NRL REPORT 4037

INTERMODULATION DISTORTION IN AMPLIFIERS

S. R. Swanson

Communications Branch
Radio Division II

September 4, 1952

NAVAL RESEARCH LABORATORY

WASHINGTON, D.C.

CONTENTS

Abstract	ii
Problem Status	ii
Authorization	ii
INTRODUCTION	1
COMBINATION FREQUENCIES	2
Theory	2
Experimental Results	5
CROSS MODULATION	6
Theory	6
Experimental Results	8
CONCLUSIONS	11
REFERENCES	11

ABSTRACT

Two components of the distortion produced in a wideband amplifier, such as a receiver multicoupler, are considered for the case when the input contains signals at two or more frequencies. These components of distortion are combination frequencies and cross modulation.

The combination-frequency (also known as sum-and-difference-frequency) signals appear at frequencies which are the sum and difference of the input frequencies and their multiples. It is shown that the amplitude of these signals is related to the amplitude of the harmonic distortion produced in a nonlinear amplifier. With two input signals of approximately equal amplitude, the combination-frequency signals will be several times greater than the corresponding harmonics.

Cross modulation or crosstalk is a result of the dependence of the amplification of one signal upon the amplitude of another signal in a nonlinear circuit, and causes modulation from one carrier to appear on another carrier. This effect is approximately proportional to the third-harmonic distortion produced by the interfering signal and is practically independent of the amplitude of the signal being interfered with.

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

AUTHORIZATION

NRL Problem R09-51
RDB Project NE 091-035

Manuscript submitted August 14, 1952

INTERMODULATION DISTORTION IN AMPLIFIERS

INTRODUCTION

Receiver multicouplers permit the operation of a number of receivers from a common antenna, thus permitting a reduction in the number of antennas required at a given receiver installation. Since satisfactory performance can not always be obtained due to such factors as the distortion produced in the multicoupler, a study has been initiated to determine the relationship of these factors to the variables of the multicoupler circuits.

A receiver multicoupler, unlike a radio receiver, must be an untuned device having a very wide bandwidth, and hence will be susceptible to all types of nonlinear distortion. The effects of this distortion will be especially noticeable when strong signals are present, such as from a nearby transmitter. In an ideal multicoupler, the undesired signals due to nonlinear distortion should be below the noise level of the receiver. Hence the amplitude of the distortion terms should be less than about one microvolt, so that a 0.1-volt input signal should give less than 0.001-percent distortion. This requirement is much more severe than is normally imposed on electron-tube circuits.

The techniques that are normally used to reduce the nonlinear distortion in amplifiers to values on the order of one percent are not suitable for obtaining the low value of nonlinear distortion required in receiver multicoupler circuits. Hence it is necessary to investigate and, if possible, develop new techniques for decreasing the nonlinear distortion. It has been found that the distortion for any given harmonic can be reduced up to 40 db by selecting the proper operating conditions for the electron-tube circuit (1).

Nonlinear distortion in the multicoupler will result in the production of various combinations of the frequencies present at the input. When the input signal is of a single frequency, the only distortion terms appearing will be harmonics of the input frequency. A number of additional effects will be present when the input contains signals of two or more frequencies.

With an input of two or more carriers, the output of a wideband amplifier may contain frequencies which are considerably higher or lower than the original input frequencies. These frequencies will be referred to as combination frequencies, since they are the sum and difference of the input frequencies and their multiples in all combinations.

Cross modulation or crosstalk is a form of intermodulation distortion in which the amplitude of one signal is dependent to some extent upon the amplitude of another signal in a nonlinear circuit. Its effect is particularly noticeable in broadband r-f amplifiers, since it causes modulation from one carrier to appear on another. Cross modulation may be especially serious in multicouplers, since signals producing cross modulation may be 100 db or more above the desired signal. Hence this phase of nonlinear distortion deserves consideration in multicoupler design.

COMBINATION FREQUENCIES

Theory

Distortion in receiver multicouplers is due to the nonlinear characteristic of the electron tubes which are used. Since the distortion must be much lower than is normally required in amplifier circuits, the analysis of nonlinear distortion is re-examined and emphasis placed on the special problem of multicoupler application.

The output current (or voltage) of a vacuum-tube amplifier having a resistive load can be written in the form of a Taylor series, using the first five terms

$$i_p = I_0 + ae_{in} + be^2_{in} + ce^3_{in} + de^4_{in}\ , \qquad (1)$$

where i_p is the instantaneous value of plate current, I_0 is the dc component of current for a fixed bias voltage (with $e_{in} = 0$), e_{in} is the instantaneous value of ac input voltage, and a, b, c, and d are circuit constants. Effects of higher order may also be present, but they are generally of less importance and will be neglected for the time being.

Let the ac input consist of two sinusoidal signals. Then

$$e_{in} = E_{m1} \sin \omega_1 t + E_{m2} \sin \omega_2 t, \qquad (2)$$

where $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$. If the input signal of Equation (2) is substituted in Equation (1) and the resulting equation expanded, the terms in Table 1 will be obtained. This is actually a series expansion, but is expressed in table form for convenience.

As can be seen in Table 1, the second-order effect includes, besides the second harmonic, two additional intermodulation terms (referred to as second-order terms) at different frequencies. Likewise, the third-order effect includes the third harmonic plus several third-order intermodulation terms.

The expansion can be simplified if E_{m1} and E_{m2} are in a known ratio. If $E_{m1} = E_{m2} = E_m$, which is an important condition, the terms in Table 2 will be obtained.

The approximate relations between combination-frequency signals and harmonics for two equal input signals are given in Table 3. This table is obtained by comparing the ratio of the first term in the coefficient of the combination frequency to that of the corresponding harmonic in Table 2 (such as b for the second-order intermodulation term and $\frac{1}{2}$b for the second harmonic, giving a ratio of 2). Since, as can be shown by a more detailed analysis (2), the terms in the coefficients beyond the first do not generally have the same ratio as the first, this comparison is valid only when the first term in the coefficient is the predominating one. From Table 3 is can be seen that, when the signals are of equal magnitude, the intermodulation terms will be larger than the corresponding harmonics. For instance, the second-order terms will be about twice the magnitude of the second harmonics and the third-order terms will be about three times the third harmonics of the individual signals.

If the input carriers are modulated, this modulation will appear on the combination-frequency signals produced by the original carriers.

Additional frequency combinations will result if the input contains three or more frequencies. The analysis presented here can be extended to determine the effects of any number of input signals by assuming additional terms in Equation (2).

TABLE 1
Output Signals from Nonlinear Amplifier Having Two Input Signals

$I_0 + \frac{1}{2}b(E_{m1}^2 + E_{m2}^2) + \frac{3}{8}d(E_{m1}^4 + E_{m2}^4) + \frac{3}{2}dE_{m1}^2E_{m2}^2$	dc term
$+ (a + \frac{3}{4}cE_{m1}^2 + \frac{3}{2}cE_{m2}^2)E_{m1}\sin\omega_1 t$	$\omega_1 t$ fundamental
$+ (a + \frac{3}{4}cE_{m2}^2 + \frac{3}{2}cE_{m1}^2)E_{m2}\sin\omega_2 t$	$\omega_2 t$ fundamental
$-(\frac{1}{2}b + \frac{1}{2}dE_{m1}^2 + \frac{3}{2}dE_{m2}^2)E_{m1}^2\cos 2\omega_1 t$	2nd harmonic of $\omega_1 t$
$-(\frac{1}{2}b + \frac{1}{2}dE_{m2}^2 + \frac{3}{2}dE_{m1}^2)E_{m2}^2\cos 2\omega_2 t$	2nd harmonic of $\omega_2 t$
$-\frac{1}{4}cE_{m1}^3\sin 3\omega_1 t$	3rd harmonic of $\omega_1 t$
$-\frac{1}{4}cE_{m2}^3\sin 3\omega_2 t$	3rd harmonic of $\omega_2 t$
$+\frac{1}{8}dE_{m1}^4\cos 4\omega_1 t$	4th harmonic of $\omega_1 t$
$+\frac{1}{8}dE_{m2}^4\cos 4\omega_2 t$	4th harmonic of $\omega_2 t$
$+(bE_{m1}E_{m2} + \frac{3}{2}dE_{m1}^3E_{m2} + \frac{3}{2}dE_{m1}E_{m2}^3)\cos(\omega_1 - \omega_2)t$	2nd-order term
$-(bE_{m1}E_{m2} + \frac{3}{2}dE_{m1}^3E_{m2} + \frac{3}{2}dE_{m1}E_{m2}^3)\cos(\omega_1 + \omega_2)t$	2nd-order term
$+\frac{3}{4}cE_{m1}E_{m2}^2\sin(2\omega_2 - \omega_1)t$	3rd-order term
$-\frac{3}{4}cE_{m1}E_{m2}^2\sin(2\omega_2 + \omega_1)t$	3rd-order term
$+\frac{3}{4}cE_{m1}^2E_{m2}\sin(2\omega_1 - \omega_2)t$	3rd-order term
$-\frac{3}{4}cE_{m1}^2E_{m2}\sin(2\omega_1 + \omega_2)t$	3rd-order term
$-\frac{1}{2}dE_{m1}^3E_{m2}\cos(3\omega_1 - \omega_2)t$	4th-order term
$+\frac{1}{2}dE_{m1}^3E_{m2}\cos(3\omega_1 + \omega_2)t$	4th-order term
$-\frac{1}{2}dE_{m1}E_{m2}^3\cos(3\omega_2 - \omega_1)t$	4th-order term
$+\frac{1}{2}dE_{m1}E_{m2}^3\cos(3\omega_2 + \omega_1)t$	4th-order term
$+\frac{3}{4}dE_{m1}^2E_{m2}^2\cos 2(\omega_1 - \omega_2)t$	4th-order term
$+\frac{3}{4}dE_{m1}^2E_{m2}^2\cos 2(\omega_1 + \omega_2)t$	4th-order term

TABLE 2
Output Signals from Nonlinear Amplifier Having Two Equal Input Signals

$I_0 + bE_m^2 + \frac{9}{4}dE_m^4$	dc term
$+ (a + \frac{9}{4}cE_m^2) E_m \sin \omega_1 t$	$\omega_1 t$ fundamental
$+ (a + \frac{9}{4}cE_m^2) E_m \sin \omega_2 t$	$\omega_2 t$ fundamental
$- (\frac{1}{2}b + 2dE_m^2) E_m^2 \cos 2\omega_1 t$	2nd harmonic of $\omega_1 t$
$- (\frac{1}{2}b + 2dE_m^2) E_m^2 \cos 2\omega_2 t$	2nd harmonic of $\omega_2 t$
$- \frac{1}{4}cE_m^3 \sin 3\omega_1 t$	3rd harmonic of $\omega_1 t$
$- \frac{1}{4}cE_m^3 \sin 3\omega_2 t$	3rd harmonic of $\omega_2 t$
$+ \frac{1}{8}dE_m^4 \cos 4\omega_1 t$	4th harmonic of $\omega_1 t$
$+ \frac{1}{8}dE_m^4 \cos 4\omega_2 t$	4th harmonic of $\omega_2 t$
$+ (b + 3dE_m^2) E_m^2 \cos (\omega_1 - \omega_2) t$	2nd-order term
$- (b + 3dE_m^2) E_m^2 \cos (\omega_1 + \omega_2) t$	2nd-order term
$+ \frac{3}{4}cE_m^3 \sin (2\omega_2 - \omega_1) t$	3rd-order term
$- \frac{3}{4}cE_m^3 \sin (2\omega_2 + \omega_1) t$	3rd-order term
$+ \frac{3}{4}cE_m^3 \sin (2\omega_1 - \omega_2) t$	3rd-order term
$- \frac{3}{4}cE_m^3 \sin (2\omega_1 + \omega_2) t$	3rd-order term
$- \frac{1}{2}dE_m^4 \cos (3\omega_1 - \omega_2) t$	4th-order term
$+ \frac{1}{2}dE_m^4 \cos (3\omega_1 + \omega_2) t$	4th-order term
$- \frac{1}{2}dE_m^4 \cos (3\omega_2 - \omega_1) t$	4th-order term
$+ \frac{1}{2}dE_m^4 \cos (3\omega_2 + \omega_1) t$	4th-order term
$+ \frac{3}{4}dE_m^4 \cos 2 (\omega_1 - \omega_2) t$	4th-order term
$+ \frac{3}{4}dE_m^4 \cos 2 (\omega_1 + \omega_2) t$	4th-order term

TABLE 3
Approximate Relations between Amplitude of Combination-Frequency Terms and Harmonics for Two Simultaneous Input Signals of Equal Amplitude (Obtained from Coefficients of Table 2)

Frequency	Relation to Harmonic
$f_1 - f_2$	2 x 2nd harmonic
$f_1 + f_2$	2 x 2nd harmonic
$2f_2 - f_1$	3 x 3rd harmonic
$2f_2 + f_1$	3 x 3rd harmonic
$2f_1 - f_2$	3 x 3rd harmonic
$2f_1 + f_2$	3 x 3rd harmonic
$3f_1 - f_2$	4 x 4th harmonic
$3f_1 + f_2$	4 x 4th harmonic
$3f_2 - f_1$	4 x 4th harmonic
$3f_2 + f_1$	4 x 4th harmonic
$2(f_1 - f_2)$	6 x 4th harmonic
$2(f_1 + f_2)$	6 x 4th harmonic

Experimental Results

Although knowledge of the existence of combination frequencies is not new, no single method has been universally adopted for measuring or comparing the magnitude of the distortion produced (3), especially at radio frequencies. Examination of the experimental procedures used by several investigators revealed about an equal number of methods for expressing the results.

A common means of measuring intermodulation is by the equivalent signal method, in which two unmodulated signals of equal amplitude and different frequency are fed in parallel to the circuit under test. This system determines the intermodulation terms individually, whereas most commercial equipments measure the total distortion. The magnitude of the individual distortion terms is more important than the magnitude of the total distortion when analyzing receiver multicoupler performance.

The objective of the experimental measurements was to verify the theoretical analysis. The l-f band was selected because of the available measuring equipment and the negligible effect of lead inductance and distributed capacitance. Test frequencies of 80 and 100 kc were selected so as to be able to measure harmonics and combination frequencies independently. These were supplied simultaneously by two variable signal generators having suitable filters and a resistive mixing network, as shown in Figure 1. The output of the test circuit, a 6AK5 tube as a conventional grounded-cathode amplifier, was fed to an RBA receiver through a high impedance resistive attenuator and a cathode follower.

The cathode follower was needed to transform the low impedance of the receiver to a high impedance load for the test circuit. With the receiver tuned to a combination frequency and the attenuator set to give a convenient output voltage, this voltage was measured by means of the Ballantine voltmeter. One of the signal generators was then set at this combination frequency and connected in place of the two parallel signal generators. Its output was adjusted to obtain the same output voltage from the receiver as previously measured. The magnitude of the combination-frequency signal entering the circuit under test is defined as the equivalent signal voltage and for convenience is expressed as a percentage of the original input signal (0.5 volt in this case).

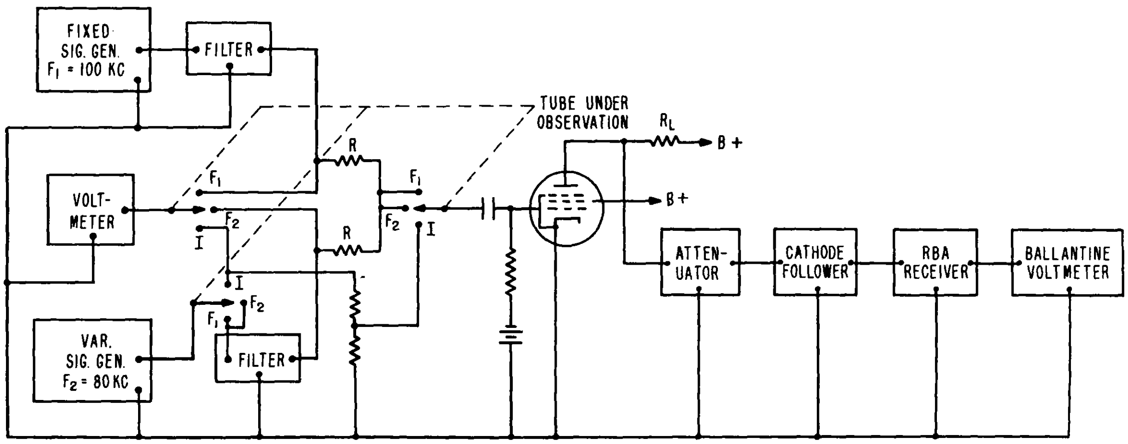

Figure 1 - Intermodulation measuring system by the equivalent signal method

The success of this method of measuring intermodulation is dependent upon the assumptions of amplification of the equivalent signal by the same amount that the original combination-frequency signal was amplified and of no appreciable distortion in the cathode follower circuit at the receiver input. These assumptions will be justified only if the input voltage to the receiver is low; hence a limit is imposed on the smallest percentage of intermodulation which can be measured with this system, since appreciable distortion in the cathode follower may occur with large input signals.

Table 4 is a sample of the combination-frequency measurements made with this system. It can be seen that the experimental relationships between the amplitude of combination frequencies and harmonics are in reasonably good agreement with the theoretical (Table 3). For instance, the individual third-order intermodulation terms average 1.85 percent, while the third harmonic distortion, multiplied by three, is about 2.05 percent.

CROSS MODULATION

Theory

Cross modulation in a receiver multicoupler may be a cause of serious interference by the generation of modulation on a relatively weak desired signal by a strong modulated signal.

If the desired signal is assumed unmodulated and is $E_{m_2} \sin \omega_2 t$ and the interfering signal is $E_{m_1} (1 + m \sin \omega_a t) \sin \omega_1 t$ where m is the index of modulation and ω_a is $2\pi f_a$, where f_a is the audio signal modulating the ω_1 carrier, then the input signal to the receiver multicoupler is the sum of these and is

$$e_{in} = E_{m1}(1 + m \sin\omega_a t) \sin\omega_1 t + E_{m2} \sin\omega_2 t. \qquad (3)$$

TABLE 4
Sample Combination-Frequency Measurements for 6AK5 Tube with Two 0.5-Volt Input Signals
f_1 = 100 kc, f_2 = 80 kc, E_c = -2.0 V., E_b = 120 V., E_{c2} = 120 V., R_L = 10,000 ohms

Frequency (kc)	Frequency Combination	$V_{equiv.}$ (volts)	Distortion (percent)	
			Experimental	Theoretical*
20	$f_1 - f_2$	0.039	7.8	7.2
40	$2f_1 - 2f_2$	0.00077	0.155	0.16
60	$2f_2 - f_1$	0.0095	1.9	2.05
80	f_2 (fund.)	0.5	-	-
100	f_1 (fund.)	0.5	-	-
120	$2f_1 - f_2$	0.0097	1.95	2.05
140	$3f_2 - f_1$	0.00032	0.064	0.11
160	$2f_2$ (2nd harm.)	0.016	3.2	-
180	$f_1 + f_2$	0.04	8.0	7.2
200	$2f_1$ (2nd harm.)	0.02	4.0	-
220	$3f_1 - f_2$	0.0005	0.1	0.11
240	$3f_2$ (3rd harm.)	0.0035	0.7	-
260	$2f_2 + f_1$	0.0094	1.9	2.05
280	$2f_1 + f_2$	0.009	1.8	2.05
300	$3f_1$ (3rd harm.)	0.0033	0.66	-
320	$4f_2$ (4th harm.)	0.00016	0.032	-
340	$3f_2 + f_1$	0.0004	0.08	0.11
360	$2f_1 + 2f_2$	0.00084	0.17	0.16
380	$3f_1 + f_2$	0.0006	0.12	0.11
400	$4f_1$ (4th harm.)	0.00011	0.022	-

* Average of the values calculated from the experimental values using the relationships of Table 3.

Substituting Equation (3) in the Taylor series expansion for plate current of Equation (1), the term obtained which is most significant in producing cross modulation is

$$3cE^2_{m1} E_{m2} m \sin\omega_a t \sin\omega_2 t.$$

This term represents modulation of the carrier which was originally unmodulated, and consists of an audio signal

$$3cE^2_{m1} E_{m2} m \sin\omega_a t$$

modulating the ω_2 carrier. The percent modulation of the ω_2 carrier is the ratio of this audio signal to the carrier and is approximately

$$d_c = \frac{3cE_{m1}^2\, m}{a} \times 100. \qquad (4)$$

The percent third harmonic (1) is

$$d_3 = \frac{cE_m^2}{4a} \times 100. \qquad (5)$$

Hence, to a good approximation for small signals, $d_c = 12\ m\ d_3$ where d_3 is the percent third harmonic produced by the carrier of the interfering modulated signal (4).

Thus if the signal from a local transmitter is strong enough to produce a third harmonic of 0.1% in a receiver multicoupler, all other stations heard will be modulated about 1.2% by the audio signal from the local station if its carrier is modulated 100%. From this it can be seen that the cross modulation will be negligible when the third-harmonic distortion is anywhere near the low values desired in receiver multicouplers.

It should be noted that the percent cross modulation is a function of the amplitude of the interfering signal, and is not (to a first approximation, at least) dependent upon the signal level of the signal interfered with.

Experimental Results

In measuring cross modulation, two simultaneous input signals were used; one of these (E_1) was modulated, the other (E_2), unmodulated. (In practice, both may be modulated, and the cross modulation on a carrier will be added to the modulation already present.) The circuit (Figure 2) is similar to that in Figure 1, except that one of the signal generators was modulated and the receiver was tuned to the desired (unmodulated) signal. The percent cross modulation was proportional to the ratio of detected audio signal to the r-f signal. The detector and amplifier were calibrated by using a signal whose percent modulation was known.

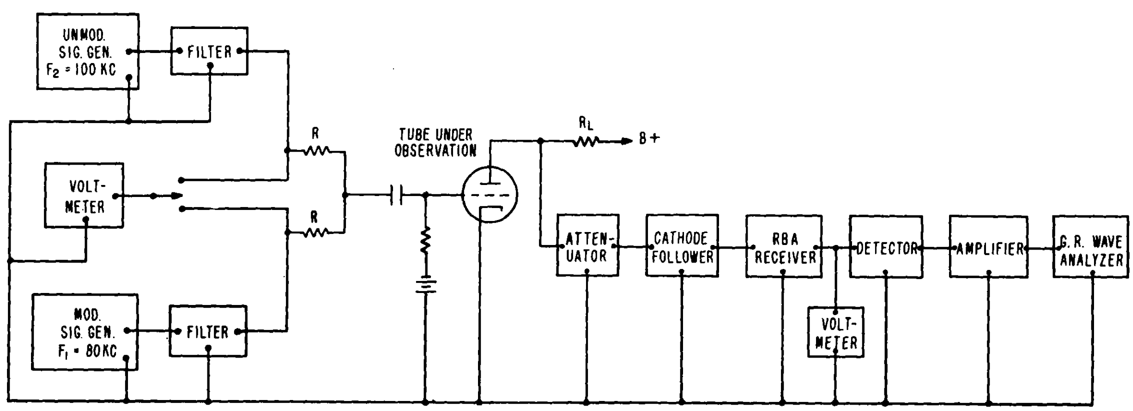

Figure 2 - Cross modulation measuring system

The experimental data plotted in Figure 3 illustrates the cross modulation on E_3 as affected by a variation of m (degree of modulation of modulated input signal E_1), (amplitude of modulated carrier), and E_2 (amplitude of unmodulated carrier). It can be seen that, as stated in the theory, the percent cross modulation is proportional to the modulation index and to the square of the carrier amplitude of the interfering modulated signal. In addition, the percent cross modulation on a carrier is fairly independent of its amplitude, so that the interference thus produced can not be appreciably reduced by increasing the field strength of the desired signal.

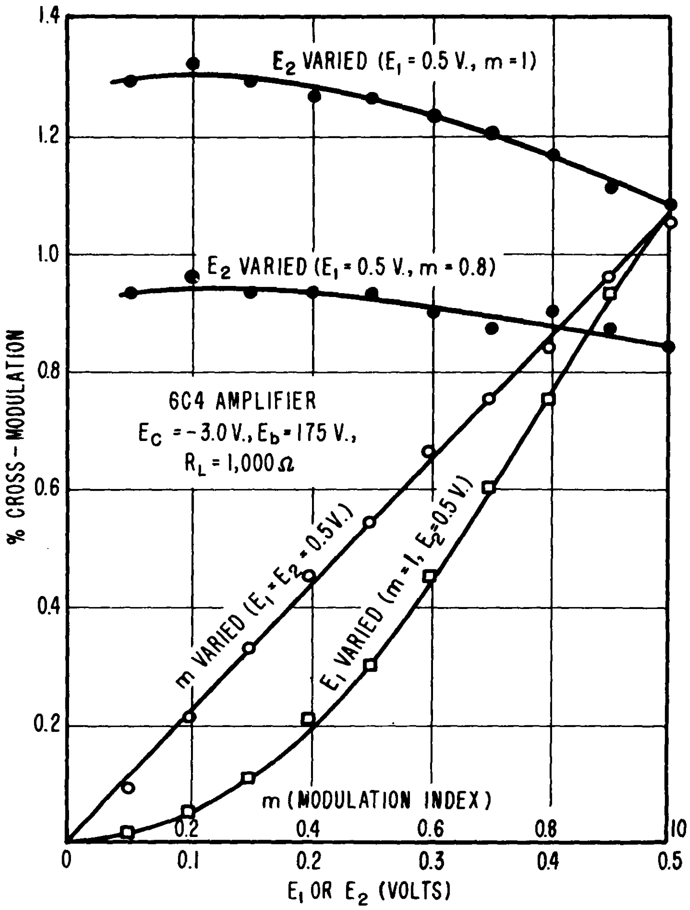

Figure 3 - Cross modulation on E_2 with m, E_1, and E_2 varied individually

Tables 5 and 6 give an idea of the percent cross modulation that can be expected in 6C4 and 6AK5 tubes. Notice that the cross modulation in the 6AK5 is on the order of over ten times that for the 6C4. This indicates that the 6AK5 would be less desirable in a receiver multicoupler under the conditions shown.

TABLE 5
Cross Modulation in
6C4 Amplifier
R_L = 5,000 ohms, m = 1,
E_{in} = 0.5 V. rms

E_c	E_b	% Cross Mod.
-3.0	100	1.1
	125	0.54
	150	0.3
	175	0.84
	200	0.78
-6.0	100	3.75
	125	0.57
	150	0.15
	175	0.15
	200	0.27
-9.0	150	1.65
	175	0.39
	200	0.15
	225	0.12
	250	0.06
-12.0	175	6.0
	200	1.05
	225	0.3
	250	0.12
	275	0.12
	300	0.09
-15.0	200	1.0
	225	2.85
	250	0.45
	275	0.18
	300	0.09
-18.0	250	4.4
	275	1.3
	300	0.1

TABLE 6
Cross Modulation in
6AK5 Amplifier
R_L = 5,000 ohms, m = 1,
E_{in} = 0.5 V. rms,
E_{c2} = 120 V.

E_c	E_b	% Cross Mod.
-1.0	60	14.5
	80	10.5
	100	7.95
	120	6.75
	140	6.3
	160	6.2
	180	6.0
-1.5	60	16.0
	80	11.5
	100	8.55
	120	7.35
	140	6.75
	160	6.45
	180	6.15
-2.0	60	17.0
	80	11.5
	100	9.0
	120	8.1
	140	7.65
	160	7.35
	180	7.35
-2.5	60	16.5
	80	11.0
	100	8.55
	120	7.95
	140	6.75
	160	6.15
	180	5.55

CONCLUSIONS

The theoretical and experimental results show that

(a) For two equal input signals, the second-order intermodulation terms $(\omega_1 \pm \omega_2)$ will be about twice the magnitude of the second harmonics of the individual signals and the third-order terms $(2\omega_1 \pm \omega_2$ and $2\omega_2 \pm \omega_1)$ will be about three times the third harmonics. Hence the combination-frequency signals may be the largest and most important interfering signals in a receiver multicoupler.

(b) With two input signals which are not equal, the second-order terms will be proportional to the product of their amplitudes and the third-order terms will be proportional to the product of the amplitude of one signal and the square of the amplitude of the other signal.

(c) The cross modulation will be proportional to the percent modulation of the interfering modulated signal, to the percent third-harmonic distortion which it produces, and to the square of its carrier amplitude.

(d) The percent cross modulation on a carrier will be fairly independent of its amplitude, and hence cannot be greatly reduced by increasing the signal strength of the interfered-with carrier.

(e) Cross modulation will be negligible when the third-harmonic distortion produced in a multicoupler is less than about one percent, and hence, will normally be insignificant compared to the harmonics and combination frequencies in producing interference to radio reception.

* * *

REFERENCES

1. Swanson, S. R., and Long, W. G., Jr., "Harmonic Distortion in Amplifiers," NRL Report 3998 (Restricted), June 1952

2. Callendar, M. V., and Matthews, S., "Relations Between Amplitudes of Harmonics and Intermodulation Frequencies in the Output from a Nonlinear Amplifier or Mixer," Electronic Eng., Vol. 23, pp. 230-232, June 1951

3. Avins, J., "Intermodulation and Harmonic Distortion Measurements," Audio Eng., Vol. 32, No. 10, pp. 17-18, 55, October 1948

4. Deketh, J., Fundamentals of Radio-Valve Technique, "Electric Valves," p. 192, N. V. Philips' Gloeilampenfabrieken, Eindhoven (Netherlands), 1949

* * *