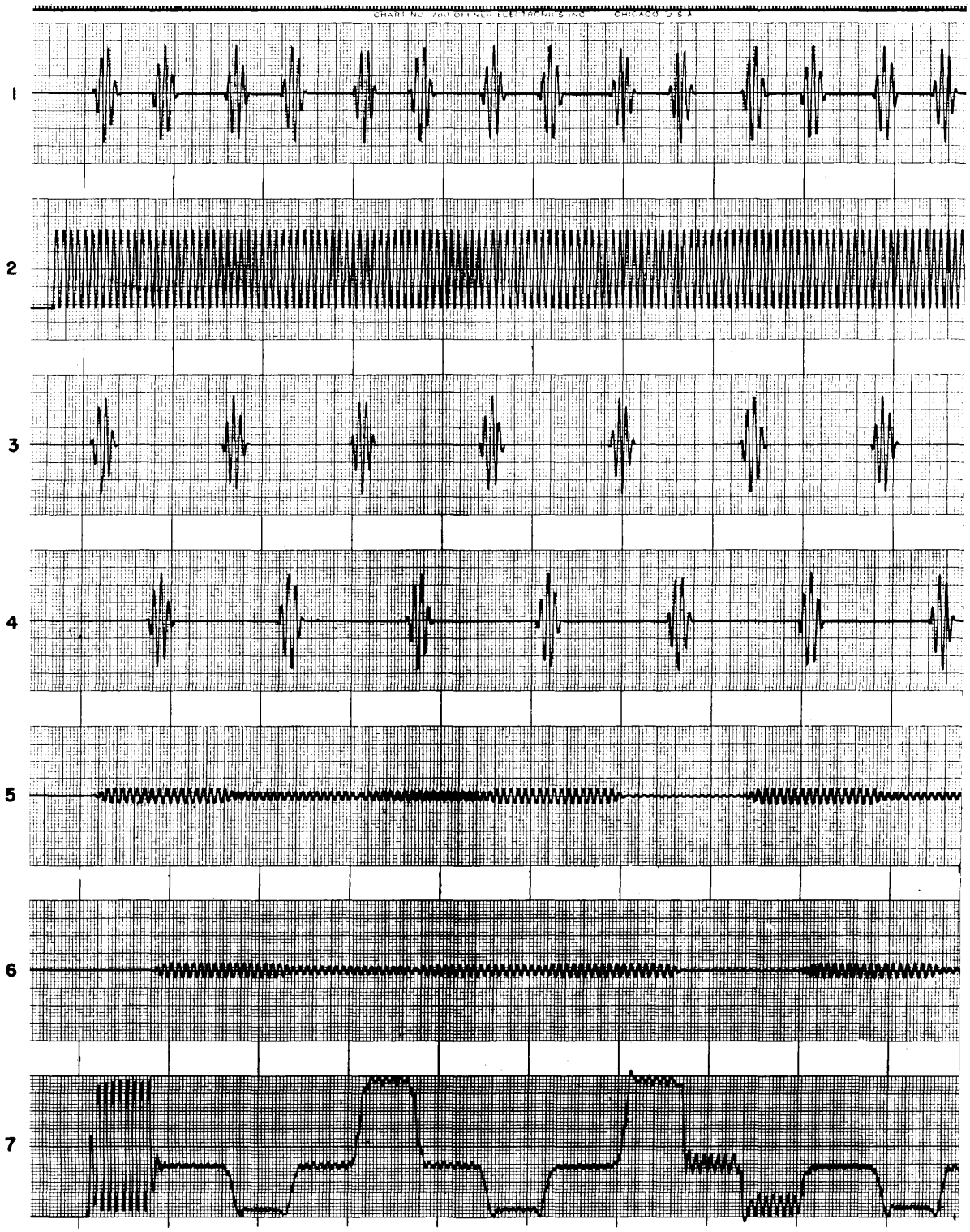

Fig. 27 - Low-prf system waveforms with doppler = 4/10 prf

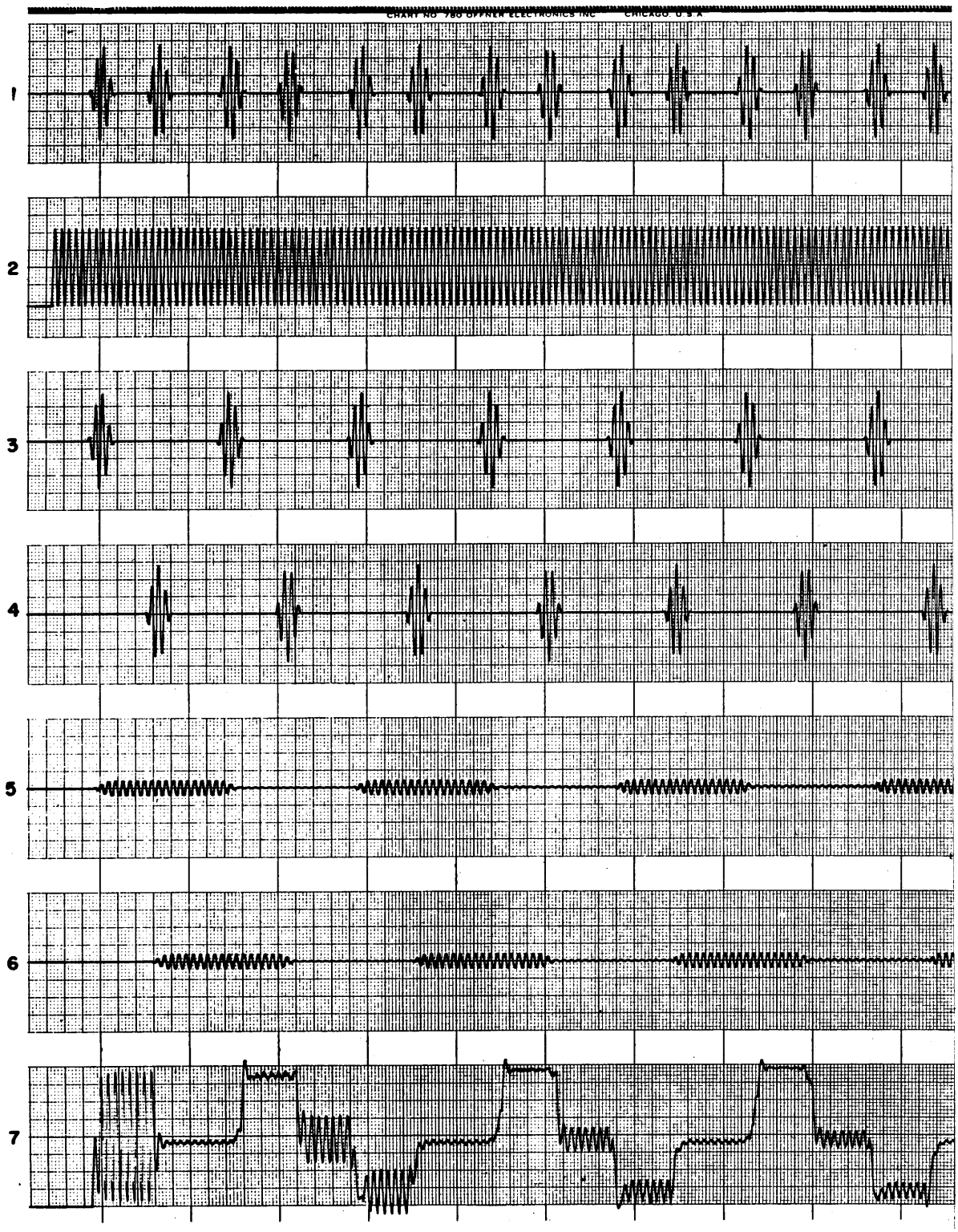

Fig. 28 - Low-prf system waveforms with doppler = 5/10 prf

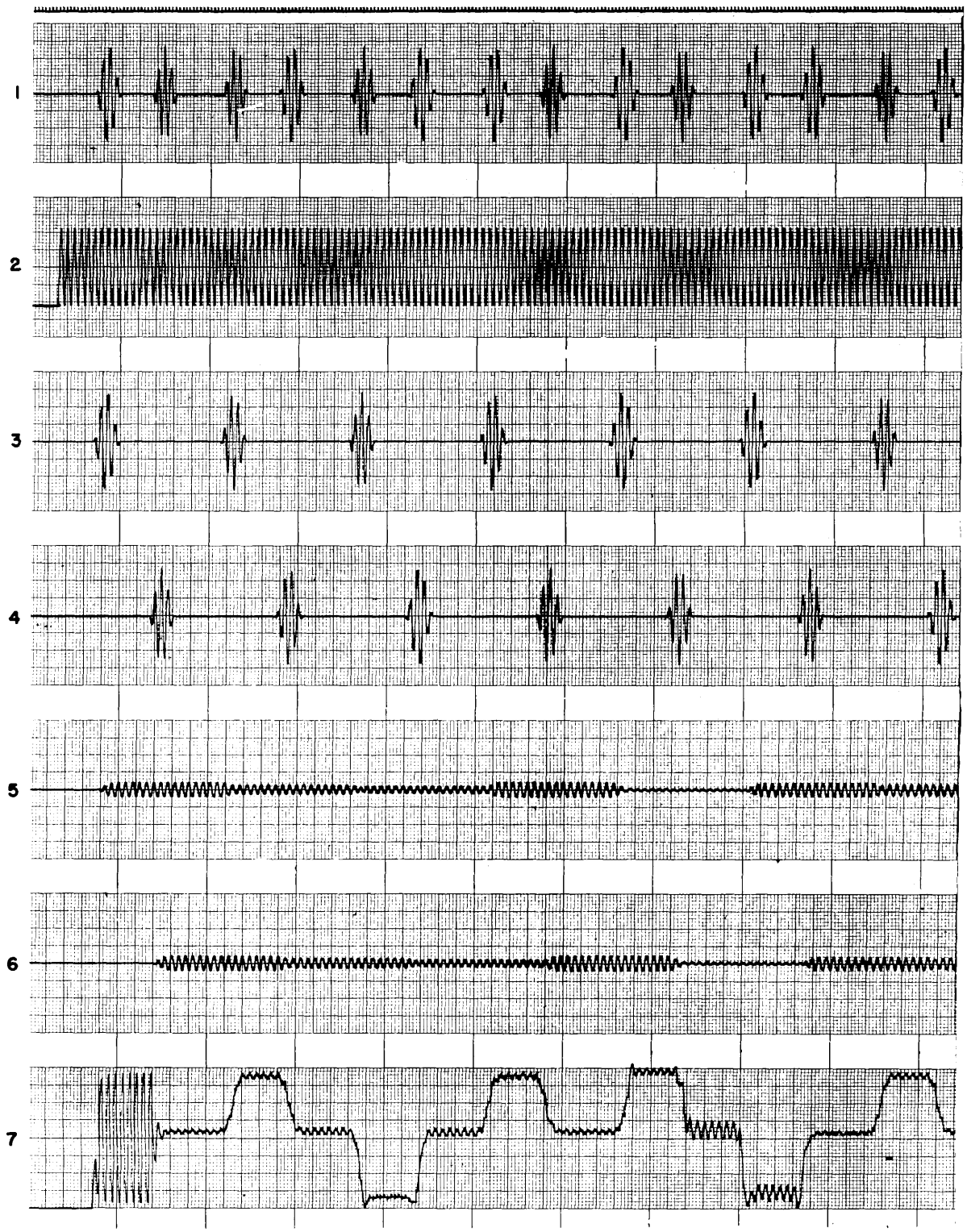

Fig. 29 - Low-prt system waveforms with doppler = 6/10 prf

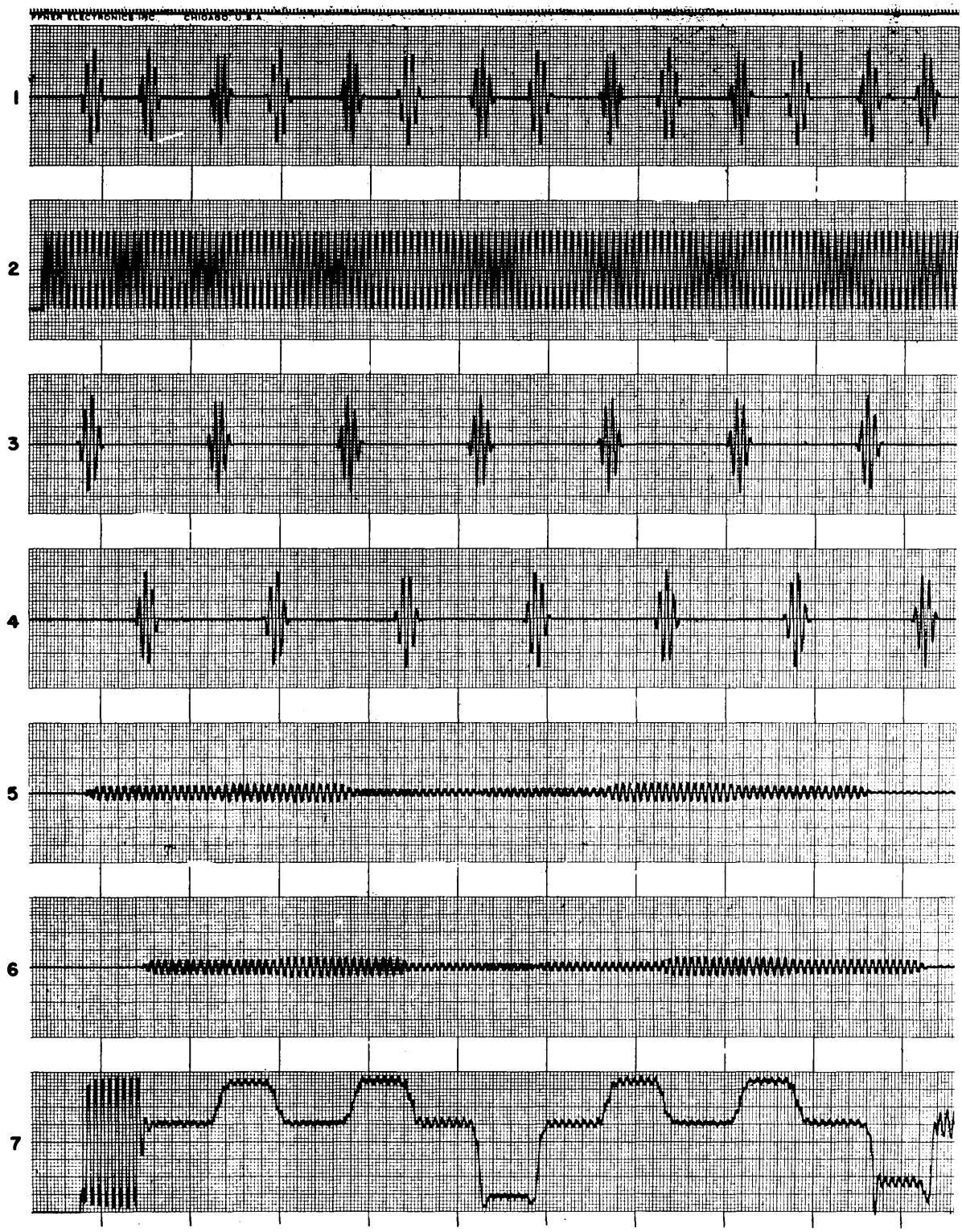

Fig. 30 - Low-prf system waveforms with doppler = 7/10 prf

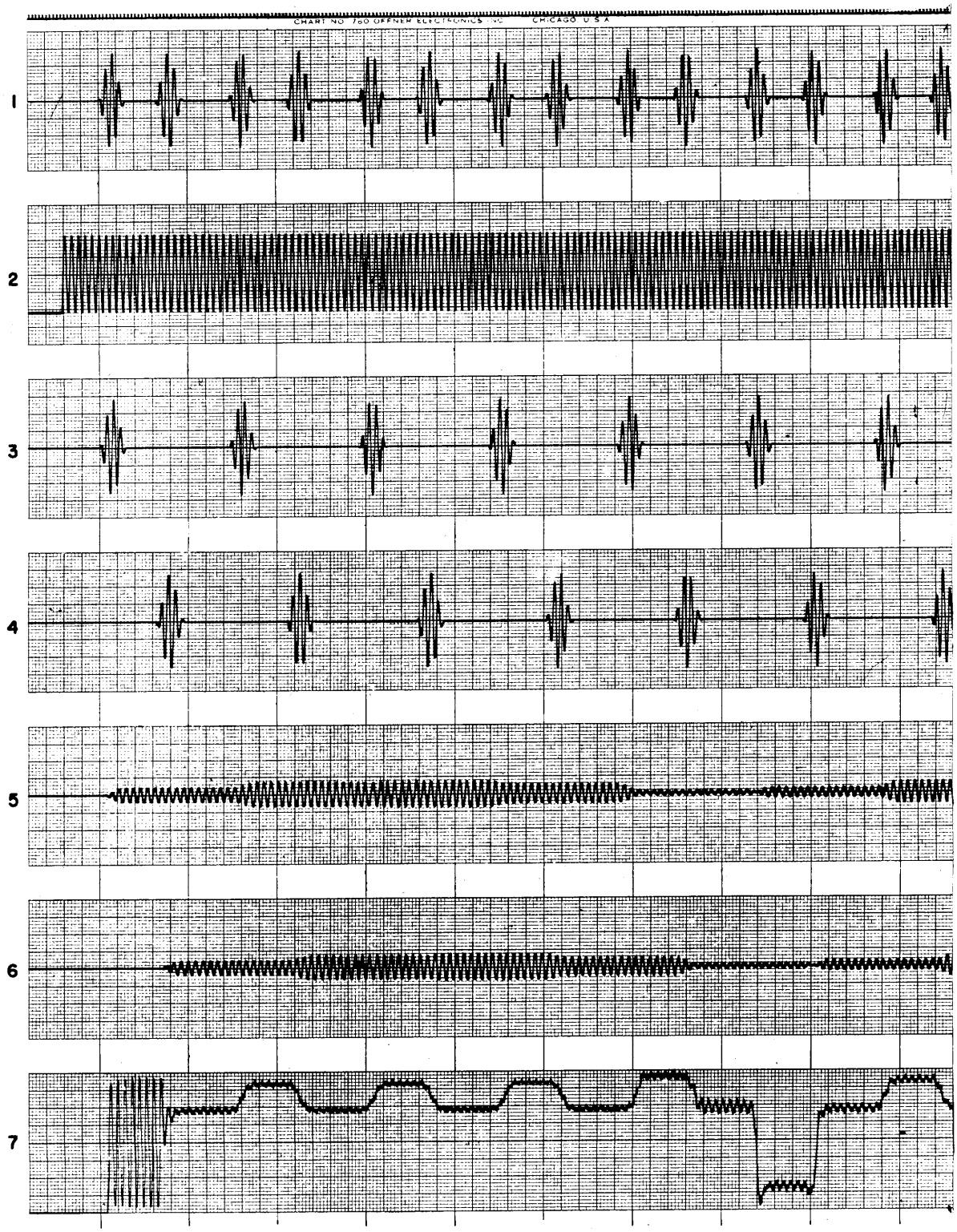

Fig. 31 - Low-prf system waveforms with doppler = 8/10 prf

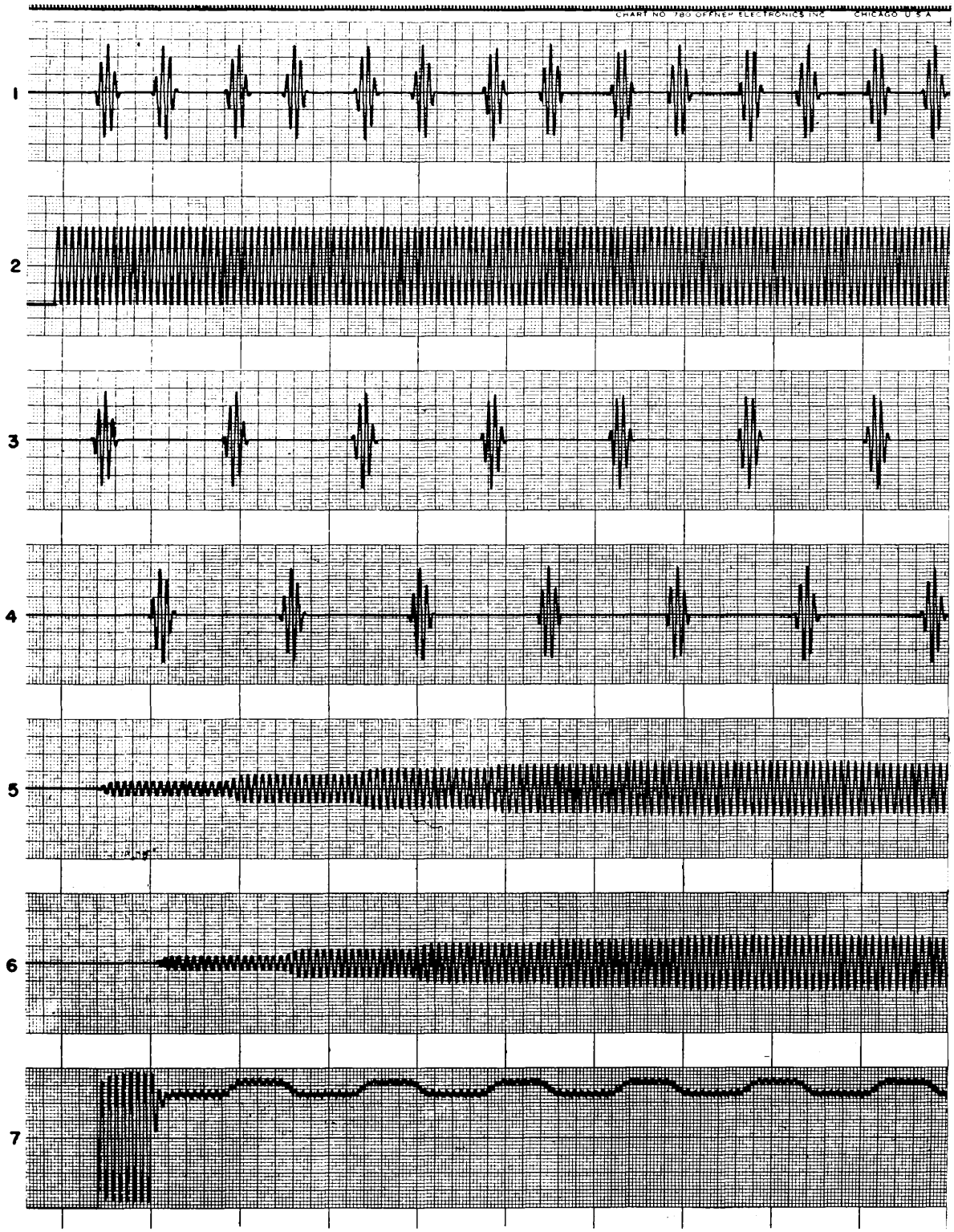

Fig. 32 - Low-prf system waveforms with doppler = 9/10 prf

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13. ABSTRACT

A method for reducing or eliminating doppler and/or range ambiguities ordinarily associated with using repetitive waveforms in radar has been developed. The basic idea calls for building two radar systems into one. Two different waveforms are impressed on one carrier, the waveforms being designed in such a fashion that they can be separated and processed in different receiver channels. The combined information of the two processed signals is used to reduce or to eliminate ambiguities. Analog computation has been used to study the feasibility of generating and processing the proposed waveforms.

A new method for measuring small frequency shifts is suggested. A pair of pulses is used; the first and the second pulse will produce ringing at the outputs of a first and a second narrowband filter, respectively, and the relative phase of the ringing oscillations is shown to be linearly related to the difference between the carrier frequency and the filter center frequency.

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