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NRL Report 6799

On the Plane Parallel Representation in Atmospheric Radiation Transfer

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December 6, 1968

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ABSTRACT

Solutions of radiative transfer have been examined with respect to the validity of approximating a spherical scattering medium by a plane parallel layer. The results show that the plane parallel approximation is poor in the case of Lyman-α scattering by geocoronal atomic hydrogen.

PROBLEM STATUS

This report is on one phase of a continuing problem.

AUTHORIZATION

NRL Problem A01-06B
Project S-20613-G

Manuscript submitted August 27, 1968.

ON THE PLANE PARALLEL REPRESENTATION IN ATMOSPHERIC RADIATION TRANSFER

Recently the problem of daytime transport of resonance radiation in the earth's atmosphere has been solved by approximating the scattering medium by a plane parallel layer (e.g. by Donahue and Fastie (1), Donahue (2), and Donahue and Meier (3). The purpose of this report is to examine the validity of this approximation.

An atmospheric distribution can be represented as plane symmetric if its scale height is small compared with the radius of the earth. Then the integrated density at a zenith angle θ and altitude z, $N(z,\theta)$, can be written as

$$N(z,\theta) = N(z) \sec \theta \tag{1}$$

where $N(z)$ is the vertical integrated density. To see when this "secant" approximation is valid, an exponential density model ($\rho(z) = e^{-z/h}$) and a power law model ($\rho(z) = z^{-n}$) were integrated numerically and compared with Eq. (1). The exponential case is illustrated in Fig. 1, where the percent error of the secant approximation is plotted as a function of scale height (in units of earth radii) and zenith angle at the surface of the earth. The power law case is shown in Fig. 2 with n as the ordinate.

These geometric limitations are, however, somewhat masked in the solution of the radiative transfer equations because of the complex functional dependence on optical depth. This can be exemplified by comparing the plane parallel source function and intensity with the $n = 3.25$ spherically symmetric power law model used by Thomas (4) to approximate the hydrogen geocorona. The total vertical optical depth (τ_ν) is 1.5. The plane and spherical initial source functions (due to direct excitation by solar radiation) are shown in Fig. 3 as a function of vertical optical depth for various zenith angles. The final source functions (including both the direct and multiply scattered excitations) are given in Fig. 4. The overhead intensities for these models are plotted in Fig. 5. All of the above functions are in units of $[(\pi F_\nu)_0 \sqrt{\pi}\, \Delta\nu_D]$, where $(\pi F_\nu)_0$ is the solar photon flux at frequency ν and $\Delta\nu_0$ is the Doppler width.

At a 60 degree solar zenith angle the plane and spherical solutions are closest. This is fortuitous because at that zenith distance the solar flux has been sufficiently attenuated to compensate for the large plane parallel optical depth. It is to be noted that Fig. 5 shows only overhead intensities. Discrepancies between plane parallel and spherical solutions are minimum in that direction.

It is clear that, even though the error in the secant approximation is tempered by the radiative transfer equations, quantitative conclusions drawn from the plane parallel representation are somewhat tenuous for extensive distributions, such as geocoronal hydrogen.

ACKNOWLEDGMENT

The author was partially supported by the National Science Foundation under Grant GP-6354 while he was an E. O. Hulburt Center Appointee.

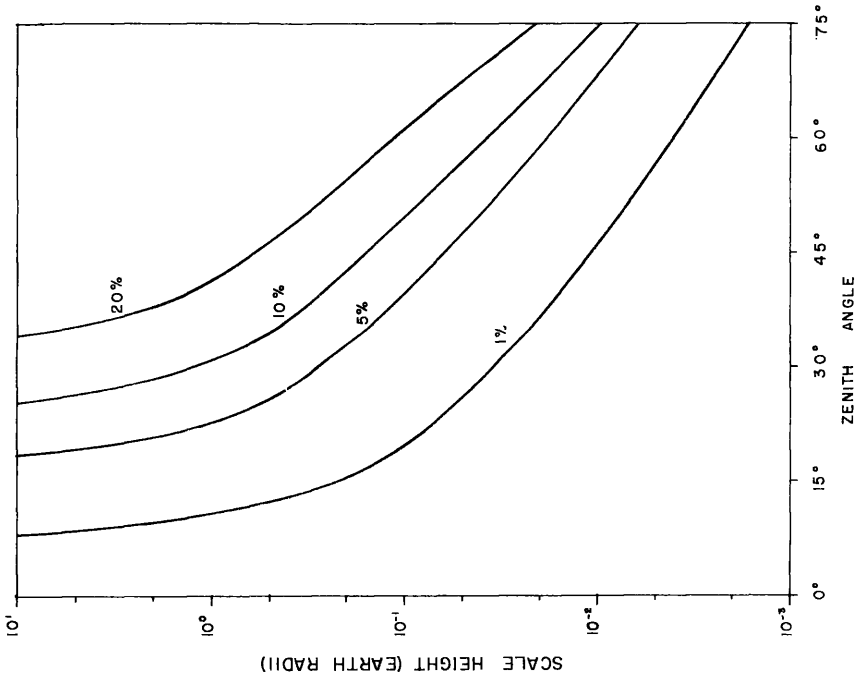


Fig. 1 - Percent error of secant approximation exponential density model

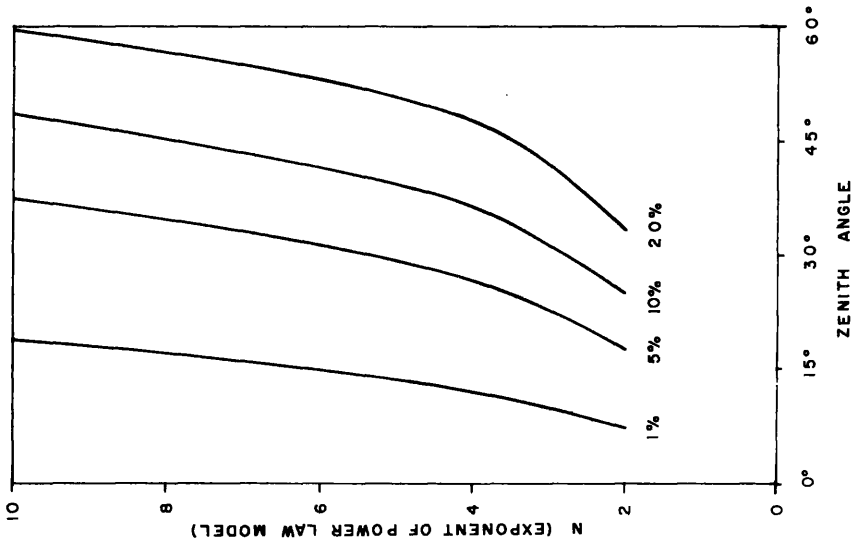


Fig. 2 - Percent error of secant approximation for a power law density model

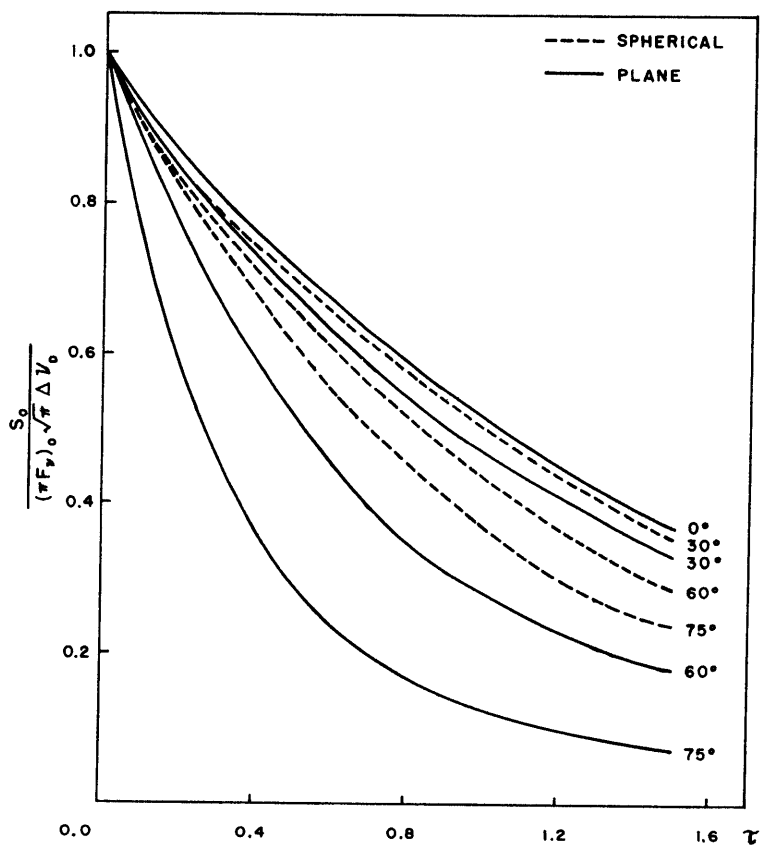
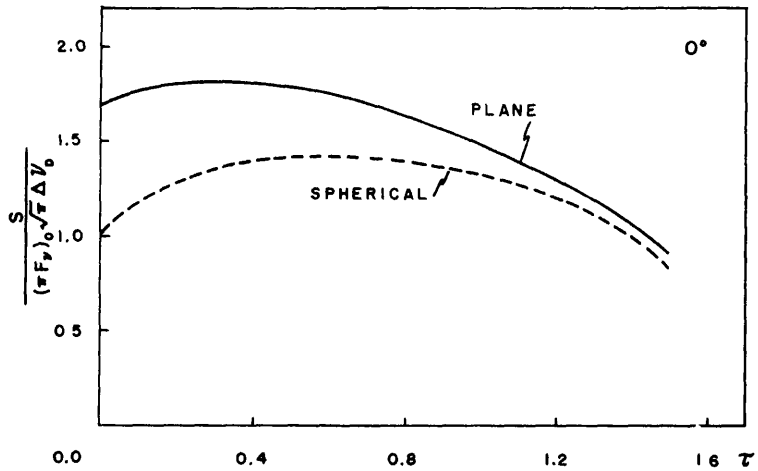
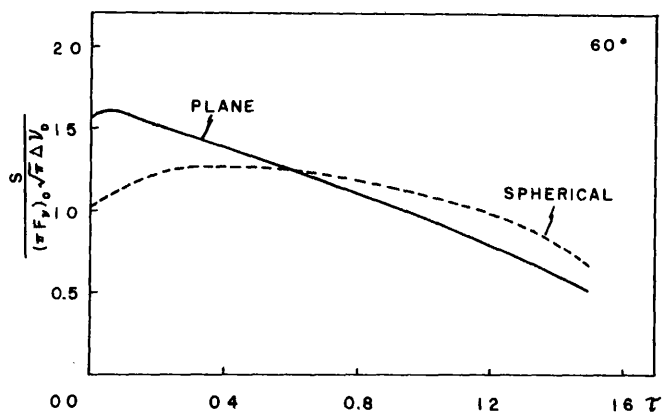


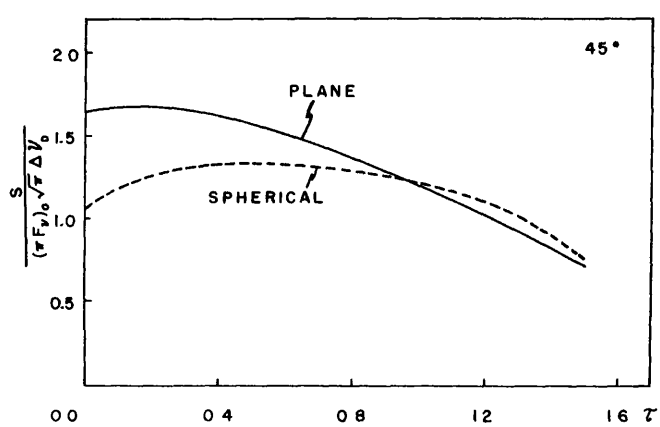
Fig. 3 - Initial source function for the plane parallel and spherical power law (n = 3.25, τ_ν = 1.5)



(a) 0 degree zenith angle (n = 3.25, τ_v = 1.5)

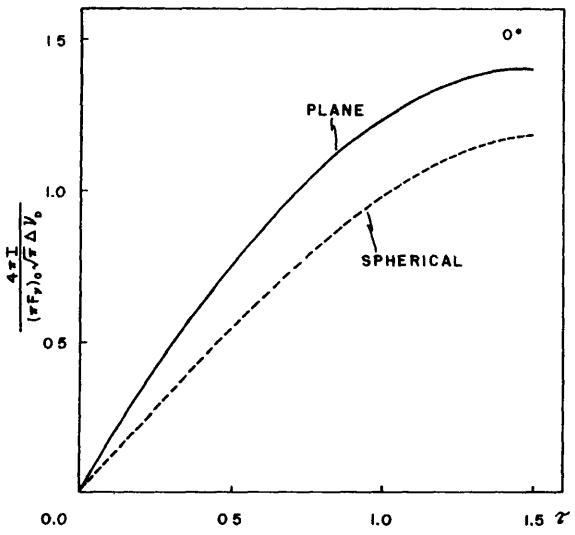


(b) 45 degree zenith angle

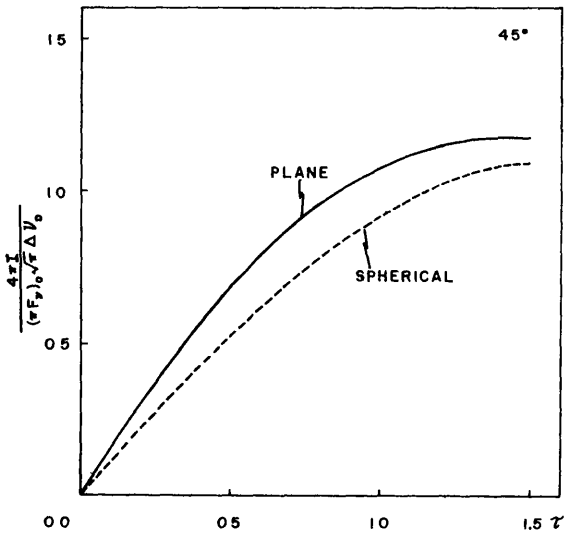


(c) 60 degree zenith angle

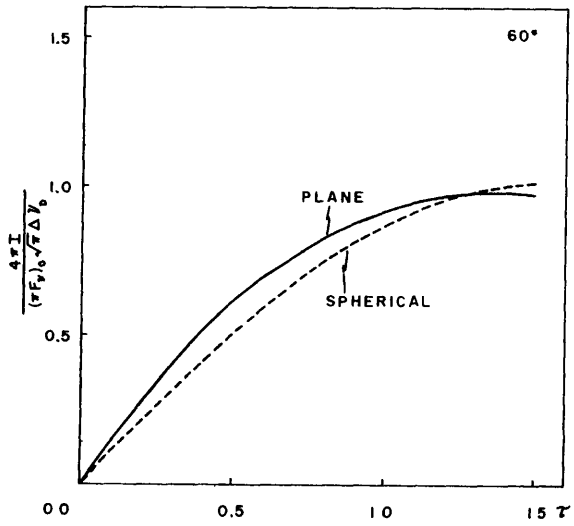
Fig. 4 - Plane parallel and spherical source functions



(a) 0 degree zenith angle



(b) 45 degree zenith angle



(c) 60 degree zenith angle

Fig. 5 - Plane parallel and spherical intensities

REFERENCES

1. Donahue, T.M., and Fastie, W.G., "Observation and Interpretation of Resonance Scattering of Lyman-α and OI (1300) in the Upper Atmosphere," Proc. Intern. Space Sci. Symp., 4th, Warsaw, 1963, P. Muller, editor, Amsterdam:North-Holland Pub. Co.; New York:Wiley, p. 304, 1964

2. Donahue, T.M., "Some Consideration Concerning Radiation Transport in the OI 1304 Triplet in the Upper Atmosphere," Planetary Space Sci. 13:871 (1965)

3. Donahue, T.M., and Meier, R.R., "Distribution of Sodium in the Daytime Upper Atmosphere as Measured by a Rocket Experiment," J. Geophys. Res. 72:2803 (1967)

4. Thomas, G.E., "Lyman-α Scattering in the Earth's Hydrogen Geocorona, 1," J. Geophys. Res. 68:2639 (1963)

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Research Laboratory Washington, D. C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

ON THE PLANE PARALLEL REPRESENTATION IN ATMOSPHERIC RADIATION TRANSFER

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

This report is on one phase of a continuing problem.

5. AUTHOR(S) *(First name, middle initial, last name)*

Robert R. Meier

6. REPORT DATE December 6, 1968	7a. TOTAL NO. OF PAGES 9	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO. NRL Problem A01-06B b. PROJECT NO. S-20613-G c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 6799 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES The National Science Foundation provided partial support under Grant-6354	12. SPONSORING MILITARY ACTIVITY Department of the Navy (Naval Ship Systems Command), Washington, D.C. 20360

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DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Security Classification

Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Earth atmosphere						
Resonance radiation						
Transport properties						
Lyman alpha radiation						
Plane parallel approximation						
Hydrogen						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

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