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The Calculation of Frequency-Modulation Distortion in Linear Networks

O. D. SLEDGE

Radar Division

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U.S. NAVAL RESEARCH LABORATORY
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ABSTRACT

A numerical method for calculating the steady-state FM distortion in a linear passive network using a sideband-type of analysis is developed to achieve economy of digital computer time and, for a given network, to find the number of significant sideband pairs associated with a given maximum allowable level of FM distortion.

To illustrate the application of the method, the FM distortion introduced by a bandpass filter is calculated over a range of signal parameters. A useful byproduct of the FM distortion calculations is a formula for the minimum ratio of the bandwidth of the filter to twice the frequency deviation of the input signal necessary to assure that the FM distortion introduced by a bandpass filter is less than 5 percent.

PROBLEM STATUS

This is a final report on one aspect of the problem. Work on the problem is continuing.

AUTHORIZATION

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THE CALCULATION OF FREQUENCY-MODULATION DISTORTION IN LINEAR NETWORKS

INTRODUCTION

It is well known that the nonflatness of the amplitude response and the nonlinearity of the phase response of a linear network cause nonlinear modulation distortion of a pure frequency-modulated (FM) wave which is transmitted through the network.

After a review of the literature on FM distortion in linear networks (for example, Refs. 1-3), it is concluded that there is a need for a numerical method of calculating FM distortion which is valid for all levels of distortion and is well adapted to economical use with a digital computer. A numerical method of the calculation of FM distortion in linear passive networks which meets this need is presented in this report.

BASIC IDEAS AND DEFINITIONS

Consider the linear, passive, four-terminal network of Fig. 1. Let the input voltage E_1 be the pure FM wave

$$E_1(t) = \sin \alpha = \sin (\omega_0 t + m \sin pt) \tag{1}$$

where ω_0 is the angular frequency of the unmodulated carrier, p is the angular frequency of modulation, and m is the deviation ratio. The instantaneous angular frequency of the FM wave is defined as

$$\omega_i = \frac{d\alpha}{dt} = \omega_0 + mp \cos pt. \tag{2}$$

The input wave may be represented by the Fourier series.

$$E_1(t) = \sum_{n=-\infty}^{\infty} J_n(m) \sin (\omega_0 + np)t \tag{3}$$

where $J_n(m)$ is the nth order Bessel function of the first kind. Equation (3) is a time representation of the familiar spectrum of an FM wave composed of a carrier and an infinite number of symmetrical sidebands spaced at intervals equal to the modulating angular frequency p. The carrier amplitude is $J_0(m)$ and the amplitude of the nth sideband is $J_n(m)$.

Because of the linearity of the network the superposition theorem may be applied to give the steady-state output voltage of the network as

$$E_2(t) = \sum_{n=-\infty}^{\infty} G(\omega_0 + np) J_n(m) \sin \left[(\omega_0 + np)t - \phi(\omega_0 + np)\right] \tag{4}$$

where $G(\omega)$ is the amplitude response and $\phi(\omega)$ is the phase retardation of the network.

Fig. 1 - Linear four-terminal network

For convenience let $G(\omega_0 + np) \equiv G_n$, $\phi(\omega_0 + np) \equiv \phi_n$, and $J_n(m) \equiv J_n$; then Eq. (4) becomes

$$E_2 = \sum_{n=-\infty}^{\infty} G_n J_n \left[\cos(npt - \phi_n) \sin \omega_0 t + \sin(npt - \phi_n) \cos \omega_0 t\right]. \tag{5}$$

The output voltage may be expressed in the form

$$E_2 = M(t) \sin\left[\omega_0 t + \psi(t)\right] \tag{6}$$

where $M(t)$ is an amplitude function and $\psi(t)$ is a phase function. From Eqs. (5) and (6), it follows that

$$M(t) = \left[N^2 + D^2\right]^{1/2}$$

and

$$\tan \psi = \frac{N}{D} \tag{7}$$

where

$$N = \sum_{n=-\infty}^{\infty} G_n J_n \sin(npt - \phi_n)$$

$$D = \sum_{n=-\infty}^{\infty} G_n J_n \cos(npt - \phi_n).$$

From Eq. (7) we obtain the variational part of the instantaneous angular frequency of E_2, namely,

$$\frac{d\psi}{dt} = \frac{DN' - ND'}{N^2 + D^2} \tag{8}$$

where

$$N' = \frac{dN}{dt} = p \sum_{n=-\infty}^{\infty} nG_n J_n \cos(npt - \phi_n)$$

$$D' = \frac{dD}{dt} = -p \sum_{n=-\infty}^{\infty} nG_n J_n \sin(npt - \phi_n).$$

If the output of the network is passed through an ideal limiter and then through an ideal, linear FM demodulator, the output of the demodulator is linearly proportional to

$d\psi/dt$. In general, the variational part of the instantaneous angular frequency of the output of the network is composed of a fundamental component at the modulation frequency as well as harmonic components. The latter constitute the nonlinear FM distortion introduced by the network.

Performing the indicated products of the series in Eq. (8) yields

$$\frac{d\psi}{dt} = \frac{p \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} G_k J_k n G_n J_n \cos\left[(n-k)pt - \Phi_n + \Phi_k\right]}{\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} G_k J_k G_n J_n \cos\left[(n-k)pt - \Phi_n + \Phi_k\right]} \tag{9}$$

where $\Phi_n = (\phi_n - \phi_0)$ is the phase shift relative to the carrier of the nth sideband.

It is observed from Eq. (9) that the FM distortion introduced by the network is a function of m, the deviation ratio of the input FM wave, and of the phase and amplitude responses of the network relative to their respective values at the carrier frequency. At this point it would seem logical and straightforward to proceed to find the FM distortion products by a numerical Fourier analysis of the expression for the variational part of the instantaneous angular frequency appearing in Eq. (9); however, such a procedure is not economical in terms of computation labor.

A METHOD OF CALCULATION OF FM DISTORTION

In what follows, the network is assumed to possess symmetrical transfer characteristics; namely, the amplitude and phase responses of the network exhibit even and odd symmetry respectively about the carrier frequency; then it follows that

$$\Phi_{-n} = -\Phi_n \tag{10}$$

$$G_{-n} = G_n. \tag{11}$$

For convenience, define

$$C_n \equiv G_n J_n. \tag{12}$$

Then from Eq. (11) and the properties of $J_n(m)$ we have

$$C_{-n} = C_n \tag{13}$$

when n is a positive even interger and

$$C_{-n} = -C_n \tag{14}$$

when n is a positive odd integer.

From Eqs. (8) and (9),

$$DN' - ND' = p \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} n C_n C_k \cos\left[(n-k)pt - \Phi_n + \Phi_k\right]. \tag{15}$$

Employing the symmetry condition of Eq. (10), the sum of the terms of Eq. (15) which constitute the rth harmonic of $(DN' - ND')$ can be shown to be

$$\text{rth harmonic of } (DN' - ND') = p \sum_{n=1}^{\infty} n \Big[(C_{n-r} C_n - C_{-n+r} C_{-n}) \cos (rpt - \Phi_n + \Phi_{n-r})$$

$$+ (C_{n+r} C_n - C_{-n-r} C_{-n}) \cos (rpt - \Phi_{n+r} + \Phi_n) \Big]. \tag{16}$$

Upon the introduction of the symmetry conditions of Eqs. (13) and (14) into (16) the right member vanishes for r an even integer; but for r an odd integer Eq. (16) becomes

$$\text{rth harmonic of } [DN' - ND'] = 2p \sum_{n=1}^{\infty} nC_n \Big[C_{n+r} \cos (rpt - \Phi_{n+r} + \Phi_n)$$

$$+ C_{n-r} \cos (rpt - \Phi_n + \Phi_{n-r}) \Big]. \tag{17}$$

Equation (17) may be written in the simpler form

$$(2k - 1)\text{th harmonic of } [DN' - ND']$$

$$= 2p \sum_{n=0}^{\infty} (2n + 1) C_{n+k} C_{n-k+1} \cos \Big[(2k - 1)pt - \Phi_{n+k} + \Phi_{n-k+1} \Big] \tag{18}$$

where k is a positive integer.

In a similar manner we find that the expression for $(N^2 + D^2)$ of Eq. (8) consists only of even harmonics of the modulation frequency. In the light of the foregoing information, we conclude from an examination of Eq. (8) that the instantaneous frequency of the output of a network with the above-described symmetrical transfer properties will contain only odd harmonics of the modulation frequency.

The Fourier series representations of the expressions for $(N^2 + D^2)$ and $(DN' - ND')$ appearing in Eq. (8) are

$$N^2 + D^2 = P_0 + \sum_{k=1}^{\infty} P_{2k} \cos 2kpt + \sum_{k=1}^{\infty} Q_{2k} \sin 2kpt \tag{19}$$

$$DN' - ND' = \sum_{k=1}^{\infty} R_{2k-1} \cos (2k - 1)pt + \sum_{k=1}^{\infty} S_{2k-1} \sin (2k - 1)pt \tag{20}$$

where

$$P_0 = C_0^2 + 2 \sum_{i=1}^{\infty} C_i^2 \tag{21a}$$

$$P_{2k} = 2C_kC_{-k} \cos 2\Phi_k + 4 \sum_{i=1}^{\infty} C_{i+k}C_{i-k} \cos (\Phi_{i+k} - \Phi_{i-k}) \tag{21b}$$

$$Q_{2k} = 2C_kC_{-k} \sin 2\Phi_k + 4 \sum_{i=1}^{\infty} C_{i+k}C_{i-k} \sin (\Phi_{i+k} - \Phi_{i-k}) \tag{21c}$$

$$R_{2k-1} = 2p \sum_{i=0}^{\infty} (2i + 1) C_{i+k}C_{i-k+1} \cos (\Phi_{i+k} - \Phi_{i-k+1}) \tag{21d}$$

$$S_{2k-1} = 2p \sum_{i=0}^{\infty} (2i + 1) C_{i+k}C_{i-k+1} \sin (\Phi_{i+k} - \Phi_{i-k+1}). \tag{21e}$$

Let the variational part of the instantaneous angular frequency of the output of the network $(d\psi/dt)$ be represented by the Fourier series

$$\frac{d\psi}{dt} = \sum_{k=1}^{\infty} A_{2k-1} \cos (2k - 1)pt + \sum_{k=1}^{\infty} B_{2k-1} \sin (2k - 1)pt. \tag{22}$$

From the substitution of the Fourier series expressions (19), (20), and (22) into (8) and imposing the condition that the expression for the instantaneous frequency of the output of the network must hold true for all time, the following equations result:

$$\sum_{k=1}^{\infty} \left[P_{2k-1+r}A_{2k-1} + Q_{2k-1+r}B_{2k-1} + P_{2k}A_{2k+r} + Q_{2k}B_{2k+r}\right] + 2P_0A_r$$

$$+ \sum_{k=1}^{(r-1)/2} \left[P_{r-2k+1}A_{2k-1} - Q_{r-2k+1}B_{2k-1}\right] = 2R_r \tag{23}$$

$$\sum_{k=1}^{\infty} \left[Q_{2k-1+r}A_{2k-1} - P_{2k-1+r}B_{2k-1} + P_{2k}B_{2k+r} - Q_{2k}A_{2k+r}\right] + 2P_0B_r$$

$$\sum_{k=1}^{(r-1)/2} \left[P_{r-2k+1}B_{2k-1} + Q_{r-2k+1}A_{2k-1}\right] = 2S_r \tag{24}$$

where r is a positive odd integer. The finite sums present in Eqs. (23) and (24) do not appear in the first harmonic equations where $r = 1$.

In the numerical computation of FM distortion components, it is necessary to determine the highest order of harmonic component of any consequence (r_m); then the

corresponding maximum value of k used in Eqs. (23) and (24) is $(r_m - 1)/2$. The approximate values of the harmonic amplitudes A_{2k-1} and B_{2k-1} are determined from the set of $(r_m + 1)$ linear equations obtained from Eqs. (23) and (24).

Probably the best way to arrive at a value for r_m is to compute the output harmonic components for each of a series of increasing values of r_m until a point is reached where the harmonic amplitudes change less than the desired limits of error with the last step-like change in r_m. Unfortunately, this process is both time-consuming and costly even with the use of a digital computer. The FM distortion calculations for the Chebyshev filter studied later (numerical results to be presented in Table 2) shows that r_m is given approximately by Table 1. The use of Table 1 to estimate r_m will result in an error of the order of 5 percent in a weak harmonic amplitude of about 1 percent of the fundamental amplitude provided the order of the harmonic does not exceed $(r_m - 2)$. It is felt that Table 1 applies to any bandpass filter with transfer characteristics similar to those of the Chebyshev filter.

Table 1
Table for Estimation of r_m

m	r_m
1	7
2	9
3	13
5	15
7	17
$\beta > 10$	2β

The method of calculation of FM distortion in a linear passive network described in this section is not limited in its application to conditions of small distortion but yields equally good results for conditions of large FM distortion. A worthy feature of this method of calculation of FM distortion is that it involves only simple mathematical operations and therefore is well adapted to use with an electronic digital computer.

LINEAR-PHASE NETWORK

The linear-phase network, though hypothetical, is of interest because this type of network introduces FM distortion only because of its nonflat amplitude transfer characteristic. The phase characteristic of the linear-phase network is

$$\Phi = \tau(\omega - \omega_0) \quad \text{or} \quad \Phi_n = \tau n p \tag{25}$$

where τ is the constant time delay of the network. The expressions for the auxiliary Fourier coefficients of Eqs. (21) now become

$$P_0 = C_0^2 + 2 \sum_{i=1}^{\infty} C_i^2 \tag{26a}$$

$$P_{2k} = \left[2C_k C_{-k} + 4 \sum_{i=1}^{\infty} C_{i+k} C_{i-k} \right] \cos 2kp\tau \tag{26b}$$

$$Q_{2k} = \left[2C_k C_{-k} + 4 \sum_{i=1}^{\infty} C_{i+k} C_{i-k} \right] \sin 2kp\tau \tag{26c}$$

$$R_{2k-1} = \left[2p \sum_{i=0}^{\infty} (2i + 1) C_{i+k} C_{i-k+1} \right] \cos (2k - 1)p\tau \tag{26d}$$

$$S_{2k-1} = \left[2p \sum_{i=0}^{\infty} (2i+1) C_{i+k} C_{i-k+1}\right] \sin(2k-1)p\tau. \tag{26e}$$

The substitution of Eqs. (26) into (19) and (20) leads to the expressions

$$N^2 + D^2 = P_0 + \sum_{k=1}^{\infty} \left[2C_k C_{-k} + 4 \sum_{i=1}^{\infty} C_{i+k} C_{i-k}\right] \cos 2kp(t-\tau) \tag{27}$$

$$DN' - ND' = 2p \sum_{k=1}^{\infty} \sum_{i=0}^{\infty} (2i+1) C_{i+k} C_{i-k+1} \cos (2k-1)p(t-\tau). \tag{28}$$

It is apparent from Eqs. (8), (27), and (28) that the Fourier series representation for $d\psi/dt$ must be of the form

$$\frac{d\psi}{dt} = \sum_{k=1}^{\infty} D_{2k-1} \cos (2k-1)p(t-\tau). \tag{29}$$

In order that the expression for the instantaneous frequency of the network output remain valid for all time, the D_{2k-1} must satisfy

$$\sum_{k=1}^{\infty} \left[M_{2k-1+r} D_{2k-1} + M_{2k} D_{2k+r}\right] + 2P_0 D_r + \sum_{k=1}^{(r-1)/2} M_{r-2k+1} D_{2k-1} = 2N_r \tag{30}$$

where

$$M_{2k} = 2C_k C_{-k} + 4 \sum_{i=1}^{\infty} C_{i+k} C_{i-k}$$

$$N_r = 2p \sum_{i=0}^{\infty} (2i+1)\, C_{i+\frac{r}{2}+\frac{1}{2}}\, C_{i-\frac{r}{2}+\frac{1}{2}}$$

r = positive odd integer.

To determine the harmonic amplitudes D_{2k-1}, one must assign a maximum value to r of r_m and then solve the set of $(r_m + 1)/2$ linear equations obtained from Eq. (30). The FM distortion introduced by the linear-phase network is independent of the time delay of the network.

THE DISTORTIONLESS NETWORK

The distortionless network has a uniform amplitude transfer characteristic and a linear-phase characteristic over all frequencies. The distortionless network transforms

its time-varying input voltage by simply delaying the input wave by $\tau = d\phi/d\omega$ seconds and multiplying the input signal by a constant amplitude response K. Hence from Eq. (1),

$$V_2 = K \sin\left[\omega_0(t - \tau) + m \sin p(t - \tau)\right] \tag{1'}$$

From Eqs. (1) and (2) the instantaneous angular frequency of the output of the network is

$$\omega_i = \omega_0 + mp \cos p(t - \tau). \tag{2'}$$

From Eqs. (27) and (28) we have

$$N^2 + D^2 = K^2\left[\left(J_0^2 + 2\sum_{i=1}^{\infty} J_i^2\right) + \sum_{k=1}^{\infty}\left(2J_kJ_{-k} + 4\sum_{i=1}^{\infty} J_{i+k}J_{i-k}\right)\cos 2k(t - \tau)\right] \tag{27'}$$

$$N'D - D'N = 2pK^2 \sum_{k=1}^{\infty}\left[\sum_{i=0}^{\infty}(2i + 1)J_{i+k}J_{i-k+1}\right]\cos(2k - 1)p(t - \tau). \tag{28'}$$

Then from Eq. (8) the variational part of the instantaneous angular frequency of the output of the distortionless network is

$$\frac{d\psi}{dt} = \frac{2p\sum_{k=1}^{\infty}\left[\sum_{i=0}^{\infty}(2i + 1)J_{i+k}J_{i-k+1}\right]\cos(2k - 1)p(t - \tau)}{\left(J_0^2 + 2\sum_{i=1}^{\infty} J_i^2\right) + \sum_{k=1}^{\infty}\left(2J_kJ_{-k} + 4\sum_{i=1}^{\infty} J_{i+k}J_{i-k}\right)\cos 2kp(t - \tau)}. \tag{31}$$

From the theory of Bessel functions (4) we have the identity

$$J_0^2(m) + 2\sum_{i=1}^{\infty} J_i^2(m) = 1. \tag{32}$$

From the distortionless property of the network it follows from Eqs. (31), (2′), and (32) that the following important summation identities in the theory of the Bessel function of the first kind must hold for all values of the argument m:

$$J_k(m)J_{-k}(m) + 2\sum_{i=1}^{\infty} J_{i+k}(m)J_{i-k}(m) = 0 \tag{33}$$

where k = 1, 2, 3, . . . , and

$$2\sum_{i=0}^{\infty}(2i + 1)J_{i+1}(m)J_i(m) = m \tag{34}$$

$$\sum_{i=0}^{\infty} (2i + 1) J_{i+k}(m) J_{i-k+1}(m) = 0 \tag{35}$$

where $k = 2, 3, 4, \ldots$.

It is interesting that one can derive the above summation identities from Eqs. (27) and (28) using the well-known properties of the distortionless network. The identity (33) is an established relation in the theory of Bessel functions (5). The identities Eqs. (34) and (35) appear to be new, for they are not found in the well-known treatises on the Bessel function; however, an elegant mathematical proof of Eqs. (34) and (35) by I. C. Chang appeared in a recent report (6).

THE NUMERICAL CALCULATION OF THE FM DISTORTION OF BANDPASS FILTERS

Example 1 - A Sharp-Cutoff Type of Bandpass Filter

A Chebyshev-response bandpass filter of a constant-K type with a 0.3-db ripple consisting of seven resonators is selected as an example of a relatively sharp cutoff filter for the distortion study. The steady-state transfer characteristics of the Chebyshev filter are presented in Fig. 2. Note that the phase characteristic exhibits considerable nonlinearity well within the passband. The FM distortion introduced by a given filter is a function of the deviation ratio of the input signal and the ratio of the half-power bandwidth of the filter to twice the frequency deviation of the input signal.

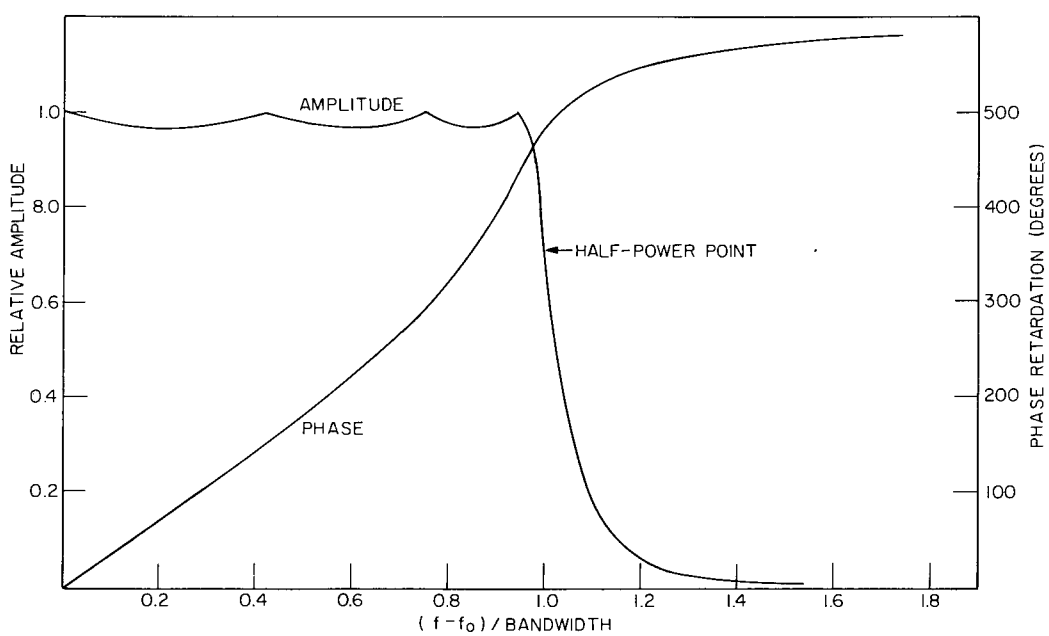

Fig. 2 - Steady-state transfer characteristics of the Chebyshev filter

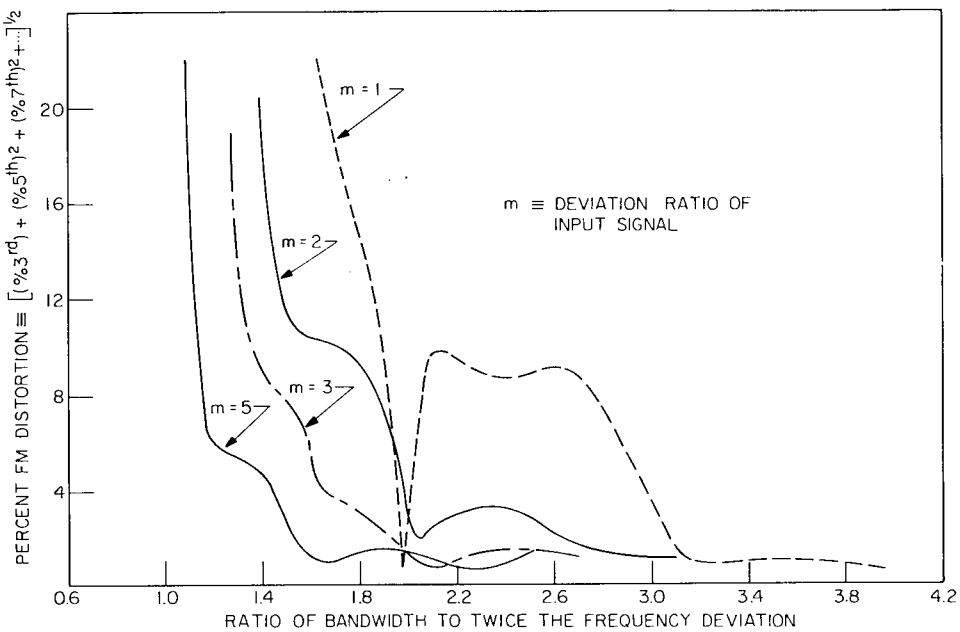

Fig. 3 - FM distortion curves for the linear-phase Chebyshev filter

The results of the calculation of the FM distortion introduced by the linear-phase Chebyshev filter with an amplitude characteristic which is that shown in Fig. 2 is presented in Fig. 3. Notice that the steplike variations in the distortion curves are smoothed out considerably as the deviation ratio m increases, because of the larger number of significant sidebands at the larger values of m. The sharp dip in the curve for $m = 1$ corresponds to a null in the third harmonic component which is dominant for unity deviation ratio. For the same reason there is a dip in the distortion curve for $m = 2$; but the dip is not as deep and as sharp as for $m = 1$ because of the presence of a fairly strong fifth harmonic component. Disregarding the dips in the curves for $m = 1$ and $m = 2$, it is observed from Fig. 3 that as the deviation ratio of the input signal is reduced, it is necessary to increase the bandwidth of the filter relative to the frequency deviation in order to keep the FM distortion at a constant level in the neighborhood of 4 percent.

For the purpose of discussion it is convenient to define the highest significant sideband pair (HSSP) as the highest order sideband pair which, if attenuated 6 db by a linear-phase filter, will result in an FM distortion of at least 4 percent (the 4 percent comes from the FM distortion study of the linear-phase Chebyshev filter). The linear-phase filter referred to in the HSSP definition must have an amplitude response resembling in general character that presented in Fig. 2. For deviation ratios lying between 1 and 5, the order of the HSSP is approximately $(m+2)$ for the Chebyshev filter.

The results of the FM distortion calculations for the nonlinear-phase Chebyshev filter possessing the transfer characteristics of Fig. 2 are presented in Fig. 4. Note the pronounced shoulder in the curve for $m = 1$ and the less pronounced shoulders in the curves for $m = 2$ and 3. The distortion curve for $m = 5$ is relatively smooth and shoulder-free because of the larger number of significant sideband pairs present in this case. The shoulder in each instance is at a point corresponding to the location of the HSSP at the point of inflection of the phase characteristic of the filter. Apparently at this point the increment in the FM distortion due to the onset of sideband attenuation is nearly offset

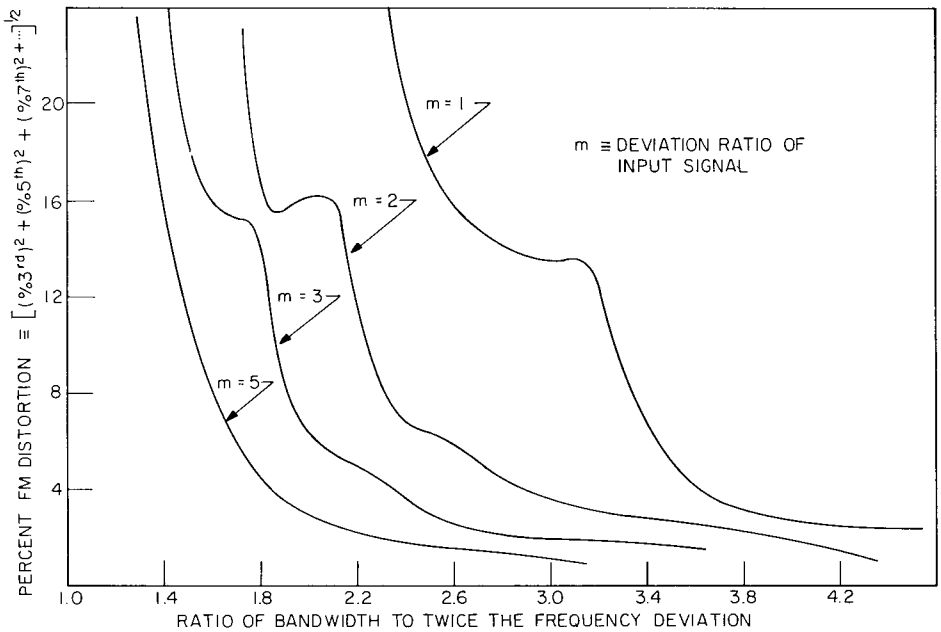

Fig. 4 - FM distortion curves for the nonlinear-phase Chebyshev filter

by a decrement in the distortion caused by the nonlinearity of the phase characteristic of the filter in the passband. For each value of m, the FM distortion level reaches 4 percent at a considerably larger bandwidth than for the linear-phase Chebyshev filter, because the HSSP suffers sizable phase distortion before it undergoes any attenuation as the ratio of the bandwidth to twice the deviation is reduced from 4 to lower values.

The ratio of the bandwidth of the filter to twice the frequency deviation of the input signal must be large enough to assure that the FM distortion introduced by the filter falls within tolerance limits. To keep the FM distortion under 5 percent the minimum value of the ratio is given by

$$\frac{\text{half-power bandwidth}}{2 \times \text{frequency deviation}} = \frac{1}{X_s}\left(0.7 + \frac{3.18}{m} - \frac{0.836}{m^2} - \frac{0.493}{m^3}\right) \tag{36}$$

where X_s is the value of $[(f - f_0)/\text{bandwidth}]$ corresponding to a point on the phase characteristic where the slope is $\sqrt{2}$ times the slope of the phase characteristic at the carrier frequency. From Fig. 2 it is seen that X_s is 0.707 for the Chebyshev filter. Equation (36) was arrived at empirically from Fig. 4; the numerical coefficients in Eq. (36) were determined at the points of 4.2-percent distortion. Although strictly speaking Eq. (36) is valid only for the Chebyshev filter under consideration or its equivalent, it is felt that Eq. (36) gives a useful approximation (±5 percent roughly) for any minimum-phase type (7) of bandpass filter with an amplitude transfer characteristic similar to that of Fig. 2, provided X_s lies between 0.6 and 0.8 and the deviation ratio m has a value between 1 and 5.

To show the energy distribution among the FM distortion components, some of the numerical results of the distortion calculations are presented in Table 2. It is observed that the FM distortion is dominantly third harmonic for $m = 1$ at all levels of distortion and for all values of m at low levels of distortion. At high levels of distortion the strength

Table 2
FM Distortion Components of the Chebyshev Filter

m	Bandwidth ÷ (2 × Deviation)	Fundamental/p	% 3rd	% 5th	% 7th	% 9th	% 11th	% 13th	% 15th	% Distortion*
1	2.50	0.980	16.8	2.8	0.5	0.1	–	–	–	17.2
1	3.33	0.975	7.9	2.2	0.1	–	–	–	–	8.2
1	3.95	0.977	2.8	1.2	0.06	–	–	–	–	3.0
2	1.77	1.963	10.4	14.5	1.0	1.0	0.7	0.1	–	18.2
2	2.18	1.987	9.2	7.4	2.7	0.1	0.2	–	–	12.2
2	3.22	1.990	3.0	0.55	0.35	0.1	–	–	–	3.1
3	1.56	2.97	10.0	6.7	11.2	1.9	0.7	0.7	0.4	16.6
3	1.87	2.99	7.0	2.5	5.4	1.9	0.2	0.1	0.1	9.3
3	2.71	3.00	2.2	0.9	0.5	0.05	0.02	–	–	2.5
5	1.35	4.95	9.7	5.7	6.8	10.0	6.2	1.5	0.7	17.7
5	1.56	4.98	6.3	2.1	3.1	3.1	3.5	1.2	0.2	8.8
5	2.50	5.00	1.4	0.6	0.5	0.5	0.1	0.04	–	1.7

*% Distortion $\equiv \left[(\%\ 3rd)^2 + (\%\ 5th)^2 + (\%\ 7th)^2 + \ldots\right]^{1/2}$.

of the harmonic components becomes more widely distributed as the deviation ratio m increases above unity. For example, for m = 5 and a bandwidth of 2.7 times the deviation of the input signal, the FM distortion is 17.7 percent. Here the strongest harmonic is the 9th, but the 3rd, 5th, 7th, and 11th are comparable in strength.

Example 2 - A Gradual-Cutoff Type of Bandpass Filter

To investigate briefly the effect on FM distortion of a gradual cutoff compared to that of a sharp cutoff in bandpass filters, the FM distortion introduced by a Butterworth-response, two-resonator bandpass filter whose transfer characteristics appear in Fig. 5 was calculated for m = 5, and the results are shown in Fig. 6. In addition to a gradual cutoff the Butterworth filter has an approximately linear phase characteristic to well past the half-power points of the amplitude response. A comparison of the distortion curves for m = 5 of Figs. 3 and 6 shows that the gradual-cutoff filter has a relatively smooth distortion curve free of the undulations which appear in the corresponding curve of the sharp-cutoff linear-phase filter. For a given frequency deviation of the input signal, Figs. 3 and 6 show that at a level of FM distortion of 3 percent in both filters the gradual-cutoff filter needs a bandwidth of only 62 percent of the bandwidth of the sharp-cutoff bandpass filter. The apparent reason for this is that as the bandwidth is reduced relative to the frequency deviation of the input signal, the gradual-cutoff filter does not clip off the sideband pairs abruptly one-by-one as does the sharp-cutoff filter, but the significant sidebands adjacent to each other in the input spectrum remain more nearly the same relative strength in the output of the network even though the sidebands may lie in the cutoff region of the filter.

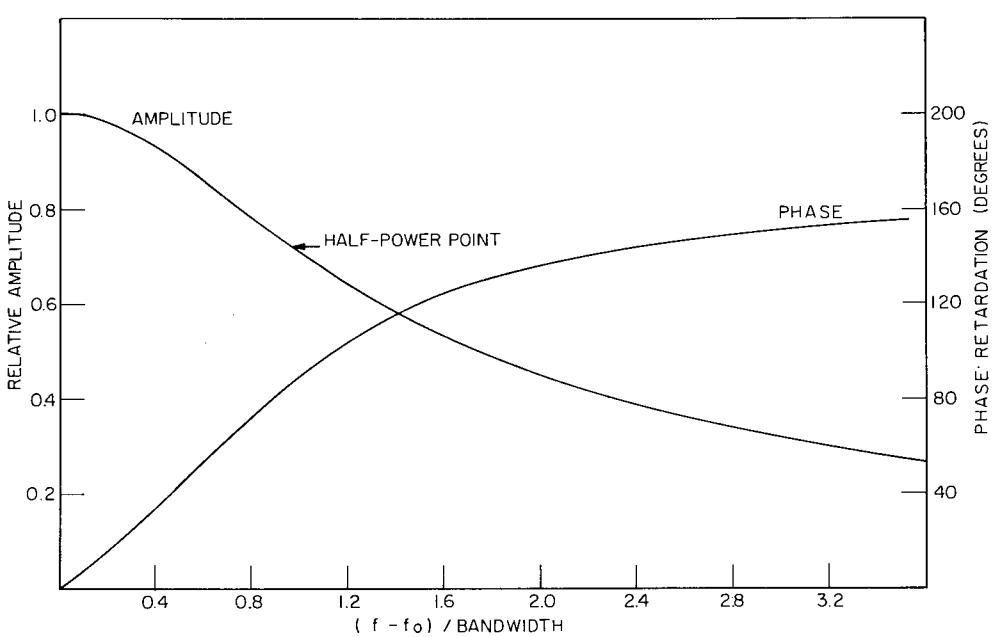

Fig. 5 - Steady-state transfer characteristics of the Butterworth filter

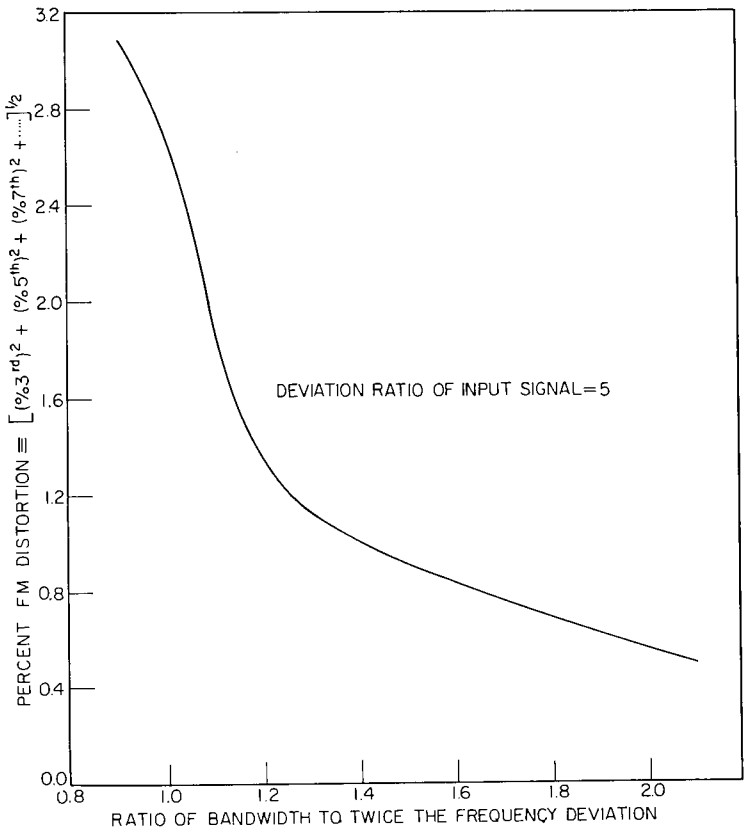

Fig. 6 - FM distortion introduced by the Butterworth filter

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A numerical method for calculating the steady-state FM distortion in a linear passive network using a sideband-type of analysis is developed to achieve economy of digital computer time and, for a given network, to find the number of significant sideband pairs associated with a given maximum allowable level of FM distortion.

To illustrate the application of the method, the FM distortion introduced by a bandpass filter is calculated over a range of signal parameters. A useful byproduct of the FM distortion calculations is a formula for the minimum ratio of the bandwidth of the filter to twice the frequency deviation of the input signal necessary to assure that the FM distortion introduced by a bandpass filter is less than 5 percent.

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