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# APPLICATION OF A BILINEAR VELOCITY PROFILE TO CONVERGENCE ZONE TRANSMISSION

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## ABSTRACT

In order to understand more fully the convergence zone transmission of underwater sound, a bilinear approximation to the velocity profile in the northern Atlantic has been made. The actual profile is approximated by one involving the surface velocity, the depth of minimum velocity, the minimum velocity, and the velocity gradient below the depth of minimum velocity. The last three of these parameters are nearly constant in the northern Atlantic. These constant values are, respectively, 4000 ft, 4900 ft/sec, and 0.016 ft/sec/ft.

A mathematical definition of the convergence zone is given and an expression for the range from the source to the convergence zone is derived. Comparison of ranges obtained by using the bilinear profile and by experiment exhibit good agreement.

Charts of the northern Atlantic have been constructed which attempt to exhibit the northern Atlantic as a medium for convergence zone transmission. Approximately 50 percent of the northern Atlantic is available for convergence zone transmission at all times, and approximately 75 percent is available during the winter.

## PROBLEM STATUS

This is an interim report; work on the general problem is continuing.

## AUTHORIZATION

NRL Problem S01-01
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# APPLICATION OF A BILINEAR VELOCITY PROFILE TO CONVERGENCE ZONE TRANSMISSION

## INTRODUCTION

The study of the propagation of sound in the ocean, with the ultimate goal the detection of submarines at long ranges, makes necessary the consideration of three possible propagation paths. These three paths are (1) the surface channel path, (2) the bottom reflection path, and (3) the skip path (convergence zone). This report is concerned with the last of these three.

It is believed that some essential features of propagation by the skip path may be obtained by approximating actual velocity profiles by bilinear ones. This simplification makes it possible to obtain useful results.

## DEFINITION OF CONVERGENCE ZONE AND DERIVATION OF RANGE TO CONVERGENCE ZONE

A bilinear velocity profile is shown in Fig. 1.

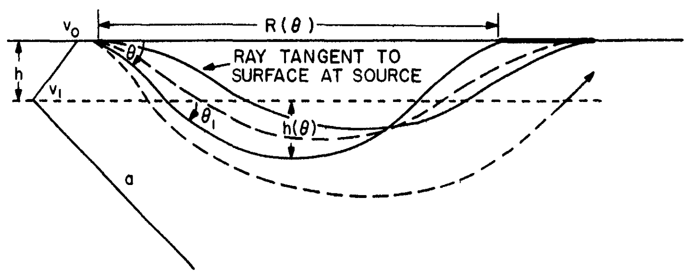


Fig. 1 - Bilinear velocity profile and ray plot

$v_0$ is surface velocity
$v_1$ is minimum velocity
h is depth of minimum velocity
a is velocity gradient below depth of minimum velocity
$\theta$ is angle ray makes with the surface
$\theta_1$ is angle at which ray $\theta$ enters the layer below the point of minimum velocity
$h(\theta)$ is depth below point of minimum velocity to which ray $\theta$ goes
$R(\theta)$ is skip range of ray $\theta$

The skip range of any ray is given by

$$R(\theta) = 2h \cot 1/2\ (\theta+\theta_1) + 2h(\theta) \cot 1/2\ \theta_1, \tag{1}$$

where $h(\theta)$ is determined by Snell's law as $\cos\theta_1 = v_1/v_{max}$, since ray $\theta$ is horizontal at its maximum depth, and $v_{max}$ is the velocity at that depth. Hence,

$$v_{max} = v_1 + ah(\theta) = v_1 \sec \theta_1 .$$

Therefore,

$$h(\theta) = \frac{v_1}{a} (\sec \theta_1 - 1) = \frac{v_0}{a}\left(\sec \theta - \frac{v_1}{v_0}\right). \tag{2}$$

As a mathematical definition of the position of the leading edge of the convergence zone, let us consider the range of ray $\theta_s$ , where

$$\left(\frac{dR}{d\theta}\right)_{\theta_s} = 0. \tag{3}$$

This is clearly the range at which the rate of change of $R$ with respect to $\theta$ is a minimum, hence the maximum intensity sould occur near this range.

It should be pointed out that, as the angle at which the ray leaves the source increases from zero, the skip range decreases until angle $\theta_s$ is reached, and then increases. In the region defined by the ray leaving the source horizontally and ray $\theta_s$ , there is double ray coverage. This is indicated by the heavy line in Fig. 1 if $\theta$ in that figure is taken to be $\theta_s$. This region is defined as the convergence zone.

One other ray should be mentioned. This is the ray leaving the source at angle $\theta_t > \theta_s$ which has the same skip range as the ray leaving the source horizontally. When this ray is determined, the ray bundle giving rise to the convergence zone will be determined.

Figure 2 is a plot of skip range versus $\theta$ for $h = 4000$ ft, $v_0 = 5051$ ft/sec, $v_1 =$ 4899 ft/sec, and $a = 0.016$ ft/sec/ft. It should be noted that for this profile, $\theta_s \simeq 4°$ and $\theta_t \simeq 10°$. This plot shows clearly that, for a given increment of $\theta$ in the neighborhood of 4° , we have a small increment of range compared to that for an increment of $\theta$ any place else.

For the bilinear gradient case, the solution of $dR/d\theta = 0$ can be obtained in simple form. Let us consider Eq. (1) in a slightly different form, readily obtained from (1) by the use of Snell's law,

$$R(\theta) = 2h \cot 1/2\ (\theta+\theta_1) + \frac{2v_1}{a} \tan \theta_1. \tag{4}$$

Differentiating with respect to $\theta$ and setting the results equal to zero gives

$$\frac{2v_1}{a} \sec^2 \theta_1 \frac{d\theta_1}{d\theta} = h \csc^2 1/2\ (\theta+\theta_1) \left(1+ \frac{d\theta_1}{d\theta}\right), \tag{5}$$

where it must be remembered that $\theta$ and $\theta_1$ are connected by Snell's law.

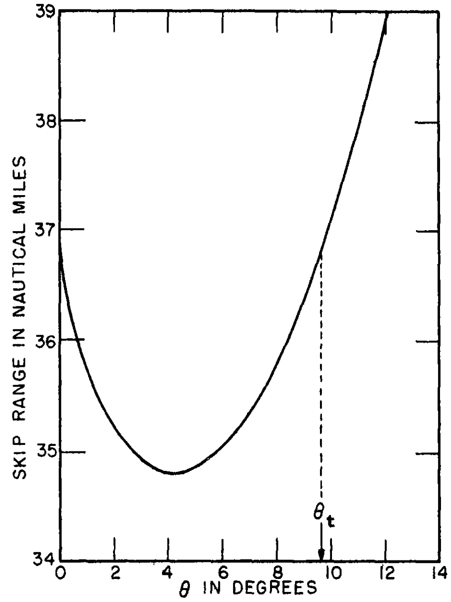


Fig. 2 - Skip range vs $\theta$

h is 4000 ft
$v_0$ is 5051 ft/sec
$v_1$ is 4899 ft/sec
a is 0.016 ft/sec/ft

Calculation of $d\theta_1/d\theta$ and substitution for $\theta_1$ in terms of $\theta$ leads to the following expression for $\theta_s$ :

$$\tan^2 \theta_s = \frac{a^2h^2}{v_0^2} \left(\frac{v_0 + v_1}{2ah + v_0 - v_1}\right).^* \tag{6}$$

It is of interest to notice that this angle will exist even if $v_0 < v_1$ . That is, this angle will exist provided $v_0 > v_1 - 2ah$. For $v_0 = v_1 - ah$ , we have a positive linear gradient all the way from the surface, and no minimum in the range is exhibited, the skip range being an increasing function of $\theta$.

For ease in calculation, Eq. (6) may be written in terms of dimensionless parameters, as follows:

$$\tan^2 \theta_s = \beta^2/n^2 \left(\frac{n + 1}{2\beta + n-1}\right), \tag{7}$$

where

$$\frac{v_0}{v_1} = n \quad \text{and} \quad \frac{ah}{v_1} = \beta.$$

*See Appendix

Figure 3 gives $\theta_s$ for values of $\beta$ from 0.002 to 0.022, and values of n from 0.980 to 1.080. These ranges are believed to cover all cases of interest.

The ordinate on this graph is $\theta_s$; for the values considered, the approximation of tan $\theta_s$ by $\theta_s$ is sufficiently accurate.

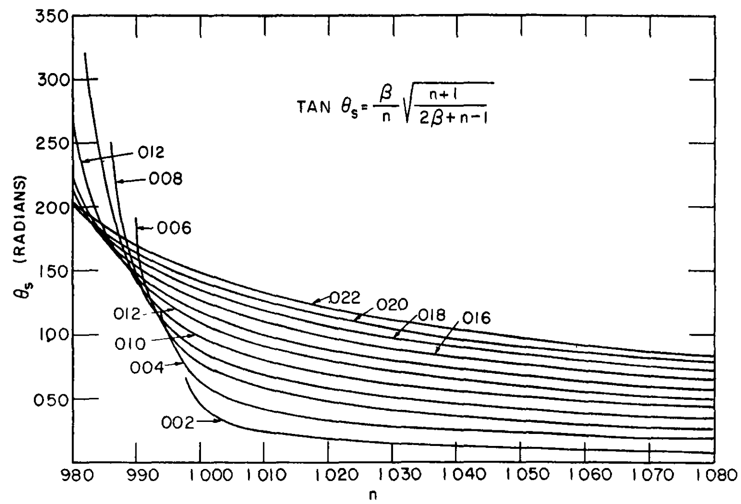


Fig. 3 - Plot of $\theta_s$ as a function of n and $\beta$

## COMPARISON OF CONVERGENCE ZONE RANGES OBTAINED BY THEORY AND BY EXPERIMENT

As an example of the agreement between the skip range as determined by a bilinear profile and the skip range for a multilinear profile, let us consider the following example from NRL Report 4658,*

$$v_0 = 5051, \; v_1 = 5056, \; v_2 = 4899, \; v_3 = 4899, \; v_4 = 4958,$$

$$h_1 = 200, \; h_2 = 2500, \; h_3 = 1100, \; h_4 = 5400, \; a_n = 0.0188,$$

where the v's are velocities in feet/sec at the top of each layer and the h's are the thicknesses of the layers in feet.

In this example, as computed from the tables, the range to the convergence zone is 207,102 ft, or 34.1 nautical miles. It should be pointed out that the 3° ray was chosen as the ray which determined the range to the convergence zone. To determine this, it was necessary to calculate ranges for several rays and to examine the results for the convergence of the rays.

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*Bryant, R. W., Pieper, A. G., and Boudreau, C. A., "Tables for the calculation of Horizontal Ranges of Rays in a Medium in Which the Velocity is a Continuous Function of Depth," NRL Report 4658, November 17, 1955

Let us approximate the foregoing profile by

$v_0 = 5051$, $v_1 = 4899$, $h = 4000$, $a = 0.016$.

We have $n = \dfrac{v_0}{v_1} = 1.031$, $\beta = \dfrac{ah}{v_1} = 0.013$.

From Fig. 3, $\theta_s = 0.075$ rad or $4° 18'$. Using Eq. (4), with

$\theta_1 = \cos^{-1}\left(\dfrac{v_1}{v_0} \cos \theta\right) = \cos^{-1}$ (0.96717) = $14°43'$, we obtain a skip range of 208,652 ft, or 34.3 nautical miles.

A comparison with the range to the convergence zone as obtained experimentally* north of Hawaii gives a calculated skip range of 33.5 nautical miles, which agrees quite closely with the observed skip range (bilinear approximation: $h = 2400$ ft, $v_0 = 5030$ ft/sec, $v_1 = 4950$ ft/sec, $a = 0.016$ ft/sec/ft).

It should be noted that the gradient below the point of minimum velocity has been taken throughout this paper as 0.016 ft/sec/ft. A literature search indicates this to be a better value than 0.018 ft/sec/ft, which is simply the pressure gradient.

## CONVERGENCE ZONE PROPAGATION IN THE NORTHERN ATLANTIC

In order for the skip path to exist, the water depth must be such that the ray leaving the source horizontally will not strike the ocean bottom; that is, the sound velocity at the bottom must be at least equal to that at the surface. This is a necessary but not a sufficient condition, since ray $\theta_s$ will go deeper than the horizontal ray. Further, the complete zone as herein defined will not be obtained unless the depth is great enough to allow ray $\theta_t$ to skip.

Figure 4 is a plot suitable for the northern Atlantic, and may be used to give some indication of the existence of the skip path. This plot contains the velocity profile below the depth of minimum velocity. Three curves are drawn, allowing for variations in minimum velocity, depth of minimum velocity, and gradient. The center curve is believed to be the most probable profile, and the others are bounds for any actual profile. These lines are curved because the velocity, plotted along the bottom, does not increase in equal increments. This was done so that the surface temperature plotted across the top would be in equal increments.

In using these curves, one measures the surface temperature, and, by knowing the ocean depth, can determine by the intersection of these lines whether convergence zone transmission is not possible, possible, probable, or assured.

Since a necessary but not a sufficient condition has been used, a generous depth allowance should be made. It is highly likely that one will not obtain the complete convergence zone in cases where these curves are used. In order to have the complete zone, as mentioned before, the ray having the same skip range, as the horizontal ray, i.e., $\theta_t$ must not be cut off by the bottom. This ray is approximately twice $\theta_s$, and hence goes quite deep.

*Pedersen, M. A., "A Long-Range, Low-Frequency, Underwater Sound Transmission Experiment in 3100 - Fathom Water," NEL Report 618 (Confidential), July 13, 1955

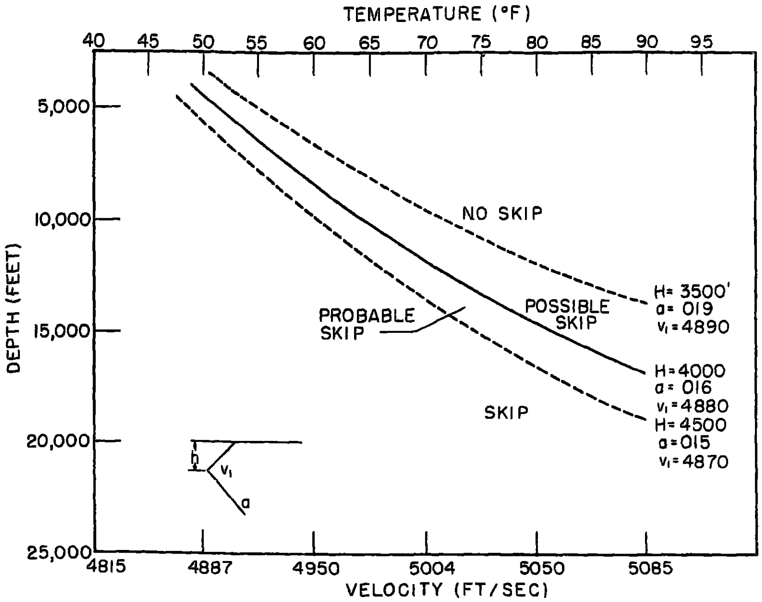


Fig. 4 - Plot for the determination of the existence of the skip path

Using these curves, and considering the skip and probable skip area, two charts of the northern Atlantic have been made. One chart is for average August surface temperature (Fig. 5), and the other for average February surface temperatures (Fig. 6). On both charts, the darker shaded area indicates where convergence zone transmission can be expected, and the lighter shaded area where such transmission is probable.

The purpose of these charts is to give a general idea of the northern Atlantic as a medium for skip transmission. In approximately 50 percent of the northern Atlantic, such propagation should be probable at all times. During the winter, this area increases to about 75 percent.

## CONCLUSIONS

The depth of the minimum velocity of the northern Atlantic appears to be stable at about 4000 ft. The gradient below that depth is approximately 0.016 ft/sec/ft. These two observations, together with the conclusion that the surface velocity and not the velocity profile in the first 4000 feet determine the range to the skip zone, gives rise to the bilinear approximation.

The skip zone has been defined as the zone between the skip range of the ray leaving the source horizontally and the minimum skip range defined by $\theta_s$. This is the region of double ray coverage. The angle greater than $\theta_s$, which has a skip range equal to that of the horizontal ray, $\theta_1$, has not been determined as yet. When this ray is determined, the ray bundle giving rise to the convergence zone will be determined.

Approximately 50 percent of the northern Atlantic is available for convergence zone transmission at all times, and approximately 75 percent is available during the winter.

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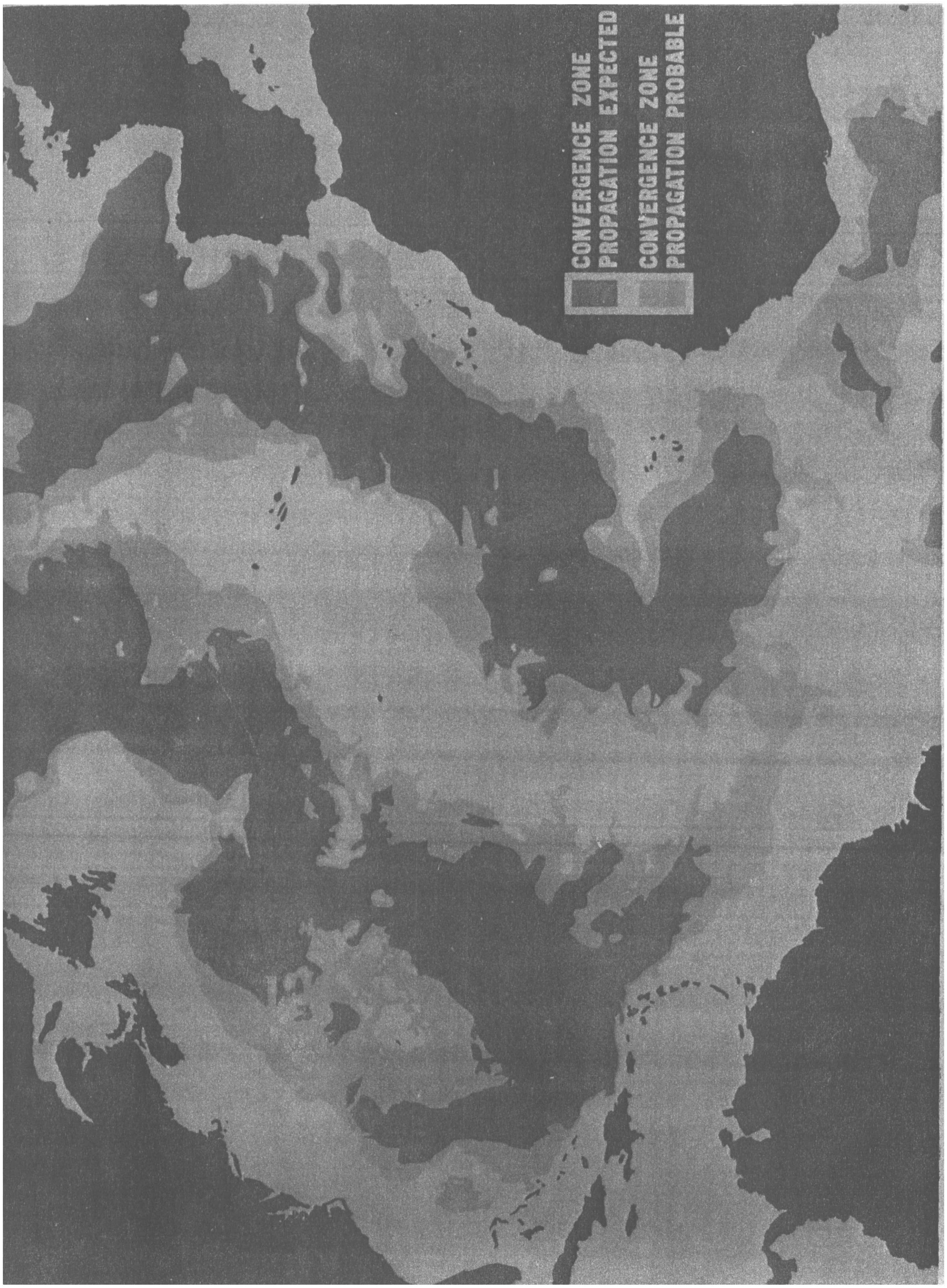


Fig. 5 - Convergence zone chart No. 1. Average August surface temperature.

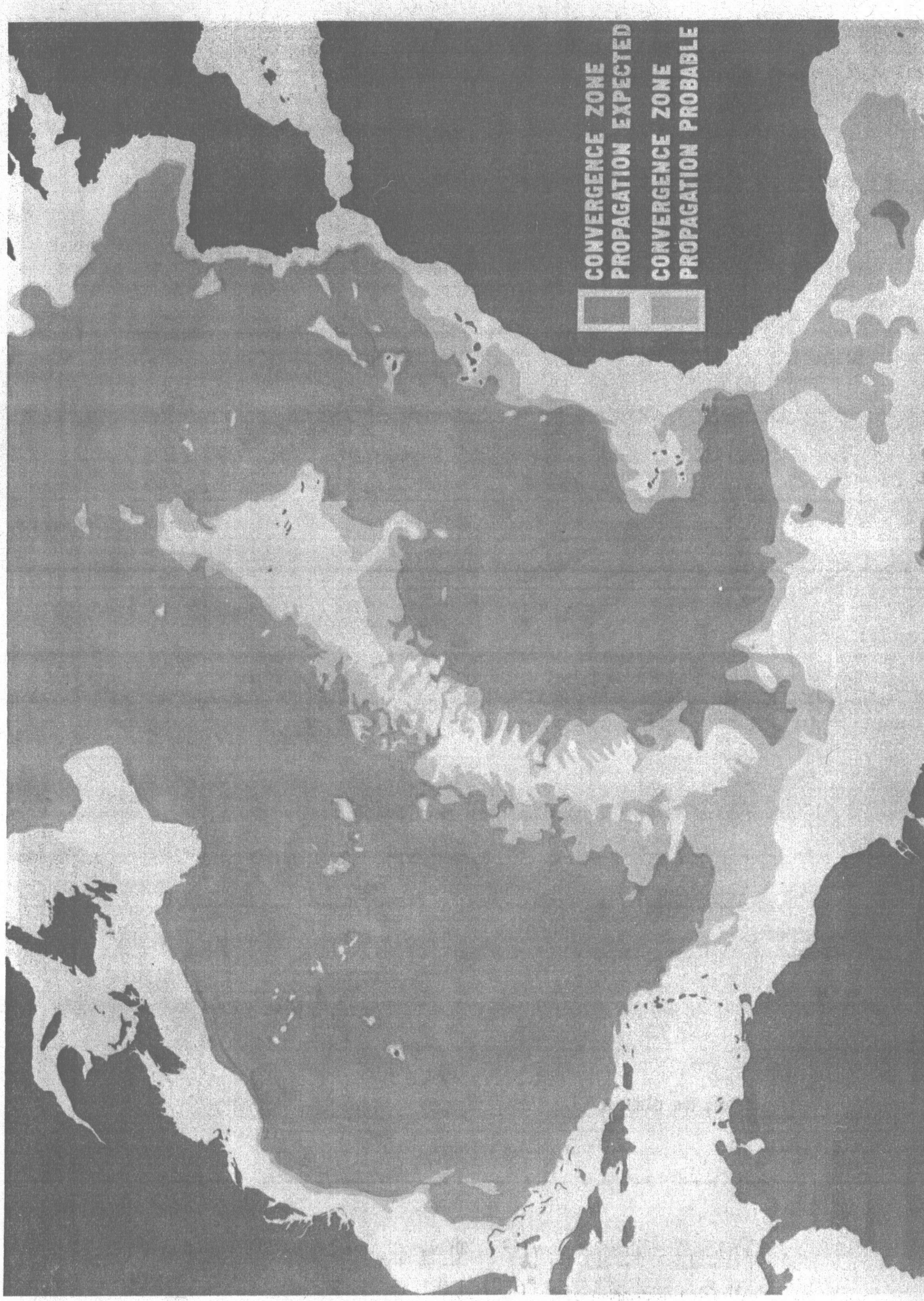


Fig. 6 - Convergence zone chart No. 2. Average February surface temperature.

## APPENDIX A
### Derivation of Equation (6)

Differentiating Eq. (4) with respect to $\theta$ gives

$$\frac{dR}{d\theta} = -2h \csc^2 \frac{1}{2}(\theta + \theta_1)\,\frac{1}{2}\left(1 + \frac{d\theta_1}{d\theta}\right) + \frac{2v_1}{a}\sec^2\theta_1\,\frac{d\theta_1}{d\theta}\,.$$

Setting this equal to zero and transposing, we obtain

$$\frac{2v_1}{a}\sec^2\theta_1\,\frac{d\theta_1}{d\theta} = h\csc^2\frac{1}{2}(\theta + \theta_1)\left(1 + \frac{d\theta_1}{d\theta}\right).$$

By Snell's law, we have

$$\frac{\cos\theta_1}{\cos\theta} = \frac{v_1}{v_0}\,,$$

hence

$$\sec\theta_1 = \frac{v_0}{v_1}\sec\theta_1\;\frac{d\theta_1}{d\theta} = \frac{v_1}{v_0}\,\frac{\sin\theta}{\sin\theta_1}\,.$$

Substituting these expressions, and using the trigonometric formula for the cosecant of a half-angle, we obtain

$$\frac{v_0}{a}\sec^2\theta\,\frac{\sin\theta}{\sin\theta_1} = \frac{h}{1-\cos(\theta+\theta_1)}\left(1 + \frac{v_1}{v_0}\,\frac{\sin\theta}{\sin\theta_1}\right),$$

or

$$\frac{v_0}{a}\sec^2\theta\,\sin\theta\,[1-\cos(\theta+\theta_1)] = h\sin\theta_1 + \frac{hv_1}{v_0}\sin\theta\,.$$

Expanding $\cos(\theta + \theta_1)$ and replacing $\cos\theta_1$ in that expansion by $\frac{v_1}{v_0}\cos\theta$ gives, with some transposing,

$$\left(\frac{v_0}{a}\tan^2\theta - h\right)\sin\theta_1 = \frac{hv_1}{v_0}\sin\theta + \frac{v_1}{a}\sin\theta - \frac{v_0}{a}\sin\theta\sec^2\theta\,.$$

Squaring both sides, we obtain

$$\left[\left(\frac{v_0}{a}\right)^2\tan^4\theta - \frac{2h\,v_0}{a}\tan^2\theta + h^2\right]\sin^2\theta_1 =$$

$$\sin^2\theta\left[\frac{h^2\,v_1^2}{v_0^2} + \frac{2h\,v_1^2}{a\,v_0} + \left(\frac{v_1}{a}\right)^2 - \frac{2h\,v_1}{a}\sec^2\theta - \frac{2\,v_0v_1}{a^2}\sec^2\theta + \left(\frac{v_0}{a}\right)^2\sec^4\theta\right].$$

Replacing $\sin^2 \theta_1$ by $1-\cos^2 \theta_1$ and using Snell's law, we obtain our equation in terms of $\theta$ only,

$$\left[\left(\frac{v_0}{a}\right)^2 \tan^4 \theta - \frac{2h\ v_0}{a} \tan^2 \theta + h^2\right]\left[1 - \left(\frac{v_1}{v_0}\right)^2 \cos^2 \theta\right]$$

$$- \sin^2 \theta \left[\frac{h^2\ v_1{}^2}{v_0{}^2} + \frac{2h\ v_1{}^2}{a\ v_0} + \left(\frac{v_1}{a}\right)^2 - \frac{2h\ v_1}{a} \sec^2 \theta - \frac{2v_0\ v_1}{a^2} \sec^2 \theta + \left(\frac{v_0}{a}\right)^2 \sec^4 \theta\right] = 0.$$

Multiplying this out, we obtain

$$\left(\frac{v_0}{a}\right)^2 \tan^4 \theta - \frac{2h\ v_0}{a} \tan^2 \theta + h^2 - \left(\frac{v_1}{a}\right)^2 \tan^4 \theta \cos^2 \theta + \frac{2h\ v_1{}^2}{a\ v_0} \tan^2 \theta \cos^2 \theta$$

$$- \frac{h^2\ v_1{}^2}{v_0{}^2} \cos^2 \theta - \frac{h^2\ v_1{}^2}{v_0{}^2} \sin^2 \theta - \frac{2h\ v_1{}^2}{a\ v_0} \sin^2 \theta - \left(\frac{v_1}{a}\right)^2 \sin^2 \theta$$

$$+ \frac{2h\ v_1}{a} \sin^2 \theta \sec^2 \theta + \frac{2v_0\ v_1}{a^2} \sin^2 \theta \sec^2 \theta - \left(\frac{v_0}{a}\right)^2 \sin^2 \theta \sec^4 \theta = 0.$$

Multiplying through by $\cos^4 \theta$ gives

$$\left(\frac{v_0}{a}\right)^2 \sin^4 \theta - \frac{2h\ v_0}{a} \sin^2 \theta \cos^2 \theta + h^2 \cos^4 \theta - \left(\frac{v_1}{a}\right)^2 \sin^4 \theta \cos^2 \theta$$

$$- \frac{h^2\ v_1{}^2}{v_0{}^2} \cos^6 \theta - \frac{h^2\ v_1{}^2}{v_0{}^2} \sin^2 \theta \cos^4 \theta$$

$$- \left(\frac{v_1}{a}\right)^2 \sin^2 \theta \cos^4 \theta + \frac{2h\ v_1}{a} \sin^2 \theta \cos^2 \theta + \frac{2v_0\ v_1}{a^2} \sin^2 \theta \cos^2 \theta$$

$$- \left(\frac{v_0}{a}\right)^2 \sin^2 \theta = 0,$$

$$\left(\frac{v_0}{a}\right)^2 \left[\sin^4\theta - \sin^2\theta\right] - \frac{2h\ v_0}{a}\sin^2\theta\cos^2\theta$$

$$-\left(\frac{v_1}{a}\right)^2 \left[\sin^4\theta\cos^2\theta + \sin^2\theta\cos^4\theta\right] + h^2\cos^4\theta$$

$$-\ h^2\left(\frac{v_1}{v_0}\right)^2 \left[\cos^6\theta + \sin^2\theta\cos^4\theta\right] + \frac{2h\ v_1}{a}\sin^2\theta\cos^2\theta$$

$$+\frac{2v_0\ v_1}{a^2}\sin^2\theta\cos^2\theta = 0,$$

$$\left(\frac{v_0}{a}\right)^2 \sin^2\theta\,(\sin^2\theta - 1) - \frac{2h\ v_0}{a}\sin^2\theta\cos^2\theta - \left(\frac{v_1}{a}\right)^2 \sin^2\theta\cos^2\theta\,(\sin^2\theta + \cos^2\theta)$$

$$+\ h^2\cos^4\theta - h^2\left(\frac{v_1}{v_0}\right)^2 \cos^4\theta\,(\cos^2\theta + \sin^2\theta) + \frac{2h\ v_1}{a}\sin^2\theta\cos^2\theta$$

$$+\frac{2v_0\ v_1}{a^2}\sin^2\theta\cos^2\theta = 0,$$

$$\left[-\left(\frac{v_0}{a}\right)^2 - \frac{2h\ v_0}{a} - \left(\frac{v_1}{a}\right)^2 + \frac{2h\ v_1}{a} + \frac{2v_0\ v_1}{a^2}\right]\sin^2\theta\cos^2\theta$$

$$+\ h^2\cos^4\theta - h^2\left(\frac{v_1}{v_0}\right)^2\cos^4\theta = 0.$$

Dividing by $\cos^4\theta$, we obtain

$$\left[-\left(\frac{v_0}{a}\right)^2 - \frac{2h\ v_0}{a} - \left(\frac{v_1}{a}\right)^2 + \frac{2h\ v_1}{a} + \frac{2v_0\ v_1}{a^2}\right]\tan^2\theta = -h^2\left[1 - \left(\frac{v_1}{v_0}\right)^2\right].$$

Hence,

$$\tan^2\theta = \frac{h^2\left[1-\left(\frac{v_1}{v_0}\right)^2\right]}{\left(\frac{v_0}{a}\right)^2 + \frac{2h\ v_0}{a} + \left(\frac{v_1}{a}\right)^2 - \frac{2h\ v_1}{a} - \frac{2v_0\ v_1}{a^2}}$$

$$= \frac{a^2h^2\left[\frac{v_0^{\ 2} - v_1^{\ 2}}{v_0^{\ 2}}\right]}{v_0^{\ 2} + 2ha\ v_0 + v_1^{\ 2} - 2ha\ v_1 - 2\ v_0\ v_1}$$

$$= \frac{a^2h^2\ (v_0^{\ 2} - v_1^{\ 2})}{v_0^{\ 2}\ [v_0^{\ 2} - 2v_0v_1 + v_1^{\ 2} - 2ah(v_1 - v_0)]}$$

$$= \frac{a^2h^2\ (v_0^{\ 2} - v_1^{\ 2})}{v_0^{\ 2}\ [(v_0 - v_1)^2 + 2ah(v_0 - v_1)]}$$

$$= \frac{a^2h^2\ (v_0 + v_1)}{v_0^{\ 2}\ [v_0 - v_1 + 2ah]}.$$

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