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Modelistic and Realistic Interactions in the 1p, 2s-1d and 2p-1f Nuclear Shells

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ABSTRACT

The excitation spectra and β-decay comparative half-lives of He^6, Li^6, O^{18}, F^{18}, Ca^{42}, and Sc^{42} are calculated using a "modelistic" interaction containing pairing plus quadrupole-quadrupole operators (with Rosenfeld exchange), single-body spin-orbit and long-ranged tensor interactions. The resulting low-lying wave functions are compared to those obtained by other workers with hard-cored "realistic" interactions and are found to overlap to a high degree.

PROBLEM STATUS

This is a final report on one phase of the problem.

AUTHORIZATION

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MODELISTIC AND REALISTIC INTERACTIONS IN THE

1p, 2s-1d AND 2p-1f NUCLEAR SHELLS

INTRODUCTION

Using a simplified interaction model, typical of many shell-model calculations, we determined the level spectra, β-decay comparative lifetimes (ft-values) and give explicit wave functions in jj-coupling for the low-lying levels of Li^6, He^6, F^{18}, O^{18}, Sc^{42}, and Ca^{42}. The configurations considered are $(1p_{3/2}, 1p_{1/2})^2$ for A=6, $(1d_{5/2}, 2s_{1/2}, 1d_{3/2})^2$ for A=18 and $(1f_{7/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2})^2$ for A=42, above the inert doubly-magic cores of He^4, O^{16}, and Ca^{40}, respectively. The interaction model consists of a variable linear combination of single-body spin-orbit, pairing (of particles to J=0) and quadrupole-quadrupole (both with exchange), as well as a long-ranged tensor interaction, all for which matrix elements are relatively simple to evaluate. Fixing the exchange mixture (Rosenfeld), we are left with 4 free parameters to adjust, yielding upon diagonalization a large number of spectra for each nucleus from which to select the one fitting optimumly with the corresponding empirical excitation spectra. Thus, we arrive at an "effective interaction" in each case, as well as the associated wave functions with which to compute observed level properties other than energies.

In recent years, semi-phenomenological potentials, such as those of Brueckner, Gammel, and Thaler[1)] and Hamada and Johnston,[2)] have been used in calculating the low-lying characteristics of <u>finite</u>

nuclei around mass numbers 6,[3] 18,[4] 42,[5] as well as the nickel[6] and tin[7] isotopes. These calculations enjoy the advantage of employing interaction parameters which have previously been fixed by two-nucleon scattering data up to ~ 310 Mev, as well as by deuteron properties. The Hamada-Johnston potential, e.g., adjusts the N-N data with 29 parameters associated with diverse hard-cored radial functions suggested by meson-theoretic one- and two-pion exchange calculations[8] with full nucleon-recoil effects.

Here we investigate two matters. First, comparison to experiment of the low-lying spectra and β-decay properties of two-valence particle nuclei in the oscillator 1p, 2s-1d and 2p-1f shells which are determined via the simple interaction model. Secondly, by the overlap method we compare the nuclear wave functions of low-lying levels obtained by the detailed, hard-cored "realistic" potentials with those of the simple interaction model.

1. INTERACTION HAMILTONIAN MODEL

We consider the hamiltonian for the two-valence-nucleons as

$$H=H_{osc}+V_{model}(1,2) \tag{1}$$

where $V_{model}(1,2)$ represents the modelistic "effective" (or "residual") interaction acting between the two nucleons in the subspace of states arising from a single major oscillator shell of completely degenerate levels. This degeneracy is broken by $V_{model}(1,2)$ which is taken to

consist of single-particle spin-orbit, central with Rosenfeld exchange and tensor interactions:

$$\sum_{i=1}^{2} \vec{\ell}_i \cdot \vec{s}_i; \qquad V_c(r)\hat{I}(\vec{\sigma}_1 \cdot \vec{\sigma}_2; \vec{\tau}_1 \cdot \vec{\tau}_2); \qquad f(T)V_t(r)\hat{S}_{12} \tag{2}$$

(s.p. spin-orbit) (central with exchange) (tensor)

in which $\hat{I}(\vec{\sigma}_1 \cdot \vec{\sigma}_2; \vec{\tau}_1 \cdot \vec{\tau}_2)$ is the exchange operator acting only on spin-isospin coordinates and $\hat{S}_{12}$ the usual tensor operator acting on the spin-orbital coordinates. The isospin-dependent factor f(T) will be chosen such that T=0 levels are bound more tightly than T=1, in accordance with deuteron properties. A typical shell-model interaction includes just those components (2).

Instead of using an arbitrary (Yukawa, Gaussian, exponential, etc.) radial function for the central potential $V_c(r)$, we employ the well-known result[9] that a linear combination of pairing $\hat{P}$ and quadrupole-quadrupole $\hat{Q}^2$ interaction operators (simulating, respectively, the short- and long-ranged central correlations) reproduces rather well the effects of a central (Gaussian) interaction potential. The complete interaction model is taken as

$$V_{model}(1,2) = -[\alpha \sum_{i=1}^{2} \vec{\ell}_i \cdot \vec{s}_i + \beta \hat{f}(1,2)\hat{Q}^2 + \gamma \hat{I}(1,2)\hat{P} + \delta f(T)V_t(r)\hat{S}_{12}]. \tag{3}$$

$$\alpha,\ \beta,\ \gamma,\ \delta \geq 0.$$

where α, β, γ, and δ are positive, adjustable parameters. The eigenvalues $E_n(\alpha, \beta, \gamma, \delta)$ of (3) will be determined by the secular equation

$$\det\left|\langle j_1 j_2 J, TM_T | V_{model}(1,2) | j'_1 j'_2 J, TM_T \rangle_a - E_n(\alpha,\beta,\gamma,\delta)\delta_{j_1 j'_1}\delta_{j_2 j'_2}\right| = 0 \tag{4}$$

involving antisymmetrized (subscript a) matrix elements, as both particles are in the <u>same</u> ocillator shell for A=6,18 and 42.

The pairing effect (after Goeppert-Mayer[10]) is given by an operator $\hat{P}$ whose matrix elements in jj-coupling are simply

$$\langle j_1 j_2 J | \hat{P} | j'_1 j'_2 J \rangle = \sqrt{(2j_1+1)(2j'_1+1)}\ \delta_{J0}\delta_{j_1 j_2}\delta_{j'_1 j'_2} \ . \tag{5}$$

Now, the two-particle exchange operator $\hat{I}(1,2)$ is, in general,

$$\hat{I}(1,2) = W + M\hat{P}^M(1,2) + B\hat{P}^B(1,2) + HP^H(1,2) \tag{6}$$

where $\hat{P}^M(1,2)$, $\hat{P}^B(1,2)$, and $\hat{P}^H(1,2)$ are the usual Majorana, Bartlett and Heisenberg exchange operators and, the mixture constants satisfy W+M+B+H=1. As $\hat{P}$ approximates the short-ranged central interaction (ideally a δ-force), it is reasonable to use here an exchange operator $\hat{I}(1,2)$ in which $\hat{P}^M(1,2)=1$, implying (since the Pauli principle requires $\hat{P}^M(1,2)\hat{P}^B(1,2)\hat{P}^H(1,2)=+1$) that

$$\hat{P}^B(1,2)=\hat{P}^H(1,2) \tag{7}$$

which simplifies (6) to

$$\hat{I}(1,2)=1-(B+H)\hat{T}^2 \tag{8}$$

since $\hat{P}^H(1,2)\equiv-\frac{1}{2}(1+\vec{\tau}_1\cdot\vec{\tau}_2)$. The pairing (with exchange) interaction in (3) will thus have antisymmetrized matrix elements

$$\begin{aligned}&{}_a\langle j_1 j_2 JT|\hat{I}(1,2)\hat{P}|j'_1 j'_2 JT\rangle_a\\ &=[1-(B+H)T(T+1)]\sqrt{(2j_1+1)(2J'_1+1)}\ \delta_{J0}\delta_{j_1 j_2}\delta_{j'_1 j'_2}\end{aligned} \tag{9}$$

Following Elliott,[11] we choose the long-ranged central interaction as the scalar product of two quadrupole operators (one for particle i, the other for particle j) summed over all i and j:

$$\begin{aligned}\hat{Q}^2&\equiv\frac{16\pi}{5}\sum_{i,j=1}\{\sum_q(-)^q r_i^2 Y_{2q}(i) r_j^2 Y_{2-q}(j)\}\\ &=\frac{32\pi}{5}\sum_q(-)^q r_1^2 Y_{2q}(1) r_2^2 Y_{2-q}(2)+4(r_1^4+r_2^4)\ .\end{aligned} \tag{10}$$

This scalar operator, <u>with</u> the self-interactions term $4(r_1^4+r_2^4)$, has been shown by Elliott and others to be diagonal in an $SU_3\supset R_3$ coupling scheme, when restricted to act within a single major oscillator shell.

If the non-negative integers k_1 and k_2 label the irreducible SU_3 representation associated with the spatial-symmetry Young partition [f] of n particles in a major oscillator shell, L the various total orbital angular momenta associated with these n particles and ω is a label distinguishing between repeated L's which may appear for a given $(k_1 k_2)$, Moshinsky's[12] formula for the matrix elements of $\hat{Q}^2$ in LS coupling is

$$\langle [f](k_1 k_2)\omega L|\hat{Q}^2|[f'](k'_1 k'_2)\omega' L'\rangle$$

$$= \delta_{ff'}\delta_{k_1 k'_1}\delta_{k_2 k'_2}\delta_{\omega\omega'}\delta_{LL'}[\tfrac{2}{3}(k_1+k_2)^2-2k_1(k_2-1)-\tfrac{1}{2}L(L+1)] \qquad (11)$$

in which the rotational, long-ranged effect L(L+1) is evident. Using (6) and (11), it is readily seen that the matrix element of $\hat{I}(1,2)\hat{Q}^2$ in (3) shall be

$$\langle [f](k_1 k_2)\omega L,S,JT|\hat{I}(1,2)\hat{Q}^2|[f'](k'_1 k'_2)\omega' L',S',JT'\rangle$$

$$=\{(W-M-B+H)+(M+B)S(S+1)+(M-H)T(T+1)-MT(T+1)S(S+1)\}\times$$
$$\times[\tfrac{2}{3}(k_1+k_2)^2-2k_1(k_2-1)-\tfrac{1}{2}L(L+1)]\delta_{ff'}\delta_{k_1 k'_1}\delta_{k_2 k'_2}\delta_{\omega\omega'}\delta_{LL'}\delta_{SS'}\delta_{TT'}. \qquad (12)$$

However, we need the matrix elements of $\hat{I}(1,2)\hat{Q}^2$ in jj-coupling. The desired transformation is directly effected by using the two-particle transformation coefficients

$$\langle SU_3 | jj \rangle \equiv \langle [f](k_1 k_2)\omega L, S, JT | j_1 j_2 JT \rangle \tag{13}$$

which are easily evaluated by explicit construction of two-particle states in both schemes, and scalar-producting. For the sd-shell, they are given in Ref.(13) and for the p-shell in Appendix I. It is then straightforward to obtain the matrices

$$\| \langle j_1 j_2 JT | \hat{I}(1,2) \hat{Q}^2 | j'_1 j'_2 JT \rangle \| . \tag{14}$$

The Rosenfeld mixture in $\hat{I}(1,2)$ (W=-0.13, M=0.93, H=-0.26, B=0.46) to be used for all three cases $(p)^2$, $(sd)^2$ and $(pf)^2$, is well known[14] to satisfy the saturation conditions as well as the empirical energy difference between the bound T=0 and unbound (virtual) T=1 states of the neutron-proton system. Through (9) and (14), it will be responsible for the <u>only</u> repulsive elements of the interaction model, as opposed to the (infinite) repulsion in all two-nucleon states represented by the hard core of most "realistic" potentials.

In order to avoid the tedious pf-shell calculation of coefficients (13), the evaluation of (14) for the pf-shell was carried out along more conventional lines: the spherical tensorial character of (10) permits the use of standard Racah algebra to deduce[15] the expression

$$\langle n_1\ell_1 n_2\ell_2 L|\hat{Q}^2|n_1'\ell_1' n_2'\ell_2' L\rangle = 8(-)^{\ell_1+\ell_2'-L}\, W(\ell_1\ell_2\ell_1'\ell_2';L2)\langle \ell_1' 200|\ell_1 0\rangle \times$$

$$\times\ \langle \ell_2' 200|\ell_2 0\rangle \sqrt{(2\ell_1'+1)(2\ell_2'+1)}\ (n_1\ell_1|r_1^2|n_1'\ell_1')(n_2\ell_2|r_2^2|n_2'\ell_2')$$

$$+4\delta_{\ell_1\ell_1'}\delta_{\ell_2\ell_2'}[(n_1\ell_1|r_1^4|n_1'\ell_1)+(n_2\ell_2|r_2^4|n_2'\ell_2)] \tag{15}$$

where the harmonic oscillator radial integrals

$$(n\ell|r^\rho|n'\ell') \equiv \int_0^\infty R_{n\ell}(r)r^\rho R_{n'\ell'}(r)r^2dr \qquad (\rho = 2,\ 4) \tag{16}$$

are evaluated by standard methods.[16] Using the exchange operator eigenvalue of the first bracketed expression on the r.h.s. of (12), one then evaluates the LS-coupling matrices

$$\|\langle n_1\ell_1 n_2\ell_2 L,S,JT|\hat{I}\hat{Q}^2|n_1'\ell_1' n_2'\ell_2' L,S,JT\rangle\| \tag{17}$$

which by 9j-coefficients[17] can be transformed to matrices (14).

In dealing with the tensor force of the interaction model (3), we first evaluate its matrix elements in LS-coupling according to the method outlined by Brody and Moshinsky,[18] who employ the Talmi transformation[19] from center-of-oscillator-well coordinates to relative- and center-of-mass coordinates. The same 9-j coefficients used to transform (17) can then be employed here to yield the jj-coupling tensor force matrix elements. The tensor interaction in (3) can be

written[20] as the scalar product of two second rank spherical tensors of the relative orbital and spin coordinates of the two particles, namely,

$$V_t(r)\hat{S}_{12} = \sqrt{\frac{32\pi}{5}}\; V_t(r)\sum_q (-)^q Y_{2q}(\theta,\phi) X_{2-q}(\vec{\sigma}_1, \vec{\sigma}_2) \tag{18}$$

in which e.g., the q = 0 component of $X_{2q}(\vec{\sigma}_1, \vec{\sigma}_2)$ is

$$X_{20} = \frac{1}{\sqrt{2}}(3S_z^2 - S^2); \qquad \vec{S} = \tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \tag{19}$$

Letting $n_i \ell_i$ be the radial and orbital angular momentum quantum numbers for particle i(=1,2) in a harmonic oscillator well, the matrix element of (18) is

$$\langle n_1\ell_1 n_2\ell_2 L,S,J|V_t(r)\hat{S}_{12}|n'_1\ell'_1 n'_2\ell'_2 L',S',J\rangle =$$

$$\sqrt{96\pi}\,(-)^{L+S-J} W(LL'SS;2J)\langle n_1\ell_1 n_2\ell_2 L\|V_t(r)Y_2(\theta,\phi)\|n'_1\ell'_1 n'_2\ell'_2 L'\rangle\,\delta_{SS'}\,\delta_{Sl}\,. \tag{20}$$

The reduced matrix element is given[21] in terms of the Moshinsky transformation brackets $\langle n\ell N\mathcal{L}|n_1\ell_1 n_2\ell_2 L\rangle$ from center- of-oscillator-well (labelled by $n_1\ell_1 n_2\ell_2 L$) to relative- ($n\ell$) center-of-mass ($N\mathcal{L}$) coordinates, by

$$\langle n_1 \ell_1 n_2 \ell_2 L \| V_t(r) Y_2(\theta,\phi) \| n'_1 \ell'_1 n'_2 \ell'_2 L' \rangle =$$

$$\sqrt{\frac{5}{4\pi}} \sum_{\substack{n\ell n'\ell' \\ N\mathcal{L}}} (-)^{\mathcal{L}-L-\ell'} \sqrt{(2L+1)(2L'+1)(2\ell+1)} \langle \ell 200 | \ell' 0 \rangle \times$$

$$\times\ W(LL'\ell\ell';2\mathcal{L}) \langle n\ell N\mathcal{L}, L | n_1 \ell_1 n_2 \ell_2, L \rangle \langle n'\ell' N\mathcal{L} L' | n'_1 \ell'_1 n'_2 \ell'_2 L' \rangle \times$$

$$\times\ (n\ell | V_t(r) | n'\ell') \tag{21}$$

where the sum in (21) is subjected to the (parity and energy conservation) restrictions

$$\ell' = \ell, \ell \pm 2 \tag{22}$$

$$n' = n + (n'_1 + n'_2 - n_1 - n_2) + \tfrac{1}{2}(\ell'_1 + \ell'_2 - \ell_1 - \ell_2) + \tfrac{1}{2}(\ell - \ell'). \tag{23}$$

The last factor in (21) is a harmonic oscillator integral of the type (16), but over the <u>relative</u> coordinate, and can be expressed in terms of the Talmi integrals

$$I_p = \frac{2}{\Gamma(p+\frac{3}{2})} \int_0^\infty r^{2p+2} e^{-r^2} V_t(r)\,dr \qquad (p = 0, 1, 2, \ldots, p_{max}) \tag{24}$$

as the linear combination

$$(n\ell | V_t(r) | n'\ell') = \sum_p B(n\ell, n'\ell', p) I_p \tag{25}$$

with coefficients $B(n\ell,n'\ell',p)$ defined by Brody, Jacob and Moshinsky[22] and tabulated in ref. (18). The integer p_{max} in (24) is given by

$$p_{max} = 2\nu \tag{26}$$

ν being the (principal quantum) number of oscillator quanta associated with each particle (ν = 1, 2, 3 respectively, for the p-, sd- and pf-shells).

Combining the above results, the matrix element (20), for configurations $(n_1\ell_1 n_2\ell_2)$ of a single major oscillator shell, reduces to

$$\langle n_1\ell_1 n_2\ell_2 L,1,J|V_t(r)\hat{S}_{12}|n'_1\ell'_1 n'_2\ell'_2 L',1,J\rangle =$$

$$= (-)^{J+1}\sqrt{120(2L+1)(2L'+1)}\ W(LL'11;2J) \sum_{n\ell\ell' N\mathcal{L}} \sqrt{2\ell+1}\ \langle \ell 200|\ell' 0\rangle \times$$

$$\times\ W(LL'\ell\ell';2L)\langle n\ell N\mathcal{L},L|n_1\ell_1 n_2\ell_2,L\rangle\langle n'\ell' N\mathcal{L},L'|n'_1\ell'_1 n'_2\ell'_2 L'\rangle \times$$

$$\times \sum_{p=0}^{2\nu} B(n\ell,n'\ell',p)I_p. \tag{27}$$

If the radial function $V_{t(r)}$ is taken as constant (long-ranged interaction), the last summation in (27) becomes $\delta_{nn'}\delta_{\ell\ell'}$ in view of (25), thus considerably simplifying the evaluation of the corresponding matrix elements. The constant remaining can be absorbed into the strength parameter δ of the interaction model (3). The matrix elements were generated by electronic computer, and then transformed to jj-coupling by 9j coefficients to give the elements

$$\langle j_1 j_2 JT | f(T)\, \hat{S}_{12} | j'_1 j'_2 JT \rangle \tag{29}$$

where the isospin factor f(T), appearing in (3), was taken as

$$f(0) = 3; \qquad f(1) = 1 \tag{30}$$

to ensure that T=0 levels are more bound than T=1 levels.

Finally, the antisymmetrized matrix elements of $\hat{I}(1,2)\hat{Q}^2$ and $f(T)\,\hat{S}_{12}$ are constructed from (14) and (29), respectively, by the usual procedures, thereby fixing the secular equation matrix elements as linear combinations of the positive parameters (α, β, γ, δ) of the interaction model.

To facilitate the parameter variation in the computer solution of the secular equation, we reparameterized (3) by factoring out an overall positive quantity $V_o \equiv (\alpha+\beta+\gamma+\delta)$ such that (3) becomes

$$V_{model}(1,2) = -V_o[A\sum_{i=1}^{2} \vec{\ell}_i \cdot \vec{s}_i + B\hat{I}(1,2)\hat{Q}^2 + C\hat{I}(1,2)\,\hat{P} \tag{31}$$

$$+(1-A-B-C)f(T)\,\hat{S}_{12}]$$

and the new parameters A,B,C are restricted by

$$A,B,C \geq 0; \; A+B+C \leq 1 \tag{32}$$

to ensure overall attraction associated with every component of the interaction model. The parameters A, B, C were varied in steps of tenths within the interval $0 \leq A, B, C \leq 1$, giving for each nucleus 286 spectra to compare against the empirical spectra and determine the optimum parameters. In some cases, specifically Li^6, He^6, O^{18}, and Ca^{42}, several parameter sets (A, B, C) fitting (in sequence) the available empirical spectra were obtained. Selection of a unique set for each mass number was carried out on the basis of the calculated β-decay ft-value compared with experiment.

2. BETA-DECAY COMPARATIVE LIFETIMES (ft-VALUES)*

According to the (vector-axialvector) V-A law of weak interaction decay, the comparative half-life for a $\beta\mp$ emission is given by [24]

$$ft = f(Z,T_o)\left[\frac{\ell n2}{\lambda}\right] = \frac{\ell n2}{(g^2/2\pi^3\hbar^7c)(mc)^5}\ \frac{1}{M^2} = \frac{6140}{M^2}\ (\text{sec}) \qquad (33)$$

where $f(Z,T_o)$ is essentially the integrated β-ray energy spectrum as a function of charge number Z and end-point kinetic energy T_o, λ is the disintegration constant and M contains the matrix elements of the operators causing the transition, evaluated between initial and final nuclear states. In the so-called "allowed approximation"

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* This section was written by E. Ley-Koo, Universidad de México, México, D. F.

(leptons are emitted with zero orbital angular momentum relative to the parent nucleus) M^2 is given by

$$M^2 = C_V^2 \langle 1 \rangle^2 + C_A^2 \langle \sigma \rangle^2 \tag{34}$$

where C_V and C_A are the vector and axialvector coupling constants, respectively (C_V=1 and C_A=1.2) and

$$\langle 1 \rangle \equiv \langle f | \sum_i \tau_\pm^i | i \rangle \qquad \text{(for } \beta^\mp \text{ emission)} \tag{35}$$

is the "singlet" or Fermi β-moment, while

$$\sigma = \frac{\langle f | \sum_i \sigma_m^i \tau_\pm^i | i \rangle}{\langle J_f 1 M_{J_f} m | J_i M_{J_i} \rangle} \qquad \text{(for } \beta^\mp \text{ emission)} \tag{36}$$

is the "triplet" or Gamow-Teller β-moment, evaluated between initial and final nuclear states. The selection rules for Fermi and Gamow-Teller transitions follow from (35) and (36):

$\langle 1 \rangle = 0$ unless $\Delta J = 0$; $\pi_i \pi_f = +1$; $\Delta T = 0$

$\langle \sigma \rangle = 0$ unless $\Delta J = 0, \pm 1$; $\pi_i \pi_f = +1$

$= 0$ for $J_i = 0 \rightarrow J_f = 0$.

The empirical data on relevant ft-values are given in Table I.

TABLE I

PARENT	DAUGHTER	EMISSION	$J_iT_i \rightarrow J_fT_f$	ft (sec)	Ref.
${}_2He^6$	${}_3Li^6$	β^-	01 → 10	808 ± 32	(24)
${}_9F^{18}$	${}_8O^{18}$	β^+	10 → 01	4,169 ± 158	(24)
${}_{21}Sc^{42}$	${}_{20}Ca^{42}$	β^+	70 → 61	15,320 ± 2,405	(25)

The evaluation of $\langle\sigma\rangle$ (taking m=0 in expression (36)) between the basis states $|j_1j_2JM_J,TM_T\rangle$ of the initial and final nuclear state wave functions in straightforward angular momentum calculus. The values involved in the aforementioned transitions are given in Tables II, III, and IV below.

TABLE II

$\langle\sum\sigma_o\tau_+\rangle$		J=0 M_J=0 T=1 M_T=1	
		$(p_{3/2})^2$	$(p_{1/2})^2$
J=1	$(p_{3/2})^2$	$\frac{\sqrt{10}}{3}$	0
M_J=1 T=0	$(p_{3/2}p_{1/2})$	$-\frac{2}{3}$	$-\frac{2\sqrt{2}}{3}$
M_T=0	$(p_{1/2})^2$	0	$-\frac{\sqrt{2}}{3}$

TABLE III

$\langle \sum \sigma_o^T - \rangle$		$J = 1 \quad M_J = 0 \quad T = 0 \quad M_T = 0$				
		$(d_{5/2})^2$	$(d_{5/2}d_{3/2})$	$(d_{3/2})^2$	$(d_{3/2}s_{1/2})$	$(s_{1/2})^2$
$J=0$	$(d_{5/2})^2$	$\frac{\sqrt{14}}{\sqrt{15}}$	$-\frac{2\sqrt{2}}{\sqrt{15}}$	0	0	0
$M_J=0$	$(d_{3/2})^2$	0	$-\frac{2}{\sqrt{5}}$	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	0
$T=1$ $M_T=1$	$(s_{1/2})^2$	0	0	0	0	$\sqrt{2}$

TABLE IV

$\langle \sum \sigma_o^T - \rangle$		$(f_{7/2})^2$ $J=7$ $M_J=6$ $T=0$ $M_T=0$
$J=6$ $M_J=6$ $T=1$ $M_T=-1$	$(f_{7/2})^2$	$\sqrt{2}/7$
	$(f_{7/2}f_{5/2})$	$2\sqrt{3}/7$

3. RESULTS. COMPARISONS WITH EMPIRICAL DATA AND WITH "REALISTIC" POTENTIAL CALCULATIONS.

As mentioned at the end of Section 2, the search for the best parameters A, B, and C of the interaction model (31) was carried out by comparing each calculated spectrum with experiment, demanding that both level sequence and spacings be reproduced as best as possible.

For those few calculated spectra satisfying the observed level sequence, we computed the mean-square-deviation

$$\chi^2 = \frac{1}{n} \sum_{i=1} (\epsilon_i - V_o E_i)^2 \qquad (37)$$

in which ϵ_i are the empirical excitation energies, E_i the corresponding calculated eigenvalues normalized to $\epsilon_o = E_o = 0$, V_o the scaling parameter appearing in (31) and n the $n^{\underline{th}}$ excited state to which the fitting was carried out for each particular mass number A. The overall strength parameter V_o in expression (3) is determined by the condition $\partial\chi^2/\partial V_o = 0$. In cases where several calculated spectra resulted reasonably acceptable from the χ^2 criterion, we chose the one best reproducing the experimental ft-value for β decay. For both mass 18 and 42 nuclei, it was found necessary to <u>reduce</u> the strength of the pairing force matrix elements (9) by a factor equal to 1/3 for the sd-shell, 1/16 for Sc^{42} and 1/50 for Ca^{42} in the pf-shell. The results are displayed in Figures 1 to 5.

a. 1 p-Shell Nuclei. He^6 and Li^6.

In Figure 1 are shown: the even parity experimental spectrum for the T=0 and 1 levels of the A=6 system taken from Ajzenberg-Selove et al,[26] the results of this calculation (labelled "modelistic") and those of Dawson and Walecka who used the BGT potential[1] (labelled "realistic"). The first excited state of He^6 is known to be $J^\pi T = 2^+1$ with $\sim$ 1.8 Mev excitation relative to its $J^\pi T = 0^+1$ ground state. It appears no other excited states below $\sim$ 12 Mev excitation have been established. Our calculation yielded the three acceptable T=0,1 spectra:

	Li^6, He^6		
A, B, C, D	0.3, 0.6, 0.0, 0.1	0.4, 0.6, 0.0, 0.0	0.4, 0.5, 0.0, 0.1
χ^2 (Mev)	0.1113	0.1575	0.3762
ft(sec)($0^+1 \rightarrow 1^+0$)	797.7	800.5	819.8

for n=5 (levels $J^\pi T = 3^+0$, 0^+1, 2^+0, 2^+1, 1^+0 respectively.) The calculated spectrum with parameter values (A, B, C, D) = (0.3, 0.6, 0.0, 0.1) ($D \equiv 1-A-B-C$) is given in Figure 1, and we notice excellent agreement in regard to sequence, and reasonable agreement in level spacings. A considerable energy gap is found above the second $J^\pi T = 1^+0$ level, in accordance with experiment. The calculated ft-value is well within the experimental margin of error (Table I). The normalized eigen wave functions of this spectrum are given in Table V.

TABLE V

Wave functions for Li^6, He^6 with interaction model parameters

(A, B, C, D) = (0.3, 0.6, 0.0, 0.1)

$J^{\pi}T$	exc	$(p_{3/2})^2$	$(p_{3/2}p_{1/2})$	$(p_{1/2})^2$
1^+0	grd	-0.675	0.719	0.166
3^+0	1	1.000		
0^+1	2	0.938		0.346
2^+0	3		1.000	
2^+1	4	0.784	0.621	
1^+0	5	-0.373	-0.526	0.764
2^+1	6	-0.621	0.784	
0^+1	7	-0.346		0.938
1^+1	8		1.000	
1^+0	9	0.637	0.454	0.623

The percentage figures given with the ground Li^6 and He^6 states of Dawson and Walecka denote the overlap between the state functions of the "modelistic" and "realistic" calculations. Both overlaps are seen to be quite large.

b. 2s - 1d Shell Nuclei. O^{18} and F^{18}.

The empirical spectra for O^{18} and F^{18} are taken respectively from Hewka et al[27] and Polletti and Warburton.[28] The recent crucial spin and parity assignments in F^{18} of Chagnon,[29] Polletti[30] and Matcus et al[31]

have been included in Figure 2. No parameter set (A, B, C, D) ($D \equiv 1-A-B-C$) was able to reproduce the correct level sequence beyond the $J^{\pi}T = 5^{+}0$ level at 1.125 Mev: in the optimum calculated spectrum several levels appeared below the $J^{\pi}T = 1^{+}0$ level (empirically at 1.70 Mev)--a puzzling feature shared also with the Kuo and Brown[4)] calculation using a Hamada-Johnston potential. On the basis of the first three excited positive parity levels, we obtained the three acceptable fits:

A, B, C, D	0.3, 0.2, 0.1, 0.4	0.2, 0.2, 0.3, 0.3	0.3, 0.2, 0.2, 0.3
χ^2	0.00003	0.0050	0.0023
ft(sec)($1^{+}0 \rightarrow 0^{+}1$)	5496	4448.88	4719.81

and in Figures 2 and 3 are given the spectra of F^{18} and O^{18} obtained with the last set. The associated nuclear state functions are shown in Tables VI and VII.

TABLE VI

Wave functions of F^{18} for interaction model parameters

(A, B, C, D) = (0.3, 0.2, 0.2, 0.3).

(Note: Pairing term rescaled by factor of 1/3.)

$J^{\pi}T$	exc.	$(d_{5/2})^2$	$(d_{5/2}d_{3/2})$	$(d_{5/2}s_{1/2})$	$(d_{3/2})^2$	$(d_{3/2}s_{1/2})$	$(s_{1/2})^2$
1^+0	grd	0.220	-0.611		0.072	0.490	0.577
3^+0	1	0.439	-0.241	0.865	0.026		
0^+1	2	0.929			0.146		0.342
5^+0	3	1.000					
2^+1	4	0.777	0.156	0.574	0.088	-0.187	
2^+0	5		0.588	0.652		0.479	
1^+0	9	-0.584	0.460		0.098	0.045	0.660

TABLE VII

Same as Table VI, but relevant to O^{18} spectrum

$J^{\pi}(T=1)$	exc.	$(d_{5/2})^2$	$(d_{5/2}d_{3/2})$	$(d_{5/2}s_{1/2})$	$(d_{3/2})^2$	$(d_{3/2}s_{1/2})$	$(s_{1/2})^2$
0^+	grd	0.929			0.146		0.342
2^+	1	0.777	0.156	0.574	0.088	-0.187	
4^+	2	0.983	0.186				
2^+	3	-0.606	-0.029	0.778	-0.008	-0.163	
0^+	4	-0.357			0.097		0.929
3^+	5		1.000	0.000			
4^+	6	-0.186	0.983				

The spectrum predictions of both "modelistic" and "realistic" calculations are similar below $\sim$ 5 Mev in O^{18} and below $\sim$ 1.5 Mev in F^{18}. In F^{18}, serious discrepancies are noted in both calculations with respect to the empirical sequence, but we notice that overlaps between "modelistic" and "realistic" calculations are rather high. Also in F^{18}, the third 0^+ and 2^+ levels (experimentally at 5.33 and 5.25 Mev) are predicted too excited in both calculations. Recent work by Federman and Talmi[33] shows that these two levels may be depressed considerably in the calculation by introducing sizeable amounts of 4 particle-2hole configurations into the wave functions.

c. 2p - 1f Shell Nuclei. Ca^{42} and Sc^{42}.

Mass 42 empirical spectra are found in Nelson et al[34] and Rivet et al.[35] Using the interaction model, the best fit to Sc^{42} (T=0 and T=1) low-lying levels was obtained for (A, B, C, D) = (0.2, 0.1, 0.4, 0.3) and to the Ca^{42}(T=1) lowest $J^{\pi} = 0^+, 2^+, 4^+, 6^+$ sequence with parameters (0.4, 0.1, 0.1, 0.4), displayed on Figures 4 and 5. The low-lying state functions are given in Tables VIII and IX. In the latter, it becomes evident that the Ca^{42} $0^+, 2^+, 4^+, 6^+$ sequence mentioned above is predominantly a pure $(1f_{7/2})^2$ configuration. The isobaric T=1 states in Sc^{42} also reveal the same feature (Table VIII). The calculated ft-value of

$$ft = 12{,}312 \text{ sec}, \qquad Sc^{42}(7^+0) \rightarrow Ca^{42}(6^+1)$$

β-transition is just slightly outside the experimental margin of error (Table I). Few electromagnetic transition rates have been reported for

TABLE VIII

Wave functions of Sc^{42} for interaction model parameters (A, B, C, D) = (0.2, 0.1, 0.4, 0.3).

(Note: Pairing term rescaled by factor of 1/16.)

$J^{\pi}T$	exc	$(f_{7/2})^2$	$(f_{7/2}f_{5/2})$	$(f_{7/2}p_{3/2})$	$(f_{7/2}p_{1/2})$	$(f_{5/2})^2$	$(f_{5/2}p_{3/2})$	$(f_{5/2}p_{1/2})$	$(p_{3/2})^2$	$(p_{3/2}p_{1/2})$	$(p_{1/2})^2$
0^+1	grd	0.966				0.076			0.237		0.070
7^+0	1	1.000									
1^+0	2	0.204	-0.483			0.026	0.453		0.302	-0.623	0.202
5^+0	3	0.640	-0.129	0.758		-0.001					
2^+1	4	0.995	0.058	0.043		0.058	-0.002	0.010	0.011	0.019	
4^+1	6	0.994	0.100	0.018	0.032	0.035	-0.018				
6^+1	7	0.9942	0.108								
3^+0	8	0.943	-0.258	-0.092	-0.122	-0.069	0.077	0.059	-0.086		

TABLE IX

Wave functions of Ca^{42} for interaction model parameters (A, B, C, D) = (0.4, 0.1, 0.1, 0.4).

(Note: Pairing term rescaled by factor of 1/50.)

$J^{\pi}(T=1)$	exc	$(f_{7/2})^2$	$(f_{7/2}f_{5/2})$	$(f_{7/2}p_{3/2})$	$(f_{7/2}p_{1/2})$	$(f_{5/2})^2$	$(f_{5/2}p_{3/2})$	$(f_{5/2}p_{1/2})$	$(p_{3/2})^2$	$(p_{3/2}p_{1/2})$	$(p_{1/2})^2$
0^+	grd	0.999				-0.017			0.045		-0.004
2^+	1	0.999	0.028	0.017		0.029	-0.002	0.004	0.004	0.008	
4^+	2	0.999	0.050	0.003	0.014	0.017	-0.007				
6^+	3	0.999	0.048								

Ca^{42} and, to our knowledge, none for Sc^{42}. Again, the gross features of the "modelistic" and "realistic" (Kuo, ref.5) spectra are similar in many respects. Worth mentioning perhaps, is that the 2.22 Mev level in Sc^{42} with $J^{\pi} = (3^+, 5^+)$ assignments is indicated to be an unresolved doublet in both calculations. In Ca^{42}, the second 0^+ and 2^+ states are predicted too high: these may be brought down to their correct positions by introducing large 4 particle-2 hole configurations, as was shown by Federman[32]. Again, overlaps between "modelistic" and "realistic" wave functions are seen to be large.

The fact that <u>two</u> distinct interaction model parameter sets were required to fit Sc^{42} and Ca^{42} may be an indication of the inadequacy of the Rosenfeld exchange mixture in this mass region. This is perhaps to be expected as Sc^{42} is one of the five or so exceptional cases throughout the periodic table in which the lowest T=1 level is <u>below</u> the lowest T=0 level.

4. CONCLUSIONS

By fitting observed low-lying spectra and β-decay moments of mass number A = 6, 18, and 42 nuclei with a typical shell-model modelistic interaction, we have essentially determined the "effective" or "residual" interaction for these nuclei as a fixed linear combination of simple <u>single-body</u> <u>spin-orbit</u>, <u>quadrupole-quadrupole</u>, <u>pairing</u> and <u>tensor</u> operators. These interactions may conceivably be used in calculations of $n > 2$ valence-particle-nuclei, with fractional-parentage or other techniques.

Since in all cases studied the parameter D fails to vanish, a definite requirement of some form of tensor interaction is indicated for these two-valence-particle nuclei, i.e., for a given mass number A, the spectra resulting with $D = 0$ were not better than those with $D \neq 0$. The tensor interaction used here is however only a long-ranged one: it would be interesting to add to the interaction model a short-ranged one (radial function $V_t(r) = \delta(r)$), but this would introduce an additional adjustable parameter.

In view of the many similarities between low-lying spectra and associated wave functions predicted by 1) the simple "modelistic" interaction and 2) calculations employing "realistic" phenomenological interactions such as the BGT and HJ potentials, and considering it ambiguous as to which of the two approaches reproduces the empirical data best, we find little sensitivity of the low-lying wave functions (if we exclude binding energy considerations) to the detailed nature of these "realistic" potentials, viz., a) repulsive hard-cores in all two-body states of relative motion, b) specific forms of radial dependencies associated with the central, two-body spin-orbit, tensor and quadratic spin-orbit interactions, and c) spin-parity dependence of the phenomenologically determined parameters of such potentials.

Finally, as either a "modelistic" or "realistic" interaction approach (restricted to the limited configurations-space of a single oscillator major shell) is clearly unable to reproduce several states observed experimentally, notably in O^{18} and Ca^{42}, and to give the correct sequence of other

states, notably F^{18} and Sc^{42} above $\sim$ 1.5 Mev, it appears generally more pressing to develop systematic methods to include more generalized configurations such as 3p - 1h, 4p - 2h, etc., than to employ complex, highly detailed phenomenological potentials fitting N - N scattering data to high energies.

APPENDIX I

Transformation coefficients* between 2-particle

SU_3 - and jj-coupling schemes in the 1p shell.

$$\langle [f](k_1 k_2)L,\ S,\ JT \,|\, j_1 j_2 JT \rangle$$

T = 0

$j_1 j_2$ \ $[f](k_1 k_2)$ LS	[2](20)01	[2](20)21	[11](11)10	[2](20)21	[2](20)21	J
$\frac{3}{2}\frac{3}{2}$	$\frac{1}{3}\sqrt{\frac{10}{3}}$	$-\frac{\sqrt{6}}{9}$	$\frac{\sqrt{5}}{3}$			
$\frac{3}{2}\frac{1}{2}$	$-\frac{4}{3\sqrt{3}}$	$-\frac{5}{9}\sqrt{\frac{3}{5}}$	$\frac{\sqrt{2}}{3}$			1
$\frac{1}{2}\frac{1}{2}$	$-\frac{1}{3\sqrt{3}}$	$\frac{10}{9}\sqrt{\frac{3}{5}}$	$\frac{\sqrt{2}}{3}$			
$\frac{3}{2}\frac{1}{2}$				1		2
$\frac{3}{2}\frac{3}{2}$					1	3

* Cf. Ref. (13) for details, as well as for similar tables for the 2s - 1d shell.

$T = 1$

$j_1 j_2$ \ $[f](k_1 k_2)_{LS}$	[2](20)00	[11](11)11	[11](11)11	[2](20)20	[11](11)11	J
$\frac{3}{2}\ \frac{3}{2}$	$\sqrt{\frac{2}{3}}$	$-\frac{1}{\sqrt{3}}$				0
$\frac{1}{2}\ \frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$				
$\frac{3}{2}\ \frac{1}{2}$			1			1
$\frac{3}{2}\ \frac{3}{2}$				$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$	2
$\frac{3}{2}\ \frac{1}{2}$				$\sqrt{\frac{2}{3}}$	$-\frac{1}{\sqrt{3}}$	

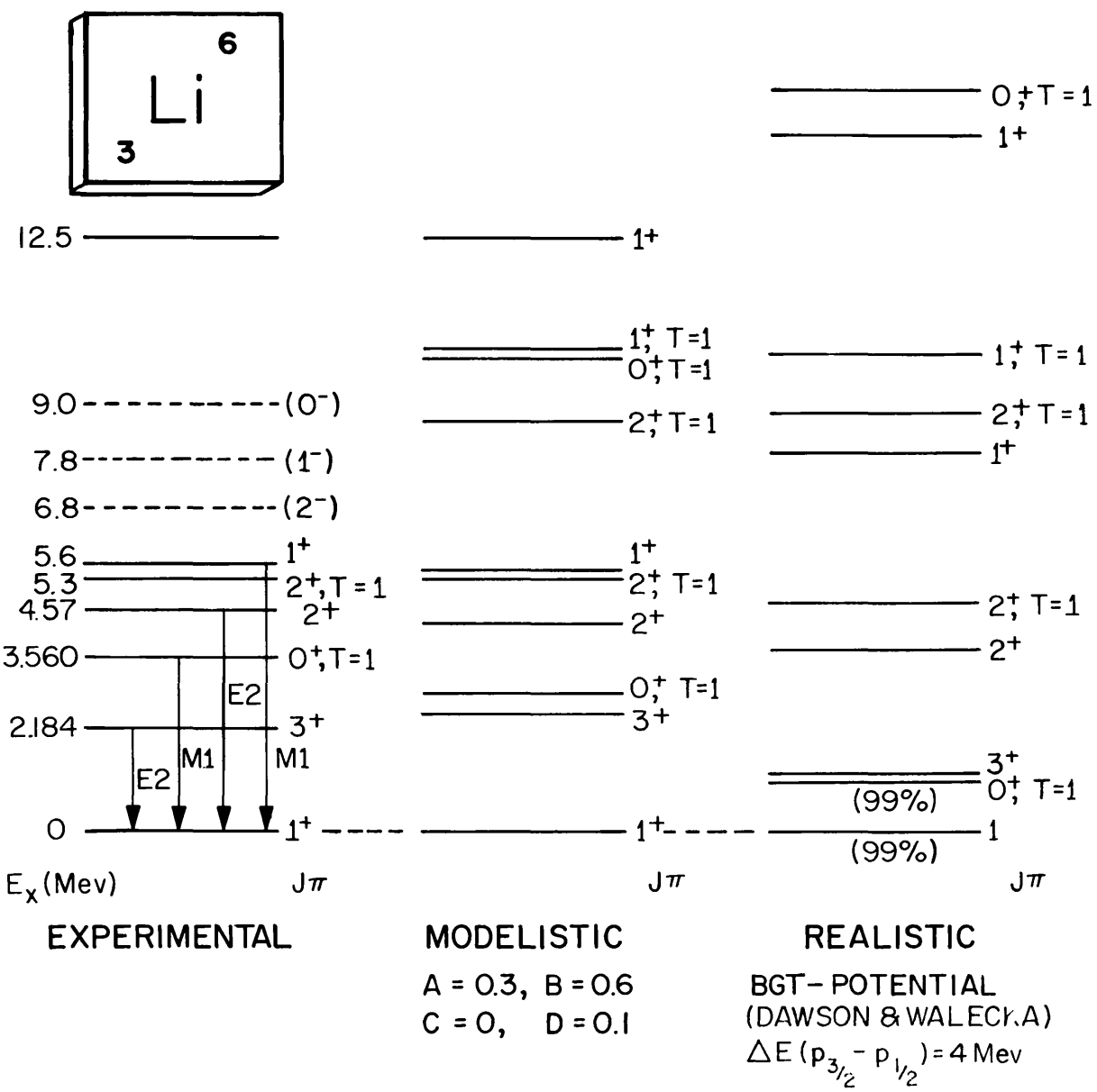
6
Li
3
12.5
9.0 (0⁻)
7.8 (1⁻)
6.8 (2⁻)
5.6 1⁺
5.3 2⁺, T = 1
4.57 2⁺
3.560 0⁺, T=1
2.184 3⁺
0 1⁺
E2
M1
M1
E2
Ex (Mev)
Jπ
EXPERIMENTAL
1⁺
1⁺, T=1
0⁺, T=1
2⁺, T=1
1⁺
2⁺, T=1
2⁺
0⁺, T=1
3⁺
1⁺
Jπ
MODELISTIC
A = 0.3, B = 0.6
C = 0, D = 0.1
0⁺, T = 1
1⁺
1⁺, T = 1
2⁺, T = 1
1⁺
2⁺, T=1
2⁺
3⁺
0⁺, T=1
(99%)
1
(99%)
Jπ
REALISTIC
BGT-POTENTIAL
(DAWSON & WALECKA)
ΔE (p3/2 - p1/2) = 4 Mev

Figure 1

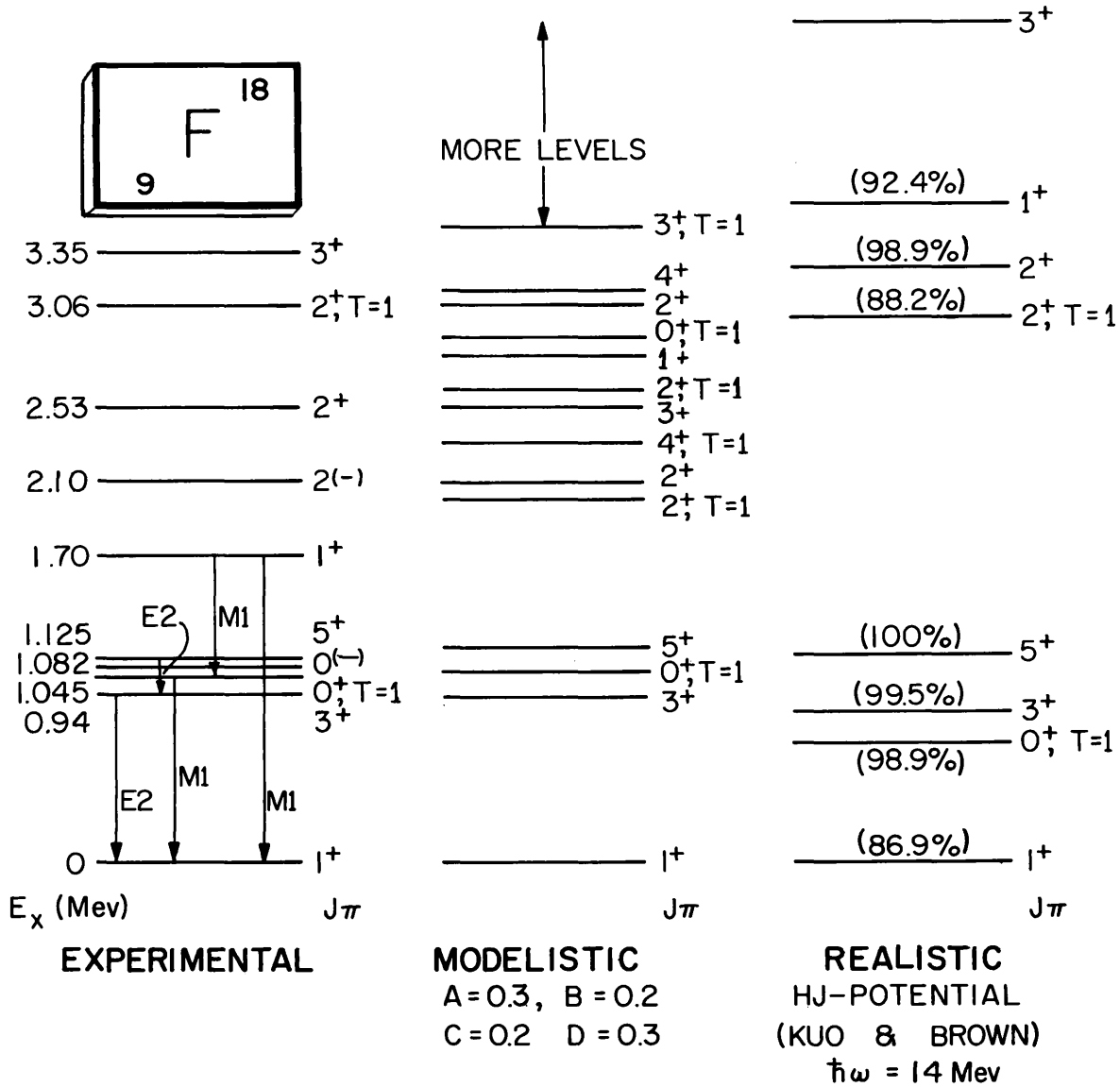
18
F
9
3.35
3+
3.06
2+; T=1
2.53
2+
2.10
2(-)
1.70
1+
E2
M1
1.125
5+
1.082
0(-)
1.045
0+; T=1
0.94
3+
E2
M1
M1
0
1+
Ex (Mev)
Jπ
EXPERIMENTAL
MORE LEVELS
3+; T=1
4+
2+
0+; T=1
1+
2+; T=1
3+
4+; T=1
2+
2+; T=1
5+
0+; T=1
3+
1+
Jπ
MODELISTIC
A=0.3, B=0.2
C=0.2 D=0.3
3+
(92.4%)
1+
(98.9%)
2+
(88.2%)
2+; T=1
(100%)
5+
(99.5%)
3+
0+; T=1
(98.9%)
(86.9%)
1+
Jπ
REALISTIC
HJ-POTENTIAL
(KUO & BROWN)
ħω = 14 Mev

Figure 2

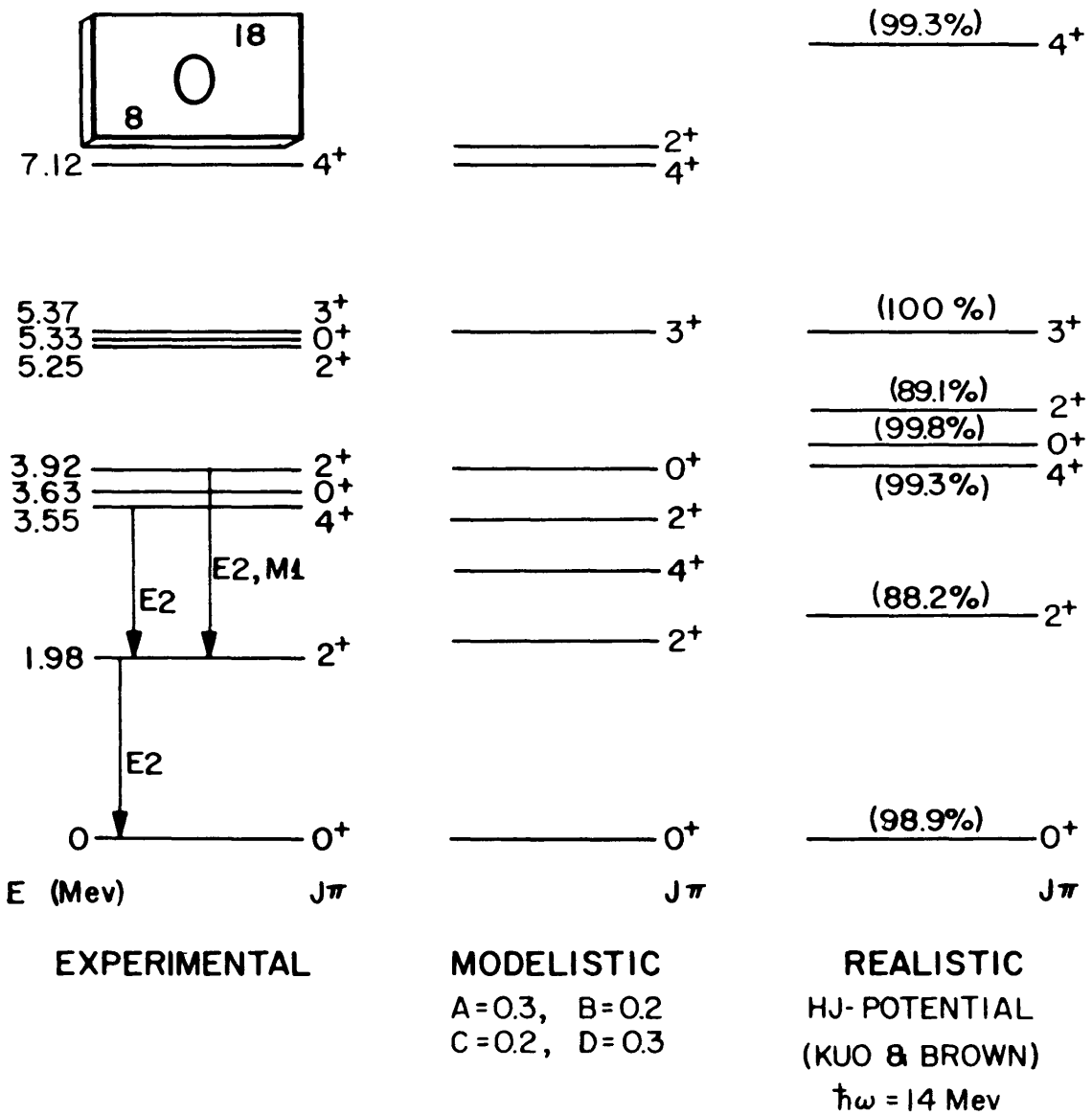
18
O
8
7.12
4+
5.37
5.33
5.25
3+
0+
2+
3.92
3.63
3.55
2+
0+
4+
E2
E2, M1
1.98
2+
E2
0
0+
E (Mev)
Jπ
2+
4+
3+
0+
2+
4+
2+
0+
Jπ
(99.3%)
4+
(100 %)
3+
(89.1%)
2+
(99.8%)
0+
4+
(99.3%)
(88.2%)
2+
(98.9%)
0+
Jπ
EXPERIMENTAL
MODELISTIC
A=0.3, B=0.2
C=0.2, D=0.3
REALISTIC
HJ-POTENTIAL
(KUO & BROWN)
ħω = 14 Mev

Figure 3

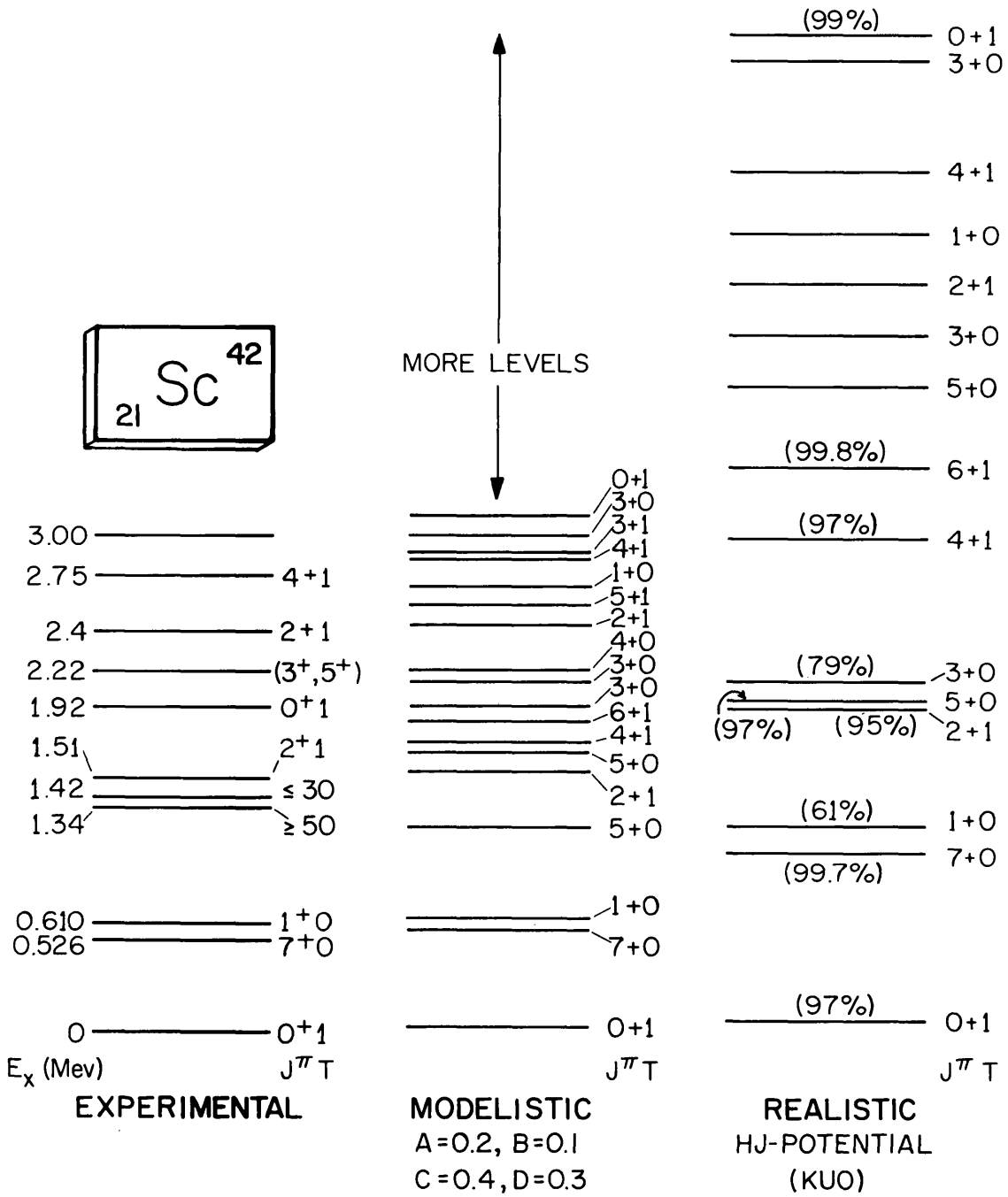
21 Sc 42
MORE LEVELS
3.00
2.75
4+1
2.4
2+1
2.22
(3+,5+)
1.92
0+1
1.51
2+1
1.42
≤ 30
1.34
≥ 50
0.610
1+0
0.526
7+0
0
0+1
Ex (Mev)
Jπ T
EXPERIMENTAL
0+1
3+0
3+1
4+1
1+0
5+1
2+1
4+0
3+0
3+0
6+1
4+1
5+0
2+1
5+0
1+0
7+0
0+1
Jπ T
MODELISTIC
A=0.2, B=0.1
C=0.4, D=0.3
(99%)
0+1
3+0
4+1
1+0
2+1
3+0
5+0
(99.8%)
6+1
(97%)
4+1
(79%)
3+0
5+0
(97%)
(95%)
2+1
(61%)
1+0
7+0
(99.7%)
(97%)
0+1
Jπ T
REALISTIC
HJ-POTENTIAL
(KUO)

Figure 4

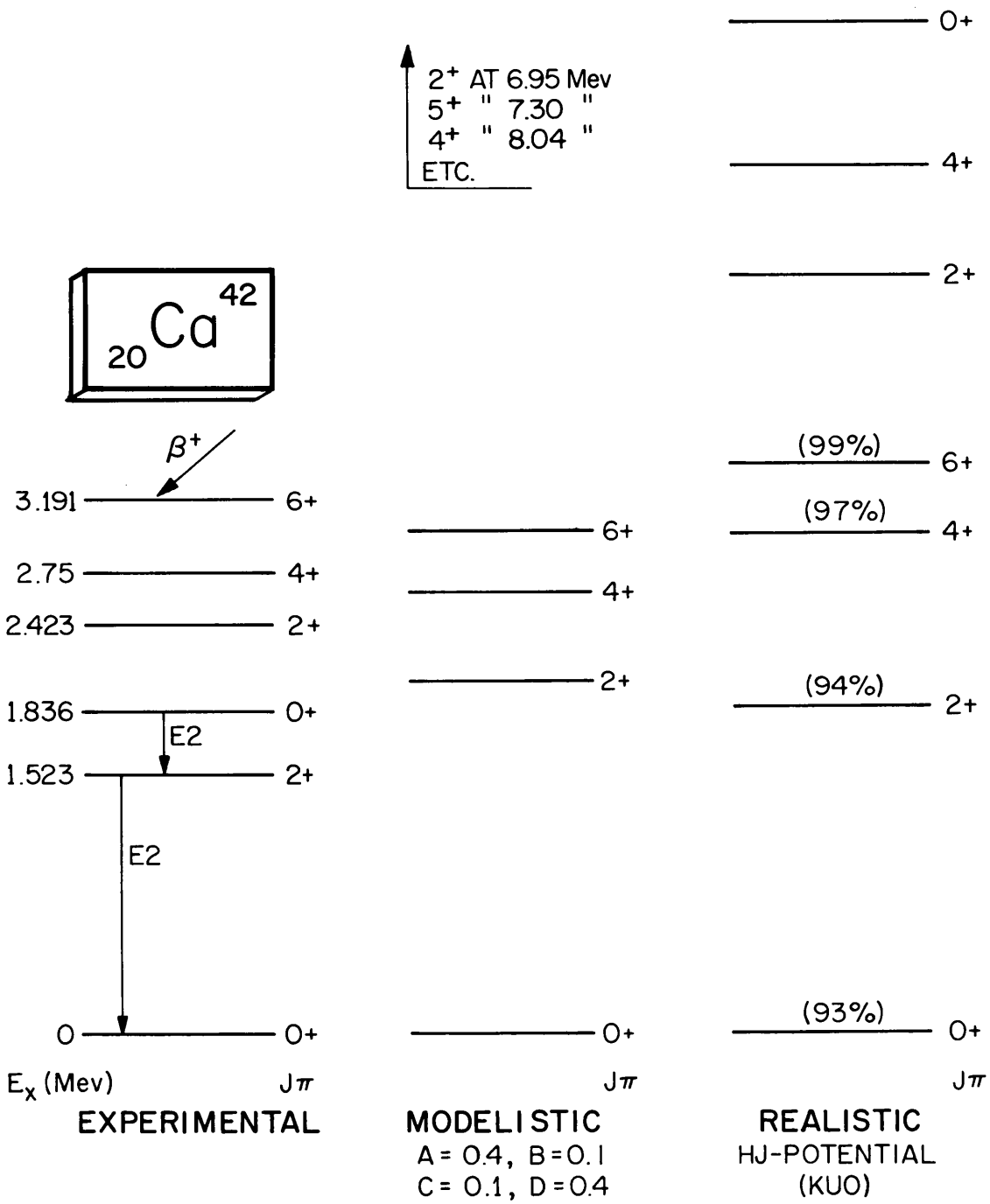
2+ AT 6.95 Mev
5+ " 7.30 "
4+ " 8.04 "
ETC.
20Ca42
β+
3.191 6+
2.75 4+
2.423 2+
1.836 0+
E2
1.523 2+
E2
0 0+
Ex (Mev)
Jπ
EXPERIMENTAL
6+
4+
2+
0+
Jπ
MODELISTIC
A = 0.4, B = 0.1
C = 0.1, D = 0.4
0+
4+
2+
(99%) 6+
(97%) 4+
(94%) 2+
(93%) 0+
Jπ
REALISTIC
HJ-POTENTIAL
(KUO)

Figure 5

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MODELISTIC AND REALISTIC INTERACTIONS IN THE 1p, 2s-1d AND 2p-1f NUCLEAR SHELLS

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13. ABSTRACT

The excitation spectra and β-decay comparative half-lives of He^6, Li^6, O^{18}, F^{18}, Ca^{42}, and Sc^{42} are calculated using a "modelistic" interaction containing pairing plus quadrupole-quadrupole operators (with Rosenfeld exchange), single-body spin-orbit and long-ranged tensor interactions. The resulting low-lying wave functions are compared to those obtained by other workers with hard-cored "realistic" interactions and are found to overlap to a high degree.

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