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An Analysis of a System of Pulse Coding for Clutter Suppression

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ABSTRACT

A system of pulse coding advanced by J. D. Mallett of the Rand Corporation to reduce the effect of ground clutter on airborne search radars has been examined and found to be of questionable value for a search radar, although other applications may exist now and in the future. The system would suppress a selected group of lines in the transmitted spectrum by means of a proper combination of the amplitudes, phases, and pulse spacings of a small group of successive pulses. The conditions for this suppression are examined in detail for an infinitely long series of pulse groups, and it is found that with a group of N pulses, $N-1$ spectrum lines may be suppressed. These lines may be any group of $N-1$, but it is shown that if they are symmetrically placed about the carrier, the rf phase angles are ϕ and $\phi+\pi$ radians. The first-order effects on the line suppression of variations from the nominal values of the parameters are analyzed, and it is shown that variations in the interpulse spacing must be kept below 1-1/2 parts in 100,000, the phase to within 0.1 degree, and the amplitude to within 1 part in 100,000 for reliable detection with a 40-db clutter-to-signal ratio. Finally, the effect of finite dwell time is examined, and it is shown that under the same conditions as above at least several thousand pulse groups need to be processed, which would require a time on target of about 18 sec.

PROBLEM STATUS

This is a final report on one phase of the problem; work on other phases of this problem is continuing.

AUTHORIZATION

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AN ANALYSIS OF A SYSTEM OF PULSE CODING FOR CLUTTER SUPPRESSION

INTRODUCTION

This report is an analysis of a technique designed to reduce the effect of ground clutter in airborne search radars by using waveforms coded by periodic variations in pulse amplitudes, phases, and spacings. The effect of ground clutter in a pulse-doppler radar can be appreciated by examining the spectrum of the echo return. For an infinite pulse train, the return from a point target consists of a line spectrum; the clutter or ground return broadens each of these prf lines by an appreciable amount (1). The effect of a target moving with constant velocity is to shift all of the spectral lines by an amount proportional to the velocity, but line broadening does not occur.

In current systems delay line cancelers are used to suppress the clutter while passing the signal from a moving target. Those targets whose corresponding doppler shifts are not equal to an integral number of prf lines are not suppressed by the delay line canceler; thus they are detected in spite of the rather poor signal-to-clutter ratio of the incoming signal. However, a serious problem with this type of canceler is that targets whose doppler shifts are approximately equal to an integral multiple of the prf will be indistinguishable from the clutter and will be suppressed. This is the so-called doppler blind speed problem.

In an effort to solve this problem a pulse coding scheme has been proposed (2) which is designed to produce a gap in the spectrum. If the gap, usually taken around the carrier frequency of the radar, is sufficiently wide, the clutter broadening of the prf lines adjacent to the gap will not fill it, but the target return will shift some one of the other lines into this clear area. The presence of this nonclutter frequency in the returned spectrum can be detected. Target velocities above a certain minimum will cause some prf line to appear in the clear area. Thus the faster targets (presumably the more interesting ones) will be separated from the stationary or at least slowly moving clutter. This desired spectrum gap is produced by suppressing one or more of the prf lines of the spectrum by the judicious choice of the amplitudes, phases, and spacings of a basic group of pulses. This group is then repeated over and over again.

This report will first derive the necessary equations to show that indeed the technique works, at least theoretically, for an infinite pulse train. Relationships are next derived which yield the deterioration in performance as a function of the tolerances allowed in the required pulse amplitudes, phases, and spacings. A specific example is used to illustrate the stringency required in maintaining these parameters under operational conditions. Finally the degradation due to a finite dwell time is considered.

SUMMARY

The first subsection of the analysis section of this report contains a derivation of the spectrum resulting from the infinite periodic coded pulse waveform proposed to reduce the clutter. Formulas for the spectral lines are determined using the methods of Fourier series, yielding for the Nth line

$$S_N = \sum_{k=0}^{m} \frac{1}{T} \int_{\sum_{i=0}^{k} \tau_i - (\ell/2)}^{\sum_{i=0}^{k} \tau_i + (\ell/2)} A_k \sin(\Omega + \phi_k)\, e^{jN\omega t}\, dt$$

where the A_k, φ_k, and τ_i are the amplitudes, phases, and spacings of the pulses in the m-pulse code group,

$$T \text{ is } \sum_{i=0}^{N} \tau_i ,$$

Ω is the carrier frequency, ℓ is the duration of the individual pulses, and $\omega = 2\pi/T$.

Employing an m-pulse code group, it is possible to cancel $m-1$ of these spectral lines. Any $m-1$ lines can be canceled, but choosing these lines symmetrically about the carrier results in easily obtained phase angles. Thus using an even length code, we cancel the carrier, $(m-2)/2$ upper sidebands, and $(m-2)/2$ lower sidebands. Working out the details for the cancellation, the phase angles for any code are $\varphi, \varphi+\pi, \varphi, \varphi+\pi, \ldots$. That is, each pulse is π radians out of phase with the preceding pulse. By solving a set of $m-1$ homogeneous simultaneous linear equations in m unknowns (the amplitudes of the pulses) we achieve a solution to the desired cancellation. By elementary linear algebra, a solution for this set of equations always exists.

For a four-pulse code with spacings τ_0, τ_1, τ_2, and τ_3, these equations are

$$A_0 - A_1 + A_2 - A_3 = 0$$

$$A_0 \sin \omega\tau_1 - A_2 \sin \omega(\tau_1 + \tau_2) + A_3 \sin \omega(\tau_1 + \tau_2 + \tau_3) = 0$$

$$A_0 - A_1 \cos \omega\tau_1 + A_2 \cos \omega(\tau_1 + \tau_2 + \tau_3) - A_3 \sin \omega(\tau_1 + \tau_2 + \tau_3) = 0$$

where, of course, ω, τ_0, τ_1, τ_2, and τ_3 are constants once a particular code has been selected. It is also shown the doppler width of the spectrum gap is essentially invariant under length of code. The width of the gap is a function of the unambiguous range only and cannot be increased by increasing the length of the code.

In Appendix A the solution for the phase angles φ_i will be shown to be unique. Thus it will be shown that a unique solution for the cancellation always exists.

The second subsection of the analysis section considers the sensitivity of the cancellation technique to perturbations in the nominal values of A_i, φ_i, and τ_i, the nominal values being those required for complete cancellation. An analysis is made of the residue resulting from deviations in the nominal values, and formulas are derived for the residues when only one parameter at a time is permitted to vary. Although the sensitivity of the system to variations does depend to some extent upon the particular code chosen, the figures quoted for the four-code, 4, 5, 6, 7, used in the illustrative example section, are representative of any code chosen. To be precise, using a 15-db signal-to-noise ratio for target detection, the ability to detect a target with a 40-db clutter-to-signal ratio requires the following tolerances:

Parameter	Tolerance (max. allowable error)
A_i	1.6 parts in 10,000
φ_i	2.0 parts in 1000
τ_i	1.5 parts in 10,000

Since τ_i is approximately 10^{-4} sec (for the necessary unambiguous range), this gives a tolerance of 10^{-8} sec for the variation in τ_i from its nominal value. The maximum allowable error in phase from nominal is about 0.1°.

In the final section the finite dwell time problem is examined with respect to this form of coded waveform. Throughout the analysis it has been assumed that the integration is carried out over an infinite train of pulse groups. In practice the integration will be over a few pulse groups, perhaps 1 to 4, at best. The spectrum will thus be continuous and not a discrete line spectrum as assumed. Based on the work of George and Zamanakos, (3) who studied the finite dwell time problem, a large number are required to achieve the approximate line spectrum necessary for adequate cancellation. Otherwise the clutter between the spectral lines S_N fills the gap and makes detection impossible. In terms of the system this means holding the beam in the same direction for many seconds.

This dwell-time requirement and the rather strict tolerances make the scheme of questionable value in a search radar, although other applications for this particular coding scheme may exist.

ANALYSIS

Coding

For purposes of illustration we shall consider that the basic group consists of four pulses; in the language of Ref. 2 this is a four-element code. The generalization of this to any number of pulses will be obvious. If we denote the time elapsed from the start of a given four-pulse group to when it starts again by T we can look on this process as being four interleaved pulse trains of the same period T but displaced one from the other by different amounts, as is shown in Fig. 1. We label, with no loss of generality, the shortest interpulse period by τ_1. The amplitudes and phases of the pulses are indicated in the figure.

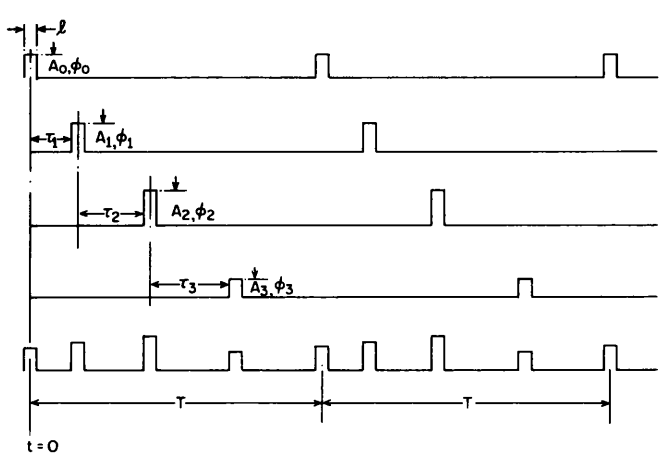

Fig. 1 - Four-element code

The spectrum of an infinite series of rf pulses is given by the Fourier series representation

$$F(t) = \sum_{-\infty}^{\infty} a_n(\omega)\, e^{jn\omega t} \tag{1a}$$

$$a_n(\omega) = \frac{1}{T}\int_{-T/2}^{T/2} F(t)\, e^{-jn\omega t}\, dt \tag{1b}$$

where $\omega = 2\pi/T$. In our case we have four different time functions:

$$F_0(t) = A_0 \sin(\Omega t + \phi_0), \quad nT - \frac{\ell}{2} \le t \le \frac{\ell}{2} + nT$$

$$F_1(t) = A_1 \sin(\Omega t + \phi_1), \quad nT + \tau_1 - \frac{\ell}{2} \le t \le \frac{\ell}{2} + \tau_1 + nT$$

$$F_2(t) = A_2 \sin(\Omega t + \phi_2), \quad nT + \tau_1 + \tau_2 - \frac{\ell}{2} \le t \le \frac{\ell}{2} + \tau_1 + \tau_2 + nT$$

$$F_3(t) = A_3 \sin(\Omega t + \phi_3), \quad nT + \tau_1 + \tau_2 + \tau_3 - \frac{\ell}{2} \le t \le \frac{\ell}{2} + \tau_1 + \tau_2 + \tau_3 + nT$$

$$F_0 = F_1 = F_2 = F_3 = 0, \quad \text{all other } t$$

where Ω is the rf frequency. We may then write

$$a_n^0(\omega) = \frac{1}{T}\int_{-T/2}^{T/2} A_0 \sin(\Omega t + \phi_0)\, e^{-jn\omega t}\, dt\ . \tag{2}$$

Replacing the sine by its equivalent exponential form,

$$a_n^0(\omega) = \frac{A_0}{T}\int_{-\ell/2}^{\ell/2} \frac{e^{j(\Omega t + \phi_0 - n\omega t)} - e^{-j(\Omega t - \phi_0 + n\omega t)}}{2j}\, dt\ . \tag{3}$$

Hence,

$$a_n^0(\omega) = \frac{A_0}{2jT}\left[e^{j\phi_0}\frac{1}{j(\Omega - n\omega)}\, e^{j(\Omega - n\omega)t}\Big|_{-\ell/2}^{\ell/2} + e^{-j\phi_0}\frac{1}{j(\Omega + n\omega)}\, e^{-j(\Omega + n\omega)}\Big|_{-\ell/2}^{\ell/2}\right]$$

or

$$a_n^0(\omega) = \frac{A_0}{2jT}\left\{e^{j\phi_0}\frac{\sin[(\Omega - n\omega)\ell/2]}{(\Omega - n\omega)} - e^{-j\phi_0}\frac{\sin[(\Omega + n\omega)\ell/2]}{(\Omega + n\omega)}\right\}. \tag{4}$$

Since the sum over n goes from $-\infty$ to $+\infty$, we define

$$C_n^{(0)} \equiv a_{|n|}^0\, e^{j|n|\omega t}$$

and

$$C_{-n}^{(0)} \equiv a_{-|n|}^0\, e^{-j|n|\omega t}$$

and form

$$K_n^{(0)}(t) = C_n^{(0)} + C_{-n}^{(0)}$$

so that

$$F_0(t) = \sum_0^\infty K_n^{(0)}(t)$$

$$K_n^{(0)}(t) = \frac{A_0}{jT}\left[e^{j(\phi_0+n\omega t)}\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)} - e^{-j(\phi_0-n\omega t)}\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega+n\omega)}\right.$$

$$\left. + e^{j(\phi_0-n\omega t)}\frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)} - e^{-j(\phi_0+n\omega t)}\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)}\right] \tag{5}$$

which simplifies to

$$K_n^{(0)}(t) = A_0\frac{e}{T}\left[\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)\ell/2}\sin(\phi_0+n\omega t) + \frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)\ell/2}\sin(\phi_0-n\omega t)\right]. \tag{6}$$

We now proceed in similar fashion with $F_1(t)$:

$$a_n^{(1)}(\omega) = \frac{1}{T}\int_{\tau_1-(\ell/2)}^{\tau_1+(\ell/2)} A_1\sin(\Omega t+\phi_1)\,e^{-jn\omega t}\,dt \tag{7}$$

$$a_n^{(1)}(\omega) = \frac{A_1}{2Tj}\int_{\tau_1-(\ell/2)}^{\tau_1+(\ell/2)}\left[e^{j(\Omega t+\phi_1-n\omega t)} - e^{-j(\Omega t+\phi_1+n\omega t)}\right]dt \tag{8}$$

$$a_n^{(1)}(\omega) = \frac{A_1}{2Tj}\left[e^{j\phi_1}\frac{1}{j(\Omega-n\omega)}e^{j(\Omega-n\omega)t}\Big|_{\tau_1-\ell/2}^{\tau_1+\ell/2} + e^{-j\phi_1}\frac{1}{j(\Omega+n\omega)}e^{-j(\Omega+n\omega)t}\Big|_{\tau_1-\ell/2}^{\tau_1+\ell/2}\right] \tag{9}$$

$$a_n^{(1)}(\omega) = \frac{A_1}{2Tj}\left\{\frac{1}{j(\Omega-n\omega)}\left[e^{j(\Omega-n\omega)\tau_1+j(\Omega-n\omega)\ell/2+j\phi_1} - e^{-j(\Omega-n\omega)\tau_1-j(\Omega-n\omega)\ell/2+j\phi_1}\right]\right.$$

$$\left. + \frac{1}{j(\Omega+n\omega)}\left[e^{-j(\Omega+n\omega)\tau_1-j(\Omega-n\omega)\ell/2-j\phi_1} - e^{j(\Omega+n\omega)\tau_1+j(\Omega+n\omega)\ell/2-j\phi_1}\right]\right\} \tag{10}$$

$$a_n^{(1)}(\omega) = \frac{A_1}{2Tj}\left\{e^{j[(\Omega-n\omega)\tau_1+\phi_1]}\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)\ell/2} - e^{-j[(\Omega+n\omega)\tau_1+\phi_1]}\frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)\ell/2}\right\} \tag{11}$$

which yields

$$K_n^{(1)}(t) = A_1\frac{\ell}{T}\left\{\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)\ell/2}\sin[n\omega t+(\Omega-n\omega)\tau_1+\phi_1]\right.$$

$$\left. + \frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)\ell/2}\sin[-n\omega t+(\Omega+n\omega)\tau_1+\phi_1]\right\}. \tag{12}$$

Similarly,

$$K_n^{(2)}(t) = A_2\frac{\ell}{T}\left\{\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)\ell/2}\sin[(\Omega-n\omega)(\tau_1+\tau_2)+\phi_2+n\omega t]\right.$$

$$\left. + \frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)\ell/2}\sin[(\Omega+n\omega)(\tau_1+\tau_2)+\phi_2-n\omega t]\right\}. \tag{13}$$

The extension to $K_n^{(3)}(t)$ (or to $K_n^{(m)}(t)$ for that matter) is obvious. Hence the nth spectrum line is given by

$$S_n^{(4)} = \sum_{m=0}^{3} K_n^{(m)}(t)$$

in our example (or by

$$S_n^{(N)} = \sum_{m=0}^{N-1} K_n^{(m)}(t)$$

for an N-element code), which we can write as

$$\begin{aligned} S_n^{(4)} = \frac{\ell}{T}\Bigg(&\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)\ell/2}\{A_0 \sin(\phi_0+n\omega t) + A_1 \sin[(\Omega-n\omega)\tau_1+\phi_1+n\omega t] \\ &+ A_2 \sin[(\Omega-n\omega)(\tau_1+\tau_2)+\phi_2+n\omega t] + A_3 \sin[(\Omega-n\omega)(\tau_1+\tau_2+\tau_3)+\phi_3+n\omega t]\} \\ &+ \frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)\ell/2}\{A_0 \sin[\phi_0-n\omega t] + A_1 \sin[(\Omega+n\omega)\tau_1+\phi_1-n\omega t] \\ &+ A_2 \sin[(\Omega+n\omega)(\tau_1+\tau_2)+\phi_2-n\omega t] + A_3 \sin[(\Omega+n\omega)(\tau_1+\tau_2+\tau_3)+\phi_3-n\omega t]\}\Bigg) . \end{aligned} \tag{14}$$

Defining $\tau_0 \equiv 0$ we can write $S_n^{(N)}$ as

$$\begin{aligned} S_n^{(N)} = \frac{\ell}{T}\Bigg\{&\frac{\sin(\Omega-n\omega)\ell/2}{(\Omega-n\omega)\ell/2}\sum_{i=0}^{N-1} A_i \sin\left[(\Omega-n\omega)\sum_{k=0}^{i}\tau_k+\phi_i+n\omega t\right] \\ &+ \frac{\sin(\Omega+n\omega)\ell/2}{(\Omega+n\omega)\ell/2}\sum_{i=0}^{N-1} A_i \sin\left[(\Omega+n\omega)\sum_{k=0}^{i}\tau_k+\phi_i-n\omega t\right]\Bigg\} . \end{aligned} \tag{15}$$

In the previous development we have incorporated the negative frequency terms in the sum and we now have to concern ourselves only with those terms for which $n \geq 0$. The $n = 0$ or dc term is of no great interest to us, as we are dealing with a carrier frequency of about 1000 megacycles and the $(\sin x)/x$ envelope will insure that terms this far from the carrier will be substantially zero. In addition the terms in $1/(\Omega+n\omega)$ will be negligible in the vicinity of $n\omega \approx \Omega$; therefore we will no longer carry these terms in our analysis.

Examining Eq. (15) we see that $S_n^{(N)}$ will be zero if

$$\sum_{i=0}^{N-1} A_i \sin\left[(\Omega-n\omega)\sum_{k=0}^{i}\tau_k+\phi_i+n\omega t\right] = 0 .$$

We will assume that we have chosen Ω and T such that $\Omega T/2\pi = n'$, an integer. We will now examine the terms in n' and $n' \pm 1$ in $S_n^{(4)}(t)$, and since we will confine our attention to this four-element-code case in what follows, we will drop the (4) superscript from S_n:

$$S_{n'}(t) = \frac{\ell}{T}[A_0 \sin(\phi_0+n'\omega t) + A_1 \sin(\phi_1+n'\omega t) + A_2 \sin(\phi_2+n'\omega t) + A_3 \sin(\phi_3+n'\omega t)] \tag{16}$$

$$S_{n'+1}(t) = \frac{\ell}{T}\left(\frac{\sin \omega\ell/2}{\omega\ell/2}\left\{A_0 \sin\left[\phi_0 + (n'+1)\omega t\right] + A_1 \sin\left[\phi_1 + (n'+1)\omega t - \omega\tau_1\right]\right.\right.$$

$$\left.\left. + A_2 \sin\left[\phi_2 + (n'+1)\omega t - \omega(\tau_1+\tau_2)\right] + A_3 \sin\left[\phi_3 + (n'+1)\omega t - \omega(\tau_1+\tau_2+\tau_3)\right]\right\}\right) \quad (17)$$

$$S_{n'-1}(t) = \frac{\ell}{T}\left(\frac{\sin \omega\ell/2}{\omega\ell/2}\left\{A_0 \sin\left[\phi_0 + (n'-1)\omega t\right] + A_1 \sin\left[\phi_1 + (n'-1)\omega t - \omega\tau_1\right]\right.\right.$$

$$\left.\left. + A_2 \sin\left[\phi_2 + (n'-1)\omega t - \omega(\tau_1+\tau_2)\right] + A_3 \sin\left[\phi_3 + (n'-1)\omega t - \omega(\tau_1+\tau_2+\tau_3)\right]\right\}\right). \quad (18)$$

These can be simplified and put in the form

$$S_{n'}(t) = \frac{\ell}{T}\left(\cos n'\omega t\ \{A_0 \sin\phi_0 + A_1 \sin\phi_1 + A_2 \sin\phi_2 + A_3 \sin\phi_3\}\right.$$

$$\left. + \sin n'\omega t\ \{A_0 \cos\phi_0 + A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3\}\right) \quad (19)$$

$$S_{n'+1}(t) = \frac{\ell}{T}\frac{\sin \omega\ell/2}{\omega\ell/2}\left(\cos (n'+1)\omega t\ \left\{A_0 \sin\phi_0 + A_1 \sin(\phi_1 - \omega\tau_1) + A_2 \sin\left[\phi_2 - \omega(\tau_1+\tau_2)\right]\right.\right.$$

$$\left. + A_3 \sin\left[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)\right]\right\} + \sin (n'+1)\omega t\ \left\{A_0 \cos\phi_0 + A_1 \cos(\phi_1 - \omega\tau_1)\right.$$

$$\left.\left. + A_2 \cos\left[\phi_2 - \omega(\tau_1+\tau_2)\right] + A_3 \cos\left[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)\right]\right\}\right) \quad (20)$$

$$S_{n'-1}(t) = \frac{\ell}{T}\frac{\sin \omega\ell/2}{\omega\ell/2}\left(\cos (n'-1)\omega t\ \left\{A_0 \sin\phi_0 + A_1 \sin(\phi_1 + \omega\tau_1) + A_2 \sin\left[\phi_2 + \omega(\tau_1+\tau_2)\right]\right.\right.$$

$$\left. + A_3 \sin\left[\phi_3 + \omega(\tau_1+\tau_2+\tau_3)\right]\right\} + \sin (n'-1)\omega t\ \left\{A_0 \cos\phi_0 + A_1 \cos(\phi_1 + \omega\tau_1)\right.$$

$$\left.\left. + A_2 \cos\left[\phi_2 + \omega(\tau_1+\tau_2)\right] + A_3 \cos\left[\phi_3 + \omega(\tau_1+\tau_2+\tau_3)\right]\right\}\right) . \quad (21)$$

We now observe that these three lines will be zero provided that the expressions in the braces are zero; i.e., we have to solve the set of six simultaneous equations

$$A_0 \sin\phi_0 + A_1 \sin\phi_1 + A_2 \sin\phi_2 + A_3 \sin\phi_3 = 0 \quad (22a)$$

$$A_0 \cos\phi_0 + A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = 0 \quad (22b)$$

$$A_0 \sin\phi_0 + A_1 \sin(\phi_1 - \omega\tau_1) + A_2 \sin\left[\phi_2 - \omega(\tau_1+\tau_2)\right] + A_3 \sin\left[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)\right] = 0 \quad (22c)$$

$$A_0 \cos\phi_0 + A_1 \cos(\phi_1 - \omega\tau_1) + A_2 \cos\left[\phi_2 - \omega(\tau_1+\tau_2)\right] + A_3 \cos\left[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)\right] = 0 \quad (22d)$$

$$A_0 \sin\phi_0 + A_1 \sin(\phi_1 + \omega\tau_1) + A_2 \sin\left[\phi_2 + \omega(\tau_1+\tau_2)\right] + A_3 \sin\left[\phi_3 + \omega(\tau_1+\tau_2+\tau_3)\right] = 0 \quad (22e)$$

$$A_0 \cos\phi_0 + A_1 \cos(\phi_1 - \omega\tau_1) + A_2 \cos\left[\phi_2 + \omega(\tau_1+\tau_2)\right] + A_3 \cos\left[\phi_3 + \omega(\tau_1+\tau_2+\tau_3)\right] = 0. \quad (22f)$$

With eight parameters in this set of six equations an infinite number of solutions exist. The symmetries lead us to try to reduce the set, and if we try $\phi_0 = 0$, $\phi_1 = \pi$, $\phi_2 = 0$, and $\phi_3 = \pi$,* we can reduce this set of six equations to a set of three equations with only the four amplitudes as unknowns. First, the set of six equations become

*In Appendix A this solution is derived rigorously and shown to be unique.

$$0 = 0 \tag{23a}$$

$$A_0 - A_1 + A_2 - A_3 = 0 \tag{23b}$$

$$A_1 \sin\omega\tau_1 - A_2 \sin\omega(\tau_1 + \tau_2) + A_3 \sin\omega(\tau_1 + \tau_2 + \tau_3) = 0 \tag{23c}$$

$$A_0 - A_1 \cos\omega\tau_1 + A_2 \cos\omega(\tau_1 + \tau_2) - A_3 \cos\omega(\tau_1 + \tau_2 + \tau_3) = 0 \tag{23d}$$

$$-A_1 \sin\omega\tau_1 + A_2 \sin\omega(\tau_1 + \tau_2) - A_3 \sin\omega(\tau_1 + \tau_2 + \tau_3) = 0 \tag{23e}$$

$$A_0 - A_1 \cos\omega\tau_1 + A_2 \cos\omega(\tau_1 + \tau_2) - A_3 \cos\omega(\tau_1 + \tau_2 + \tau_3) = 0 . \tag{23f}$$

We observe that Eq. (23a) drops out, that Eq. (23f) is identical to Eq. (23d), and that Eq. (23e) when multiplied through by -1 is identical to Eq. (23c). Hence we have the set of three equations

$$A_0 - A_1 + A_2 - A_3 = 0 \tag{24a}$$

$$A_1 \sin\omega\tau_1 - A_2 \sin\omega(\tau_1 + \tau_2) + A_3 \sin\omega(\tau_1 + \tau_2 + \tau_3) = 0 \tag{24b}$$

$$A_0 - A_1 \cos\omega(\tau_1) + A_2 \cos\omega(\tau_1 + \tau_2) - A_3 \cos\omega(\tau_1 + \tau_2 + \tau_3) = 0 . \tag{24c}$$

These three equations will give us values for A_1, A_2, and A_3 in terms of A_0 such that the carrier and the adjacent upper and lower sidebands will be suppressed.* Table 1 shows amplitudes computed for various sets of pulse spacings. In Appendix B we list the computed spectra for these same pulse trains exclusive of the $(\sin x)/x$ envelope, which is determined solely by the pulse length ℓ and code period T. These same values are plotted in Fig. 2.

Table 1
Amplitudes for Various Sets of Pulse Spacings Such that the Carrier and Adjacent Upper and Lower Sidebands will be Suppressed

Pulse Spacing Ratio Code				Resulting Amplitudes			
τ_1	τ_2	τ_3	τ_4	A_0	A_1	A_2	A_3
4	5	6	7	1.00	1.23259	0.91899	0.68639
4	5	7	6	1.00	1.11870	0.74167	0.62297
4	7	5	6	1.00	0.90760	0.74167	0.83407
20	21	22	23	1.00	1.07018	0.99466	0.92448
20	21	22	24	1.00	1.10394	1.02685	0.92290
20	21	23	24	1.00	1.10273	0.99235	0.88962
20	22	21	24	1.00	1.06687	1.02685	0.95998
20	23	21	22	1.00	1.03233	1.03233	1.00000
20	23	21	24	1.00	1.03194	0.99235	0.96041
20	23	22	24	1.00	1.03156	0.95612	0.92456

We have shown that with a four-pulse code the carrier and the adjacent upper and lower sidebands will be suppressed; hence we will have a gap in the spectrum between the lines at $\Omega - 2\omega$ and $\Omega + 2\omega$. This is a gap of 4ω radians. If we had employed a higher order code of say N pulses, we would have a gap of $N\omega$ radians. Now $\omega = 2\pi/T$, where T is the period of the code, i.e., $T = \tau_1 + \tau_2 + \tau_3 + \cdots + \tau_N$. If we fix the unambiguous range by

*We note that with a four-element code we can cancel three prf lines; the general rule is that in any N-element code $N-1$ lines can be canceled.

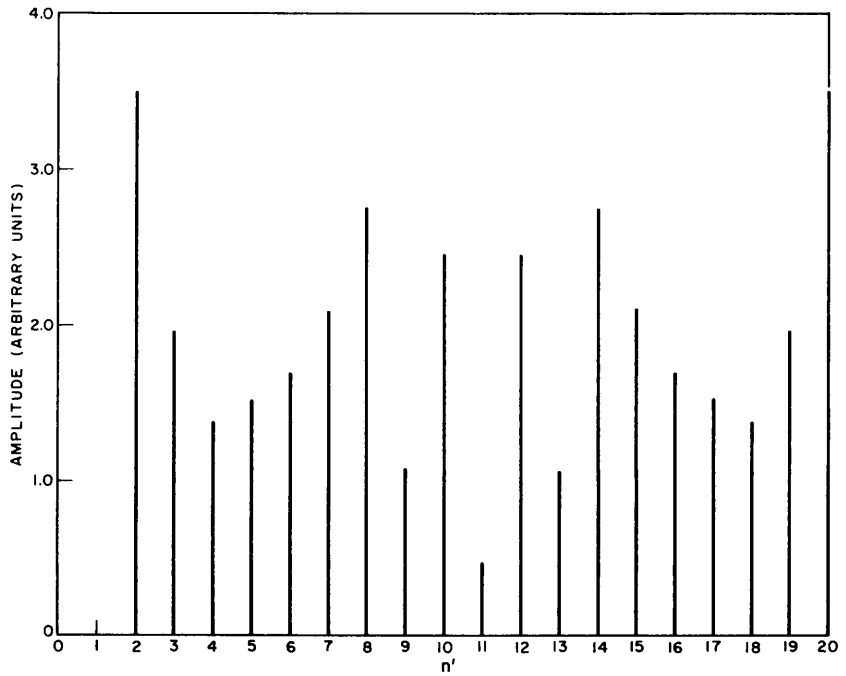

Fig. 2 - A representative spectrum (τ_i = 4, 5, 6, and 7; see Tables 1 and B1)

making τ_1, the shortest pulse spacing, a constant, we have $T > N\tau_1$; hence the total gap width W is given by

$$W < N\frac{2\pi}{N\tau_1} = \frac{2\pi}{\tau_1} . \tag{25}$$

So we see that the gap width is set by the unambiguous range and that increasing the complexity of the code does not widen the gap — in fact it diminishes it. The advantages to increasing the order of the code are that (a) the residual spectrum can be much smoother with these longer codes (2) and (b) the more closely packed prf lines mean that the filter bandwidth can be narrower; i.e., the clutter modulation on the nearest uncanceled lines can spread farther into the gap without degrading the performance.

System Sensitivity

We now wish to consider the sensitivity of this coding system to errors in the parameters. A priori it seems incontrovertible that variations from the nominal values in the amplitudes, phases, and interpulse spacings will result in at least the partial reappearance of those spectrum lines which were to be canceled. The question is how much variation can be tolerated before this unsuppressed signal is too big vis-a-vis the target return to permit us to make reliable detections of targets.

We will assume that we are employing synchronous detection and derive the expressions for the carrier and adjacent sidebands after detection. We will also assume that we require a 15-db signal-to-noise ratio for reliable detection and will compute for different ratios of target-to-clutter cross sections how large an error can be tolerated before this signal-to-noise-ratio criterion is no longer met. Further we will not consider multiple errors but merely the maximum value the error could attain if it were the only

error; i.e., we will use a simple Taylor series expansion of the expression for the canceled lines in terms of a small shift in one of the parameters. Repeating Eq. (19),

$$S_{n'}(t) = \frac{\ell}{T}\Big(\cos n'\omega t\ \{A_0 \sin\phi_0 + A_1 \sin\phi_1 + A_2 \sin\phi_2 + A_3 \sin\phi_3\}$$
$$+ \sin n'\omega t\ \{A_0 \cos\phi_0 + A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3\}\Big)$$

where $n' = \Omega/\omega$. Beating this against an oscillator whose frequency is Ω yields

$$S_{n'}(t) \sin\Omega t = \frac{\ell}{T}\left(\cos n'\omega t \sin\Omega t \sum_{i=0}^{3} A_i \sin\phi_i + \sin n'\omega t \sin\Omega t \sum_{i=0}^{3} A_i \cos\phi_i\right) \tag{26}$$

$$S_{n'}(t) \sin\Omega t = \frac{\ell}{T}\left[\left(\sum_{i=0}^{3} A_i \cos\phi_i\right)\left(\frac{-\cos 2\Omega t}{2}\right) + \left(\sum_{i=0}^{3} A_i \sin\phi_i\right)\left(\frac{\sin 2\Omega t}{2}\right)\right]. \tag{27}$$

Filtering out the terms in $2\Omega t$ we have

$$\Xi_0(t) = \frac{\ell}{2T}(A_0 \cos\phi_0 + A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3). \tag{28}$$

Repeating Eq. (20),

$$S_{n'+1}(t) = \frac{\ell}{T}\frac{\sin\omega\ell/2}{\omega\ell/2}\Big(\cos(n'+1)\omega t\ \{A_0 \sin\phi_0 + A_1 \sin(\phi_1 - \omega\tau_1) + A_2 \sin[\phi_2 - \omega(\tau_1+\tau_2)]$$
$$+ A_3 \sin[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)]\} + \sin(n'+1)\omega t\ \{A_0 \cos\phi_0 + A_1 \cos(\phi_1 - \omega\tau_1)$$
$$+ A_2 \cos[\phi_2 - \omega(\tau_1+\tau_2)] + A_3 \cos[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)]\}\Big)$$

so that

$$S_{n'+1}(t) \sin\Omega t = \frac{\ell}{T}\frac{\sin\omega\ell/2}{\omega\ell/2}\left(\frac{1}{2}[\sin(2\Omega+\omega)t - \sin\omega t] \times \left\{\sum_{i=0}^{3} A_i \sin\left[\phi_i - \sum_{k=0}^{i}\omega(\tau_0 + \cdots + \tau_k)\right]\right\}\right.$$
$$\left. - \frac{1}{2}[\cos(2\Omega+\omega)t - \cos\omega t] \times \left\{\sum_{i=0}^{3} A_i \cos\left[\phi_i - \omega\sum_{k=0}^{i}(\tau_0 + \cdots + \tau_k)\right]\right\}\right) \tag{29}$$

which reduces after filtering to

$$\mathcal{S}_{n'+1}(t) = \frac{\ell}{T}\frac{\sin\omega\ell/2}{\omega\ell/2}\Big(\cos\omega t\ \{A_0 \cos\phi_0 + A_1 \cos(\phi_1 - \omega\tau_1) + A_2 \cos[\phi_2 - \omega(\tau_1+\tau_2)]$$
$$+ A_3 \cos[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)]\} - \sin\omega t\ \{A_0 \sin\phi_0 + A_1 \sin(\phi_1 - \omega\tau_1)$$
$$+ A_2 \sin[\phi_2 - \omega(\tau_1+\tau_2)] + A_3 \sin[\phi_3 - \omega(\tau_1+\tau_2+\tau_3)]\}\Big). \tag{30}$$

Similarly Eq. (21) will become

$$\mathcal{S}_{n'-1}(t) = \frac{\ell}{T}\frac{\sin\omega\ell/2}{\omega\ell/2}\Big(\cos\omega t\ \{A_0 \cos\phi_0 + A_1 \cos(\phi_1 + \omega\tau_1) + A_2 \cos[\phi_2 + \omega(\tau_1+\tau_2)]$$
$$+ A_3 \cos[\phi_3 + \omega(\tau_1+\tau_2+\tau_3)]\} + \sin\omega t\ \{A_0 \sin\phi_0 + A_1 \sin(\phi_1 + \omega\tau_1)$$
$$+ A_2 \sin[\phi_2 + \omega(\tau_1+\tau_2)] + A_3 \sin[\phi_3 + \omega(\tau_1+\tau_2+\tau_3)]\}\Big). \tag{31}$$

Finally, we combine these two equations and define

$$\Xi_1(t) \equiv \mathcal{S}_{n'+1}(t) + \mathcal{S}_{n'-1}(t) \tag{32}$$

$$\begin{aligned}\Xi_1(t) \equiv \frac{\ell}{T}\frac{\sin \omega\ell/2}{\omega\ell/2} \{\cos \omega t\ [A_0 \cos \phi_0 + A_1 \cos \phi_1 \cos \omega\tau_1 + A_2 \cos \phi_2 \cos \omega(\tau_1 + \tau_2) \\ + A_3 \cos \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3)] + \sin \omega t\ [A_1 \cos \phi_1 \sin \omega\tau_1 \\ + A_2 \cos \phi_2 \sin \omega(\tau_1 + \tau_2) + A_3 \cos \phi_3 \sin \omega(\tau_1 + \tau_2 + \tau_3)]\} \ .\end{aligned} \tag{33}$$

It is the expressions for Ξ_0 and Ξ_1 that we wish to examine as the A's, ϕ's, τ's, and T are varied. In general

$$\begin{aligned}\Xi(a + \delta a,\ b + \delta b, \ldots,\ z + \delta z) = \Xi(a, b, \ldots, z) + \delta a \frac{\partial \Xi(a, b, \ldots, z)}{\partial a} + \delta b \frac{\partial \Xi}{\partial b} + \ldots + \delta z \frac{\partial \Xi}{\partial z} \\ + \frac{1}{2}(\delta a)^2 \frac{\partial^2 \Xi}{\partial a^2} + \frac{1}{2}(\delta b)^2 \frac{\partial^2 \Xi}{\partial a^2} + \ldots + \frac{1}{2}(\delta z)^2 \frac{\partial^2 \Xi}{\partial z^2} + \delta a\, \delta b \frac{\partial \Xi}{\partial a}\frac{\partial \Xi}{\partial b} + \ldots\end{aligned} \tag{34}$$

where the expressions on the right side are to be evaluated at the nominal values of the parameters. We will assume that we need only to retain the first-order terms; since these will be small, the second-order terms (including cross products) will be negligible. In one case, however, we will find that the first-order term is zero and will retain the second-order term.

We consider $\Xi_0(t)$ first, from Eq. (28):

$$\frac{\partial \Xi_0}{\partial A_i} = \frac{\ell}{2T} \cos \phi_i$$

$$\frac{\partial \Xi_0}{\partial \phi_i} = -\frac{\ell}{2T} A_i \sin \phi_i\ ; \quad \frac{\partial^2 \Xi}{\partial \phi_i^2} = -\frac{\ell}{2T} A_i \cos \phi_i$$

$$\frac{\partial \Xi_0}{\partial T} = -\frac{\ell}{2T^2} (A_0 \cos \phi_0 + A_1 \cos \phi_1 + A_2 \cos \phi_2 + A_3 \cos \phi_3) .$$

Hence, remembering that Ξ_0 evaluated at the nominal values of the parameters is zero, and that $\cos \phi_i = (-1)^i$,

$$\begin{aligned}\Xi_0(A_i + \delta A_i) &= \Xi_0(A_i) + \delta A_i \frac{\ell}{2T} \cos \phi_i \\ &= \delta A_i \frac{\ell}{2T} (-1)^i\end{aligned} \tag{35}$$

$$\begin{aligned}\Xi_0(\phi_i + \delta \phi_i) &= \Xi(\phi_i) - \delta\phi_i \frac{\ell}{2T} A_i \sin \phi_i - \frac{1}{2}(\delta\phi_i)^2 \frac{\ell}{2T} A_i \cos \phi_i \\ &= -\frac{1}{2} (\delta\phi_i)^2 A_i \frac{\ell}{2T} (-1)^i\end{aligned} \tag{36}$$

$$\Xi_0(T + \delta T) = \Xi_0(T) - \delta T \frac{1}{T} \Xi_0(T) + \cdots = 0. \tag{37}$$

In the case of Ξ_1, from Eq. (32) we have

$$\frac{\partial \Xi_1}{\partial A_i} = \frac{\ell}{T} \frac{\sin \omega \ell/2}{\omega \ell/2} \left[\cos \omega t \left(\cos \phi_i \cos \omega \sum_{k=0}^{i} \tau_k \right) + \sin \omega t \left(\cos \phi_i \sin \omega \sum_{k=0}^{i} \tau_k \right) \right] \tag{38}$$

where τ_0 has been defined to be equal to zero.

The magnitude, written $|\partial \Xi_1 / \partial A_i|$, is obtained by taking the positive square root of the sum of the squares of the in-phase and quadrature components:

$$\left| \frac{\partial \Xi_1}{\partial A_i} \right| = \frac{\ell}{T} \left| \frac{\sin \omega \ell/2}{\omega \ell/2} \cos \phi_i \right| = \frac{\ell}{T} \left| \frac{\sin \omega \ell/2}{\omega \ell/2} \right| \tag{39}$$

$$\frac{\partial \Xi_1}{\partial \phi_i} = \frac{\ell}{T} \left[\cos \omega t \left(- \sin \phi_i A_i \cos \omega \sum_{k=0}^{i} \tau_k \right) + \sin \omega t \left(- \sin \phi_i A_i \sin \omega \sum_{k=0}^{i} \tau_k \right) \right] \tag{40}$$

$$\left| \frac{\partial \Xi_1}{\partial \phi_i} \right| = \frac{\ell}{T} \left| \frac{\sin \omega \ell/2}{\omega \ell/2} A_i \sin \phi_i \right| = 0 \tag{41}$$

$$\frac{\partial^2 \Xi_1}{\partial \phi_i^2} = \frac{\ell}{T} \frac{\sin \omega \ell/2}{\omega \ell/2} \left[\cos \omega t \left(- \cos \phi_i A_i \cos \omega \sum_{k=0}^{i} \tau_k \right) + \sin \omega t \left(- \cos \phi_i A_i \sin \omega \sum_{k=0}^{i} \tau_k \right) \right] \tag{42}$$

$$\left| \frac{\partial^2 \Xi_1}{\partial \phi_i^2} \right| = \frac{\ell}{T} \left| \frac{\sin \omega \ell/2}{\omega \ell/2} A_i \right| . \tag{43}$$

As a variation in any of the τ_i's will appear also as a variation in the period T, we will replace T in Ξ_1 by its equal

$$\sum_{k=0}^{4} \tau_k$$

and then take the appropriate derivatives. After replacing the $\cos \phi$ terms by ± 1 as appropriate, we have

$$\begin{aligned} \frac{\partial \Xi_1}{\partial \tau_1} = \frac{\ell}{T} \frac{\sin \omega \ell/2}{\omega \ell/2} \Bigg\{ & \cos \omega t \left[A_1 \omega \left(\frac{\tau_2 + \tau_3 + \tau_4}{T} \right) \sin \omega \tau_1 - A_2 \omega \left(\frac{\tau_3 + \tau_4}{T} \right) \sin \omega (\tau_1 + \tau_2) \right. \\ & \left. + A_3 \omega \frac{\tau_4}{T} \sin \omega (\tau_1 + \tau_2 + \tau_3) \right] + \sin \omega t \left[-A_1 \omega \left(\frac{\tau_2 + \tau_3 + \tau_4}{T} \right) \cos \omega \tau_1 \right. \\ & \left. + A_2 \omega \left(\frac{\tau_3 + \tau_4}{T} \right) \cos \omega (\tau_1 + \tau_2) - A_3 \omega \frac{\tau_4}{T} \cos \omega (\tau_1 + \tau_2 + \tau_3) \right] \Bigg\} \end{aligned} \tag{44}$$

$$\begin{aligned} \frac{\partial \Xi_1}{\partial \tau_2} = \frac{\ell}{T} \frac{\sin \omega \ell/2}{\omega \ell/2} \Bigg\{ & \cos \omega t \left[-A_1 \omega \frac{\tau_1}{T} \sin \omega \tau_1 - A_2 \omega \frac{\tau_3 + \tau_4}{T} \sin \omega (\tau_1 + \tau_2) \right. \\ & \left. + A_3 \omega \frac{\tau_4}{T} \sin \omega (\tau_1 + \tau_2 + \tau_3) \right] + \sin \omega t \left[A_1 \frac{\tau_1}{T} \cos \omega \tau_1 \right. \\ & \left. + A_2 \omega \frac{\tau_3 + \tau_4}{T} \cos \omega (\tau_1 + \tau_2) - A_3 \omega \frac{\tau_4}{T} \cos \omega (\tau_1 + \tau_2 + \tau_3) \right] \Bigg\} \end{aligned} \tag{45}$$

$$\frac{\partial \Xi_1}{\partial \tau_3} = \frac{\ell}{T}\frac{\sin \omega\ell/2}{\omega\ell/2}\left\{\cos \omega t\left[-A_1\omega\frac{\tau_1}{T}\sin \omega\tau_1 + A_2\omega\frac{\tau_1+\tau_2}{T}\sin \omega(\tau_1+\tau_2)\right.\right.$$

$$\left. + A_3\omega\frac{\tau_4}{T}\sin \omega(\tau_1+\tau_2+\tau_3)\right] + \sin \omega t\left[A_1\omega\frac{\tau_1}{T}\cos \omega\tau_1\right.$$

$$\left.\left. - A_2\omega\frac{\tau_1+\tau_2}{T}\cos \omega(\tau_1+\tau_2) - A_3\omega\frac{\tau_4}{T}\cos \omega(\tau_1+\tau_2+\tau_3)\right]\right\} \tag{46}$$

$$\frac{\partial \Xi_1}{\partial \tau_4} = \frac{\ell}{T}\frac{\sin \omega\ell/2}{\omega\ell/2}\left\{\cos \omega t\left[-A_1\omega\frac{\tau_1}{T}\sin \omega\tau_1 + A_2\omega\frac{\tau_1+\tau_2}{T}\sin \omega(\tau_1+\tau_2)\right.\right.$$

$$\left. - A_3\omega\frac{\tau_1+\tau_2+\tau_3}{T}\sin \omega(\tau_1+\tau_2+\tau_3)\right]$$

$$+\sin \omega t\left[A_1\frac{\omega\tau_1}{T}\cos \omega\tau_1 - A_2\omega\frac{\tau_1+\tau_2}{T}\cos \omega(\tau_1+\tau_2)\right.$$

$$\left.\left. + A_3\omega\frac{\tau_1+\tau_2+\tau_3}{T}\cos \omega(\tau_1+\tau_2+\tau_3)\right]\right\}. \tag{47}$$

In summary, then,

$$|\Xi_1(A_i + \delta A_i)| = \frac{\ell}{T}\left|\frac{\sin \omega\ell/2}{\omega\ell/2}\right| \delta A_i \tag{48}$$

$$|\Xi_1(\phi_i + \delta\phi_i)| = \frac{1}{2}\frac{\ell}{T}\left|\frac{\sin \omega\ell/2}{\omega\ell/2} A_i\right| (\delta\phi_i)^2 \tag{49}$$

$$|\Xi_1(\tau_i + \delta\tau_i)| = \left|\frac{\partial \Xi_1}{\partial \tau_i}\right| \delta\tau_i . \tag{50}$$

There seems to be no advantage in writing out a general term for $|\partial\Xi_1/\partial\tau_i|$; it will be evaluated in a specific example later in the next section in this report.

With the mathematical expressions in hand we now want to consider the physics of the problem. Assuming that one of the parameters is in error, a signal ϵ_D will be transmitted instead of a zero in the blanked spectral region. At some range R it will be reflected from a clutter region of cross section σ_c and received. If at this same range there is a target whose cross section is σ_T traveling at some velocity V_T, it will doppler shift some one of the uncanceled spectrum lines into this blanked region; let this line be of strength L_n. Since both signals have traveled the same distance, their ratio will depend only on the relative cross section of the targets and their relative strength on transmission, that is,

$$R_a = \frac{L_n}{\epsilon_D}\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} . \tag{51}$$

In Appendix B are values of computed spectra for various interpulse spacings. This collection demonstrates the wide variability of these lines. It would seem that any reasonable criterion for maximum allowable error should be based on the smallest line, which is 0.2 $A_0\ell/T$ in the group selected. Using other codes, line strengths as low as 0.03 $A_0\ell/T$ have been computed. In this analysis we will assume that the smallest line is 0.1 $A_0\ell/T$ on transmission. This is a reasonable number and allows for simple conversion to other line strengths.

If we are to avoid blind speeds, and also maintain coverage at the lower velocities, the filter in the receiver should have a width somewhat in excess of 1 prf; hence ϵ_D will be the rms of Ξ_0 and Ξ_1. Examination of Eqs. (42) through (44) shows that these can be evaluated only for specific values of ω's, τ's, etc. We shall work out one specific example to show the degree of control required over the variations in the parameters; although this is only one example, the values are typical. There may be some differences, but certainly order of magnitude differences between these values and those for any other example or system are not expected.

ILLUSTRATIVE EXAMPLE

Let us take the τ's in the ratio of: 4:5:6:7, which is the example given in Table B1 and Fig. 2. Then $A_0 = 1$, $A_1 = 1.23259$, $A_2 = 0.91899$, $A_3 = 0.68639$, and

$$T = \sum_{i=0}^{4} \tau_i = 22 .$$

We will take $\tau_1 = 10^{-3}$ sec, i.e., $R = 161$ naut mi to make conversion to other ranges easier. Hence $T = 5.5 \times 10^{-3}$. We will assume that we require a target-to-clutter ratio (power) of 15 db for a reliable decision that a target is present. Since all our expressions are in volts, we convert this to a voltage ratio, 5.62. Hence on the basis of Eq. (51) and our assumption that the smallest line is 0.1 $A_0 \ell / T$ on transmission we require

$$5.62 = \frac{0.1\, A_0 \ell / T (\sigma_T)^{1/2}}{\epsilon_D (\sigma_c)^{1/2}} .$$

Error Due to Amplitude Variations

From Eqs. (35) and (48) we have

$$|\Xi_0(A_i + \delta A_i)| = \delta A_i \frac{\ell}{2T}$$

and

$$|\Xi_1(A_i + \delta A_i)| = \delta A_i \frac{\ell}{T} \left| \frac{\sin \omega \ell / 2}{\omega \ell / 2} \right| .$$

We have stated that ϵ_D should be the rms of Ξ_0 and Ξ_1; hence

$$\epsilon_{Di} = \delta A_i \frac{\ell}{T} \sqrt{\frac{1}{4} + \frac{\sin^2 \omega \ell / 2}{(\omega \ell / 2)^2}} . \tag{52}$$

If $\ell \approx 10^{-6}$ sec, then with our other assumptions the term

$$\frac{\sin^2 \omega \ell / 2}{(\omega \ell / 2)^2}$$

is essentially unity; hence

$$5.62 = \frac{0.1\ A_i\ \frac{\ell}{T}}{\delta A_i\ \frac{\ell}{T}\sqrt{\frac{5}{4}}}\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2}$$

or

$$\frac{\delta A_i}{A_i} = 0.0159\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2}. \tag{53}$$

Figure 3 shows $\delta A/A$ for a range of values of signal-to-clutter ratio. To cite just one value from this figure we see that for a signal-to-clutter ratio of -40 db, $\delta A/A = 1.6 \times 10^{-4}$. This means that if all parameters are held fixed at their nominal values except the amplitude of one of the pulses, then variations in the amplitude of this pulse must be held to within 1-1/2 parts in 10,000 if the spurious signal is to be well below threshold.

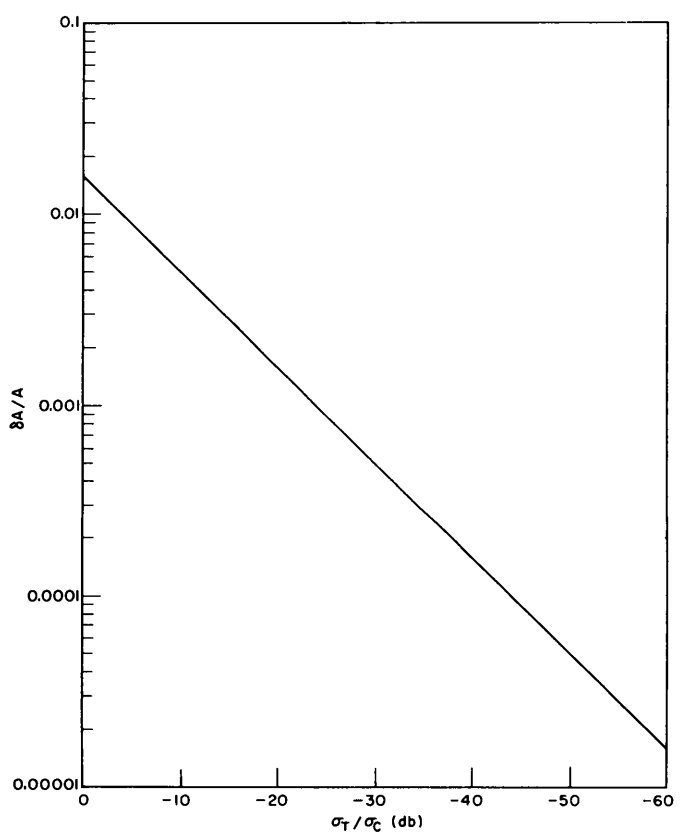

Fig. 3 - $\delta A/A$ as a function of the signal-to-clutter ratio

However, we know that all the values will vary somewhat from nominal, and as we are concerned only with first-order effects (Eq. (32)), the errors will add linearly; we have eleven parameters to consider — three amplitudes, four phases, and four pulse spacings — and we can assume a uniform error budget. Therefore we will reduce the maximum allowable error by an order of magnitude in order that their sum will be sufficiently small. Hence $\delta A/A$ is to be held within about 1 part in 100,000.

Error Due to Phase Variations

From Eqs. (36) and (49) we have

$$|\Xi_0(\phi_i + \delta\phi_i)| = \frac{1}{2}(\delta\phi_i)^2 A_i \frac{\ell}{2T}$$

and

$$|\Xi_1(\phi_i + \delta\phi_i)| = \frac{1}{2}(\delta\phi_i)^2 A_i \frac{\ell}{T}\left|\frac{\sin \omega\ell/2}{\omega\ell/2}\right| .$$

Hence

$$\epsilon_{Di} = \frac{1}{2}(\delta\phi_i)^2 \frac{\ell}{T} A_i \sqrt{\frac{1}{4} + \frac{\sin^2 \omega\ell/2}{(\omega\ell/2)^2}} . \tag{54}$$

Therefore

$$5.62 = \frac{0.1\, A_0 \frac{\ell}{T}}{\frac{1}{2}(\delta\phi_i)^2 A_i \frac{\ell}{T}\sqrt{\frac{5}{4}}}\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2}$$

using the same simplifying assumptions we made above; also $A_0/A_i \approx 1$, so that we have

$$5.62 = \frac{0.1}{\frac{\sqrt{5}}{4}(\delta\phi_i)^2}\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} .$$

Hence

$$\delta\phi_i = 0.2006\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} . \tag{55}$$

Figure 4 shows $\delta\phi/\phi$ vs σ_T/σ_c, and we see that for a target-to-clutter ratio of -40 db the maximum possible error is 0.020 radians. Again this is for the case of the total error budget being taken up by this one error; a more realistic number would be an order of magnitude less or 0.002 radians; i.e., the phase of the rf carrier must be 0 ± 2.0 mils or $\pi \pm 2.0$ mils.

Error Due to Variations in Pulse Spacings

To determine $\delta\tau_i/\tau_i$ we first evaluate the magnitude of Eq. (44):

$$\left|\frac{\partial\Xi}{\partial\tau_1}\right| = 1.186 \times 10^3 .$$

Then

$$\delta\tau_1 = \frac{0.1}{5.62} \times \frac{1}{1.186} \times 10^{-3}\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2}$$

$$= 1.500 \times 10^{-5}\left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} \text{sec}$$

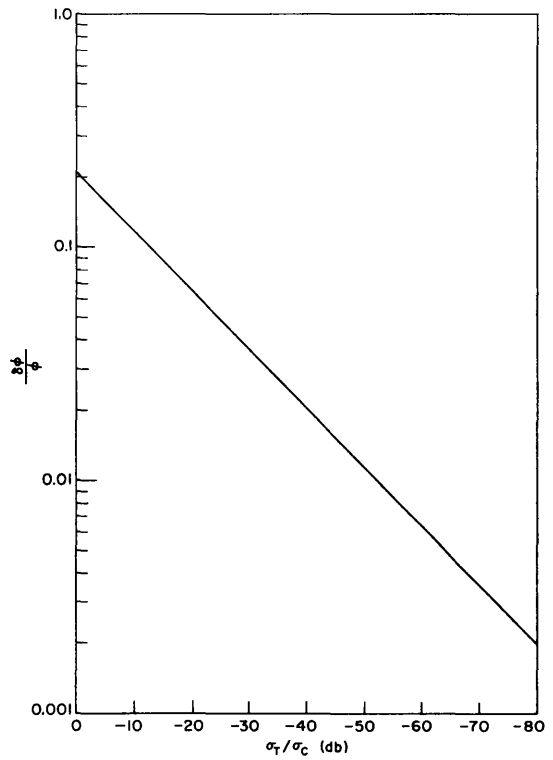

Fig. 4 - $\delta\phi/\phi$ as a function of the signal-to-clutter ratio

and

$$\frac{\delta\tau_1}{\tau_1} = 1.5 \times 10^{-2} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} . \tag{56}$$

To determine $\delta\tau_2/T_2$ we first evaluate the magnitude of Eq. (45):

$$\left| \frac{\partial \Xi}{\partial \tau_2} \right| = 0.759 \times 10^3 .$$

Then

$$\delta\tau_2 = 2.344 \times 10^{-5} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} \text{sec}$$

and

$$\frac{\delta\tau_2}{\tau_2} = 1.871 \times 10^{-2} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} . \tag{57}$$

To determine $\delta\tau_3/\tau_3$ we first evaluate the magnitude of Eq. (46):

$$\left| \frac{\partial \Xi}{\partial \tau_3} \right| = 0.5876 \times 10^3 .$$

Then

$$\delta\tau_3 = 3.028 \times 10^{-5} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} \text{sec}$$

and

$$\frac{\delta\tau_3}{\tau_3} = 2.02 \times 10^{-2} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} . \tag{58}$$

To determine $\delta\tau_4/\tau_4$ we first evaluate the magnitude of Eq. (47):

$$\left| \frac{\partial \Xi}{\partial \tau_4} \right| = 0.645 \times 10^3 .$$

Then

$$\delta\tau_4 = 2.758 \times 10^{-5} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} \text{sec}$$

and

$$\frac{\delta\tau_4}{\tau_4} = 1.57 \times 10^{-2} \left(\frac{\sigma_T}{\sigma_c}\right)^{1/2} . \tag{59}$$

Two of these four ratios are plotted in Fig. 5 vs σ_T/σ_c; the other two lines would fall between the two shown. For a -40-db target-to-clutter ratio we see that $\delta\tau/\tau$ is on the order of 1.5×10^{-4}, i.e., a variation of not more than 1-1/2 parts in 10,000. Under our

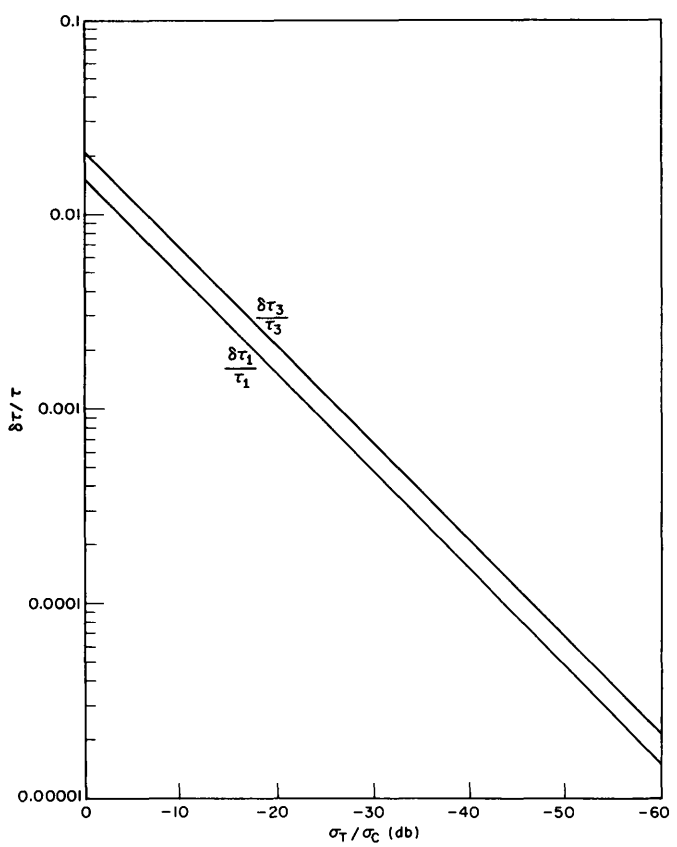

Fig. 5 - $\delta\tau/\tau$ as a function of the signal-to-clutter ratio

uniform error budget we need to control the pulse spacing to 1-1/2 parts in 100,000. As the pulse spacings are about 1 msec, we need to control the timing to 10^{-8} sec or 10 nsec.

EFFECTS OF FINITE TIME ON TARGET

Finally, it should be noted that the same assumption has been made here as was made in Ref. 2, that we are dealing with an infinitely long series of pulses. Such a series yields the familiar line spectrum. However, in any real application there will be, of course, only a finite number of pulses available. The problem of the effect of the number of pulses in a simple pulse train on the appearance of the spectrum was studied by George and Zamanakos (3). They showed (Eq. (10), p. 1161) that the spectrum of a finite sequence of pulses consists of the sum of $(\sin mx)/mx$ functions, each one centered at one of the prf lines, where m is the number of pulses. In our case, we have looked on our pulse groups as being the sum of relatively displaced series; therefore we can carry over the work of Ref. 3 unchanged.

The cancellation will still take place, as it is the results of summing the coefficients of the $(\sin mx)/mx$ terms and therefore independent of the behavior with changes in m. However, the first uncanceled line, which is usually the largest and will be returned from the clutter will have its envelope spreading into the canceled region. The problem, therefore, is to determine how large m must be if these side lobes are to be reduced to an acceptable level. The first point to be considered is that all of the side-lobe energy in the region between the canceled lines, i.e., in the region covered by the bandpass filter competes with the signal, so that we must determine the integral over this region of the square of this $(\sin mx)/mx$ function and compare it with the energy received from the target. As 90 percent of the total energy is within the first lobe centered about the prf line ω_0, we may evaluate this integral, via the mean value theorem, taking as the value of the function its value at the center of the interval, since the region of integration is many lobes away from the center:

$$I = A^2 \frac{\ell^2}{T^2} \int_{-2(2\pi/T)}^{-(2\pi/T)} \left[\frac{\sin \frac{mT}{2} (\omega - \omega_0)}{\frac{mT}{2} (\omega - \omega_0)} \right]^2 d(\omega - \omega_0) \tag{60}$$

or

$$I = A^2 \frac{\ell^2}{T^2} \left[\frac{1}{\frac{3}{2} \pi m} \right]^2 \frac{2\pi}{T} . \tag{61}$$

In the illustrative example of the preceding section the target was represented by a line of strength $0.1\ A_0 \ell / T$. In this case we will assume that the main lobe and adjacent side lobes are in the filter, so that the received energy from the target is still $(0.1\ A_0 \ell / T)^2 \sigma_T$ times the common propagation factors.

As we are now concerned with a power ratio, we convert 15 db to a ratio of 31.63 and following the same rationale of the preceding section we have

$$31.63 = \frac{\left(0.1\ A_0 \frac{\ell}{T}\right)^2 \sigma_T}{\left(A \frac{\ell}{T}\right)^2 \left[\frac{1}{\frac{3}{2} \pi m} \right]^2 \frac{2\pi}{T} \sigma_c} . \tag{62}$$

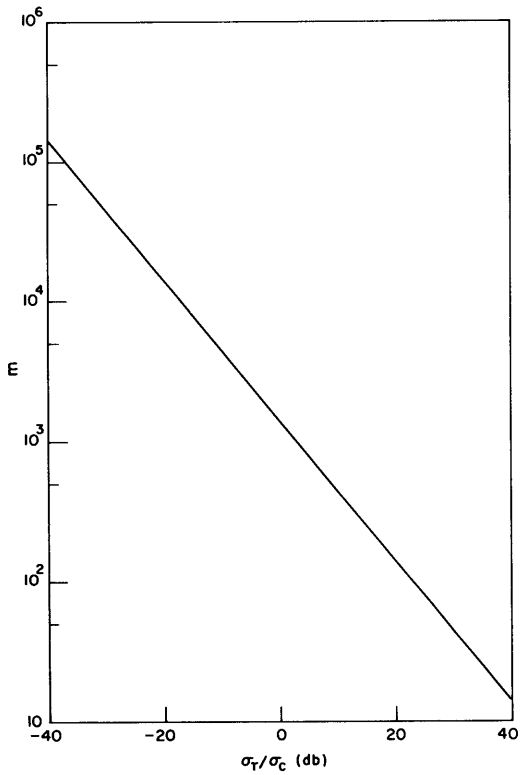

Fig. 6 - Required number of pulse groups as a function of the signal-to-clutter ratio, with single filtering

Using the same conditions as in the preceding section we replace T by 5.5×10^{-3} sec and A by 3.49 A_0; hence

$$m^2 = 1.982 \times 10^6 \left(\frac{\sigma_c}{\sigma_T}\right)$$

or

$$m = 1.408 \times 10^3 \left(\frac{\sigma_c}{\sigma_T}\right)^{1/2} . \qquad (63)$$

This expression is plotted in Fig. 6, which shows that for a clutter-to-target ratio of 40 db we must have $m = 1.408\times10^5$ pulse groups. With our period of 5.5×10^{-3} sec this requires a time on target of t = 774 sec. However, if the system is designed to process precisely m pulses, then this time can be reduced. The single filter covering the bandwidth equivalent to 1 prf would be replaced by a bank of contiguous filters each of width $(2/m)(2\pi/T)$, so that the interval over which we must integrate Eq. (60) is $2/m$ as wide as before, which results in a new expression for m:

$$m^3 = 3.964 \times 10^6 \, (\sigma_c/\sigma_T)$$

or

$$m = 1.58 \times 10^2 \, (\sigma_c/\sigma_T)^{1/3} . \qquad (64)$$

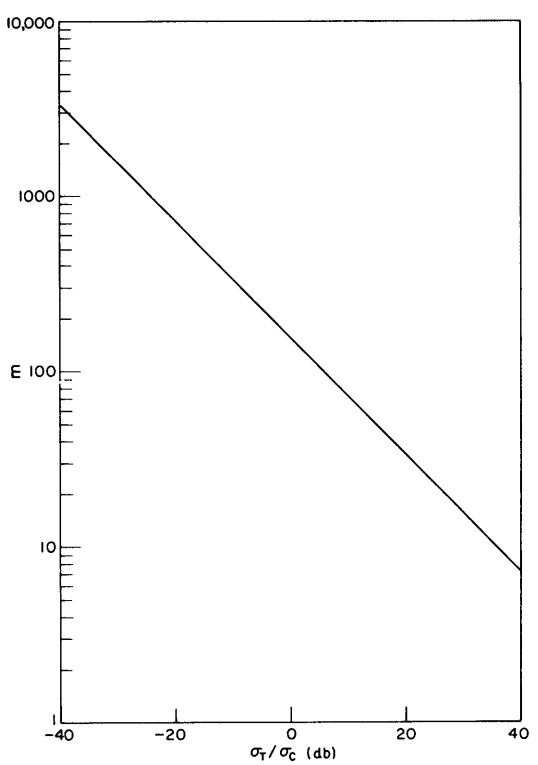

Fig. 7 - Required number of pulse groups as a function of the signal-to-clutter ratio, with comb filtering

This is plotted in Fig. 7. For a -40-db target-to-clutter ratio we see that $m = 3350$ pulse groups, which requires a total time on target of 18.4 sec. This requirement of a time on target of 18.4 sec makes this pulse coding system of questionable value in an airborne search radar, although other applications for this coding scheme may exist.

ACKNOWLEDGMENT

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APPENDIX A

SOLUTION FOR THE PHASE ANGLES IN EQS. (22)

Repeating the six equations for cancellation (Eqs. (22)) in expanded form

$$A_0 \sin \phi_0 + A_1 \sin \phi_1 + A_2 \sin \phi_2 + A_3 \sin \phi_3 = 0 \tag{A1a}$$

$$A_0 \cos \phi_0 + A_1 \cos \phi_1 + A_2 \cos \phi_2 + A_3 \cos \phi_3 = 0 \tag{A1b}$$

$$\begin{aligned} A_0 \sin \phi_0 &+ A_1 \sin \phi_1 \cos \omega\tau_1 - A_1 \cos \phi_1 \sin \omega\tau_1 \\ &+ A_2 \sin \phi_2 \cos \omega(\tau_1 + \tau_2) - A_2 \cos \phi_2 \sin \omega(\tau_1 + \tau_2) \\ &+ A_3 \sin \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3) - A_3 \cos \phi_3 \sin \omega(\tau_1 + \tau_2 + \tau_3) = 0 \end{aligned} \tag{A1c}$$

$$\begin{aligned} A_0 \cos \phi_0 &+ A_1 \cos \phi_1 \cos \omega\tau_1 + A_1 \sin \phi_1 \sin \omega\tau_1 \\ &+ A_2 \cos \phi_2 \cos \omega(\tau_1 + \tau_2) + A_2 \sin \phi_2 \sin \omega(\tau_1 + \tau_2) \\ &+ A_3 \cos \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3) + A_3 \sin \phi_3 \sin \omega(\tau_1 + \tau_2 + \tau_3) = 0 \end{aligned} \tag{A1d}$$

$$\begin{aligned} A_0 \sin \phi_0 &+ A_1 \sin \phi_1 \cos \omega\tau_1 + A_1 \cos \phi_1 \sin \omega\tau_1 \\ &+ A_2 \sin \phi_2 \cos \omega(\tau_1 + \tau_2) + A_2 \cos \phi_2 \sin \omega(\tau_1 + \tau_2) \\ &+ A_3 \sin \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3) + A_3 \cos \phi_3 \sin \omega(\tau_1 + \tau_2 + \tau_3) = 0 \end{aligned} \tag{A1e}$$

$$\begin{aligned} A_0 \cos \phi_0 &+ A_1 \cos \phi_1 \cos \omega\tau_1 - A_1 \sin \phi_1 \sin \omega\tau_1 \\ &+ A_2 \cos \phi_2 \cos \omega(\tau_1 + \tau_2) - A_2 \sin \phi_2 \sin \omega(\tau_1 + \tau_2) \\ &+ A_3 \cos \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3) - A_3 \sin \phi_3 \sin \omega(\tau_1 + \tau_2 + \tau_3) = 0 . \end{aligned} \tag{A1f}$$

A more useful set of equations is formed by Eq. (A1a), Eq. (A1b), Eq. (A1c) plus and minus Eq. (A1e), and Eq. (A1d) plus and minus Eq. (A1f):

$$A_0 \sin \phi_0 + A_1 \sin \phi_1 + A_2 \sin \phi_2 + A_3 \sin \phi_3 = 0 \tag{A2a}$$

$$A_0 \cos \phi_0 + A_1 \cos \phi_1 + A_2 \cos \phi_2 + A_3 \cos \phi_3 = 0 \tag{A2b}$$

$$\begin{aligned} A_0 \sin \phi_0 &+ A_1 \sin \phi_1 \cos \omega\tau_1 + A_2 \sin \phi_2 \cos \omega(\tau_1 + \tau_2) \\ &+ A_3 \sin \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3) = 0 \end{aligned} \tag{A2c}$$

$$\begin{aligned} A_0 \cos \phi_0 &+ A_1 \cos \phi_1 \cos \omega\tau_1 + A_2 \cos \phi_2 \cos \omega(\tau_1 + \tau_2) \\ &+ A_3 \cos \phi_3 \cos \omega(\tau_1 + \tau_2 + \tau_3) = 0 \end{aligned} \tag{A2d}$$

$$A_1 \cos\phi_1 \sin\ \omega\tau_1 + A_2 \cos\phi_2 \sin\ \omega(\tau_1+\tau_2) + A_3 \cos\phi_3 \sin\ \omega(\tau_1+\tau_2+\tau_3) = 0 \qquad \text{(A2e)}$$

$$A_1 \sin\phi_1 \sin\ \omega\tau_1 + A_2 \sin\phi_2 \sin\ \omega(\tau_1+\tau_2) + A_3 \sin\phi_3 \sin\ \omega(\tau_1+\tau_2+\tau_3) \quad 0. \qquad \text{(A2f)}$$

For any given code, the τ's are constant; therefore we replace the sines and cosines of the $\omega\tau$ terms as follows:

$$\cos\omega\tau_1 = C_1 \qquad \sin\omega\tau_1 = S_1$$

$$\cos\omega(\tau_1+\tau_2) = C_2 \qquad \sin\omega(\tau_1+\tau_2) = S_2$$

$$\cos\omega(\tau_1+\tau_2+\tau_3) = C_3 \qquad \sin\omega(\tau_1+\tau_2+\tau_3) = S_3.$$

The A's and ϕ's are variables, and we introduce new variables to replace them:

$$X_0 = A_0 \sin\phi_0 \qquad Y_0 = A_0 \cos\phi_0$$

$$X_1 = A_1 \sin\phi_1 \qquad Y_1 = A_1 \cos\phi_1$$

$$X_2 = A_2 \sin\phi_2 \qquad Y_2 = A_2 \cos\phi_2$$

$$X_3 = A_3 \sin\phi_3 \qquad Y_3 = A_3 \cos\phi_3 .$$

Now the equations read

$$X_0 + X_1 + X_2 + X_3 = 0 \qquad \text{(A3a)}$$

$$Y_0 + Y_1 + Y_2 + Y_3 = 0 \qquad \text{(A3b)}$$

$$X_0 + C_1X_1 + C_2X_2 + C_3X_3 = 0 \qquad \text{(A3c)}$$

$$Y_0 + C_1Y_1 + C_2Y_2 + C_3Y_3 = 0 \qquad \text{(A3d)}$$

$$S_1Y_1 + S_2Y_2 + S_3Y_3 = 0 \qquad \text{(A3e)}$$

$$S_1X_1 + S_2X_2 + S_3X_3 = 0 . \qquad \text{(A3f)}$$

We can make these more symmetric in form by introducing two new constants $C_0 = 1$ and $S_0 = 0$, and we finally have

$$X_0 + X_1 + X_2 + X_3 = 0 \qquad \text{(A4a)}$$

$$Y_0 + Y_1 + Y_2 + Y_3 = 0 \qquad \text{(A4b)}$$

$$C_0X_1 + C_1X_1 + C_2X_2 + C_3X_3 = 0 \qquad \text{(A4c)}$$

$$C_0Y_1 + C_1Y_1 + C_2Y_2 + C_3Y_3 = 0 \qquad \text{(A4d)}$$

$$S_0X_0 + S_1X_1 + S_2X_2 + S_3X_3 = 0 \qquad \text{(A4e)}$$

$$S_0Y_0 + S_1Y_1 + S_2Y_2 + S_3Y_3 = 0 . \qquad \text{(A4f)}$$

Now we note that Eqs. (A4a), (A4c), and (A4e) involve only the X's and that Eqs. (A4b), (A4d), and (A4f) involve only the Y's; furthermore the constants are the same in each set, so that the solution found for one is the same for the other. Each set comprises a set of three equations in four unknowns; if we consider the zeroth term to be arbitrary, we can solve in terms of this parameter by the ordinary methods.

The matrix of coefficients is

$$\begin{vmatrix} 1 & 1 & 1 \\ C_1 & C_2 & C_3 \\ S_1 & S_2 & S_3 \end{vmatrix}$$

which is nonzero. Therefore the solution exists and is unique; e.g.,

$$X_1 = \frac{\begin{vmatrix} -1 & 1 & 1 \\ -C_0 & C_2 & C_3 \\ -S_0 & S_2 & S_3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ C_1 & C_2 & C_3 \\ S_1 & S_2 & S_3 \end{vmatrix}} X_0 \tag{A5}$$

with a similar expression for Y_1 in terms of Y_0. In fact

$$\frac{X_0}{Y_0} = \frac{X_1}{Y_1} = \frac{X_2}{Y_2} = \frac{X_3}{Y_3} .$$

But $X_i / Y_i = \tan \phi_i$; therefore we have

$$\tan \phi_0 = \tan \phi_1 = \tan \phi_2 = \tan \phi_3 .$$

Therefore the ϕ's are equal, modulo π. If we require that the A_i be greater than or equal to zero, then the solution for the ϕ's is

$$\phi_0 = \phi , \quad \phi_1 = \phi + \pi , \quad \phi_2 = \phi , \quad \phi_3 = \phi + \pi .$$

In the main body of the report, ϕ was taken to be zero for convenience.

APPENDIX B

COMPUTED SPECTRA FOR THE SETS OF PULSE SPACINGS GIVEN IN TABLE 1

Table B1
Spectrum for a τ Ratio Code of 4, 5, 6, 7 and Amplitudes 1.00000, 1.23259, 0.91899, and 0.68639

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-2.28621	2.63843	3.49114
3	1.06351	1.65486	1.96714
4	1.20493	0.67129	1.37931
5	0.55421	1.42227	1.52643
6	-0.77857	-1.49611	1.68657
7	-1.23516	1.67954	2.08482
8	0.36899	2.71047	2.73547
9	-0.42315	0.97592	1.06371
10	2.24241	0.97592	2.44558
11	0.00000	-0.46519	0.46519
12	-2.24241	0.97592	2.44558
13	0.42315	0.97592	1.06371
14	-0.36899	2.71047	2.73547
15	1.23516	1.67954	2.08482
16	0.77857	-1.49611	1.68658
17	-0.55421	1.42227	1.52643
18	-1.20493	0.67129	1.37931
19	-1.06351	1.65486	1.96714
20	2.28621	2.63843	3.49114

Table B2
Spectrum for a τ Ratio Code of 4, 5, 7, 6 and Amplitudes 1.00000, 1.23259, 0.74167, and 0.62297

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-1.69562	2.63843	3.13631
3	0.48262	1.92015	1.97987
4	0.88360	0.14944	0.89615
5	1.28458	1.17847	1.74326
6	-0.86667	-0.24478	0.90058
7	-2.00463	1.12082	2.29669
8	0.98562	1.70904	1.97289
9	0.34632	2.02223	2.05167
10	0.80195	1.24787	1.48334
11	0.00000	-1.48334	1.48334
12	-0.80195	1.24787	1.48334
13	-0.34632	2.02223	2.05167
14	-0.98562	1.70904	1.97289
15	2.00463	1.12082	2.29669
16	0.86667	-0.24478	0.90058
17	-1.28458	1.17847	1.74326
18	-0.88360	0.14944	0.89615
19	-0.48262	1.92015	1.97987
20	1.69562	2.63843	3.13631

Table B3
Spectrum for a τ Ratio Code of 4, 7, 5, 6 and Amplitudes 1.00000, 0.90760, 0.74167, and 0.83407

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.92090	3.13631	3.26871
3	-0.50300	0.78268	0.93037
4	1.34929	1.16917	1.78537
5	1.12103	0.04101	1.12178
6	-1.12103	1.52435	1.89218
7	-1.34929	-0.31417	1.38539
8	0.50300	2.26602	2.32117
9	0.92090	1.65297	1.89218
10	0.00000	1.48334	1.48334
11	0.00000	-1.48334	1.48334
12	0.00000	1.48334	1.48334
13	-0.92090	1.65297	1.89218
14	-0.50300	2.26602	2.32117
15	1.34929	-0.31417	1.38539
16	1.12103	1.52435	1.89219
17	-1.12103	0.04101	1.12178
18	-1.34929	1.16917	1.78537
19	0.50300	0.78268	0.93037
20	0.92090	3.13631	3.26871

Table B4
Spectrum for a τ Ratio Code of 20, 21, 22, 23 and Amplitudes 1.00000, 1.07018, 0.99466, and 0.92448

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.72026	3.89944	3.96540
3	0.56009	0.14643	0.57891
4	0.29782	0.02348	0.29875
5	0.53939	0.18330	0.56969
6	-1.98363	3.21503	3.77773
7	0.95393	0.58296	1.11796
8	0.61776	0.11308	0.62802
9	0.88193	0.62690	1.08204
10	-2.76264	2.02083	3.42285
11	1.04605	1.17241	1.57123
12	0.95169	0.31482	1.00240
13	0.92018	1.17701	1.49402
14	-2.87793	0.61805	2.94355
15	0.79832	1.72406	1.89992
16	1.24542	0.67193	1.41512
17	0.64989	1.64775	1.77428
18	-2.32259	-0.64752	2.41116
19	0.28402	2.05609	2.07561
20	1.40597	1.18975	1.84181

Table B5
Spectrum for a τ Ratio Code of 20, 21, 22, 24 and Amplitudes 1.00000, 1.10394, 1.02685, and 0.92290

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.93356	3.90408	4.01415
3	0.73954	0.09645	0.74580
4	0.41435	0.06992	0.42021
5	0.55003	0.38184	0.66958
6	-2.43029	2.80037	3.70788
7	1.29071	0.70629	1.47132
8	0.80610	0.29640	0.85887
9	0.66159	0.97117	1.17511
10	-2.97259	1.04477	3.15085
11	1.33621	1.59894	2.08376
12	1.10977	0.70558	1.31508
13	0.33647	1.40924	1.44885
14	-2.37715	0.65466	2.46565
15	0.78207	2.38062	2.50580
16	1.21238	1.27813	1.76167
17	-0.18034	1.44102	1.45226
18	-0.92876	-1.63684	1.88197
19	-0.18601	2.67892	2.68535
20	1.00066	1.90411	2.15104

Table B6
Spectrum for a τ Ratio Code of 20, 21, 23, 24 and Amplitudes 1.00000, 1.10273, 0.99235, and 0.88962

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.97355	3.81432	3.93660
3	0.78854	0.29411	0.84161
4	0.32718	-0.02622	0.32823
5	0.70991	0.38486	0.80752
6	-2.45787	2.58429	3.56647
7	1.10504	1.10848	1.56520
8	0.83006	0.03112	0.83064
9	0.86974	1.14231	1.43574
10	-2.80689	0.74704	2.90460
11	0.70170	1.91481	2.03933
12	1.43557	0.44872	1.50406
13	0.40683	1.72531	1.77262
14	-1.95353	-0.80990	2.11476
15	-0.18601	2.16910	2.17706
16	1.78529	1.35256	2.23980
17	-0.31453	1.73141	1.75974
18	-0.42729	-1.42604	1.48868
19	-0.98827	1.68322	1.95190
20	1.46398	2.49941	2.89660

Table B7
Spectrum for a τ Ratio Code of 20, 22, 21, 24 and Amplitudes 1.00000, 1.06687, 1.02685, and 0.95998

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.79408	3.94554	4.02465
3	0.47081	-0.02787	0.47164
4	0.66646	0.23429	0.70644
5	0.32873	0.18688	0.37814
6	-2.14967	3.13074	3.79771
7	0.97896	0.20872	1.00096
8	1.04644	0.84150	1.34282
9	0.50574	0.40984	0.65095
10	-2.88281	1.75136	3.37311
11	1.35187	0.73191	1.53728
12	0.96768	1.57228	1.84620
13	0.59538	0.59236	0.83986
14	-2.80290	0.22362	2.81181
15	1.41635	1.44513	2.02348
16	0.44162	2.12182	2.16729
17	0.68029	0.73634	1.00249
18	-1.97341	-1.00829	2.21608
19	1.07815	2.14859	2.40392
20	-0.33915	2.25018	2.27560

Table B8
Spectrum for a τ Ratio Code of 20, 23, 21, 22 and Amplitudes 1.00000, 1.03233, 1.03233, and 1.00000

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.28054	3.83299	3.84324
3	-0.04087	0.27773	0.28073
4	0.55064	0.10171	0.55995
5	0.12045	-0.25848	0.28516
6	-0.80035	3.63400	3.72109
7	-0.23448	0.49865	0.55103
8	1.02066	0.38932	1.09239
9	0.37686	-0.42539	0.56832
10	-1.20391	3.27023	3.48479
11	0.53993	0.59780	0.80559
12	1.34417	0.81357	1.57121
13	0.71480	-0.94638	0.84273
14	-1.43747	2.80367	3.15070
15	0.89184	0.53413	1.03956
16	1.48202	1.30265	1.97314
17	1.06113	-0.29579	1.10158
18	-1.47822	2.31254	2.74463
19	-1.21290	0.29872	1.24915
20	1.42868	1.77561	2.27902

Table B9
Spectrum for a τ Ratio Code of 20, 23, 21, 24 and Amplitudes 1.00000, 1.03194, 0.99235, and 0.96041

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.70254	3.89389	3.95676
3	0.27600	0.06004	0.28245
4	0.79757	0.27608	0.84400
5	0.29274	0.02338	0.29367
6	-1.91795	3.20738	3.73709
7	0.51425	0.18920	0.54795
8	1.27580	1.00717	1.62544
9	0.57454	0.13696	0.59064
10	-2.62192	2.03351	3.31808
11	0.70170	0.34682	0.79319
12	1.22210	1.93339	2.28726
13	0.81457	0.33668	0.88140
14	-2.64912	0.70885	2.74039
15	0.83230	0.58459	1.01708
16	0.60304	2.71695	2.78307
17	0.98383	0.60637	1.15569
18	-2.02930	-0.39649	2.06767
19	0.90461	0.81877	1.22013
20	-0.42094	3.05287	3.08176

Table B10
Spectrum for a τ Ratio Code of 20, 23, 22, 24 and Amplitudes 1.00000, 1.03156, 0.95612, and 0.92456

n'	In-Phase Component	Quadrature Component	RMS Amplitude
0	0.00000	0.00000	0.00000
1	0.00000	0.00000	0.00000
2	-0.74935	3.81118	3.88415
3	0.35352	0.23002	0.42176
4	0.65775	0.22743	0.69596
5	0.51366	-0.02554	0.51430
6	-2.03266	3.04896	3.66440
7	0.50270	0.51603	0.72041
8	1.04108	0.81961	1.32499
9	1.04379	0.25385	1.07421
10	-2.73806	1.75434	3.25185
11	0.50295	0.73408	0.88985
12	0.98166	1.54058	1.82676
13	1.39481	0.83138	1.62379
14	-2.68464	0.31125	2.70262
15	0.46513	0.83879	0.95912
16	0.48312	2.09860	2.15349
17	1.39685	1.57488	2.10509
18	-1.92464	-0.86711	2.11095
19	0.48882	0.86711	1.00310
20	-0.27832	2.25781	2.27490

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13. ABSTRACT

A system of pulse coding advanced by J. D. Mallett of the Rand Corporation to reduce the effect of ground clutter on airborne search radars has been examined and found to be of questionable value for a search radar, although other applications may exist now and in the future. The system would suppress a selected group of lines in the transmitted spectrum by means of a proper combination of the amplitudes, phases, and pulse spacings of a small group of successive pulses. The conditions for this suppression are examined in detail for an infinitely long series of pulse groups, and it is found that with a group of N pulses, $N-1$ spectrum lines may be suppressed. These lines may be any group of $N-1$, but it is shown that if they are symmetrically placed about the carrier, the rf phase angles are ϕ and $\phi+\pi$ radians. The first-order effects on the line suppression of variations from the nominal values of the parameters are analyzed, and it is shown that variations in the interpulse spacing must be kept below 1-1/2 parts in 100,000, the phase to within 0.1 degree, and the amplitude to within 1 part in 100,000 for reliable detection with a 40-db clutter-to-signal ratio. Finally, the effect of finite dwell time is examined, and it is shown that under the same conditions as above at least several thousand pulse groups need to be processed, which would require a time on target of about 18 sec.

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Line suppression						
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