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Some Preliminary Calculations on the Direct Transmission of Light Through a Plane Water Surface and Its Return by a Submerged Retroreflector

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CONTENTS

ABSTRACT

When light passes through a plane water surface from the atmosphere and is reflected by a retroreflector having a flat glass entrance face, reflective losses occur at two interfaces: air-water and water-glass. In this report the general problem of computing round-trip transmission values for arbitrary relative orientations of the incidence planes at the water surface and the retrosurface is solved, and numerical computations of the effective transmissions for unpolarized light are presented for the coplanar case. The transmission depends on the angle between the direction of incident light and the normal to the interface and on the degree of polarization.

The results show that high transmissions can exist, provided the light does not graze either the water surface or the entrance face of the retroreflector. When the light is incident normal to the water surface, the round-trip transmission is approximately 95 percent even when the incident angle at the retroreflector face is as much as 30 degrees; but this value is decreased by about one-half when the incident angle at the retroreflector is 80 degrees. Similarly, the transmission remains high when the direction of incident light at the water surface is inclined up to 30 degrees to the normal. Even when this angle is 85 degrees, the transmission is about 20 percent, provided the retroreflector is inclined properly.

Light incident on the water surface from a given direction will be returned most efficiently by a retroreflector when its face is normal to the direction of the light after being refracted at the water surface. For many combinations of angles between the directions of light and the normals to the interfaces, almost equal fractions of the parallel and perpendicular components of the light are returned. However, the parallel component is always greater than the perpendicular component except that the components are equal when the direction of light is normal to both the water surface and the retroreflector entrance face.

PROBLEM STATUS

This is a final report on one phase of the problem; work is continuing on the problem.

AUTHORIZATION

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SOME PRELIMINARY CALCULATIONS ON THE DIRECT TRANSMISSION OF LIGHT THROUGH A PLANE WATER SURFACE AND ITS RETURN BY A SUBMERGED RETROREFLECTOR

R.L. Denningham and G.L. Stamm

INTRODUCTION

Plane waves of light incident on a plane interface between two media with different indices of refraction are both transmitted and reflected. If the angle of incidence is not 0 degrees, the transmitted portion of the light is refracted in a direction different from the direction of the incident light, and when the light is initially unpolarized, both the transmitted and reflected light are polarized. The transmission and reflection of incident unpolarized plane waves of light at a plane air-water interface and a plane water-glass interface of a glass retroreflector is discussed in this report.

A property characteristic of an ideal retroreflector is the reflection of light on a path parallel to the paths of the incident light (Fig. 1). The fraction of the total incident light returning to the air depends on, among other things, the total round-trip transmission, equal to the product of four transmittances: two at the air-water interface and two at the water-glass interface. For this reason values for the round-trip transmission by consideration of the transmissions at the individual interfaces are obtained for preliminary calculations. The interfaces are assumed to transmit light without diffusion.

The calculations presented in the body of the report are termed preliminary because they are given for a special case of the problem: the retroreflector is oriented so that the plane of incidence at the retroreflector face is coplanar with the plane of incidence at

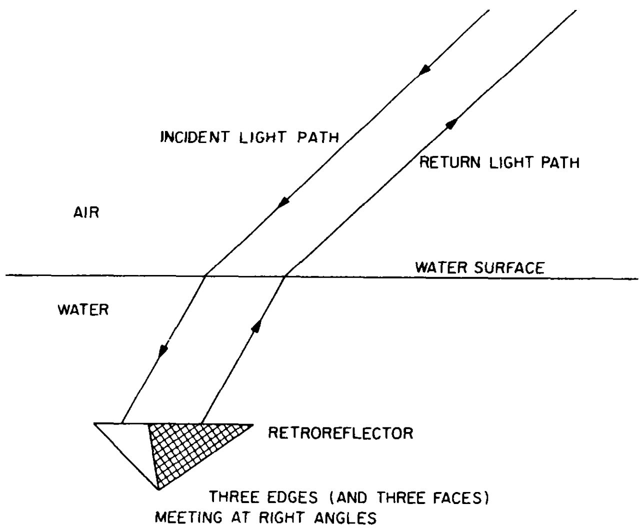

Fig. 1 - A submerged retroreflector illuminated by a light source over a plane water surface

the water surface. Appendix A presents the detailed analytic solution for the general problem, and if the problem continues to be of concern, the calculations will be extended.

The interface transmission is only one of several factors which determine the amount of light returned by a submerged retroreflector. Other factors include distances, atmospheric and water attenuation, and, under certain conditions, the effective acceptance aperture of the retroreflector. In certain real situations (for example, at sea when the surface is disturbed and uneven) the air-water interface transmission factor for a light beam is not easily calculated, since the angles of incidence vary from one small area to another and are constantly changing with time. Also, when there are breaking waves, light is scattered out of a transmitted beam by tiny air bubbles suspended in a stratum of water near the surface, resulting in a transmission loss. Furthermore, a rough water surface seriously impairs the return of light from an illuminated underwater retroreflector. Thus the calculations presented in this report pertain only to the special conditions described and are not likely to apply to conditions to be found at sea.

REFRACTION

Let n_1 and n_2 be the indices of refraction of two homogeneous media (Fig. 2), and suppose that light composed of plane waves in medium 1 is incident at an angle ϕ_1, measured from the normal at the plane interface between the two media. Snell's Law states that

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 \tag{1}$$

where the angle ϕ_2 is the angle of refraction, that is, the angle between the normal to the refracted plane waves and the normal to the interface. The incident light path defines a plane with the normal to the interface known as the plane of incidence. The refracted light path is also in this plane.

If the second medium is optically less dense than the first $(n_2 < n_1)$, and the light is incident on the interface at a particular angle called the critical angle ϕ_c, the angle of

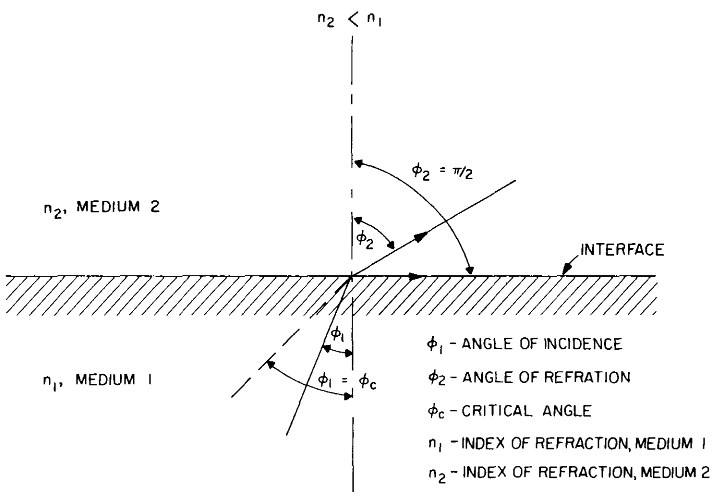

Fig. 2 - Refraction of a plane light wave at the plane interface between two media having different indices of refraction

refraction will be $\pi/2$. This is shown in Fig. 2, where the light emerges tangent to the interface. By setting ϕ_2 equal to $\pi/2$ in Eq. (1), the critical angle can be calculated:

$$\sin \phi_c = n_2/n_1 . \tag{2}$$

For all angles of incidence equal to or greater than the critical angle, no light is transmitted through the interface.

TRANSMISSION

If the two media in which the light travels are of zero conductivity, i.e., have real indices of refraction, the direct transmittances of the interface for polarized light are calculable from the Fresnel formulas as

$$\tau_{r_{\parallel}} = \frac{\sin 2\phi_1 \sin 2\phi_2}{\sin^2(\phi_1 + \phi_2) \cos^2(\phi_1 - \phi_2)} \tag{3}$$

and

$$\tau_{r_{\perp}} = \frac{\sin 2\phi_1 \sin 2\phi_2}{\sin^2(\phi_1 + \phi_2)} . \tag{4}$$

$\tau_{r_{\parallel}}$ and $\tau_{r_{\perp}}$ are the direct intensity transmittances (1) for the two polarized components of light, referred to as the parallel and perpendicular components, having their electric vectors parallel and perpendicular respectively to the plane of incidence.

In the case of unpolarized incident light (natural light), the parallel and perpendicular components are equal in magnitude. Thus, the intensity transmittance τ_r for unpolarized light at a dielectric surface is given by one-half of Eq. (3) plus one-half of Eq. (4):

$$\tau_r = \frac{1}{2} \frac{\sin 2\phi_1 \sin 2\phi_2}{\sin^2(\phi_1 + \phi_2) \cos^2(\phi_1 + \phi_2)} + \frac{1}{2} \frac{\sin 2\phi_1 \sin 2\phi_2}{\sin^2(\phi_1 + \phi_2)} . \tag{5}$$

The first term gives the portion of the incident light transmitted as a parallel component and the last term gives the portion transmitted as a perpendicular component.

For normal incidence (ϕ_1 = 0 degrees) no distinction is made between the parallel and perpendicular components, both being transmitted equally. The direct transmittance for both components in this case is

$$\tau_{r_{\parallel}}(0) = \tau_{r_{\perp}}(0) = \frac{4n_1 n_2}{(n_1 + n_2)^2} . \tag{6}$$

A second important angle of incidence, called ϕ_p, satisfies the condition $\phi_p + \phi_2 = \pi/2$. When ϕ_p is substituted in Eq. (3), $\tau_{r_{\parallel}}$ becomes unity, signifying that the parallel component of the incident light is completely transmitted. This angle, known as the polarizing (or Brewster's) angle, can be solved for in Eq. (1):

$$\tan \phi_p = n_2/n_1 . \tag{7}$$

When the polarizing angle is known, $n_1 \tan \phi_p$ can be substituted for n_2 in Eq. (6) to calculate the direct transmittance at normal incidence:

$$\tau_{r_{\parallel}}(0) = \tau_{r_{\perp}}(0) = \frac{4 \tan \phi_p}{(\tan \phi_p + 1)^2} \,. \tag{8}$$

TRANSMISSION THROUGH A PLANE AIR-WATER INTERFACE

Calculations of the transmittance values for a plane air-water interface have been made from Eq. (5) for various incident angles different from zero and have been made from Eq. (6) for an incident angle equal to zero. In the graphs to follow, three curves are plotted for the values of each of the two parts of Eq. (5) and their sum: the parallel component, the perpendicular component, and the parallel plus perpendicular components. The index of refraction of air was taken to be 1.00 and that of water to be 1.34, which is close to that of sea water.

Figures 3a and 3b show the percentage of incident unpolarized light transmitted through a plane air-water interface for light incident from the air and from the water respectively. Light is transmitted from air into water at all angles of incidence (see Fig. 3a) except at grazing incidence (ϕ_1 = 90 degrees). However, light is transmitted from water to air only at angles smaller than the critical angle of 48.3 degrees (Fig. 3b); for angles greater than this, all light is reflected back into the water. Angles of polarization are shown for light incident from the air and from the water to be 53.3 degrees and 36.7 degrees respectively.

The effective transmission of light after a two-way passage through an air-water interface is determined in the following way. Assume that unpolarized light is incident on the interface from the air, is transmitted through the interface to a perfect underwater retroreflector, and is reflected back to the air along a path parallel to that of the incident light (Fig. 4). Data for the curves in Fig. 4, showing the transmission for the two-way passage, were calculated by multiplying the air-to-water transmittances plotted in Fig. 3a by the appropriate water-to-air transmittances plotted in Fig. 3b. In the two-way transmission being discussed, the angle of incidence for light striking the water surface is equal to the angle of refraction for light being returned into the air, and the angle of refraction for light transmitted into the water is equal to the angle of incidence for the returned light in the water.

The transmission is appreciable even at large angles of incidence, being a maximum of 96 percent for normal incidence and reduced by a factor of 2 for an incident angle of 79 degrees, and by a factor of 10 for an angle of 87 degrees. At normal incidence, the returned light is equally divided between the parallel and perpendicular components. The parallel and perpendicular components are reduced by a factor of 2 from their values at normal incidence at 82 degrees and 74 degrees respectively, and by a factor of 10 at 88 degrees and 85 degrees respectively. At the air-water polarizing angle, 53.3 degrees, all of the incident parallel component is returned after its two-way passage through the interface.

TRANSMISSION THROUGH A PLANE WATER-GLASS INTERFACE

In the previous section a perfect retroreflector was assumed; but in practice such a glass corner cube type retroreflector might be considered, which approximates the ideal. The corner cube is made from a block of glass having three mutually perpendicular sides which meet at a point. A flat entrance face is cut diagonally across the block such that the point is on a line perpendicular to the center of the face. Within a certain range of

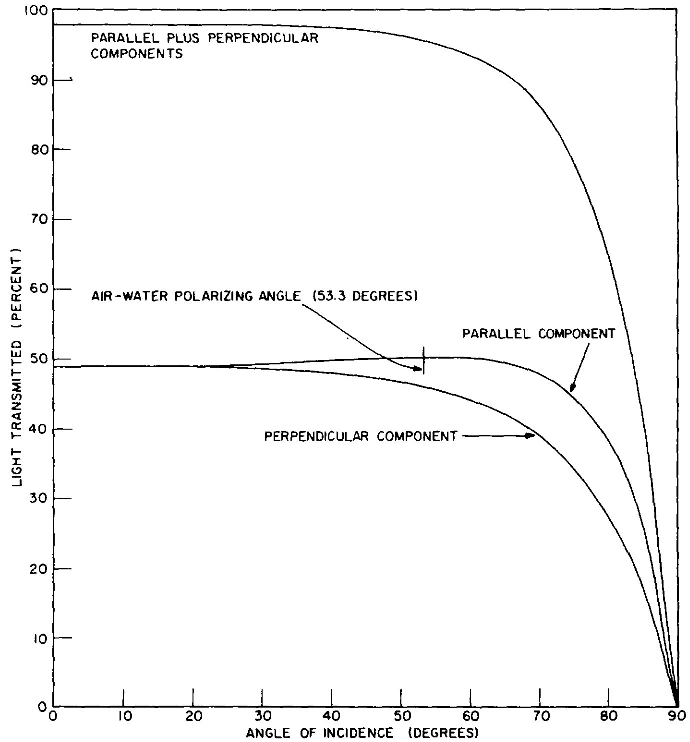

Fig. 3a - Transmission of unpolarized light through a plane air-water interface as a function of incident angle with the light incident from the air

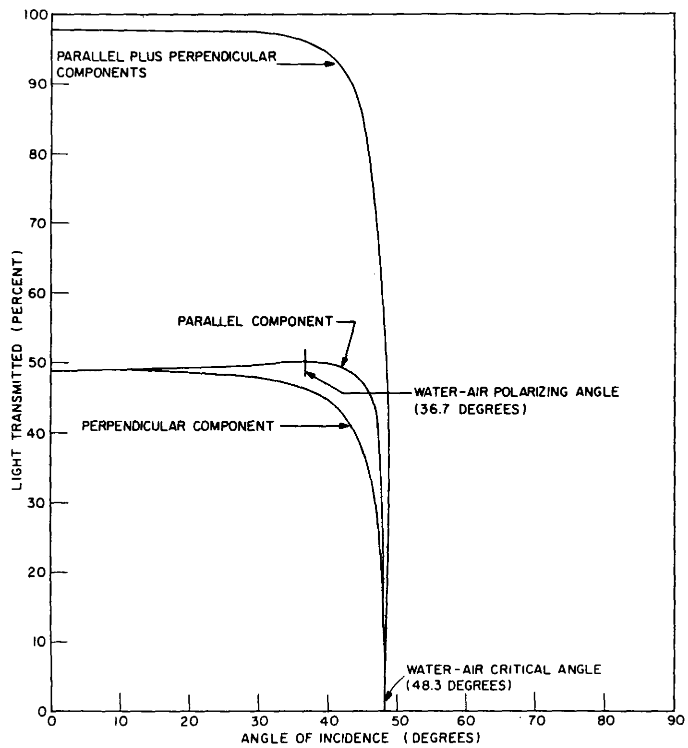

Fig. 3b - Transmission of unpolarized light through a plane air-water interface as a function of incident angle with the light incident from the water

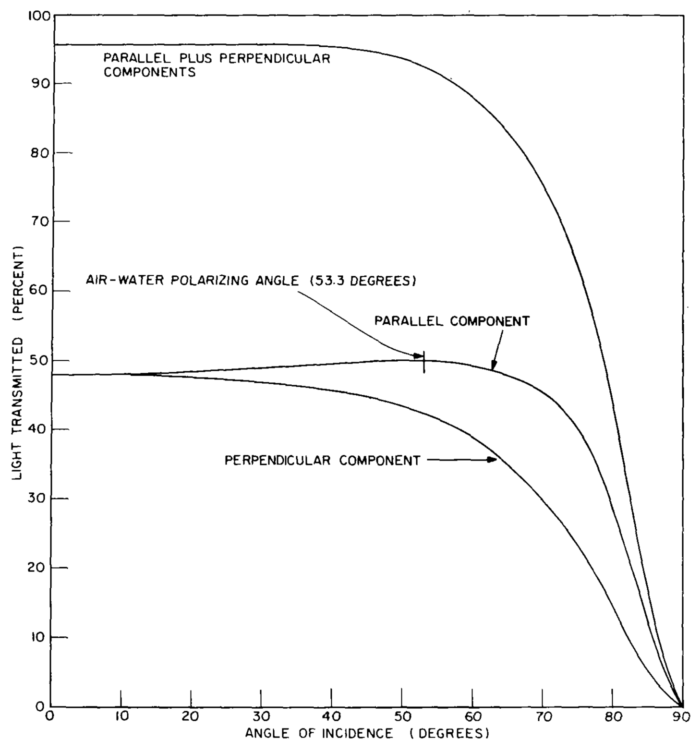

Fig. 4 - Transmission of unpolarized light for a two-way passage through a plane air-water interface as a function of incident angle with the light incident originally from the air

incident angles, the incident and reflected light paths will be parallel, but light will be lost by reflection at the face; this loss and thus the transmittance will vary with the angle of incidence.

Some glass retroreflectors (2) have other designs, and the internal reflections at their surfaces may modify the light differently. Some real retroreflector configurations introduce internal polarization shifts. Therefore, in order to simplify the problem, it is assumed that the retroreflector is made of glass and that the light enters and exits through a flat face and is reflected internally without loss or change.

The percentage of incident unpolarized light transmitted through the plane water-glass interface of a submerged retroreflector for different incident angles was calculated using Eqs. (5) and (6), as described in the case of the air-water interface. The indices of refraction of water and glass were taken to be 1.34 and 1.52 respectively, since many typical crown glasses have indices of refraction close to the latter value.

Figure 5a shows the transmission through a water-glass interface when unpolarized light is incident on it from the water. As before, two curves show the percentage of light transmitted as parallel and perpendicular components, and the third curve gives the sum of the two transmitted components. At normal incidence, 99 percent of the incident light is transmitted; half as a parallel component and half as a perpendicular component. The polarizing angle is shown at 48.6 degrees.

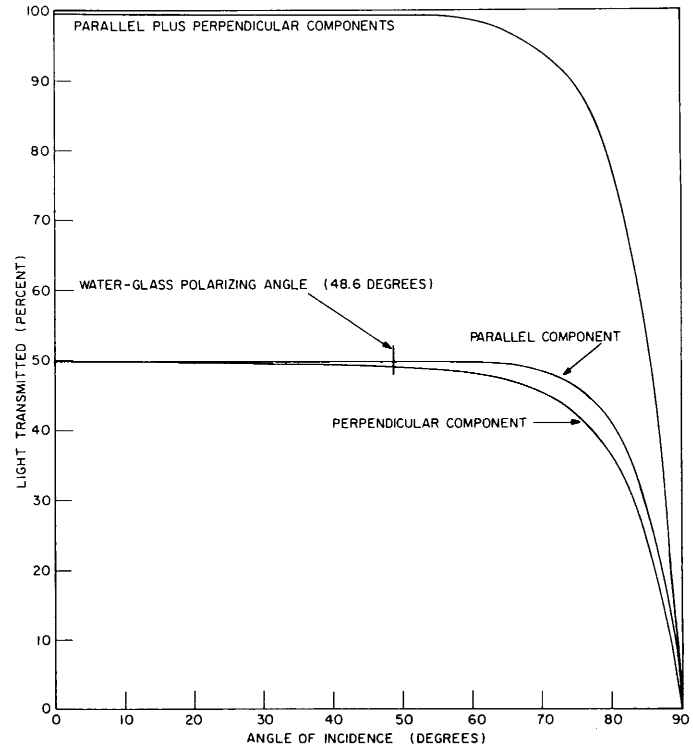

Fig. 5a - Transmission of unpolarized light through a plane water-glass interface as a function of incident angle with the light incident from the water

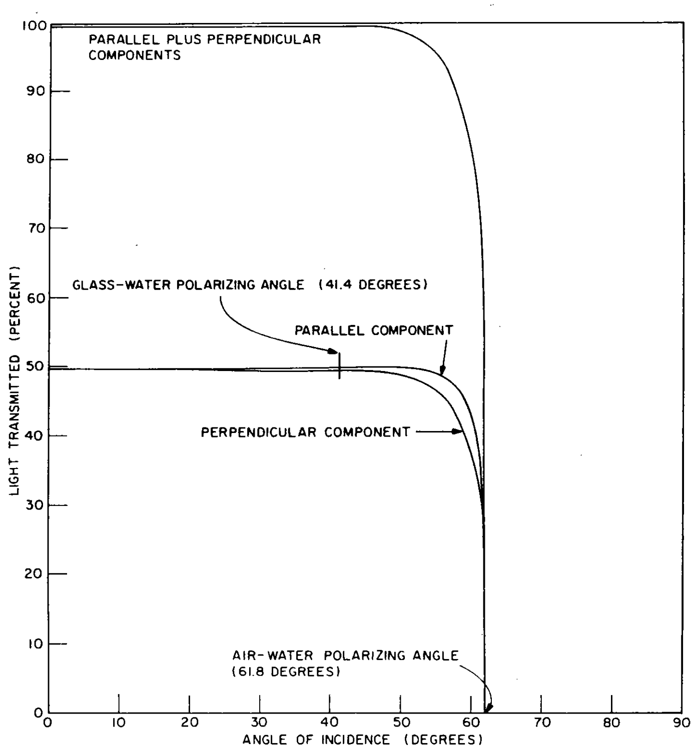

Fig. 5b - Transmission of unpolarized light through a plane water-glass interface as a function of incident angle with the light incident from the glass

Figure 5b shows similar curves for light incident from the glass side at a water-glass interface. Here the polarizing angle is 41.4 degrees, and because the light is being refracted into a medium of smaller refractive index, there is a critical angle at 61.8 degrees.

The data represented by the curves in Figs. 5a and 5b were combined to calculate the two-way transmission (Fig. 6) for light incident on the interface from the water side, reflected internally unchanged, and returned through the interface. At normal incidence, the two-way transmittance is 99 percent; it is reduced by a factor of 2 at an incident angle of 82 degrees and by a factor of 10 at 88 degrees. Equal fractions of the incident light are transmitted as parallel and perpendicular components at normal incidence; these fractions are reduced by a factor of 2 at 82 degrees and 81 degrees and by a factor of 10 at 89 degrees and 88 degrees, respectively.

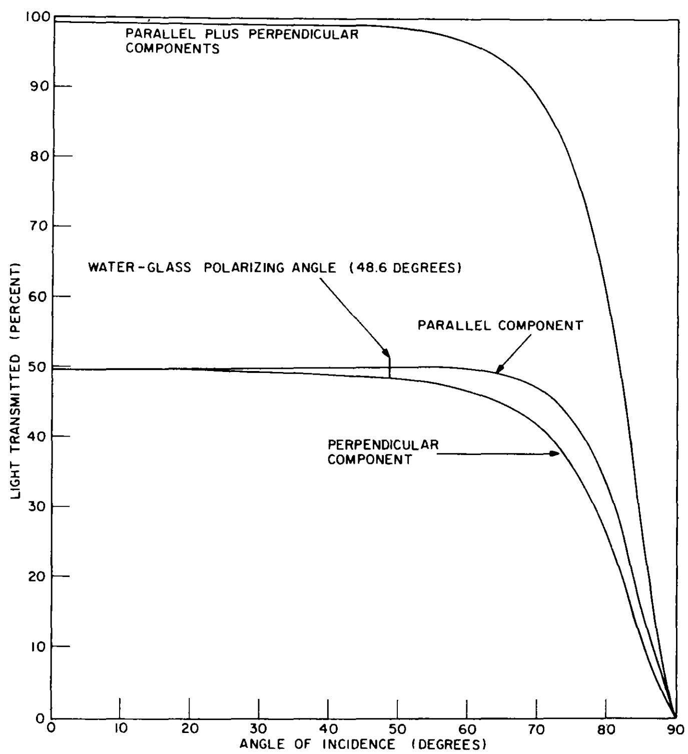

Fig. 6 - Transmission of unpolarized light for a two-way passage through a plane water-glass interface as a function of incident angle with the light incident originally from the water

LIGHT RETURNED BY A SUBMERGED RETROREFLECTOR

When the glass retroreflector is submerged in still water and is illuminated by a source over the water, the light returned to the air is diminished by both reflective light losses which occur during the two-way transmission through the air-water interface and the water-glass interface. Figure 7 is an explanatory diagram showing the angular relationships involved when a submerged retroreflector is oriented so that the plane of incidence at the retroreflector is coplanar with the plane of incidence at the water surface (Appendix A gives an analysis which does not restrict the planes of incidence to be coplanar).

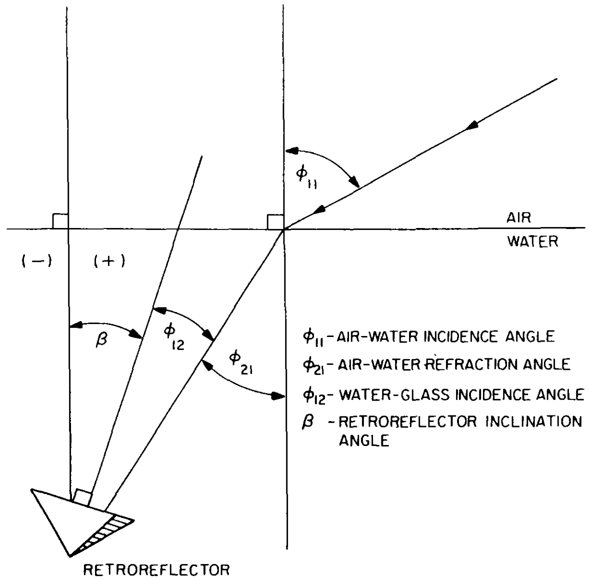

Fig. 7 - Angular relationships for a submerged retroreflector illuminated by light originating in the air. The planes of incidence at the air-water interface and at the retroreflector face are coplanar.

Figure 8a shows the transmission of the parallel component for constant air-water incident angles while retroreflector inclination angles are varied. At any given angle of incidence the transmittances are nearly constant over a wide range of retroreflector inclination angles. The maximum values for each curve occur for those retroreflector inclination angles β which correspond to an incident angle (ϕ_{12}) at the water-glass interface equal to the polarizing angle (ϕ_{p2} = 48.6 degrees). There are two maximum points for each angle of incidence at the water surface, one near each shoulder of the curve. When the incident angle at the air-water interface is the polarizing angle (53.3 degrees), there are two retroreflector inclination angles (-11.9 and 85.3 degrees) corresponding to the two polarizing angles at the entrance face of the retroreflector where the transmittances increase to a maximum of 50 percent.

Figure 8b shows a family of curves for the transmission of the perpendicular component of the light after its two-way passage through the interfaces. The maximum point of each curve is at the retroreflector inclination angle β, equal to the angle of refraction at the air-water interface ϕ_{21}. The maximum value for the individual curves decreases with increasing angle of incidence; the overall maximum, 48 percent, occurs when both the incident angle at the water surface and the retroreflector angles are 0 degrees. As seen by comparing the curves in Figs. 8a and 8b the transmittances for the parallel component are higher than for the perpendicular component for all angles of incidence and retroreflector inclination angles except at 0 degrees, where the transmittances are equal.

In Fig. 8c a family of curves is plotted to show the transmission of incident unpolarized light as a function of retroreflector inclination angle for various air-water incident angles. The overall maximum, 95 percent, occurs at an incident angle of 0 degrees and remains close to this value for retroreflector inclination angles between -30 and +30 degrees. Increasing the angle of incidence results in decreasing the maximum transmission, which is obtained when the retroreflector inclination angle β equals the angle of

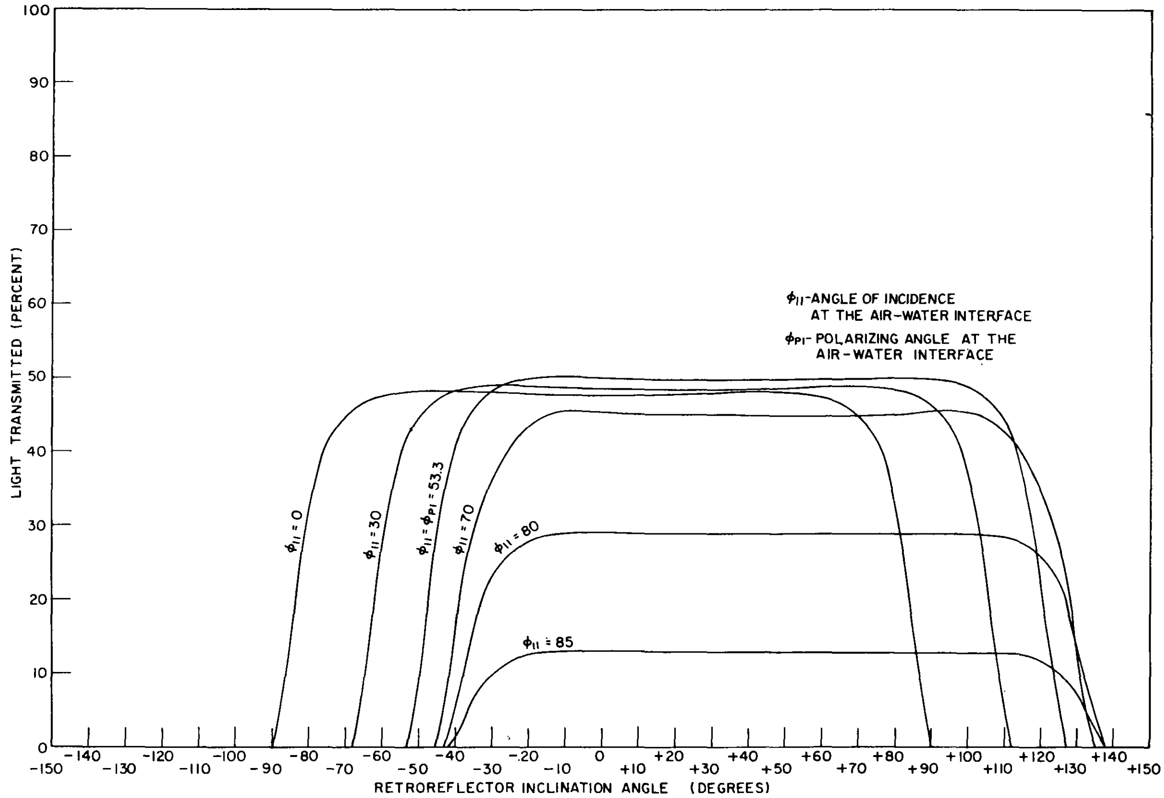

Fig. 8a - Transmission of the parallel component of unpolarized light for a two-way passage through a system consisting of a plane air-water interface and a plane water-glass interface as a function of the retroreflector inclination angle with the light incident originally from the air at various constant angles

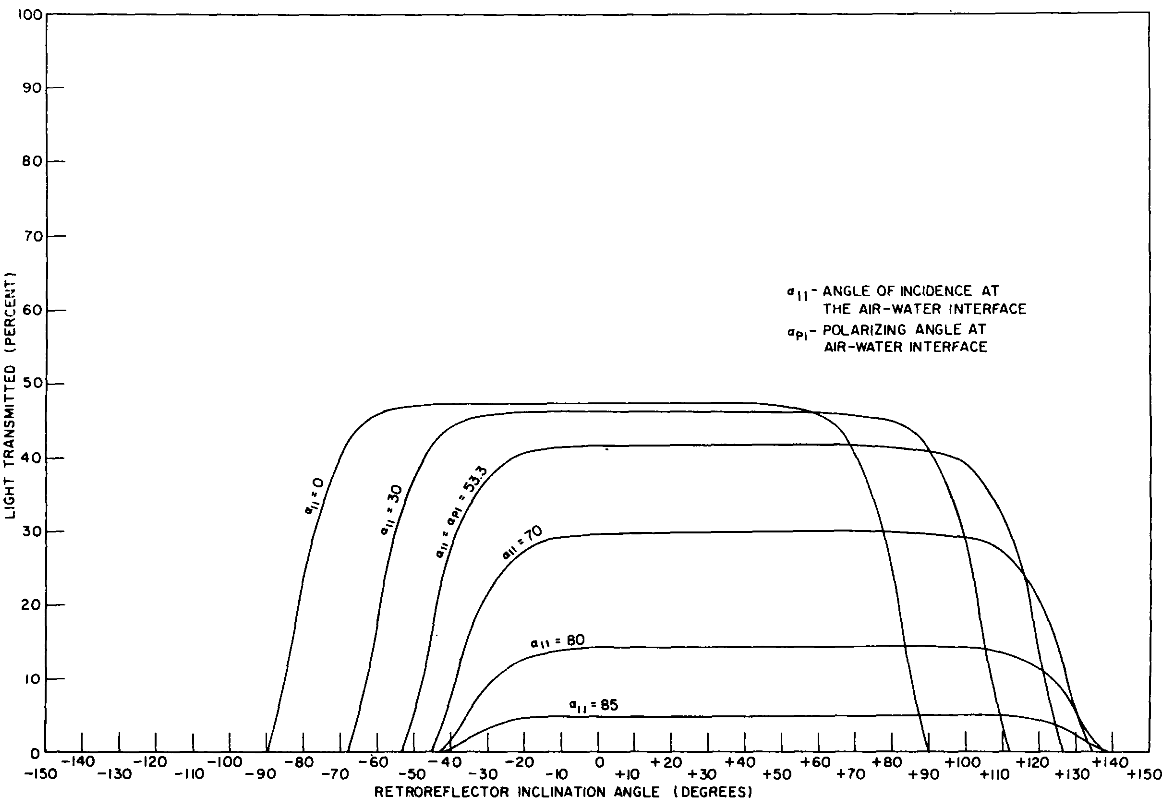

Fig. 8b - Transmission of the perpendicular component of unpolarized light for a two-way passage through a system consisting of a plane air-water interface and a plane water-glass interface as a function of the retroreflector inclination angle with the light incident originally from the air at various constant angles

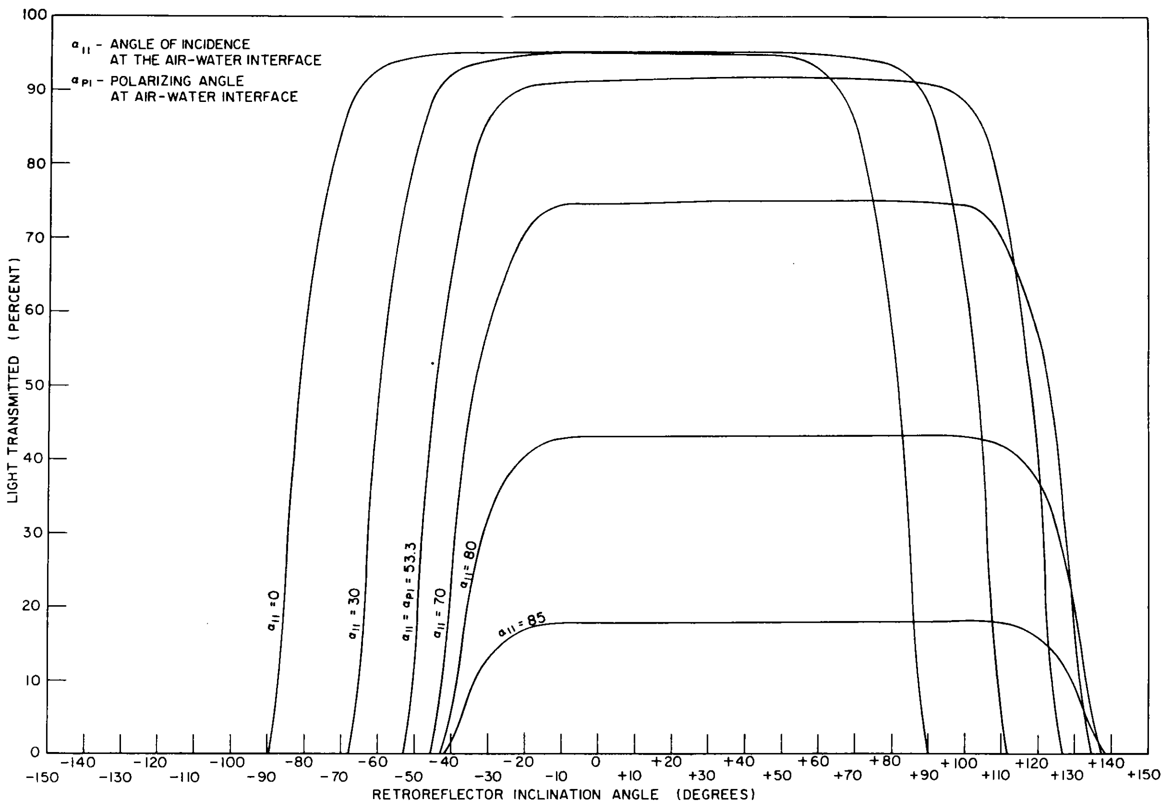

Fig. 8c - Transmission of the total unpolarized light for a two-way passage through a system consisting of a plane air-water interface and a plane water-glass interface as a function of the retroreflector inclination angle with the light incident originally from the air at various constant angles

refraction ϕ_{21} at the air-water interface. The plateaus of the curves ϕ_{11} = 0 degrees and ϕ_{11} = 30 degrees nearly coincide between retroreflector angles of -10 degrees and +40 degrees, indicating that there is little increase in light loss when the angle of incidence is increased up to 30 degrees for these retroreflector inclination angles. Even at ϕ_{11} = 85 degrees, close to the grazing incidence, the transmittance is almost 20 percent.

Provided that the planes of incidence at the water surface and at the retroreflector face are coplanar and that the retroreflector inclination angle is adjusted properly, the two-way transmission for light incident on a plane water surface from the air can be relatively large for a wide range of angles. This is shown in Fig. 9, where the maximum transmission for incident unpolarized light is plotted versus the angle of incidence ϕ_{11} for the parallel and perpendicular components individually and together. For the perpendicular component and the combination of parallel and perpendicular components, the inclination angle corresponding to maximum transmission is equal to the angle of refraction ϕ_{21} at the air-water interface. For the parallel component, this angle has two values: the polarizing angle ϕ_{p2} = 48.6 degrees plus and minus the angle of refraction ϕ_{21} ($\phi_{p2} \pm \phi_{21}$). When $\beta = \phi_{21}$, light enters the retroreflector normal to its face, and when $\beta = \phi_{p2} \pm \phi_{21}$, light is incident at the retroreflector face at the polarizing angle ϕ_{p2}.

The overall maximum transmittance is 95 percent at an incident angle of 0 degrees, and the overall maxima for the parallel and perpendicular components are 50 percent and 48 percent respectively at incident angles of 53.3 degrees and 0 degrees respectively.

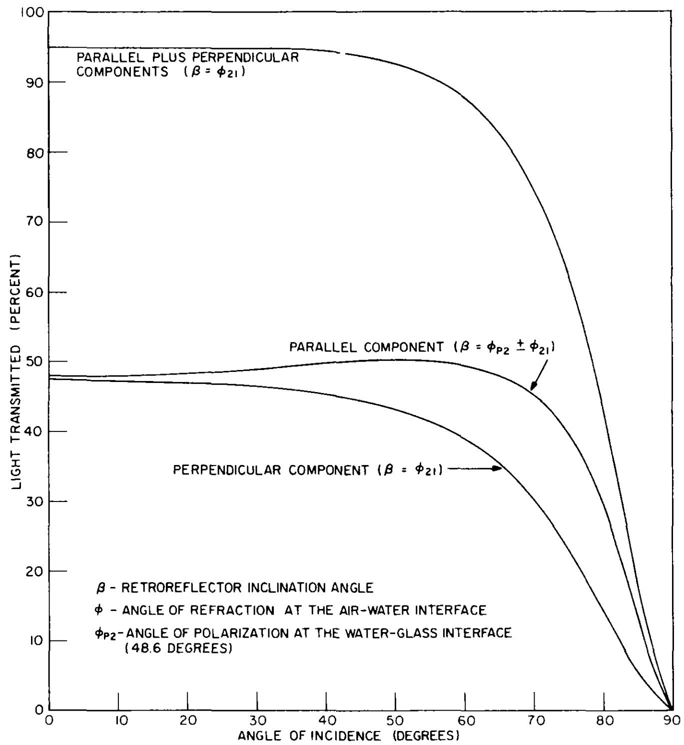

Fig. 9 - Maximum transmission of unpolarized light for a two-way passage through a system consisting of a plane air-water and plane water-glass interface as a function of the angle of incidence with the light incident originally from the air

The overall maximum values for the parallel and perpendicular components and for the combination of these two components are reduced by a factor of 2 at 81, 74, and 79 degrees respectively and by a factor of 10 at 88, 85, and 87 degrees respectively.

The retroreflector inclination angle β, as well as the incident angle, can be varied greatly before the transmission decreases significantly. Each curve of Fig. 8 is approximately flat over most of its range, and the inclination angle can be varied over a range of approximately 160 degrees before the transmission is reduced to half its maximum. For certain retroreflector inclination angles, two angles of incidence at the water surface result in the same transmission. For example, at an inclination angle of 74 degrees (Fig. 8c) the transmittance is 75 percent for light incident at angles of 0 and 70 degrees.

Curves given in this report indicate that a submerged retroreflector can be used effectively to return light incident on a plane water surface when the planes of incidence at the water surface and the retroreflector face are coplanar. Only a small percentage of the light is lost at the air-water and water-glass interfaces for a wide range of angles. Numerical results have not been obtained for the case where the two planes of incidence are not coplanar (Appendix A), but it is expected that the above conclusion will not be changed drastically.

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Appendix A

A GENERAL ANALYSIS OF THE SUBMERGED RETROREFLECTOR PROBLEM

L.F. Drummeter, Jr.

A dielectric surface serves as a partial polarizer for radiation striking it at an incidence angle ϕ_1. If we consider the incidence plane to be the plane of reference, initially unpolarized light may be resolved into two components half perpendicular to the plane of incidence and half parallel to the plane of incidence. Because the surface is a partial polarizer, the transmission factors for the two components are different. They are

$$K_1 = \frac{\sin 2\phi_1 \sin 2\phi_2}{\sin^2(\phi_1 + \phi_2) \cos^2(\phi_1 - \phi_2)} \qquad \text{(parallel)} \qquad \text{(A1)}$$

and

$$K_2 = \frac{\sin 2\phi_1 \sin 2\phi_2}{\sin^2(\phi_1 + \phi_2)} \qquad \text{(perpendicular)} \qquad \text{(A2)}$$

where ϕ_1 and ϕ_2 are the incident and refractive angles. Equations (A1) and (A2) are the Fresnel equations, corresponding to Eqs. (3) and (4) in the body of this report. The symbols for transmission factors are changed from those used earlier because the internationally approved symbols used in the body of the report become too cumbersome in the present analysis and because the K symbolism is the most common one in the mathematics of polarization analysis. For each surface n, let K_{1n} be the K_1 factor, and K_{2n} be the K_2 factor.

The most powerful tool available for handling problems which involve a series of polarization-altering devices is the Mueller calculus.* In this calculus a plane dielectric surface is equivalent to a transmission polarizer and is represented by the polarization matrix operator P_{10}:†

$$P_{10} \doteq \frac{1}{2}\begin{bmatrix} K_{11}+K_{21} & K_{11}-K_{21} & 0 & 0 \\ K_{11}-K_{21} & K_{11}+K_{21} & 0 & 0 \\ 0 & 0 & 2\sqrt{K_{11}K_{21}} & 0 \\ 0 & 0 & 0 & 2\sqrt{K_{11}K_{21}} \end{bmatrix} \qquad \text{(A3)}$$

The subscript index 1 on the operator indicates the first surface; the zero indicates that the incidence plane is the plane of reference (which will be clarified later).

*W.A. Shurcliff, "Polarized Light, Production and Use," Cambridge:Harvard University Press, 1962, p. 109 ff.
†Ibid., p. 168.

The initially unpolarized light is represented by the normalized Stokes column vector $\mathbf{M}$:*†

$$\mathbf{M} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A4}$$

The transmitted light is described by the vector $\mathbf{M}_1$, which results from the matrix operation

$$\mathbf{M}_1 = P_{10}\mathbf{M} \tag{A5}$$

which gives

$$\mathbf{M}_1 = \begin{bmatrix} K_{11} + K_{21} \\ K_{11} - K_{21} \\ 0 \\ 0 \end{bmatrix} . \tag{A6}$$

as the Stokes vector for the emerging light. The first term in the vector is the intensity, which is $K_{11} + K_{21}$. The second term has been assigned the name "horizontal preference." A positive value for this term indicates that the light is linearly, horizontally polarized. A negative value indicates the orthogonal (here the vertical) polarization form. The next term in the vector, which happens to be zero for the vector which describes the incident light but which will have nonzero values in subsequent calculations, is called the "plus-45-degree preference." The sign convention described for the horizontal preference term also applies to this term. The fourth term, named the "right circular preference," is of no concern here, since it will be zero throughout this discussion. This procedure is heavyhanded for the result obtained, since the intensity result is essentially axiomatic. The advantage of this approach will only appear in subsequent operations; although we are primarily interested in the intensity, we shall use the full calculus to determine the complete Stokes vectors.

If now we allow the light described by $\mathbf{M}_1$ to fall on a second dielectric plane interface we have a second polarization operation to contend with. For complete generality we must allow the normal to the second surface to be oriented at any arbitrary angle to the light $\mathbf{M}_1$ (arbitrary angle of incidence); thus we must permit the plane of incidence at the second surface to be oriented arbitrarily relative to our reference plane (the original plane of incidence).

Let θ be the angle between the second plane of incidence and the reference plane. This angle is shown in Fig. A1, which illustrates analytically how the general orientation of the retroreflector should be described. The polarization matrix for this second plane is given by‡

*The first component of the vector is the intensity; hence the 1 indicates the choice of unit incident intensity.
†Ibid., p. 23.
‡Ibid., p. 116.

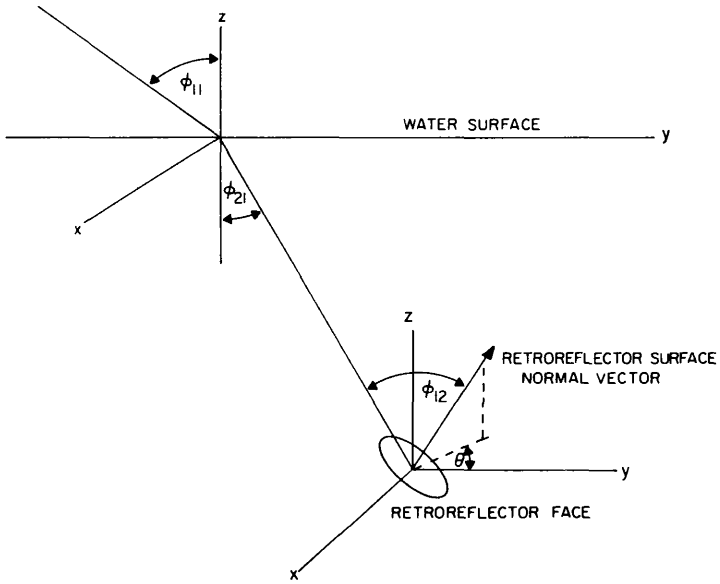

Fig. A1 - Analytical description of the orientation of the retroreflector. The yz plane is the initial plane of incidence and is the reference plane. The angle θ is the angle between the plane of incidence at the retroreflector surface and the plane of reference.

$$P_{2\theta} = [R(-2\theta)]\,[P_{10}]\,[R(2\theta)] \tag{A7}$$

where

$$R(2\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C & S & 0 \\ 0 & -S & C & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A8}$$

which is a rotator matrix, and $C = \cos 2\theta$ and $S = \sin 2\theta$. The matrix $R(-2\theta)$ is the same as $R(2\theta)$ except the algebraic signs of the sine elements are changed. Thus Eq. (A7) becomes

$$P_{2\theta} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C & -S & 0 \\ 0 & S & C & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} K_{12}+K_{22} & K_{12}-K_{22} & 0 & 0 \\ K_{12}-K_{22} & K_{12}+K_{22} & 0 & 0 \\ 0 & 0 & 2\sqrt{K_{12}K_{22}} & 0 \\ 0 & 0 & 0 & 2\sqrt{K_{12}K_{22}} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C & S & 0 \\ 0 & -S & C & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A9}$$

which may be multiplied out to give

$$P_{2\theta} = \frac{1}{2}\begin{bmatrix} K_{12}+K_{22} & C(K_{12}-K_{22}) & S(K_{12}-K_{22}) & 0 \\ C(K_{12}-K_{22}) & C^2(K_{12}+K_{22}) + 2S^2\sqrt{K_{12}K_{22}} & CS(K_{12}+K_{22}) - 2SC\sqrt{K_{12}K_{22}} & 0 \\ S(K_{12}-K_{22}) & CS(K_{12}+K_{22}) - 2SC\sqrt{K_{12}K_{22}} & S^2(K_{12}+K_{22}) + 2C^2\sqrt{K_{12}K_{22}} & \\ 0 & 0 & 0 & 2\sqrt{K_{12}K_{22}} \end{bmatrix} \tag{A10}$$

as the second surface polarization matrix. Note the second index on the K's is 2, denoting the second surface. K_{12} and K_{22} are determined from the Fresnel equations and the angle of incidence for the perpendicular and parallel components in the second plane of incidence.

Multiplying to get

$$\mathbf{M}_2 = P_{2\theta}\,\mathbf{M}_1 \tag{A11}$$

we obtain

$$\mathbf{M}_2 = \frac{1}{2}\begin{bmatrix} (K_{11}+K_{21})(K_{12}+K_{22}) + C(K_{12}-K_{22})(K_{11}-K_{21}) \\ C(K_{12}-K_{22})(K_{11}+K_{12}) + (K_{11}-K_{21})\left[C^2(K_{12}+K_{22}) + 2S^2\sqrt{K_{12}K_{22}}\right] \\ S(K_{12}-K_{22})(K_{11}+K_{12}) + (K_{11}-K_{21})\left[CS(K_{12}+K_{22}) - 2SC\sqrt{K_{12}K_{22}}\right] \\ 0 \end{bmatrix} \tag{A12}$$

from which the intensity is seen to be

$$I_2 = \frac{1}{2}\left[(K_{11}+K_{21})(K_{12}+K_{22}) + C(K_{12}-K_{22})(K_{11}-K_{21})\right]. \tag{A13}$$

If we allow the two incidence planes to be parallel, $C = 1$ and

$$I_2 = \frac{1}{2}\left[(K_{11}+K_{21})(K_{12}+K_{22}) + (K_{12}-K_{22})(K_{11}-K_{21})\right] \tag{A14}$$

which by a little algebra is readily reduced to

$$I_2 = K_{11}K_{12} + K_{21}K_{22} \tag{A15}$$

which, for this special case, is easily written down by inspection. Note that the general vector $\mathbf{M}_2$ gives, at the expense of relatively little effort, all the information about the light, which in this case has finite horizontal and 45-degree preferences.

If now we reverse the direction of the light $\mathbf{M}_2$ and send it back through the second surface with no change in polarization or direction, the passage through the second surface in the opposite direction is equivalent to passage through a third surface or through a third polarizer. This third polarizer is oriented exactly as the second insofar as the principal azimuth is concerned; hence the net effect is as though polarizers 2 and 3 were combined to give equivalent transmittances of $K_{12}K_{13}$ and $K_{22}K_{23}$ for a single polarizer oriented at an angle θ relative to the reference plane. Thus the net effect of surfaces 2 and 3 can be obtained by rewriting Eq. (A10) and replacing K_{12} by $K_{12}K_{13}$ and K_{22} by $K_{22}K_{23}$ to obtain

$$P_{3\theta} = \frac{1}{2}\begin{bmatrix} K_{12}K_{13} + K_{22}K_{23} & C(K_{12}K_{13} - K_{22}K_{23}) \\ C(K_{12}K_{13} - K_{22}K_{23}) & C^2(K_{12}K_{13} + K_{22}K_{23}) + 2S^2\sqrt{K_{12}K_{13}K_{22}K_{23}} \\ S(K_{12}K_{13} - K_{22}K_{23}) & CS(K_{12}K_{13} + K_{22}K_{23}) - 2SC\sqrt{K_{12}K_{13}K_{22}K_{23}} \\ 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} S(K_{12}K_{13} - K_{22}K_{23}) & 0 \\ CS(K_{12}K_{13} + K_{22}K_{23}) - 2SC\sqrt{K_{12}K_{13}K_{22}K_{23}} & 0 \\ S^2(K_{12}K_{13} + K_{22}K_{23}) + 2C^2\sqrt{K_{12}K_{13}K_{22}K_{23}} & 0 \\ 0 & \sqrt{K_{12}K_{13}K_{22}K_{23}} \end{bmatrix} . \tag{A16}$$

The light transmitted through the third surface would thus be described by

$$\mathbf{M}_3 = \frac{1}{2}\begin{bmatrix} (K_{11} + K_{21})(K_{12}K_{13} + K_{22}K_{23}) + C(K_{12}K_{13} - K_{22}K_{23})(K_{11} - K_{21}) \\ C(K_{12}K_{13} - K_{22}K_{23})(K_{11} + K_{12}) + (K_{11} - K_{21})\,[C^2(K_{12}K_{13} + K_{22}K_{23}) \\ + 2S^2\sqrt{K_{12}K_{13}K_{22}K_{23}}] \\ S(K_{12}K_{13} - K_{22}K_{23})(K_{11} + K_{12}) + (K_{11} - K_{21})\,[CS(K_{12}K_{13} + K_{22}K_{23}) \\ - 2SC\sqrt{K_{12}K_{13}K_{22}K_{23}}] \\ 0 \end{bmatrix} . \tag{A17}$$

If the second/third surface is in fact that of a retroreflector, the above equations are not strictly correct. The transmittances K_{13} and K_{23} are not quite those given by the Fresnel equations because of successive reflections within the prism; but the errors should be small enough that the Fresnel expressions are acceptable.

Finally, in the problem at hand, the light described by $\mathbf{M}_3$ is transmitted back through the first surface, which becomes the fourth surface in our train of polarizers. The operator is

$$P_{4\theta} = \frac{1}{2}\begin{bmatrix} K_{14} + K_{24} & K_{14} - K_{24} & 0 & 0 \\ K_{14} - K_{24} & K_{14} + K_{24} & 0 & 0 \\ 0 & 0 & 2\sqrt{K_{14}K_{24}} & 0 \\ 0 & 0 & 0 & 2\sqrt{K_{14}K_{24}} \end{bmatrix} \tag{A18}$$

and the exiting beam is given by

$$\mathbf{M}_4 = P_{4\theta}\,\mathbf{M}_3 . \tag{A19}$$

After the equations are algebraically manipulated, the result is

$$M_4 = \frac{1}{4}\begin{bmatrix} \begin{array}{l} (K_{14}+K_{24})\,[(K_{11}+K_{21})(K_{12}K_{13}+K_{22}K_{23}) \;+\; C(K_{11}-K_{21})(K_{12}K_{13}-K_{22}K_{23})] \\ +\;(K_{14}-K_{24})\,\{C(K_{11}+K_{21})(K_{12}K_{13}-K_{22}K_{23}) \;+\; (K_{11}-K_{21})\,[C^2(K_{12}K_{13}+K_{22}K_{23}) \\ +\;2S^2\sqrt{K_{12}K_{13}K_{22}K_{23}}]\} \end{array} \\[2ex] \begin{array}{l} (K_{14}-K_{24})\,[(K_{11}+K_{21})(K_{12}K_{13}+K_{22}K_{23}) \;+\; C(K_{11}-K_{21})(K_{12}K_{13}-K_{22}K_{23})] \\ +\;(K_{14}+K_{24})\,\{C(K_{11}+K_{12})(K_{12}K_{13}-K_{22}K_{23}) \;+\; (K_{11}-K_{21})\,[C^2(K_{12}K_{13}+K_{22}K_{23}) \\ +\;2S^2\sqrt{K_{12}K_{13}K_{22}K_{23}}]\} \end{array} \\[2ex] \begin{array}{l} 2\sqrt{K_{14}K_{24}}\;\{S(K_{12}K_{13}-K_{22}K_{23})(K_{11}+K_{12}) \;+\; (K_{11}-K_{21})\,[CS(K_{12}K_{13}+K_{22}K_{23}) \\ -\;2SC\sqrt{K_{12}K_{13}K_{22}K_{23}}]\} \end{array} \\[2ex] 0 \end{bmatrix} \tag{A20}$$

As a check, we set $\theta = 0$; i.e., we set the incidence planes into coincidence. This gives, for the intensity factor:

$$I_4 = K_{11}K_{12}K_{13}K_{14} + K_{21}K_{22}K_{23}K_{24}\,. \tag{A21}$$

which is what is expected.

Accordingly, the first term of the vector M_4 in Eq. (A20) gives the general expression for the intensity or transmission for an arbitrary angle of incidence and an arbitrary orientation of the retrosurface. It is possible to rewrite the intensity in Eq. (A20) through a series of tedious but straightforward operations. These include the following stages (which omit the intermediate algebra):

$$\begin{aligned} 4I_4 = {} & (K_{14}+K_{24})\,[(1+C)(K_{11}K_{12}K_{13}+K_{21}K_{22}K_{23}) \;+\; (1-C)(K_{11}K_{22}K_{23}-K_{21}K_{12}K_{13})] \\ & +\;(K_{14}-K_{24})\,[C(1+C)(K_{11}K_{12}K_{13}-K_{21}K_{22}K_{23}) \;+\; (1-C)(K_{21}K_{12}K_{13}-K_{11}K_{22}K_{23}) \\ & +\;2S^2(K_{11}-K_{21})\sqrt{K_{12}K_{13}K_{22}K_{23}}] \end{aligned} \tag{A22}$$

$$\begin{aligned} 4I_4 = {} & (1+C)^2\,(K_{11}K_{12}K_{13}K_{14}+K_{21}K_{22}K_{23}K_{24}) \\ & +\;(1-C)^2\,(K_{11}K_{22}K_{23}K_{14}+K_{21}K_{12}K_{13}K_{24}) \\ & +\;S^2\,[K_{14}(K_{21}K_{22}K_{23}+K_{21}K_{12}K_{13}) \;+\; K_{24}(K_{11}K_{12}K_{13}+K_{11}K_{22}K_{23}) \\ & +\;2(K_{14}-K_{24})(K_{11}-K_{21})\sqrt{K_{12}K_{13}K_{22}K_{23}}] \end{aligned} \tag{A23}$$

$$4I_4 = (4\cos^4\theta)A + (4\sin^4\theta)B + 4(\sin^2\theta\,\cos^2\theta)D \tag{A24}$$

where A, B, and D are written for the terms involving the K factors.

If Eq. (A24) is differentiated with respect to θ, the result after algebraic manipulation is

$$\frac{dI_4}{d\theta} = 2 \sin\theta \cos\theta[(\cos^2\theta)(D-2A) - (\sin^2\theta)(D-2B)] \tag{A25}$$

which has values of zero at $\theta = 0, \pm\pi/2, \pm\pi, \pm 3\pi/2$, etc., and one at

$$\theta = \pm \tan^{-1}\sqrt{\frac{D-2A}{D-2B}} . \tag{A26}$$

$D-2A$ and $D-2B$ must have the same sign; if they are both negative the following relations hold:

$\theta = 0$, maximum

$\theta = \pm\pi/2$, maximum

$\theta = \pm\pi$, maximum

$\theta = \pm 3\pi/2$, maximum

$\theta = \pm\tan^{-1}\sqrt{(D-2A)/(D-2B)}$, minimum.

If they are both positive the preceding maxima and minima are interchanged. Note that A and B will be reasonably close in value; hence

$$\tan^{-1}\sqrt{\frac{D-2A}{D-2B}} \approx \tan^{-1}(1)$$

and $\theta \approx \pm\pi/4$.

It is not possible to determine precisely which minimum corresponds to the lowest possible value for I_4 without knowing the magnitudes of the K coefficients and the corresponding constants A, B, and D. However if we consider the ratio of

$$\frac{I_{4\theta}}{I_{4\pi}} = \cos^4\theta\left[1 + \frac{B}{A}\tan^4\theta + \frac{D}{A}\tan^2\theta\right] \tag{A27}$$

and consider that $\theta \approx \pi/4$ at $\theta = \tan^{-1}\sqrt{(D-2A)/(D-2B)}$ for a large range of incidence values, then

$$\frac{I_{4\theta}}{I_{4\pi}} \approx \cos^4\theta\left[\frac{A+B+D}{A}\right]$$

$$= \frac{A+B+D}{4A} \quad \text{for } \theta = \frac{\pi}{4}$$

which suggests that the value for the angle $\theta = \tan^{-1}\sqrt{(D-2A)/(D-2B)}$ will correspond to the lowest value of the function in those cases where $A \approx B \leq D$.

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13. ABSTRACT

When light passes through a plane water surface from the atmosphere and is reflected by a retroreflector having a flat glass entrance face, reflective losses occur at two interfaces: air-water and water-glass. In this report the general problem of computing round-trip transmission values for arbitrary relative orientations of the incidence planes at the water surface and the retrosurface is solved, and numerical computations of the effective transmissions for unpolarized light are presented for the coplanar case. The transmission depends on the angle between the direction of incident light and the normal to the interface and on the degree of polarization.

The results show that high transmissions can exist, provided the light does not graze either the water surface or the entrance face of the retroreflector. When the light is incident normal to the water surface, the round-trip transmission is approximately 95 percent even when the incident angle at the retroreflector face is as much as 30 degrees; but this value is decreased by about one-half when the incident angle at the retroreflector is 80 degrees. Similarly, the transmission remains high when the direction of incident light at the water surface is inclined up to 30 degrees to the normal. Even when this angle is 85 degrees, the transmission is about 20 percent, provided the retroreflector is inclined properly.

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Light incident on the water surface from a given direction will be returned most efficiently by a retroreflector when its face is normal to the direction of the light after being refracted at the water surface. For many combinations of angles between the directions of light and the normals to the interfaces, almost equal fractions of the parallel and perpendicular components of the light are returned. However, the parallel component is always greater than the perpendicular component except that the components are equal when the direction of light is normal to both the water surface and the retroreflector entrance face.