

NRL Report 5236

EFFECT UPON SHOCK SPECTRA OF THE DYNAMIC REACTION OF STRUCTURES

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December 16, 1958



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Washington, D.C.

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ABSTRACT

Attempts to use fiducial limit curves of a set of classes of shock spectra as a basis for the design of structures have shown that the design spectra obtained by the combinatorial analysis of many shock spectra tend to be overconservative. This interim report presents a possible explanation for this. It exhibits some experimental evidence to show that the values of interest in a shock spectrum plot tend to lie in the valleys of that plot and not upon the peaks, whereas fiducial limit curves are controlled by the peaks of the individual shock spectra.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing.

AUTHORIZATION

NRL Problem F03-04
Project NS 711-105

Manuscript submitted October 9, 1958

EFFECT UPON SHOCK SPECTRA OF THE DYNAMIC REACTION OF STRUCTURES

INTRODUCTION

An analysis of shock motions, in terms of their damage potential to single-degree-of-freedom mechanical systems, is obtained by observing the maximum response of each of these systems to the shock motions. A plot of these maximum responses as a function of the natural frequencies of the systems for a particular shock motion may be defined as a response spectrum. A sufficient number of systems are assumed to exist so as to cover the frequency range of interest. If there is no damping specified, it is assumed that the systems are undamped.

The response can be stated in terms of acceleration, velocity, or displacement; however for many applications the relative displacement response is most useful. As this displacement response may not be easily derivable from velocity or acceleration responses, particularly when damping terms are included, and as it is frequently desirable to express the spectrum in units of velocity or acceleration, these latter terms are arbitrarily expressed as $X\omega$ and $X\omega^2$ respectively. Displacement, velocity, or acceleration shock spectra for a given shock motion are therefore arbitrarily defined by curves representing X , $X\omega$, or $X\omega^2$ respectively as a function of frequency, where X is the maximum relative displacement response of a single-degree-of-freedom system to the given shock motion and ω represents the natural angular frequencies of the systems.

The shock spectra to be used with normal-mode theory of structural analysis are those which are valid for the foundation of the structure. Shock spectra may be obtained elsewhere, but these can be used only with difficulty.

The concept of shock spectra to aid in the description and analysis of mechanical shock has been used with varying degrees of success for many years (1-3). Some difficulty has been experienced by workers in this field in applying these spectra for design purposes because of the overconservative assumptions generally made when some sort of combinatorial analysis has been used to provide a spectrum representing many shocks to roughly similar equipments and foundations. This spectrum is frequently taken as the envelope of the maximum values of the individual spectra.

The major assumption currently employed in the use of shock spectra is that the spectra for the same classes of shock are unaffected by variations of the stiffness of a structure being subjected to shock as long as the weight of the structure remains reasonably close to that for which the experimental shock spectra were obtained. It is also generally assumed that over a wide range of possible shocks to these structures the severest condition to which a structure would ever be subjected is the envelope of the maxima of all possible shock spectra or some lesser value determined by the use of statistical fiducial limit curves (4). These assumptions usually define a shock spectrum which is so severe that few structures can withstand the shock described by the spectrum.

A popular type of combinatorial analysis is first to form classes of shock as to excitation and subdivide these classes into groups according to weight and location. These spectral classes then form loosely correlated groups from which it might be possible to define more specific sets of shock spectra. Often maxima of individual groups are plotted on separate graphs (fly-speck style) and fiducial limit curves drawn. It has been noted

in the past (4) that shock spectra obtained this way define a shock which is extremely severe (for, say, a 90-percent fiducial limit curve) and that few structures* could withstand such a shock. It is obvious that something is wrong with such a type of analysis because most structures which were in place during field trials for which shock spectra were obtained survived, whereas methods of design that made use of this shock spectra data indicated that they should have failed.

Attention was therefore focused upon this problem, and it was noted that structures on nonrigid foundations feed back forces into the foundation which affect this motion in such a fashion that the spectrum values of major interest for a shock tend to lie in the region of a valley rather than in the vicinity of a peak of the plotted spectrum (5). This has been called "shock spectrum dip," and experiments to verify its presence have been made, and are the subject for the major portion of this report.

A rough understanding as to why the valleys represent the most significant shock spectrum values can be obtained by noting that the valleys normally occur at frequencies corresponding to normal modes of vibration of the equipment under test. The equipment acts as a dynamic vibration absorber for shock motions having these frequency components and therefore causes shock spectrum dips at these frequencies. However, the responses of the equipment to the shock motion are small except at these frequencies. It is therefore inappropriate to use envelopes of maxima of shock spectra for design of heavy items and to apply the envelope values at normal-mode frequencies. To do so often results in strength and relative displacement requirements that are obviously absurdly great.

It is a secondary purpose of this report to show that, since high values have a great effect in controlling the position of fiducial limit curves, a type of analysis which gives equal weight to all points on the shock spectra will produce an overconservative result.

PLAN OF EXPERIMENT

This experiment was performed to study the effect of changes in structural parameters such as mass and stiffness upon shock spectra. Hence a model structure (Figs. 1 and 2) was designed and built such that it was possible to markedly change the stiffness of this structure without changing its mass, so spectra could be provided for the same structural weight, and also such that it was possible to change the mass while retaining the same stiffness, so the effects of varying this parameter could be obtained.

The experimental records obtained from velocity pickups were converted to velocity shock spectra. It was believed that sharp local downward variations in shock spectrum values would occur in the region of fixed-base† natural frequencies of the structure. It was the purpose of this experiment to expose the spectrum dips, to correlate them with fixed-base natural frequencies, and to show that the dips rather than the peak values of shock spectra are of major importance in design techniques using shock spectra.

DESCRIPTION OF EQUIPMENT

The structure was mounted on the Navy Medium-Weight High-Impact Shock Machine (6,7). The shock machine hammer was dropped from given heights so that the primary

*The degree of overdesign increases with effective weight of the equipment. For light component parts this technique is quite appropriate.

†Fixed-base natural frequencies are those that would exist if the foundation (Fig. 2) were infinitely stiff and heavy.

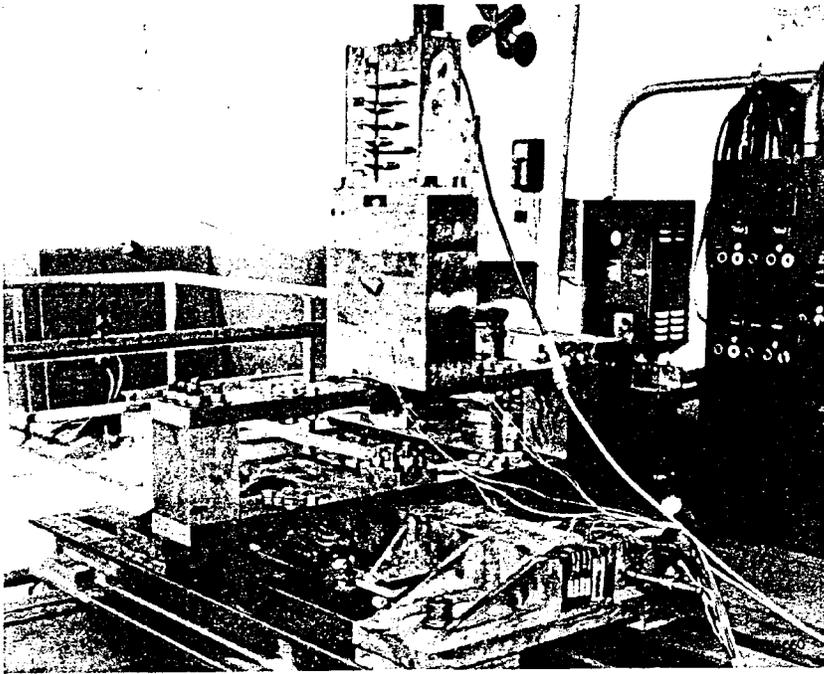


Fig. 1 - Model structure

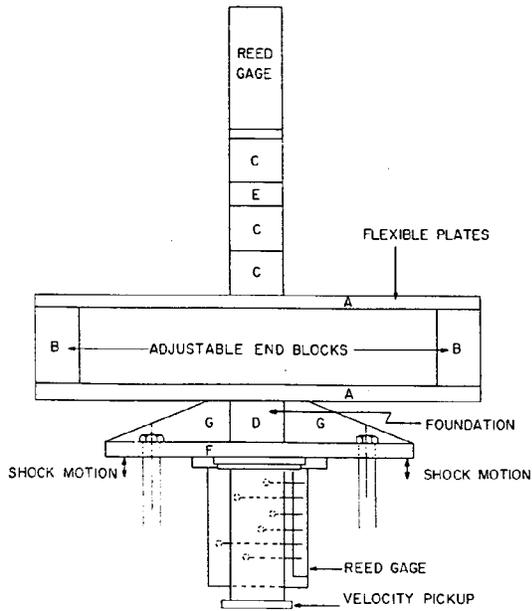


Fig. 2 - Approximate scale drawing of model structure

shock excitation given to the anvil was controlled. The resulting data were analyzed on the Narec (a digital computer). Both facilities are located at NRL. Besides the shock machine, the test equipment consisted of a model structure, velocity meters, oscilloscopes and camera recording equipment. The reed gages and strain gages visible in Fig. 1 were for purposes not connected with this report.

The model structure (Figs. 1 and 2) was mounted on a solid steel block, D, which was the foundation of the test structure. This block was in turn mounted on car building channels which were connected to the shock machine table. It was possible to change the "load" carried by the structure by using different combinations of parts C and E. It was also possible to change the "stiffness" of the structure by moving blocks B into an intermediate position between the ends of beams A and the center of the structure. Table 1 is a list of part sizes and weights for the frame. All the material was steel. There are some minor parts along with nuts, bolts, taper pins, etc., which are not delineated in this report.

Table 1
Major Structural Part Sizes and Purposes

Part	Size (in.)	Approx. Weight (lb)	Purpose
A	31 x 4 x 1/2 Plate	18	To give flexibility
B	5 x 4 x 3 Block	18	To control flexibility
C	9-1/2 x 4 x 3 Block	34	To control mass
D	31-1/2 x 4 x 3 Block	113	Foundation
E	9-1/2 x 4 x 1-1/2 Block	17	To control weight
F		38	Mounting pad
G		2	Gussett

The velocity-meter signal was presented on a cathode-ray oscilloscope and photographed by a moving-film type camera to give a trace of foundation velocity as a function of time.

DESCRIPTION OF EXPERIMENT

The shock machine was used on two occasions to create two sets of a series of twelve shocks; the excitation for each set was maintained at uniform intensity by dropping the hammer from the same height for each test. A series of light shocks was applied to the frame by dropping the hammer only 3 inches each time. A series of heavy shocks was applied to the frame by dropping the hammer 12 inches for each shock of a set. For each shock of each set the weight and stiffness of the test frame were changed so that a range of frame conditions, i.e., stiffness and mass, could be subjected to approximately the same hammer drop.

For the remainder of this report "Configuration I" refers to blocks B (Fig. 2) in the extended position, and "Configuration II" refers to these blocks in the intermediate or stiffer position (where the short bolts are shown on beams A in Fig. 1).

The weight series was varied by changing the "load" on top beam A as follows:

Weight 1 consisted of Block C_1 ,

Weight 2 consisted of Blocks C_1 and E_1 ,

Weight 3 consisted of Blocks C_1 and C_2 ,

Weight 4 consisted of Blocks C_1 , C_2 , and E_1 ,

Weight 5 consisted of Blocks C_1 , C_2 , and C_3 ,

Weight 6 consisted of Blocks C_1 , C_2 , C_3 , and E_1 .

In addition a reed gage (approximately 18 lb) was on top of the stack of weights for each blow. The weights of C_1 , C_2 , and C_3 are each equal to C , and E_1 is approximately one-half the weight of C . In the description of these tests it is important to note that "Configuration I, Weight 2" and "Configuration II, Weight 2," for example, both have the same mass, but different stiffnesses.

The total change in weight from Weight 1 to Weight 6 only amounted to approximately 85 lb. The frame and foundation (including velocity meters and reed gages) had a minimum weight of about 440 lb and therefore a maximum weight of about 525 lb. The total weight including the shock machine table, side rails, car building channels, and frame, was in excess of 5000 lb.

The shock was transmitted to the complete structure by means of the hammer impact upon the anvil (Fig. 3).

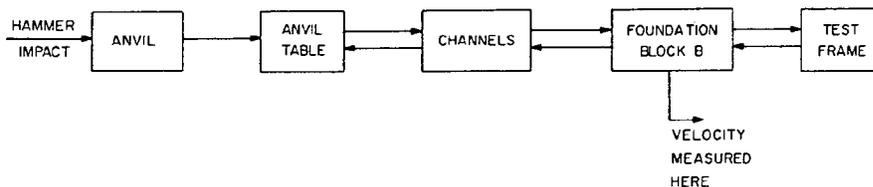


Fig. 3 - Schematic of shock transfer system

ANALYSIS OF MODEL STRUCTURE

The structure was analyzed, with the aid of the Narec, by the method of influence coefficients and matrix iteration (8,9) for the first four mode shapes and natural frequencies of each of the twelve configuration and weight combinations. Figure 4 shows a schematic representation of the lumped mass system which was used to approximate the real system. By taking advantage of the symmetry, $M_1 = M_2$, $M_3 = M_7$, $M_4 = M_6$, it was only necessary to calculate 16 influence coefficients for either position.

The response analysis (3,5,10) showed that the participation factors were zero for the symmetric modes II and IV in all cases, so that they could not be excited by the type of motion assumed for this experiment. It was decided that the natural frequencies of the fundamental modes for the frame with all weights and positions should be found experimentally. The system foundation was welded to a steel bar buried in a concrete block in

an effort to secure as rigid a foundation as possible, and the natural frequency of each configuration and weight combination was found by examination of the record of the response to a light mallet blow.

The effective mass acting in each mode was calculated (5,11), and it was noted that if effective mass in a mode was the cause of the shock spectrum dip, then perhaps only mode I would show it, because in all cases over 92 percent of the mass was acting in that mode, whereas in the next mode only 6 percent of the mass was effective. Table 2 is a brief summary of the analysis results.

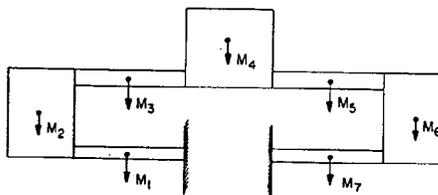


Fig. 4 - Schematic of lumped mass system

Table 2
Summary of Mode Analysis Results

Configuration and Weight Combination	Mode I			Mode III	
	Nat. Freq., Test (cps)	Nat. Freq., Calculated (cps)	Mass in Mode (percent)	Nat. Freq., Calculated (cps)	Mass in Mode (percent)
Configuration I					
Weight 1	83	84	92.1	242	5.6
Weight 2	77	78	92.0	235	5.9
Weight 3	72	73	92.1	230	6.0
Weight 4	66	69	92.3	227	5.9
Weight 5	63	65	92.6	224	5.8
Weight 6	60	62	92.8	223	5.6
Configuration II					
Weight 1	186*	151	92.9	421	5.7
Weight 2	177*	143	92.6	409	6.1
Weight 3	164*	134	92.6	401	6.2
Weight 4	153*	125	92.8	395	6.1
Weight 5	142*	119	92.9	391	6.1
Weight 6	134*	113	93.2	389	5.9

* In the test records for the 1st natural frequency it became apparent that Configuration II was extremely sensitive to vibration in the other orthogonal directions and a torsional mode seemed to appear. Great difficulty was encountered in getting these values, and little claim is made to any great degree of precision. It was also felt that if the shock machine anvil table rotated slightly during the shock motion, then these modes would become excited, thereby altering the spectrum. It is also to be noted that the finite size of the separating blocks B caused larger errors to appear in the computation of the modal parameters for Configuration II than for Configuration I.

EXPERIMENTAL RESULTS OF 3-IN. BLOW

Spectrum Values of Interest

If the purpose of each of the calculated shock spectra (Figs. 5 through 10) was to find an approximation to the stress at any location, then in each case there would have been only two values of interest on each spectra. This is because the structure was essentially a system with two degrees of freedom (see Table 2). The two values of interest would be the fixed-base natural frequencies of the structure. These spectrum values (indicated by dot marks on each of the spectra, Figs. 5 through 12) are compiled and presented in Table 3. It is seen that these spectrum values vary from a maximum of 3.48 ft/sec down to 1.00 ft/sec and tend to fall off in magnitude as the frequency increases.

Table 3
Shock Spectrum Values

Configuration and Weight Combination	Mode I		Mode III	
	Nat. Freq. (cps)	Spectrum Value (ft/sec)	Nat. Freq. (cps)	Spectrum Value (ft/sec)
Configuration I				
Weight 1	83	3.48	242	2.60
Weight 2	77	3.43	235	2.83
Weight 3	72	2.71	230	2.97
Weight 4	66	2.63	227	2.62
Weight 5	63	2.41	224	3.13
Weight 6	60	2.50	223	2.91
Configuration II				
Weight 1	186	2.47	421	1.00
Weight 2	177	2.37	409	2.64
Weight 3	164	2.51	401	1.65
Weight 4	153	2.24	395	1.96
Weight 5	142	2.16	391	1.41
Weight 6	134	2.22	389	1.33

Explanation of Shock Spectrum Graphs

In all the spectra of this report the values of the ordinates are in feet per second, and the abscissa have scales of cycles per second. The symbols are as follows: f_{11} is the 1st fixed-base natural frequency for Configuration I, f_{31} is the 3rd fixed-base natural frequency for Configuration I, f_{12} is the 1st fixed-base natural frequency for Configuration II, f_{32} is the 3rd fixed-base natural frequency for Configuration II, and the dots at these frequencies represent the spectrum values as given in Table 3.

Recall that Configurations I and II are identical in mass for a given weight number but differ in stiffness, and this is the major reason for the differences in the respective shock spectra. The assumption that stiffness need not be considered in predicting shock spectra can lead to grave inaccuracy, as the following example illustrates. Suppose the shock spectrum of Configuration I in Fig. 8 were taken as the basis for prediction of the shock spectrum of a structure having the same mass but a greater stiffness, stiffness causing the frequency of Mode I to be 153 cps, and suppose that this greater stiffness was considered unimportant to such a prediction. Then it would be predicted that the shock

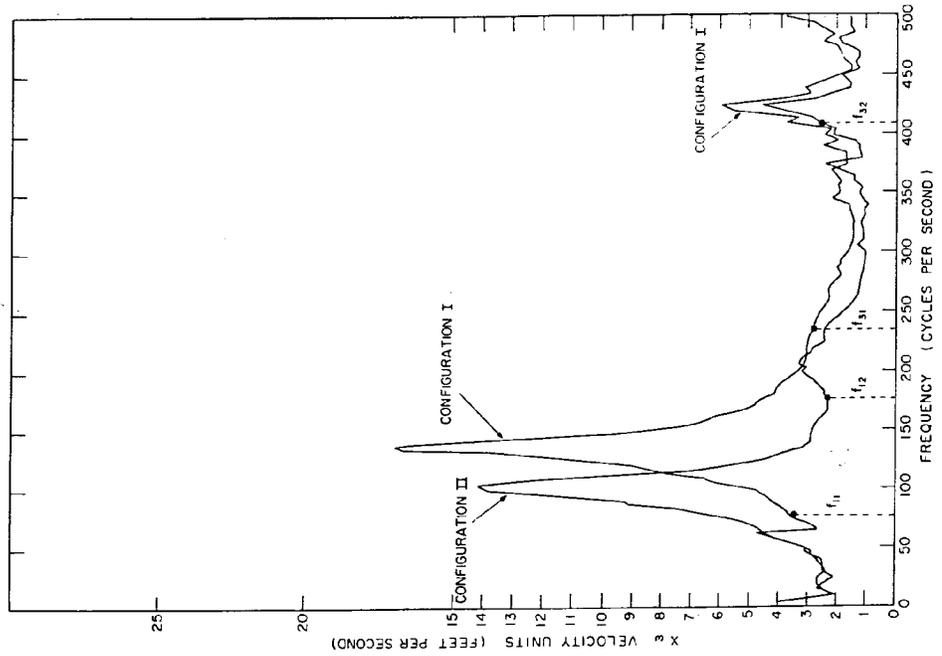


Fig. 6 - Shock spectra of Configurations I and II, Weight 2, 3-in. blow

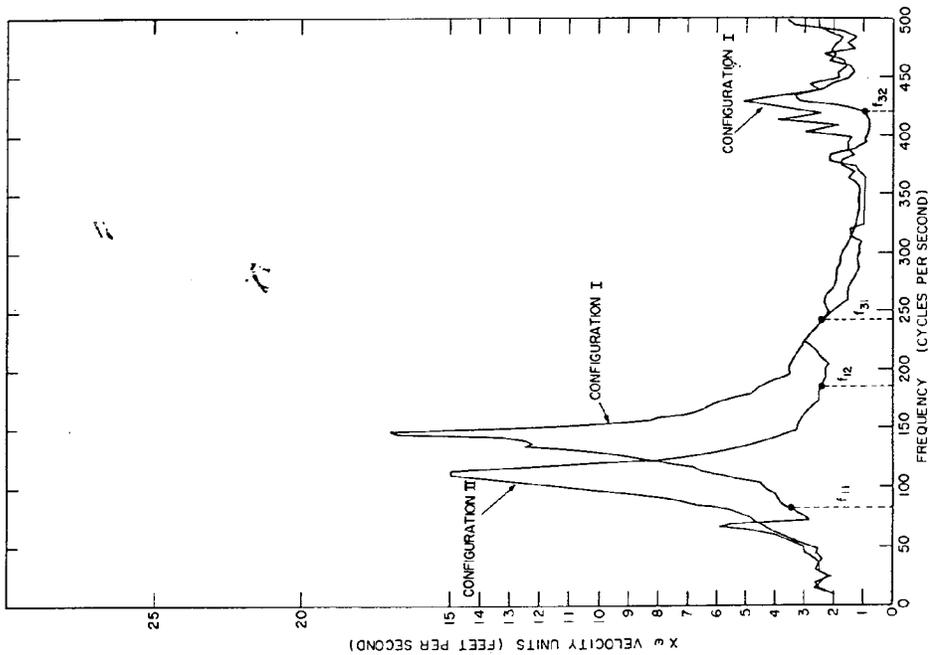


Fig. 5 - Shock spectra of Configurations I and II, Weight 1, 3-in. blow

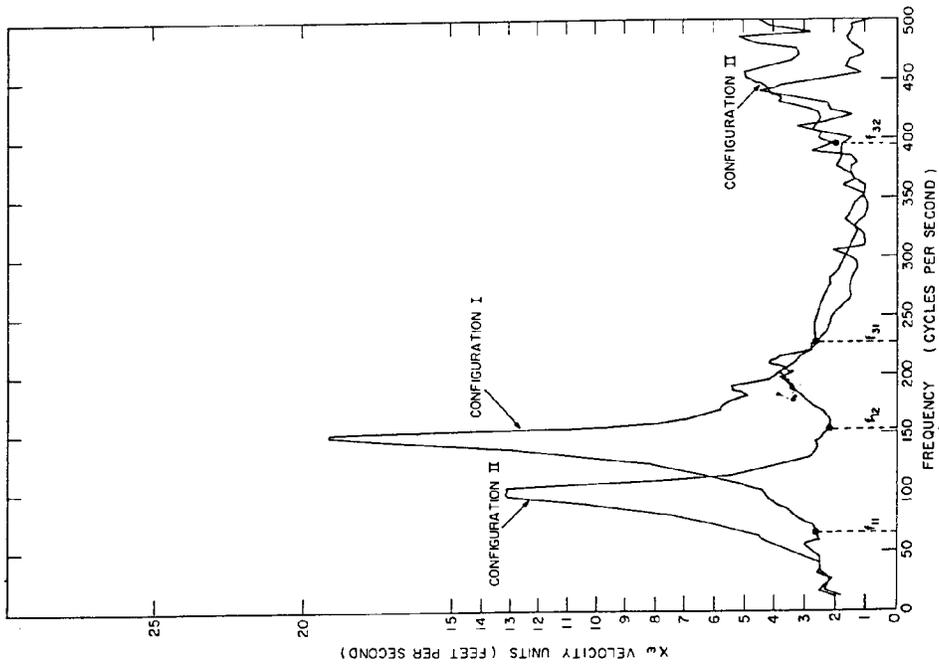


Fig. 8 - Shock spectra of Configurations I and II, Weight 4, 3-in. blow

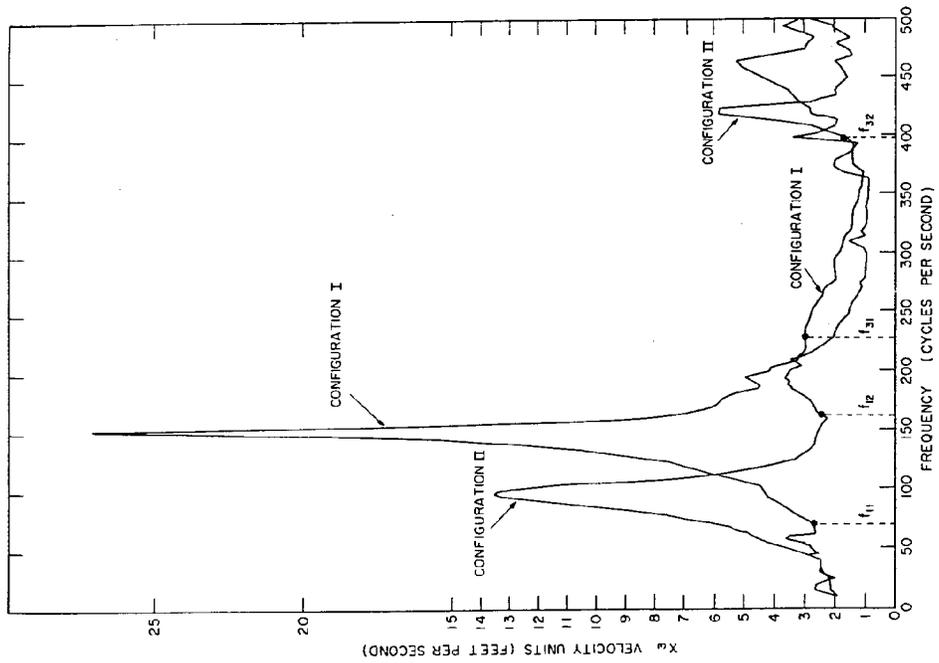


Fig. 7 - Shock spectra of Configurations I and II, Weight 3, 3-in. blow

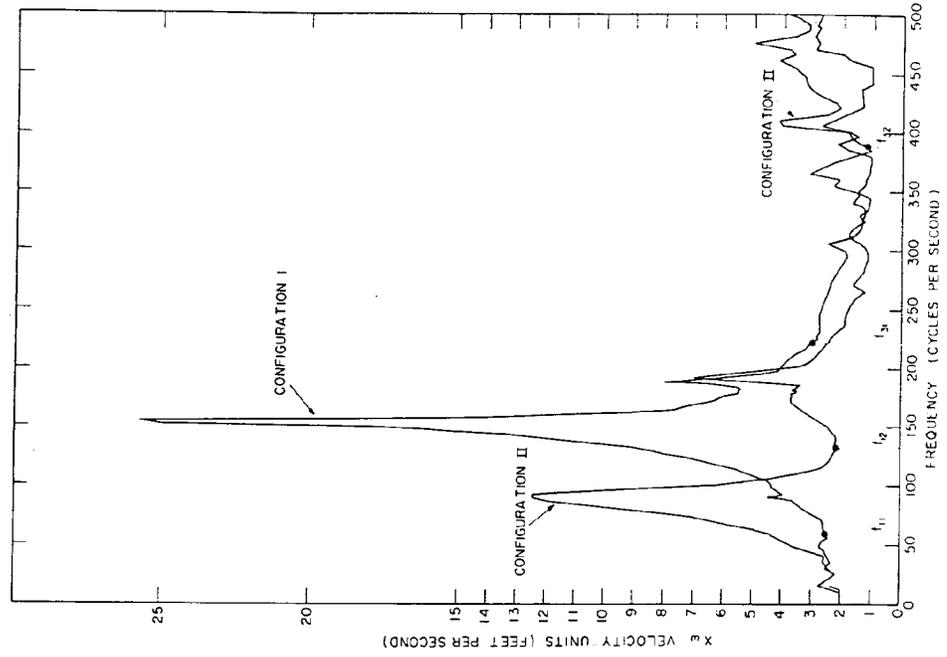


Fig. 10 - Shock spectra of Configurations I and II, Weight 6, 3-in. blow

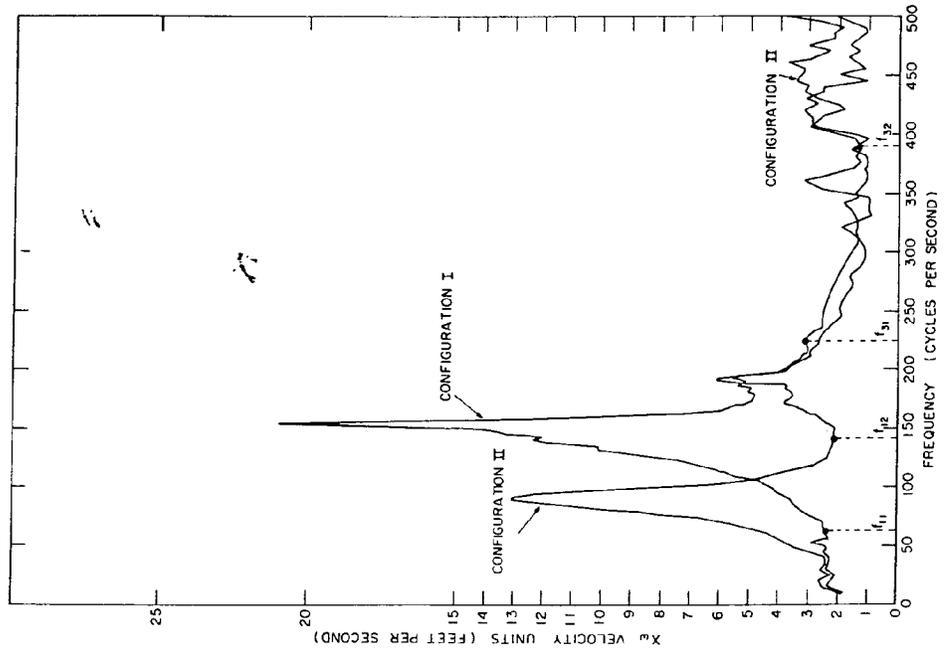


Fig. 9 - Shock spectra of Configurations I and II, Weight 5, 3-in. blow

spectrum of the stiffer system should have an ordinate of 15.5 ft/sec at 153 cps. The experimental shock spectrum for Configuration II provides the test for the correctness of this prediction. It has an ordinate of 2.2 ft/sec instead of the predicted 15.5 ft/sec at 153 cps. A structural member of Configuration II made according to the prediction would have been oversized by the factor of $15.5/2.2 = 7$.

Again, if the shock spectrum of Configuration II had been taken as the basis for design and prediction of the shock spectrum of a less stiff assembly, specifically one with 66 cps as f_{11} , and if the stiffness difference had been assumed unimportant as above, the predicted ordinate would have been 4.7 ft/sec as compared with the observed 2.6 ft/sec at 66 cps, giving an overdesign factor of 1.8.

If each figure is examined in this fashion, it will be noted that this effect of overdesign is present for any system having a different fundamental frequency than that of the system present when the shock spectrum data were obtained. It will also be noted that the spectrum values of interest, i.e., those for frequencies corresponding to fixed-base frequencies of the system tested, tend to lie in valleys and not on peaks of the shock spectra.

Figures 11 and 12 are composites of all shock spectra for Configuration I and for Configuration II respectively. The dots represent the velocity shock spectrum values of interest in stress calculations. Qualitatively speaking it is possible to say that these values of interest did not occur on large peaks.

Points of interest on Fig. 11 include:

1. The range of velocity spectrum values of major interest for the fundamental natural frequency was between 60 and 83 cps.
2. There is a set of large peaks of velocity in the region 125 to 190 cps which are 6 or 7 times higher than most of the shock spectrum values of interest.
3. The spectra had the same general shape but tended to be displaced frequencywise.

Points of interest on Fig. 12 include:

1. All spectrum values corresponding to the first fixed-base natural frequency of the frame were between 134 and 186 cps, and the spectrum values for the other natural frequency were between 389 and 421 cps.
2. There are two sets of individual peaks, one between 50 and 150 cps and one between 175 and 225 cps.
3. The spectra have the same general shape but tend to be displaced frequencywise.

Figure 13 is a representation of a type of analysis of these spectra which might be made. The upper curve is the envelope of the maximum values of all the shock spectra for this "class of equipment," and the lower line is the envelope of the minimum values. It can be easily seen that the shock spectrum values of interest tend to lie nearer the minimum line than the maximum one, so a combinatorial analysis that takes the peaks into account would produce spectra which are too severe.

EXPERIMENTAL RESULTS OF 12-IN. BLOW

The results of the 12-in. blow were plotted in the same fashion as for the 3-in. blow, but only the composites are shown (Figs. 14-16). The results do not clearly illustrate shock spectrum dips, especially for Configuration II. However, they do point out quite clearly that the large set of peaks in Configuration I was replaced by a deep valley in Configuration II when the fixed-base natural frequency came into that region.

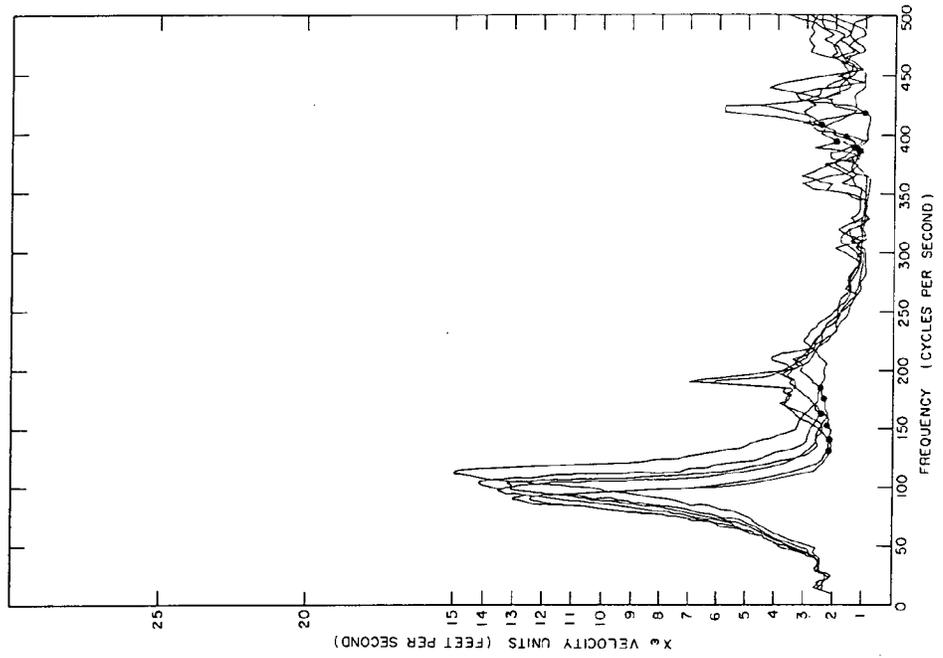


Fig. 12 - Composite shock spectra of Configuration II, 3-in. blow

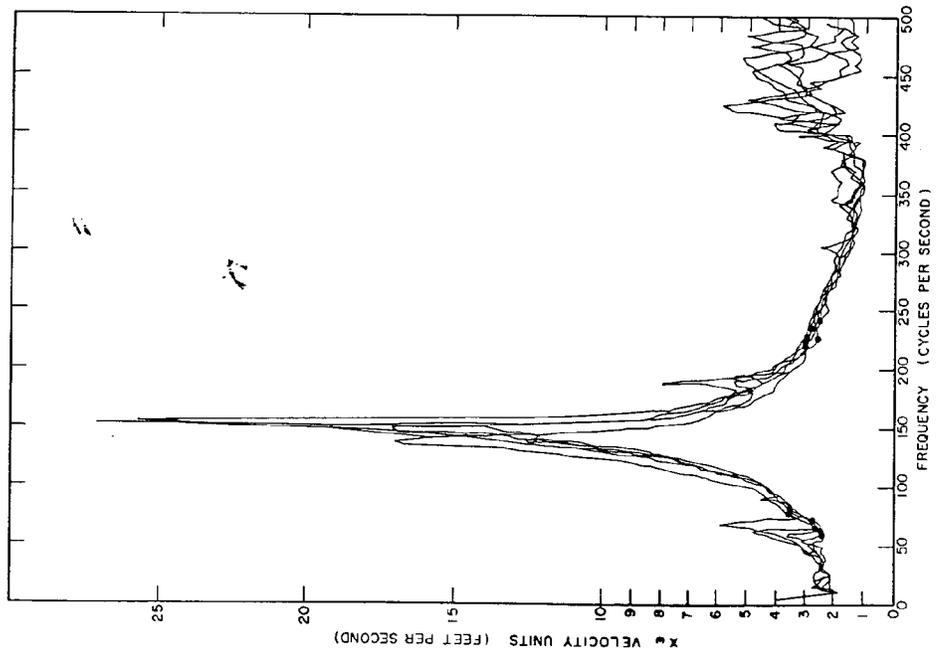


Fig. 11 - Composite shock spectra of Configuration I, 3-in. blow



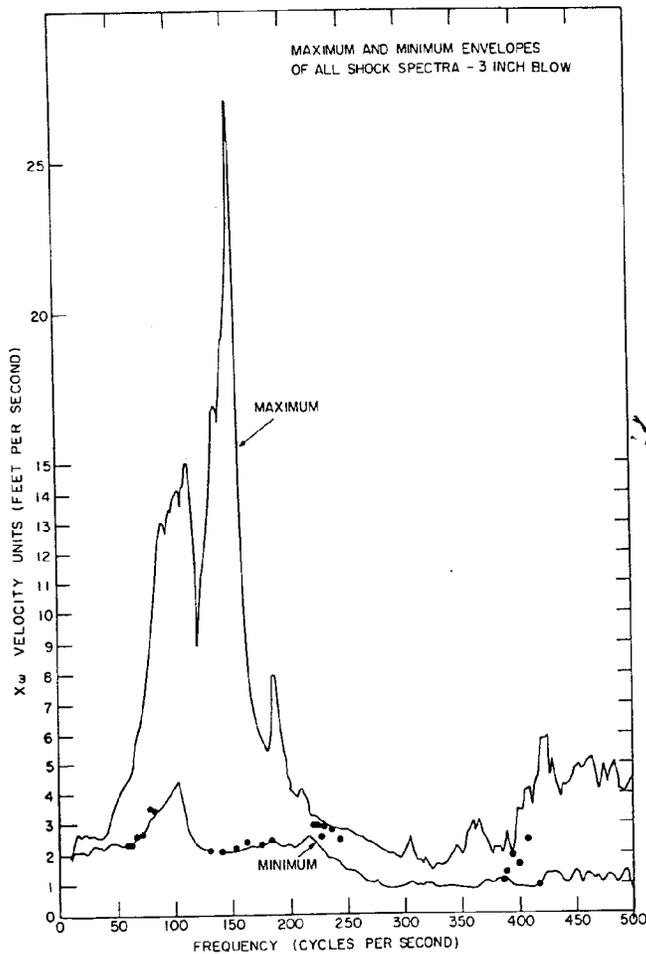


Fig. 13 - Maximum and minimum envelopes of all shock spectra, 3-in. blow

There are many reasons why the heavy blow did not give the anticipated results as clearly as did the lighter blow. Some are enumerated below:

1. The system was nonlinear and the amplitudes became large enough during the heavy blow to affect the results.
2. The shock table rotated during the test, undoubtedly exciting other types of motion than were considered in the frame analysis. This rotation also caused the anvil table to suffer three distinct blows (the hammer blow and stops on first one side and then the other).
3. Maximum stresses were in the order of the yield point, which introduced more damping into the structure.

Figure 14 again points out that, even with the spectrum values for f_{12} in doubt, an envelope of maximum values creates a false impression of the severity of the shock.

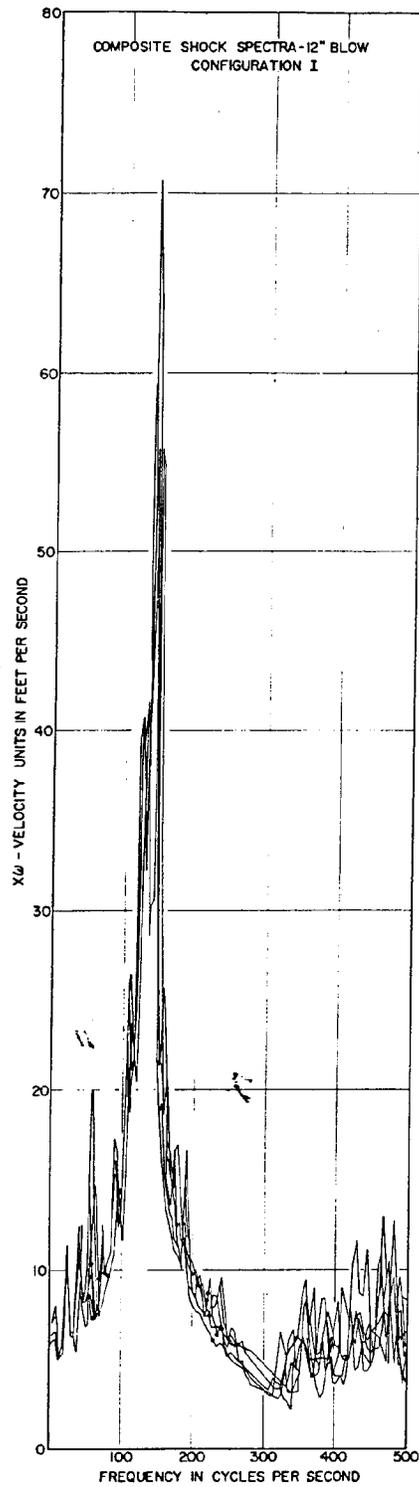


Fig. 14 - Composite shock spectra of Configuration I, 12-in. blow

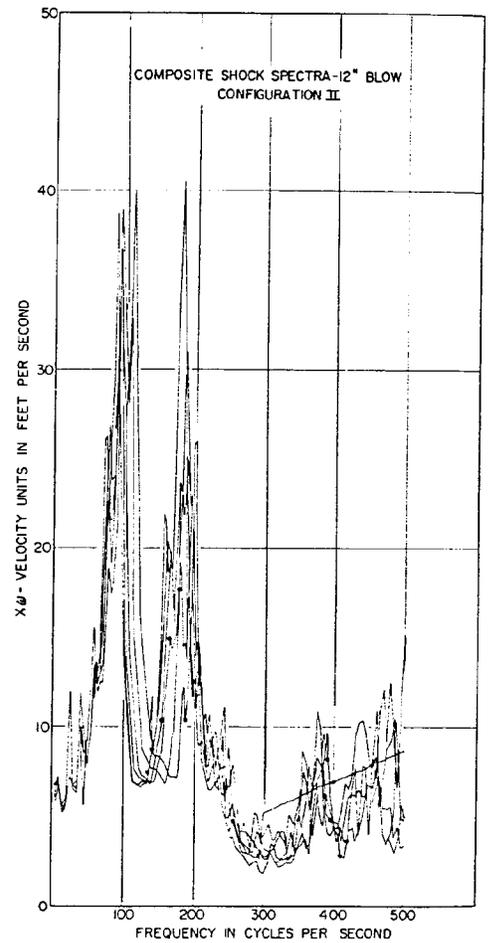


Fig. 15 - Composite shock spectra of Configuration II, 12-in. blow

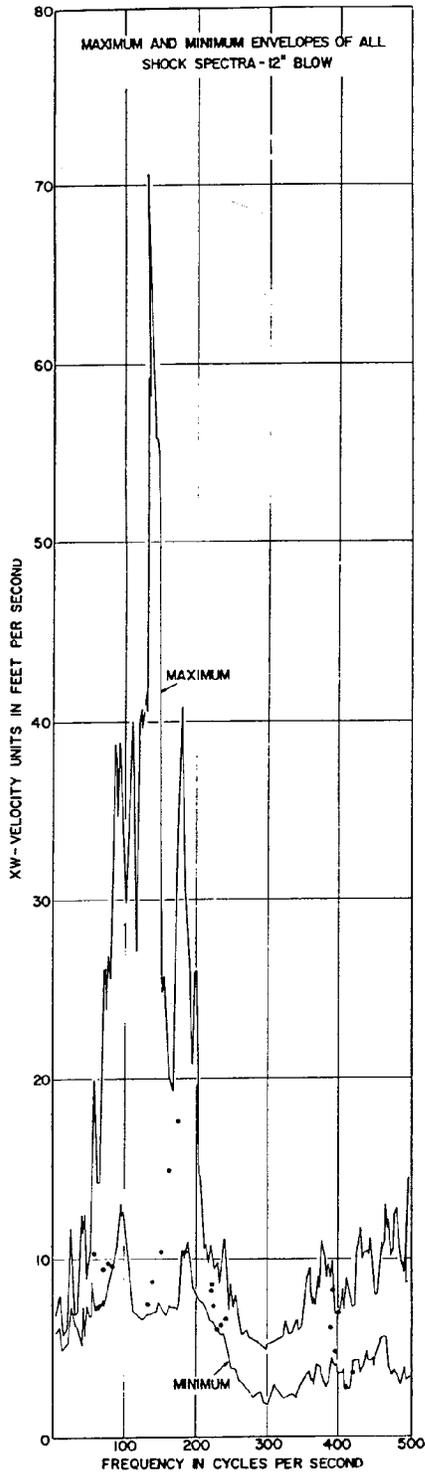


Fig. 16 - Maximum and minimum envelopes of all shock spectra, 12-in. blow

DISCUSSION

It has been demonstrated in this series of experimental tests that even for the same shock-generating system and weight of test structure, the shock spectra obtained are extremely sensitive to the frequencies of the test structure being considered. The shock spectrum peaks are undoubtedly caused by the natural frequencies of the system as a whole (11), and these do not correspond to the fixed-base natural frequencies of the structure for which the spectra represent the foundation motion effect.

Figures 17 and 18 represent two of the typical foundation velocity records obtained during the test. The top record in each case is the one in question. As can be seen after the initial impulse there is a type of motion which suggests that natural frequencies could be obtained from the records. It must be remembered that these were obtained while the whole system (anvil, table, channels, model, etc.) was in motion, and will therefore show the coupled natural frequencies. Hence, the observed frequencies will not in general correspond to the fixed-base natural frequencies of the model structure. If $\dot{Z}(T)$ is the velocity-meter record, the Duhamel integral

$$X_{\omega} = -\omega \int_0^t \dot{Z}(T) \cos \omega [t - T] dT$$

will have large values at those frequencies which coincide with the frequencies present in $\dot{Z}(T)$. Table 4 presents a comparison of the predicted frequencies where these peaks would occur and where they actually did occur.

It has also been shown that valleys tend to appear in the region of spectrum locations for fixed-base natural frequencies, and that these valleys rather than the peaks tend to control the stresses in a structure. Analysis of a large group of spectra wherein the peaks are statistically combined with the rest of the data as a point of equal weight or more, would give a false impression of the severity of a set of shocks. That is, when fiducial limit curves are computed using values regardless of whether they correspond to the tested structure or not, then the high peaks practically determine such a fiducial limit curve, and therefore the resulting curve is not at all representative of the spectrum values useful in design.

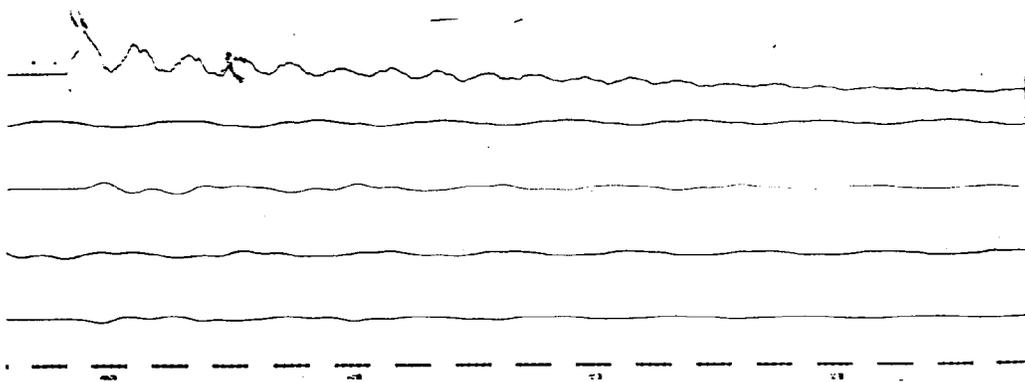


Fig. 17 - Velocity-meter record - Configuration I

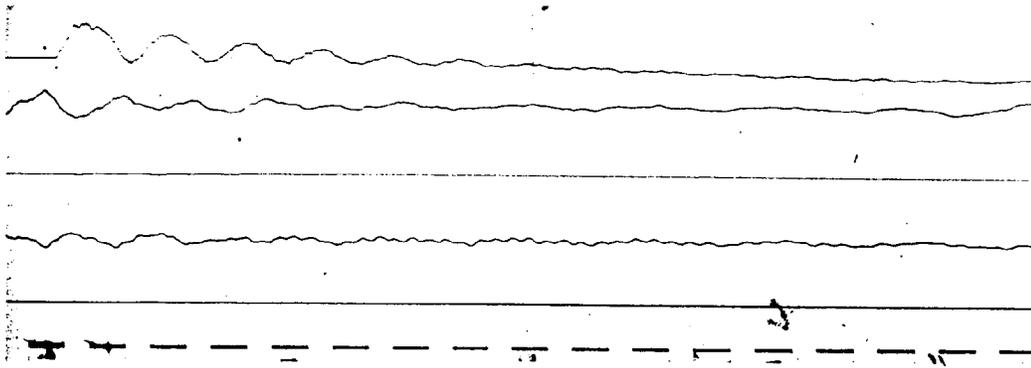


Fig. 18 - Velocity-meter record - Configuration II

Table 4
Comparison of Recorded and Predicted Peaks
of Shock Spectra for 3-in. Blow

Configuration and Weight Combination	Frequency of Major Component in Velocity Record (cps)	Peak Frequency in Shock Spectrum (cps)
Configuration I		
Weight 1	136	147
Weight 2	140	138
Weight 3	143	152
Weight 4	143	147
Weight 5	140	154
Weight 6	143	154
Configuration II		
Weight 1	103	112
Weight 2	100	104
Weight 3	95	97
Weight 4	97	102
Weight 5	88	92
Weight 6	87	91

In the analysis of the data of this experiment not much has been said of the upper frequency range, but attention has been focused upon the 1st fixed-base natural frequency of the frame in each instance. The reasons for doing this are the following:

1. Only about 6 percent of the available mass was in mode III, making it relatively ineffective in reducing the spectra level.
2. The size of time increment chosen for the numerical integration technique and the appearance of Figs. 11 and 12 seem to indicate that perhaps only spectrum values below 350 or 375 cps are reliable.
3. It is difficult to calculate natural frequencies in the upper frequency range with the precision required for discerning a valley. For example ± 5 -percent error at 40 cps is only ± 2 cps, for a bandwidth of possible values of 4 cps for that frequency, while ± 5 per cent of 400 cps is ± 20 cps, for a bandwidth of possible values of 40 cps.
4. The system was slightly nonlinear for large amplitudes.

CONCLUSIONS

1. The potentially extreme overconservatism in design resulting from incorrect usage of shock spectra was verified by a series of controlled shock experiments in the laboratory.
2. The distinction between the shock-spectrum values at fixed-base natural frequencies of equipment and at natural frequencies of the complete equipment-foundation system was demonstrated.

Work is continuing on this problem and a report to be published will present some mathematical explanation for the phenomena as well as the results of another set of experiments which have been planned using the knowledge gained from these tests.

ACKNOWLEDGMENT

The author would like to express his appreciation to Mr. John Bachman of NRL who aided in all phases of the experiment by instrumentation, data reduction, and plotting.

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