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SIGNAL PROCESSING IN THE SECTOR SCAN INDICATOR

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CONTENTS

Abstract	ii
Problem Status	ii
Authorization	ii
INTRODUCTION	1
SECTOR SCAN INDICATION	1
The SSI Circuit and Display	1
Phase Relationships	2
THE SSI AS A CORRELATOR	3
Coherent Signals and Noise Interference	3
Low-Frequency Listening	3
Reception of Echoes	4
Balancing Out Coherent-Noise Interference	5
Effectiveness of SSI	7
APPENDIX A - MATHEMATICS OF SSI	9
Assumptions	9
Formulation of Signal Voltages	9
SSI Processing	10
Coherent Signal in a Noise Background	12
Signal at a Single Frequency	13
Discussion of Coherence	14
APPENDIX B - COHERENCE OF WATER NOISE	15
Nature of Interference	15
Coherence of Water Noise at Two Transducers	15
Coherent-Noise Processing	16
General Discussion of the Result	17

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ABSTRACT

The Sector Scan Indicator is known as a device for indicating the position in a sound beam of objects from which sonar echoes are received. The centering of the sound beam on a target submarine and the presentation of the submarine's aspect have appeared to be the principal uses for this equipment. It is now evident that this device is also particularly adapted to the reception of a weak noise signal in a background of ambient noise.

SSI uses two superheterodyne channels with different local oscillator frequencies, g_1 and g_2, and concentrates most of the signal at the difference frequency $g_2 - g_1$. In a background of noncoherent noise, a coherent noise signal produces a spike at $g_2 - g_1$, the center frequency of a band. A narrow bandpass filter accepts the spike and rejects most of the remainder of the band. If the power spectra of signal and noise at the input are respectively P and Q, the signal-to-noise ratio at the output is $1 + (P^2/Q^2)(W/w)$ where W is the frequency acceptance bandwidth and w is the bandwidth of the post-detector filter.

In the reception of echoes, SSI has a sensitivity about equal to that of the ear. However, electronic treatment of the output to extract remaining information offers promise of further improvement.

The concept that ambient noise produces coherent inputs to SSI is developed. For the ideal case, omnidirectional noise of spectral level P has a coherent component $(Pc/2\pi hf) \sin 2\pi hf/c$ where h is the separation of two point hydrophones, c is the velocity of sound waves reaching the transducers, and f is frequency.

PROBLEM STATUS

This is a final report on one phase of the problem; other phases are completed and will be reported separately.

AUTHORIZATION

NRL Problem S07-12
RDB Projects NE 050-961-9 and NR 527-120

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SIGNAL PROCESSING IN THE SECTOR SCAN INDICATOR

INTRODUCTION

The Sector Scan Indicator (SSI) measures the phase difference between two signal inputs, but particular emphasis is placed on the phase difference between the inputs from the two halves of a split transducer. Since the signal frequency out of the SSI detector is constant in spite of Doppler shifts in the echo, there has existed the possibility of channeling the detector output through an extremely sharp bandpass filter to secure a reduction of background noise. In experimental studies of optimum passbands, it has been observed that coherent inputs receive preferential treatment, and theoretical considerations have demonstrated that signal processing by the SSI is a correlation technique. The object of this report is to present and interpret the results of the signal processing which the SSI provides in terms of the signal-to-noise ratio out, as a function of signal-to-noise ratio in, and of design parameters.

SECTOR SCAN INDICATION

The SSI Circuit and Display

A skelton diagram of the circuit is shown in Figure 1; operation is as follows:

(a) A current having a linear saw-tooth waveform is applied to the Y-axis of the cr tube. In echo ranging, this waveform is synchronized with a keying pulse in order that upward displacement may correspond to range.

(b) Each of the two incoming signals traverses its own superheterodyne channel. The local oscillators in the two channels are separated in frequency, preferably by at least one bandwidth of the passband. When heterodyned against each other, these local oscillators produce a voltage at the difference frequency that may be regarded as a reference. When inverted, this voltage synchronizes a saw-tooth waveform applied to the X-axis of the cr tube.

(c) The i-f outputs are combined and detected to produce a difference frequency which is distorted into periodic spikes and used for Z-axis modulation of a cr tube.

In operation, a small phase shift of the reference frequency by a screw-driver control centers the cr spot when the two channels are receiving the same (in-phase) inputs. Thereafter, a phase difference of the inputs displaces the crt spot in such proportion to the phase difference that 180° causes a displacement to the edge of the raster.

The signal processing with which this report is concerned from this point on uses only those parts of the circuit that are shown outside the broken line in Figure 1.

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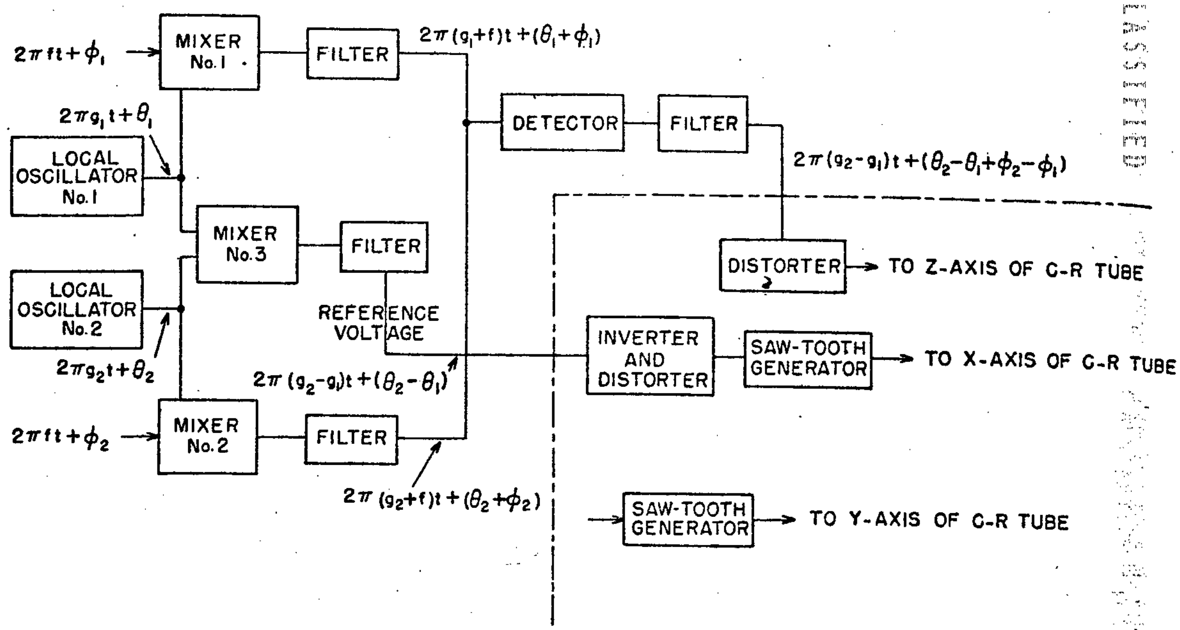

Figure 1 - Functional block diagram of SSI

Phase Relationships

The phase relations obtained in the signal processing are indicated in Figure 1 at different points in the circuit. The frequencies of local oscillators 1 and 2 are represented by g_1 and g_2, which are known quantities. The difference frequency used as a reference is $g_2 - g_1$. Let the phase of local oscillator 1 be represented by $2\pi g_1 t + \theta_1$ and the phase of local oscillator 2 by $2\pi g_2 t + \theta_2$. Since phases subtract in the difference frequency obtained from a square-law detector,* the reference phase obtained at the difference frequency when g_1 and g_2 are heterodyned against each other is $2\pi(g_2 - g_1)t + (\theta_2 - \theta_1)$.

Consider, for the moment, a frequency pair of signal components consisting of inputs to both channels at the same frequency. Let the phases into the two channels be represented by $2\pi ft + \phi_1$ and $2\pi ft + \phi_2$. The intermediate frequency in each channel has a phase which is assumed here to be the sum of the phases of the local oscillator and the input frequency, as indicated in Figure 1. (Either sum or difference may be used.) Likewise, the phase out of the detector at the difference frequency of the two local oscillators (Figure 1) is the difference in phase of the two intermediate frequencies combined to produce a difference frequency. Comparing the phase of the output signal with the reference phase, the former is seen to lead the latter by $\phi_2 - \phi_1$, which is exactly the phase difference between the two input signals. On this relationship rests the operation of SSI. As a particular case, the output is in phase with the reference frequency when the two input signals are in phase with each other.

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* When two signals, $A \cos(\omega_1 t + \theta_1)$ and $B \cos(\omega_2 t + \theta_2)$, are combined in a square-law device, its output is $K[A \cos(\omega_1 t + \theta_1) + B \cos(\omega_2 t + \theta_2)]^2$, which, when expanded, contains a cross-product term

$$2KAB \cos(\omega_1 t + \theta_1) \cos(\omega_2 t + \theta_2) = KAB\left\{\cos[(\omega_2 - \omega_1)t + (\theta_2 - \theta_1)] + \cos[(\omega_2 + \omega_1)t + (\theta_2 + \theta_1)]\right\}.$$

At the difference frequency, then, phase $\omega_1 t + \theta_1$ is subtracted from phase $\omega_2 t + \theta_2$.

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When the signals are bands of frequency components (the same frequency band in each channel), a particular component in one channel and that in the other channel at the same frequency together constitute a "frequency pair." Since every frequency pair from dead ahead has $\phi_2 - \phi_1 = 0$, all input frequencies in a noise band from dead ahead produce signal outputs in the reference phase and therefore in phase with each other. As a consequence of this relationship, the output amplitudes are directly added. Omnidirectional interference, unless coherent, receives no such favorable treatment.

THE SSI AS A CORRELATOR

Coherent Signals and Noise Interference

Coherence means "sameness." Coherent signals may be identical inputs to the two channels of SSI.* In the laboratory, the same signal, either cw or noise, may be put into both channels. In the field, the signals will be coherent if they arise from a sound wave reaching the two channels simultaneously.

Noncoherent noise interferences are those which are interrelated only by chance; such interferences are, for example, thermal noises arising in independent isolated amplifiers. In the laboratory, independent generators provide noncoherent noise interferences. In the field, the ambient noise may give rise to noncoherent interferences in the two SSI channels (Appendix B).

If continuous waves uniformly distributed in angle constituted the noise interference, the SSI response would depend upon directivity and spacing in wavelengths of the two hydrophones. For this kind of interference, the degree of coherence versus spacing in wavelengths of two point hydrophones is derived in Appendix B and shown in Figure B1. Perfect coherence must result from ambient noise, or from any other disturbance, in two identical superimposed hydrophones (zero spacing). One point on the derived curve is, therefore, certain. As spacing is increased, agreement with the derived curve of Appendix B becomes poorer because of space variation in path characteristics and because of finite wave-train lengths. However, except for large hydrophone spacing, ambient noise from the hydrophones generally exhibits considerable coherence; otherwise a sharp beam for noise rejection could not be produced. The coherent part of interfering noise gives rise to an output voltage in a phase equal to that of the reference-frequency voltage; these two can be balanced against each other. Some consideration, however, should be given to the design of transducers for reducing the coherent part of the ambient noise.

Components of ship's self-noise come from definite directions. Any particular noise source on the ship gives rise to coherent signals. The phase and amplitude of self-noise response may be determined relative to the reference frequency by means of Lissajous figures displayed on an oscilloscope. A record may be kept of the bearings of offending noise sources until they are identified and possibly eliminated.

Low-Frequency Listening

When listening to broadband noise, the signal-to-noise ratio out of the SSI post-detector filter in terms of SSI parameters, along with signal and noise input powers, is shown by Equation (110.1) of Appendix A to be

$$\text{Signal Power/Noise Power} = 1 + (P^2/Q^2)(W/w). \qquad (1)$$

*A more general case is treated under "Discussion of Coherence" in Appendix A.

[redacted]

P and Q are the input spectral levels of coherent signals and noncoherent noises respectively. W is bandwidth before detection, and w is bandwidth after detection. The signal appears as a sharp spike at the middle of the difference frequency band. No other band contains such a spike.

The larger W/w, the higher will be the signal-to-noise ratio out, subject, of course, to the limitation in w that it must permit buildup in the interval during which the signal is received.

Whenever W/w is quadrupled, the ratio of signal-to-noise power output is quadrupled. However, if then the value of P/Q is reduced to restore the original output ratio, it (P/Q) need be only halved. In other words, quadrupling W/w will permit a 6-db improvement in output for the same input or will permit the same output with 3-db poorer signal-to-noise ratio at the input.

A reasonable value of W/w is 2500/0.5 = 5000 which permits a 37-db gain in output for the same input or an 18.5-db decrease in input for the same output signal-to-noise ratio.

As a consequence of the improvement in signal-to-noise ratio in utilizing broadband reception with SSI, this type of signal selection and processing presents a promising alternative to the use of narrow bands for the reception of discrete frequencies as an approach to long-range submarine detection.

A further comparison of the alternatives is provided by consideration of the following facts:

(a) Various reports have given conflicting data concerning the ambient noise background and submarine spectra at low frequencies. From reports and discussions, it has been concluded by NRL scientists that the signal-to-noise ratio versus frequency is nearly independent of frequency between 100 and 1000 cps. Below 100 cps, the average signal-to-noise ratio is about the same as above 100 cps but discrete-frequency spikes exist at levels several decibels above the average level in 4-cps bands. This fact, if accurate, results in an advantage to very low discrete-frequency reception.

(b) For the same size array, an increase in directivity gives an advantage to the higher frequency. Broadband reception utilizing SSI is therefore favored.

(c) Filters narrower than 4 cps (or their equivalent) are feasible in some processing schemes for discrete frequencies and also in SSI for broadband reception. A 1-cps filter tunable to a discrete frequency below 100 cps gains 4 fold relative to a 4-cps band. SSI with W = 400 cps (100 to 500 cps) and with w = 0.1 cps (fixed tuned in this case) gains 63-fold. This gain achieved in processing results in an advantage to broadband reception with SSI.

(d) The highest frequency (500 cps) proposed for very long range with SSI loses 1-1 relative to 100 cps in 300 miles. This loss appears negligible.

Reception of Echoes

Echoes of say 100-millisecond duration approach continuous waves in character since their energy is concentrated in a narrow band. For cw, the limiting case, with background

[redacted]

noise still assumed noncoherent, Equation (1) is replaced by

$$\frac{\text{Signal Power}}{\text{Noise Power}} = 1 + \frac{P^2}{Q^2 Ww} \qquad (2)$$

taken from Equation (111.1). In W and w, the SSI provides adjustable constants which can be manipulated to advantage.

The mechanism of the human ear has associated with it certain time constants which are not adjustable. Experience has shown that for Ww = 100, the SSI detector output displayed on an A-scan is as good as the ear. The SSI gains over the ear if Ww is less than $100/\text{sec}^2$. Whether the SSI can achieve this gain depends upon the restriction placed on W by the necessity for accommodating echoes with Doppler shifts and the restriction placed on w by the requirement of adequate buildup in the duration of an echo. Long pulses and low frequency are desirable to permit a small Ww. Additional SSI gain may be obtainable by the form of presentation. In fact, the SSI narrow-filter output contains information about phase that the ear could not normally use but which the SSI display is designed to use.

A comparison of Equations (1) and (2) shows that a total noise-signal power, PW, is treated equally as well as a total single-frequency power P. On the other hand, reverberation fluctuates much less if spread out in a fairly broadband rather than being confined to a band in a few cycles per second. It follows that the use of a noise signal in echo ranging would improve echo reception when the SSI provides signal processing.

Balancing Out Coherent-Noise Interference

The interference noises from the two pickup transducers generally possess a degree of coherence (Appendix B). In this section, a technique is proposed for giving the coherent parts of ambient noises a chance to balance themselves out while several signals received at different times from different directions relative to transducer heading are added in phase.

All reception is amplified and recorded on the same magnetic tape; this process uses a separate channel for each half of the transducer. The two recording heads are placed side by side as are paired playback heads. Returns from pings on successively different frequencies are accepted consecutively.

The route traversed by any cross section of the recorder tape (Figure 2) is past the two recording heads at R_1 and R_2, then consecutively past the pairs of playback heads at 1c and 2c, at 1b and 2b, and at 1a and 2a, then past the erase heads, and finally back to the recording heads, thus closing the loop. The transit time from 1c to 1b and also from 1b to 1a is a ping interval which may be controlled by varying tape speed. The heads at 1a and 2a play back the first of a sequence of returns from three consecutive pings simultaneously with the playbacks at 1b and 2b of the second, and also at 1c and 2c of the third of the sequence. In all three reception intervals of the sequence, all echoes received from any fixed range are rendered simultaneous at the recording and playback heads. The artist's conception of a sound track indicates the dying out of reverberations and the increase in level at the echo after each ping.

The train rate is synchronized with the ping rate in a manner that provides 120° change in the phase relation at the two halves of the transducer between two successive echoes from the same target. This procedure provides for the reception of three echoes within the sound beam. Under these condtions and when training clockwise, the echo received nearest

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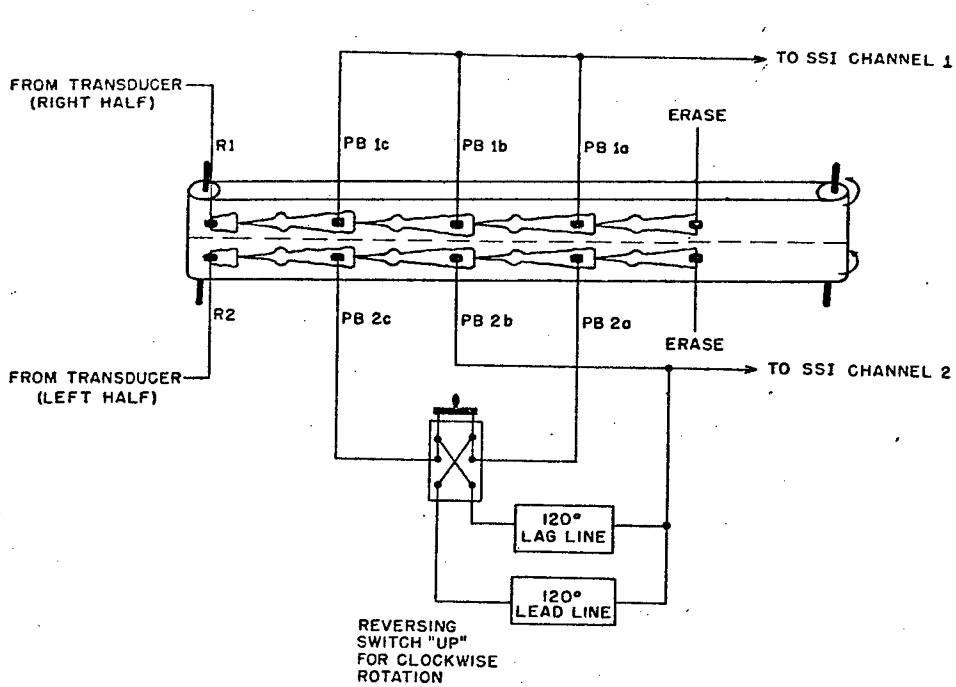

Figure 2 - Plan for recording and playback

the transducer axis will have a phase lead of some γ at the right half of the transducer relative to the left half. The preceding echo must then have arrived with a phase lead $\gamma + 120^0$, and the following echo will be expected with a phase lead of $\gamma - 120^0$. In Figure 3(a), vectors show the zero-phase relation between the coherent noises in the tw channels at each pair of heads, while other vectors show the phase relations in echoes received from one target in all three intervals of the sequence.

Now, in the left channel only, a phase lead of 120° is inserted in the 2a playback hea and a phase lag of 120° is inserted in the 2c playback. These insertions change the phas relationships of Figure 3 (a) to those of Figure 3 (b).

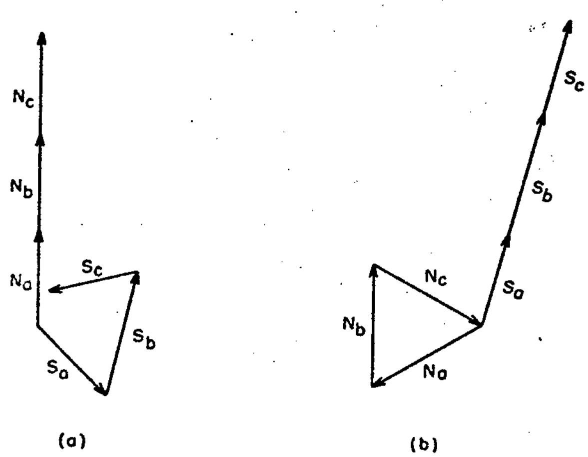

Figure 3 - Vector relationships between samples of signal and noise
(a) Without phase shifts
(b) With phase shifts

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In order that the phase relationship in each pair of recorder outputs be reproduced between the reference and output voltages of an SSI, thereby permitting the three signal outputs to add in phase, and the three noise outputs to balance out, each pair of recorder outputs must be made to lie in a unique frequency band. The balance may be accomplished by pinging at three slightly different frequencies or by translating two outputs in frequency before adding all three outputs to form the SSI input.

The amount of gain by this procedure is not readily estimated since the extent of noise coherence at two pickup points has not been measured. However, for a reasonable separation of transducer halves (about 1-1/2 to 2-1/2 wavelengths), the coherent part of the noise might conceivably contribute 5 percent of the interference background at the SSI input and 99 percent at its output. In this case, a gain in the order of 20 db would be theoretically possible from noise balancing; at the same time, a further gain of nearly 10 db might be achieved by adding three in-phase echoes. These two contributions add to a total gain of 30 db. Even a 10-db actual gain would be worth the effort of incorporating this scheme.

This plan enhances point sources of noise relative to omnidirectional noise. It is, therefore, applicable to passive detection as well as to echo ranging. On the other hand, it it ineffectual against self-noise.

Effectiveness of SSI

The effectiveness of the SSI for a noise signal may be assessed by comparing the signal-to-noise ratio out of it to that obtained from a square-law detector. The low-frequency output of the detector is a direct power (zero frequency) of $(P + Q)^2W^2$ plus a random power $2\ (P + Q)^2\ Ww$ for the same bands (W and w) considered with SSI. In either case, $R = (1 + P/Q)^2$. The use of sharper low-pass filters results in a greater reduction of fluctuation together with a longer buildup time, but does not change R. R is the output ratio of signal to background noise.

Comparison of the preceding power ratio with Equation (1) indicates that SSI enjoys a considerable advantage when W/w is large and the noise background is noncoherent. Physically, this comes about from the rejection of intrachannel terms by SSI. If $P = Q$, and $W/w = 1000$, $R = 4$ for the square-law detector and $R = 1001$ for SSI. This is an advantage of 24 db. If, now, the input signal-to-noise ratio is decreased until $R = 4$ for SSI, this input ratio is decreased 12.5 db, which is the realizable advantage of SSI for detection in this case.

In reception of a single-frequency signal, SSI also gains relative to the square-law detector by approximately the same factor as for the noise signal in so far as the ratio of direct powers is concerned. The fluctuation is unimproved.

No conflict has been found between experimental data taken in the laboratory and the theoretical results. In the field, SSI designed for wideband noise reception has not been used. The SSI of XDG submarine equipment, though narrow band and tuned to the relatively high frequency of 25 kc, has received noise signals from surface vessels out to ranges of 20 kyds. The SSI of the 10-kc LRS equipment was once used for listening to the noise of a submarine operating on batteries at 3.5 knots. On this occasion the target was tracked to 17-kyd range where SSI still had something to spare. Such pieces of evidence were frequent enough so that NRL felt bound to build a listening SSI for trial with the JT transducer in the frequency range of 1 to 3 kc. This device will soon be ready.

In echo ranging, a well-designed SSI appears to be slightly better than the ear. This been confirmed time and time again but particularly in the 10-kc LRS system. Phase gates with counting circuits are believed capable of providing further inprovement, and work is progressing along this line. Also, the scheme for balancing out coherent noise and adding echoes described herein appears promising. On the whole, the prospects of SSI as a detection device with active sonar are bright.

* * *

APPENDIX A
MATHEMATICS OF SSI

Assumptions

A restricted mathematical treatment of the signal processing performed suffices to bring out its salient features, and the following assumptions are justified. A rectangular passband accepting white noise of spectral level P is assumed. The phase shift caused by filtering is assumed to be the same function of signal frequency in both channels so that it may be dismissed with the remark that it cancels out in the difference frequency of the SSI where phases subtract. Coherent signals are assumed identical in the two channels.

Formulation of Signal Voltages

Let the bandwidth be W centered at a frequency f_0 in each channel. Let this be divided into 2n + 1 narrow bands of width δf. Then

$$W = (2n + 1)\ \delta f. \qquad (101)$$

Let the narrow bands be labelled δf_{i_1} in channel 1 with i ranging through integral values from -n to +n and δf_{j_2} in channel 2 with j likewise ranging through integral values from -n to +n.

The instantaneous voltages are represented by a set of 2n + 1 discrete frequencies, one in each narrow band.[1] Voltages in channels 1 and 2 are, respectively:

$$V = \sum_{i=-n}^{+n} \sqrt{2P\delta f}\ \cos\left[2\pi(f_0 + i\delta f)\, t + \phi_{i_1}\right] \quad \text{and} \qquad (102.1)$$

$$V = \sum_{j=-n}^{+n} \sqrt{2P\delta f}\ \cos\left[2\pi(f_0 + j\delta f)\, t + \phi_{j_2}\right] \qquad (102.2)$$

where P is power spectrum level and ϕ_{i_1} and ϕ_{j_2} are epoch angles.

The power P may be measured by observing the output of a narrow-band filter on an rms voltmeter and averaging over a considerable time. During the averaging interval, the voltmeter reading fluctuates. Rice[1] points out that the amplitude actually has a normal distribution and the $\sqrt{2P\delta f}$ is its standard deviation from zero. An amplitude wherever given is the standard deviation from zero or rms value of many samples, and it is as representative as any simple quantity that can be presented.

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[1] Rice, S. O., "Mathematical Analysis of Random Noise," BSTJ 23, 282-332, July 1944 (See particularly section 1.7)

SSI Processing

The local oscillator voltages are designated by

$$\text{Oscillator Voltage}_1 = \cos(2\pi g_1 t + \theta_1) \quad \text{and} \tag{103.1}$$

$$\text{Oscillator Voltage}_2 = \cos(2\pi g_2 t + \theta_2)\,. \tag{103.2}$$

Assuming unity gain, interaction between the voltages of Equations (102.1) and (103.1) in a mixer gives

$$V_{if_1} = \sum_{i=-n}^{+n} \sqrt{2P\delta f}\, \cos\left[2\pi(g_1 + f_0 + i\delta f)\,t + (\theta_1 + \phi_{i1})\right]\,. \tag{104.1}$$

The bands of width δf are allowed to retain the same subscripts when translated in frequency in the SSI. The upper sideband is selected for convenience although the lower one would serve as well and is more commonly used. In like manner, interaction between the voltages of Equations (102.2) and (103.2) gives

$$V_{if_2} = \sum_{i=-n}^{+n} \sqrt{2P\delta f}\, \cos\left[2\pi(g_2 + f_0 + j\delta f)\,t + (\theta_2 + \phi_{j2})\right]. \tag{104.2}$$

Interaction between the voltages of Equations (104.1) and (104.2) in the square-law detector produces an output voltage

$$V_{out} = \left\{ \sum_{i=-n}^{+n} \sqrt{2P\delta f}\, \cos\left[2\pi(g_1 + f_0 + i\delta f)t + (\theta_1 + \phi_{i1})\right] + \sum_{j=-n}^{+n} \sqrt{2P\delta f}\, \cos\left[2\pi(g_2 + f_0 + j\delta f)\,t + (\theta_2 + \phi_{j2})\right] \right\}^2 . \tag{105}$$

The terms of Equation (105) expanded give rise to five frequency bands. Three of these, one adjacent to zero frequency of width W, and two of width 2W centered at $2(g_1 + f_0)$ and $2(g_2 + f_0)$ respectively, arise from intrachannel interactions, that is, from interactions between terms in the same summations in Equation (105). They do not involve interactions of the two summations with each other. These bands are labelled by mid-frequency (in one case, 0 frequency) in Figure A1. All these intrachannel bands of frequency are rejected in the SSI by a filter.

The interchannel bands are of width 2W centered at the sum frequency $g_2 + g_1 + 2f_0$ and at the difference frequency $g_2 - g_1$. This latter band is the one selected and used by SSI. By design, the central portion of this band is separated in frequency from any of the other bands (Figure 5). This requires $(g_2 - g_1) > W$.

What SSI does is to separate the product $V_1 V_2$ (or its equivalent $V_{if_1} V_{if_2}$) from the terms V_1^2 and V_2^2 which arise in square-law detection. This separation is accomplished without the use of clipping or balancing techniques.

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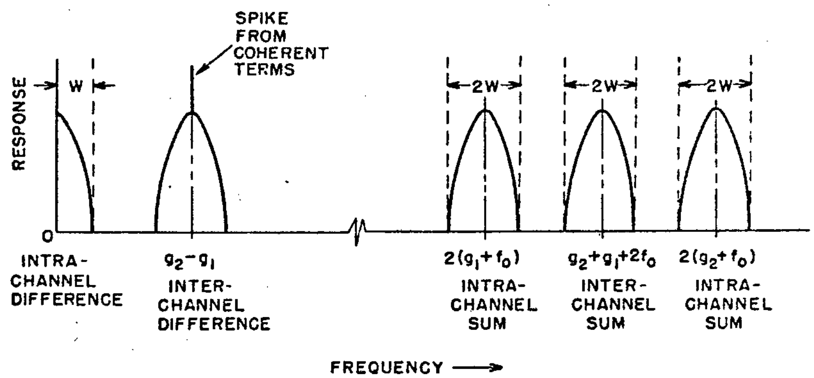

Figure A1 - Frequency spectrum of detector output

The interchannel difference-frequency terms are next extracted from Equation (105). A typical cross product term is

$$4P\delta f \cos\left[2\pi(g_1 + f_0 + i\delta f)t + (\theta_1 + \phi_{i1})\right] \cos\left[2\pi(g_2 + f_0 + j\delta f)t + (\theta_2 + \phi_{j2})\right] .$$

The trigonometrical expansion

$$2 \cos C \cos D = \cos(C + D) + \cos(C - D)$$

is now employed and the $\cos(C - D)$ term for the case in which $j = i + k$ is

$$2P\delta f \cos\left[2\pi(g_2 - g_1 + k\delta f)t + (\theta_2 - \theta_1 + \phi_{(i+k)2} - \phi_{i1})\right] .$$

For any given k, the frequency for all values of i is

$$g_2 - g_1 + k\delta f.$$

Let us consider the summation of terms at such a frequency. Denoting the summation by V_k there is obtained, for positive k

$$V_k = \sum_{i=-n}^{+n-k} 2P\delta f \cos\left[2\pi(g_2 - g_1 + k\delta f)t + (\theta_2 - \theta_1 + \phi_{(i+k)2} - \phi_{i1})\right] . \quad (106)$$

For negative k, Equation (106) still applies except that the summation is from

$$i = -n - k \text{ to } i = +n.$$

When $k \neq 0$, $\phi_{(i+k)2}$ and ϕ_i are randomly related. Hence, the terms add on the average by power.[2] There are $2n - |k| + 1$ terms so that the resultant amplitude is the $2P\delta f$ of one term multiplied by

$$\sqrt{2n - |k| + 1}.$$

RMS voltage U_k is then

$$U_k = P\delta f \sqrt{2(2n - |k| + 1)} . \quad (107.1)$$

A narrow filter tuned to $g_2 - g_1$ may be used to reject most of this band.

[2] Rice, R., loc. cit.

[redacted]

When $k = 0$, the noncoherent and coherent cases have to be distinguished. In the former, Equation (107.1) still holds with k put equal to zero. In the latter $\phi_{i_1} = \phi_{i_2}$, and hence all voltage components at the frequency, $g_2 - g_1$, are in phase and add by amplitudes, giving

$$U_0 = \sqrt{2}\, P\delta f\ (2n + 1). \tag{107.2}$$

This particular case is selectively emphasized by the SSI processing as shown by Equation (107.2) as compared to Equation (107.1) and by the use of a narrow post-detector filter tuned to $g_2 - g_1$, the frequency of U_0. The component U_0 accounts for the spike at $g_2 - g_1$ (Figure A1); this spike is present only when coherence exists.

Coherent Signal in a Noise Background

In a practical sense, it is sufficiently accurate to describe a coherent signal at the two halves of the transducer as a signal reaching the two halves simultaneously even though a signal off the axis may also be coherent (see Discussion of coherence). Ambient noise is expected to be partially coherent at the two halves of a transducer. To be presently considered, then, is a coherent signal from dead ahead in a noise background containing both coherent and noncoherent voltage sets.

Let the power spectral levels of noise be Q_c (coherent) and Q_n (noncoherent), and let the signal power spectrum be P. In the equation, which for this case replaces Equation (105), each summation includes three generalized cosine terms to be summed with respect to i or j.

In the cross product between summations, the cross product of any one of the three generalized terms (P, Q_c, or Q_n) in one summation with its mate in the other summation yields a term of the form of Equations (107.1) or (107.2). The three results of the type of Equation (107.1) add together by power, since there is random phase among them, to give

$$\delta f\ \sqrt{2(P^2 + Q_c^{\,2} + Q_n^{\,2})\,(2n - |k| + 1)}\ .$$

The two results of the type of Equation (107.2) resulting from the coherent signals add by amplitudes to give

$$\sqrt{2}\,(P + Q_c)\,(2n + 1)\,\delta f\ .$$

In addition to the terms of the last paragraph, there are the cross products between P and Q_c terms, P and Q_n terms, and Q_c and Q_n terms, all noncoherent and therefore together contributing

$$\delta f\ \sqrt{4(PQ_c + PQ_n + Q_c\,Q_n)\,(2n - |k| + 1)}$$

to a formula of the type of Equation (107.1), but contributing nothing to the type of Equation (107.2).

Collecting terms from the preceding two paragraphs Equation (107.1) goes over into

$$U_k = (P + Q_c + Q_n)\,\delta f\,\sqrt{2\,(2n - |k| + 1)}\ , \tag{108.1}$$

and Equation (107.2) goes over into

$$U_0 = (P + Q_c)\,\delta f\,(2n + 1)\,\sqrt{2}\ . \tag{108.2}$$

Let w be the bandwidth of a post-detector filter tuned to $g_2 - g_1$, that is, to the term U_0 given by Equation (108.2). This filter passes, in addition to U_0, a number of noncoherent terms U_k for different k values from $-w/2\delta f$ to $+w/2\delta f$, that is, $w/\delta f$ terms. These add with each other and with U_0 by power. By use of Equation (101) and neglecting $|k| \ll n$ for terms within the narrow filter, there is obtained

$$\text{Power out} = (P + Q_c)^2\, 2W^2 + (P + Q_c + Q_n)^2\, 2Ww\,. \qquad (109)$$

A more important relationship is the ratio of power out in the presence of a signal to power out in the absence of a signal. If the ratio of signal plus noise to noise is denoted by R, there results

$$R = \frac{(P + Q_c)^2 W + (P + Q_c + Q_n)^2 w}{Q_c^2 W + (Q_c + Q_n)^2 w},$$

which, for practical purposes when w is small, becomes

$$R = 1 + \frac{(P^2 + 2PQ_c)W}{Q_c^2 W + Q_n^2 w}\,. \qquad (110)$$

If the noise is completely noncoherent ($Q_c = 0$), the case of chief interest involves the further assumption that $P \ll Q_n$. Equation (110) then becomes

$$R = 1 + \frac{P^2}{Q_n^2}\frac{W}{w}\,. \qquad (110.1)$$

If the noise is completely coherent ($Q_n = 0$), Equation (110) becomes

$$R = \left(1 + \frac{P}{Q_c}\right)^2 \qquad (110.2)$$

For the case of completely coherent noise, the potential advantage of SSI seems by inspection of Equation (110.2) to have disappeared. A little thought, however, reveals that there are approaches to balancing out Q_c. One approach is to design the transducer to make $Q_c = 0$ at or near the midfrequency. That this can be done for point transducers in a homogeneous ambient noise field is suggested by the results in Appendix B. For other transducers, the problem requires further study. By sufficient separation of hydrophones, Q_c may also be reduced without critical spacing. Another approach is to balance the output caused by the coherent part of ambient noise against the SSI reference-frequency voltage which has the same phase.

Signal at a Single Frequency

In active sonar (echo ranging), a signal at a single frequency is usually employed. If the total power of the signal in this case is taken as P and if again coherent noise is made up of a frequency set of amplitudes $\sqrt{2Q_c\,\delta f}$ and noncoherent noise is made up of a frequency set of amplitudes $\sqrt{2Q_n\,\delta f}$, the same reasoning as before yields Equations (111), (111.1), and (111.2) in place of Equations (110), (110.1), and (110.2) respectively.

For the general case,

$$R = 1 + \frac{P^2 + 2P(Q_c W + Q_n w)}{Q_c^2 W^2 + Q_n^2 W w}; \tag{111}$$

for the noncoherent case,

$$R = 1 + \frac{2P}{Q_n W} + \frac{P^2}{Q_n^2 W w}; \tag{111.1}$$

and for the coherent case,

$$R = \left(1 + \frac{P}{Q_c W}\right)^2. \tag{111.2}$$

Equation (111.2) shows the signal faring poorly relative to coherent noise. Some sort of balancing technique, either in the transducer spacing or by the use of the reference frequency, or both, is desirable to suppress the coherent part of the noise background. Either Equation (111.1) or Equation (111.2) shows improvement in R when W is made small. A smaller w improves the R of Equation (111.1). In deriving Equation (111.2), w has been assumed small in comparison with W.

Discussion of Coherence

The assumption of signal coming from dead ahead was not essential to the treatment. If the signal comes from any single direction, it possesses coherence at the two hydrophones because a particular translation in time of the output of one hydrophone relative to that of the other can then compensate for the time translation in the water and produce identical signals.

The SSI provides a continuous shift in phase rather than in time. At a single frequency, the continuous phase shift is equivalent to a continuous time shift. An oscilloscopic presentation of the narrow-filter output of SSI with a single-frequency input would be a presentation of the correlation coefficient for the time shift from $-T_0/2$ to $+T_0/2$, where T_0 is a period for the single frequency.

A band of frequencies arriving at the two hydrophones from a direction other than dead ahead undergoes a fixed time lag at one transducer relative to the other. The frequencies of the band are then shifted by SSI through the time T, where T is a different period for each frequency. Since the instant when SSI compensates in time varies with frequency, the different spectral components in the two hydrophones are not brought into precise phase coincidence with each other except when the signal arrives at both hydrophones in phase, or at a split transducer from dead ahead. For narrow-frequency bands, this effect is of no consequence, T being nearly constant over the band; response to bands off beam center is as good as on center except for the attenuation of the beam patterns. For wide-frequency bands, the SSI action has the effect of sharpening the beam pattern, relative to the effect of time compensation independent of frequency, by providing imperfect alignment of frequency components in the SSI output when the signal is off beam center. This action may be an advantage. If desired, more perfect alignment of frequency components from a signal off the axis may be achieved by causing the effective transducer spacing to vary with frequency.

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APPENDIX B
COHERENCE OF WATER NOISE

Nature of Interference

Water noise is composed of a multitude of wave trains arriving from all directions. Whether each wave train has the same spectral distribution cannot be determined by measurement, but it is highly probable that differences do occur between wave trains. However, the resultant of all the wave trains from any direction is noise having a spectral level which falls off 6db per octave increase in frequency. This noise is assumed converted to constant spectral level by filtering and is further assumed isotropic.

Coherence of Water Noise at Two Transducers

Since coherence is a measure of sameness, every wave train reaching a single transducer is coherent with itself, as is the sum of all wave trains. If, however, two transducers are gradually separated, the coherence between the noises picked up by the two is expected to vary and eventually vanish because:

(a) The phase relation between the signal picked up at the two transducers becomes a function of angle as well as frequency,

(b) at any instant, one transducer receives fewer of the same wave trains that the other transducer is receiving as a result of the limited length of wave trains,

(c) different parts of the same wave train are brought into simultaneity, thus requiring for coherence at the two transducers that there be coherence in a train which does not necessarily exist, and

(d) noise from a given direction travels to the two transducers over different paths which have less likelihood of being identical when more separated.

If the last three of the preceding causes for deterioration are ignored, less than actual deterioration of coherence is calculated as the spacing is increased. Perfect coherence is, nevertheless, assumed at any given frequency and direction as if all the wave trains from that direction and at that frequency were replaced by a single c-w signal and only the first of the four reasons for deterioration with spacing is considered. When deviation from this ideal case has been determined experimentally at some later date, it will be time to attempt explanation in terms of some of the last of these listed causes.

What happens on the preceding assumptions as two point transducers are separated is qualitatively as follows. At zero transducer spacing, each noise component* contributes to the SSI detector output in the same phase so that amplitudes of the contributions add. If the transducers are spaced a small fraction of a wavelength, say, 1/8 at the highest frequency, contributions at different frequencies and from different directions contribute to the SSI output in slightly different phases. Hence, when amplitudes are added vectorially (permissible since phase relations are known), the resultant is not quite so great as for zero

* A noise component is defined as all noise coming from a particular direction at a particular frequency.

spacing when all components are in phase. As the spacing is increased further, say to 1/2 wavelength, some of the contributions in the SSI output are 180° out of phase with each other, and the resultant response is greatly reduced or zero.

Coherent-Noise Processing

Let two point hydrophones be spaced equidistant from the origin of coordinates along the Y axis (Z axis is vertical). Let their separation from each other be h. It proves convenient to measure direction of noise arrival by an angle θ with the Y axis, since then all noise arriving along elements of a cone obtained by rotating the line at angle θ about the Y axis reaches the two transducers in the fixed-phase relationship

$$\phi_2 - \phi_1 = \frac{2\pi h \cos\theta}{\lambda} . \tag{201}$$

It is apparent that except when $\theta = 90°$, for which case $\phi_2 - \phi_1 = 0$, this phase relation depends on wavelength.

Let a narrow band of frequencies δf at f_j and a narrow range of directions $\delta\theta$ at θ_i be chosen, thereby fixing the frequency and the relative phase relationship at the two transducers. This restriction defines a noise component. Considering the solid angle, $2\pi \sin\theta_i \delta\theta$ between the cones in comparison with the total solid angle, 4π, this noise component introduces into each SSI channel a power

$$P' \delta f = \frac{P \sin\theta_i \delta\theta_i \delta f_j}{2} , \tag{202}$$

where P is the spectral level of noise from all directions together.

The amplitude associated with the power $P'\delta f$ is $\sqrt{2P'\delta f}$ or using Equation (202)

$$\text{Amplitude} = \sqrt{P \sin\theta_i \delta\theta \delta f} . \tag{203}$$

The instantaneous voltages derived from all such amplitudes in channels 1 and 2 respectively are

$$u_1 = \sum_{i=1}^{m} \sum_{j=-n}^{+n} \sqrt{P \sin\theta_i \delta\theta\delta f} \cos(2\pi f_j t + \phi_{ij}) \quad \text{and} \tag{204.1}$$

$$u_2 = \sum_{i=1}^{m} \sum_{j=-n}^{+n} \sqrt{P \sin\theta_i \delta\theta\delta f} \cos\left(2\pi f_j t + \phi_{ij} + \frac{2\pi h \cos\theta_i}{\lambda_j}\right). \tag{204.2}$$

Upon translation to intermediate frequency, there are obtained the voltages

$$v_1 = \sum_{i=1}^{m} \sum_{j=-n}^{+n} \sqrt{P \sin\theta_i \delta\theta\delta f} \cos\,[2\pi(g_1 + f_j)t + \alpha_1 + \phi_{ij}] \quad \text{and} \tag{205.1}$$

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$$v_2 = \sum_{i=1}^{m} \sum_{j=-n}^{+n} \sqrt{P \sin\theta_i\, \delta\theta\delta f}\, \cos\left[2\pi(g_2+f_j)t+\alpha_2+\phi_{ij}+\frac{2\pi h\cos\theta_i}{\lambda_j}\right] \quad (205.2)$$

In these equations, g_1 and g_2 are the frequencies and α_1 and α_2 are the epoch angles of the local oscillators in channels 1 and 2 respectively.

Upon interaction of v_1 with v_2 in a square-law detector, let the coherent terms first be extracted from the cross product obtained. These terms will occur only when the same i and j occur simultaneously in the two channels and will result in the output voltage at the difference frequency

$$w_c = P \sum_{i=1}^{m} \sum_{j=-n}^{+n} \sin\theta_i \cos\left[2\pi(g_2-g_1)t+\alpha_2-\alpha_1+\frac{2\pi h\cos\theta_i}{\lambda_j}\right]\delta\theta\delta f. \quad (206)$$

The limit is now taken, $\delta\theta$ being replaced by the infinitesmal $d\theta$. The summation with respect to i then becomes integration between $\theta = 0$ and $\theta = 180°$. Let the integration with respect to θ be carried out. There is obtained

$$w_c = \frac{P}{2\pi h} \sum_{j=-n}^{+n} \lambda_j \left\{\sin\left[2\pi(g_2-g_1)t+\alpha_2-\alpha_1+\frac{2\pi h}{\lambda_j}\right] \sin\left[2\pi(g_1-g_2)t+\alpha_2-\alpha_1-\frac{2\pi h}{\lambda_j}\right]\right\},$$

or

$$w_c = \frac{P}{\pi h}\cos\left[2\pi(g_2-g_1)t+\alpha_2-\alpha_1\right]\delta f \sum_{j=-n}^{+n} \lambda_j \sin\frac{2\pi h}{\lambda_j}. \quad (207)$$

General Discussion of the Result

A brief discussion from the physical point of view is in order. Every product entering Equation (206) possesses a phase differing from the reference phase of $2\pi h\cos\theta_i/\lambda_j$. As θ_i is varied from 0 to 180°, a negative value of phase angle exists for every positive value somewhere in this range. If equal positive and negative values are paired, the resultant of the two is in phase with the reference frequency (or 180° out). The summation or integration, therefore, can produce only a vector in phase with the reference frequency. In other words, the coherent part of the noise appears to come from the direction for which there is no phase difference at the two transducers (or 180° phase difference when $\sin 2\pi h/\lambda_j$ is negative). Equation (207) exhibits this physically understandable phase relation.

Comparing the amplitude of a term of Equation (207) with a term of Equation (106) when $k = 0$, reveals that the omnidirectional noise is equivalent for this case to a total noise of spectral level $(P\lambda_j/2\pi h)\sin 2\pi h/\lambda_j$ at each transducer, and in phase at the two transducers. (A negative $\sin 2\pi h/\lambda_j$ means that the output phase is shifted 180°.) This function is plotted against separation of hydrophones in Figure B1.

The summation of terms indicated by Equation (207) can be carried out by approximate methods for particular cases as they arise. For a narrow band, making λ_j substantially constant, δf may be replaced by W, the bandwidth, when the summation is performed. For a wider band,

$$\delta f \sum_{j-n}^{+n} \lambda_j \sin\frac{2\pi h}{\lambda_j}$$

may be replaced by the integral

$$\int_{f_1}^{f_2} \frac{c}{f} \sin \frac{2\pi hf}{c}\, df$$

and the integration may be performed, for example, graphically.

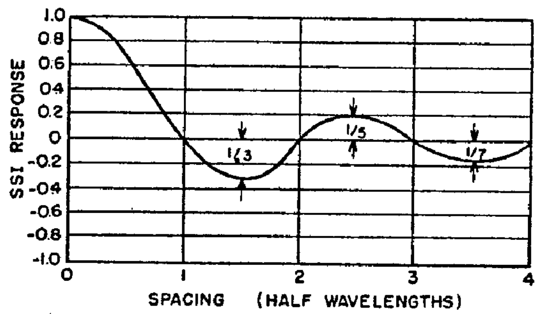

Figure B1 - Relative SSI response as a function of hydrophone spacing

The cross-product terms between different frequencies provide the band centered at the difference frequency. Since these terms have no coherence, the band is independent of the coherence at the inputs; only the central spike is dependent. Therefore, the noncoherent noise power is simply the total noise power minus the coherent noise power or

$$P\left(1 - \frac{\lambda_j}{2\pi h} \sin \frac{2\pi h}{\lambda_j}\right)$$

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