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Aircraft Aspect Solution in Dynamic Radar Area Measurements

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CONTENTS

ABSTRACT

The radar cross section of a target varies with the aspect angle of the radar beam relative to the target. Among the problems in radar cross-section measurement of full-size aircraft in flight is the difficulty associated with correlating the actual measured values with the aircraft aspect angles. Since the only aspect information available from the radar consists of target locations in spherical coordinates, a solution is desired from these data alone. By imposing restrictive conditions on the aircraft flight courses, target-oriented coordinates may be specified. When these are then combined with the radar beam position, the desired solution is established.

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

AUTHORIZATION

NRL Problem R07-04
Project RR 008-01-41-5552

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AIRCRAFT ASPECT SOLUTION IN DYNAMIC RADAR AREA MEASUREMENTS

INTRODUCTION

The radar cross-section measurement of a full-size aircraft in flight involves two basic problems. First, since all measurements must be time related, the actual radar cross-section measurement σ is a function of time, or $[\sigma = f(t)]$. Next, the radar beam position must be determined with respect to a coordinate system about the tracked aircraft. Two angles, azimuth and elevation, are required; these too are time related $[A, E = g(t)]$. Having determined these functions, it is then possible to plot the required function $\sigma = h(A, E)$.

REQUIREMENTS

The means available for solution, in the case of the NRL radar-area-measurement system, were restricted, in that no cooperative measurement was permissible within the target aircraft. This restriction ruled out measurement of roll, pitch, and yaw, and forced two constraints, if a solution was to be obtained in terms of the information available (the spherical coordinates of the target measured at the radar).

TARGET CONSTRAINTS

The aircraft-oriented coordinate system, in which a solution for the radar beam position is desired (Fig. 1), consists of the mutually perpendicular axes QU, QV, and QW. The conditions under which a problem solution is given depend upon two assumptions:

1. The QU axis coincides with the aircraft velocity vector

2. The QV axis is parallel to the ground plane XOY of the tracking radar.

The solution to be considered in this report therefore ignores any aircraft crab or roll angle. In order to approach these conditions, measurements will need to be performed under favorable weather conditions and along straight flight courses. The problems associated with actual measurements in this respect will be discussed in a later section.

PROBLEM SOLUTION

The radar continuously generates the range, azimuth, and elevation to the target. If these are converted to linear coordinates:

$$X = R \cos \varphi \cos \theta , \quad (1a)$$

$$Y = R \cos \varphi \sin \theta , \quad (1b)$$

$$Z = R \sin \varphi . \quad (1c)$$

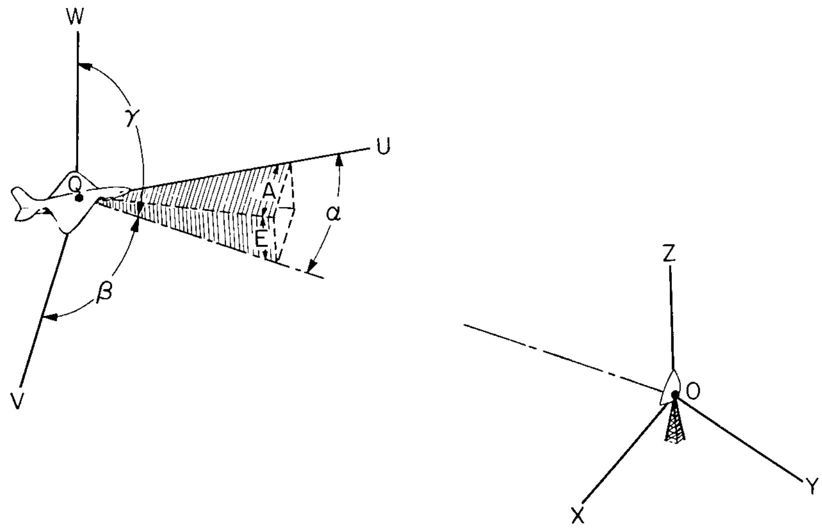


Fig. 1 - Aircraft and radar coordinate systems. The direction angles α, β, and γ are formed by the radar beam and the aircraft coordinates. The quantities A and E are the radar beam azimuth and elevation defined in the problem solution.

The quantities R, θ, φ are the range, azimuth, and elevation of the target in the radar coordinate system. Lower-case letters x, y, z will represent the magnitudes of the distances along axes OX, OY, OZ. In accordance with the first assumption in the previous section, the QU axis of the aircraft coordinate system coincides with the aircraft velocity vector. Denoting this by a unit vector **U**, its components are given by

$$U = \frac{1}{\dot{s}}\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix} \tag{2}$$

$$\dot{s}^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 .$$

The second assumption in the previous section implies that $\mathbf{V} \perp \mathbf{Z}$. Since the aircraft coordinate system forms an orthogonal set, $\mathbf{V} \perp \mathbf{U}$. With **V** further defined as a unit vector, we may write:

$$\mathbf{V} \cdot \mathbf{Z} = 0 \tag{3a}$$

$$\mathbf{V} \cdot \mathbf{U} = 0 \tag{3b}$$

$$\mathbf{V} \cdot \mathbf{V} = 1 . \tag{3c}$$

Using Eq. (2), the components of **V** are found to be

$$V = \frac{1}{\sqrt{\dot{x}^2 + \dot{y}^2}} \begin{pmatrix}-\dot{y}\\ \dot{x}\\ 0\end{pmatrix}. \tag{4}$$

The remaining vector **W** may be found by imposing the conditions

$$\mathbf{W} \cdot \mathbf{U} = 0 \tag{5a}$$

$$\mathbf{W} \cdot \mathbf{V} = 0 \tag{5b}$$

$$\mathbf{W} \cdot \mathbf{W} = 1 \tag{5c}$$

from which

$$W = \frac{1}{\dot{s}\sqrt{\dot{x}^2 + \dot{y}^2}} \begin{pmatrix} \dot{x}\,\dot{z} \\ \dot{y}\,\dot{z} \\ -(\dot{x}^2 + \dot{y}^2) \end{pmatrix}. \tag{6}$$

Once the location of each of the aircraft axes has been determined, the required coordinate transformation may be completed. With reference to Fig. 1, the angles between the radar beam and each of the axes may be found by using the law of cosines. If the radar beam position is written as a vector **B** of unit length, then

$$\cos\alpha = \mathbf{B} \cdot \mathbf{U} \tag{7a}$$

$$\cos\beta = \mathbf{B} \cdot \mathbf{V} \tag{7b}$$

$$\cos\gamma = \mathbf{B} \cdot \mathbf{W}. \tag{7c}$$

Denoting the coordinates of **B** by $\{b_x, b_y, b_z\}$, from Eq. (1):

$$B = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} \cos\varphi\cos\theta \\ \cos\varphi\sin\theta \\ \sin\varphi \end{pmatrix}. \tag{8}$$

Using Eqs. (2), (4), (6), and (8), the solutions to Eqs. (7a, b, c) are

$$\cos\alpha = \frac{\dot{x}b_x + \dot{y}b_y + \dot{z}b_z}{\dot{s}} \tag{9a}$$

$$\cos\beta = \frac{-\dot{y}b_x + \dot{x}b_y}{\sqrt{\dot{x}^2 + \dot{y}^2}} \tag{9b}$$

$$\cos\gamma = \frac{\dot{z}(\dot{x}b_x + \dot{y}b_y) - (\dot{x}^2 + \dot{y}^2)b_z}{\dot{s}\sqrt{\dot{x}^2 + \dot{y}^2}}. \tag{9c}$$

With reference to Fig. 1, the required angles A and E are given by

$$A = \tan^{-1}\frac{\cos\beta}{\cos\alpha} \tag{10a}$$

$$E = 90^\circ - \gamma. \tag{10b}$$

From Eq. (9), the angles are

$$A = \tan^{-1}\left[\sqrt{1 + \frac{\dot{z}^2}{\dot{x}^2 + \dot{y}^2}}\left(\frac{\dot{x}b_y - \dot{y}b_x}{\dot{x}b_x + \dot{y}b_y + \dot{z}b_z}\right)\right] \tag{11a}$$

$$E = 90^\circ - \cos^{-1}\left\{\frac{1}{\sqrt{1 + \frac{\dot{z}^2}{\dot{x}^2 + \dot{y}^2}}}\left[\frac{\dot{z}(\dot{x}b_x + \dot{y}b_y)}{\dot{x}^2 + \dot{y}^2} - b_z\right]\right\} . \tag{11b}$$

The use of the positive root causes an ambiguity in the azimuth angle A about 90 degrees. This is resolved by examining successive pairs of the sampled range function R. Thus if the range is increasing $A > 90$ degrees, whereas with decreasing range, $A < 90$ degrees (Fig. 1), let

$$\Delta R = R_{i+1} - R_i ; \tag{12}$$

then if ΔR is +, $A > 90$ degrees and if ΔR is -, $A < 90$ degrees.

COMPUTATIONAL PROCEDURE

The data used for the present computation consists of values for R, θ, φ sampled once each second. Since numerical differentiation is required, the answers are very sensitive to random variations in the input data. To remove these variations, the input data are smoothed using a five-point linear interpolative routine, repeated three times. The defining equations are*

$$R_{-2} = \frac{1}{5}[3f_{-2} + 2f_{-1} + f_0 - f_2] \tag{13a}$$

$$R_{-1} = \frac{1}{10}[4f_{-2} + 3f_{-1} + 2f_0 + f_1] \tag{13b}$$

$$R_0 = \frac{1}{5}[f_{-2} + f_{-1} + f_0 + f_1 + f_2] . \tag{13c}$$

Following smoothing and conversion to rectangular coordinates, the velocities are obtained using a simple three-point formula.* For one-second samples, this becomes:

$$\dot{x}_{-1} = \frac{1}{2}(-3x_{-1} + 4x_0 - x_1) \tag{14a}$$

$$\dot{x}_0 = \frac{1}{2}(-x_{-1} + x_1) \tag{14b}$$

$$\dot{x}_1 = \frac{1}{2}(x_{-1} - 4x_0 + 3x_1) . \tag{14c}$$

The complete computational procedure has been programmed for the NAREC and uses input data on tape.†

*"Introduction to Numerical Analysis," F.B. Hildebrand, New York:McGraw-Hill, p. 82, p. 295, 1956.
†NAREC Tape B5004-E-2000.

RUN 226

TIME	A	E	VELOCITY	OLD RANGE	NEW RANGE	OLD PHI	NEW PHI
			OLD THETA	NEW THETA	X	Y	Z
1	178.814	2.070	157,99	2378.0	2383.3	9.800	9,872
			79.800	79.839	414.22	2311,22	408,62
2	178.735	2.207	158,00	2529.0	2541.2	10.000	10,010
			79,800	79.760	444.87	2462,64	441,69
3	178.672	2.285	158,11	2719.0	2699.0	10.300	10,147
			79.700	79.681	475.92	2613.84	475,50
4	178.634	2.240	158,43	2880.0	2857.1	10.300	10,278
			79,600	79,603	507.33	2765.07	509,78
5	178.616	2.088	158,81	3000.0	3015.6	10.400	10,397
			79,500	79.528	539.10	2916.65	544.20
6	178.619	1.885	159.17	3153.0	3174.4	10.600	10,500
			79.400	79.456	571.16	3068.53	578,46
7	178.629	1.636	159,54	3328.0	3333.6	10.600	10,587
			79,400	79.388	603.46	3220.85	612,50
8	178.625	1.334	159,43	3489.0	3493.2	10.700	10,658
			79.300	79.323	636.02	3373,55	646.04
9	178.613	1.035	158,78	3673.0	3652.3	10.800	10,710
			79,300	79.261	668,71	3525.84	678,77
10	178.597	.757	158,29	3834.0	3810.7	10.800	10,749
			79.200	79.201	701.47	3677,50	710,70
11	178.579	.489	158,20	3969.0	3968.8	10.800	10,774
			79.100	79,142	734.41	3829.01	741,93
12	178.566	.271	158,43	4110.0	4127.0	10.800	10,789
			79,100	79.086	767.59	3980.67	772,52
13	178.572	.131	159,15	4260.0	4285.5	10.800	10,796
			79,000	79.030	801.05	4132.76	802.73
14	178.580	.048	159,85	4445.0	4445.2	10.800	10,799

Fig. 2 - Computer line printer output

RESULTS

The results of computation are available in both printed and punched-tape form. A sample of the printed form is shown in Fig. 2. The printout consists of the input spherical coordinate and their smoothed values, the target rectangular coordinates, target velocity, and the required azimuth and elevation angles. The punched tape is used to obtain a plot of azimuth versus elevation for the entire run. The results obtained from the digital plotter are shown in Fig. 3.

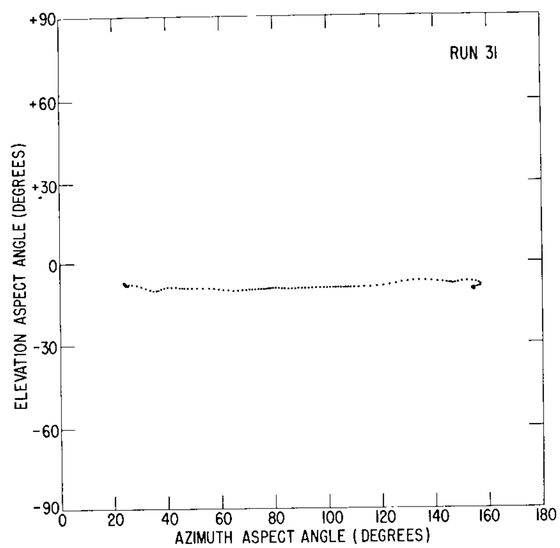
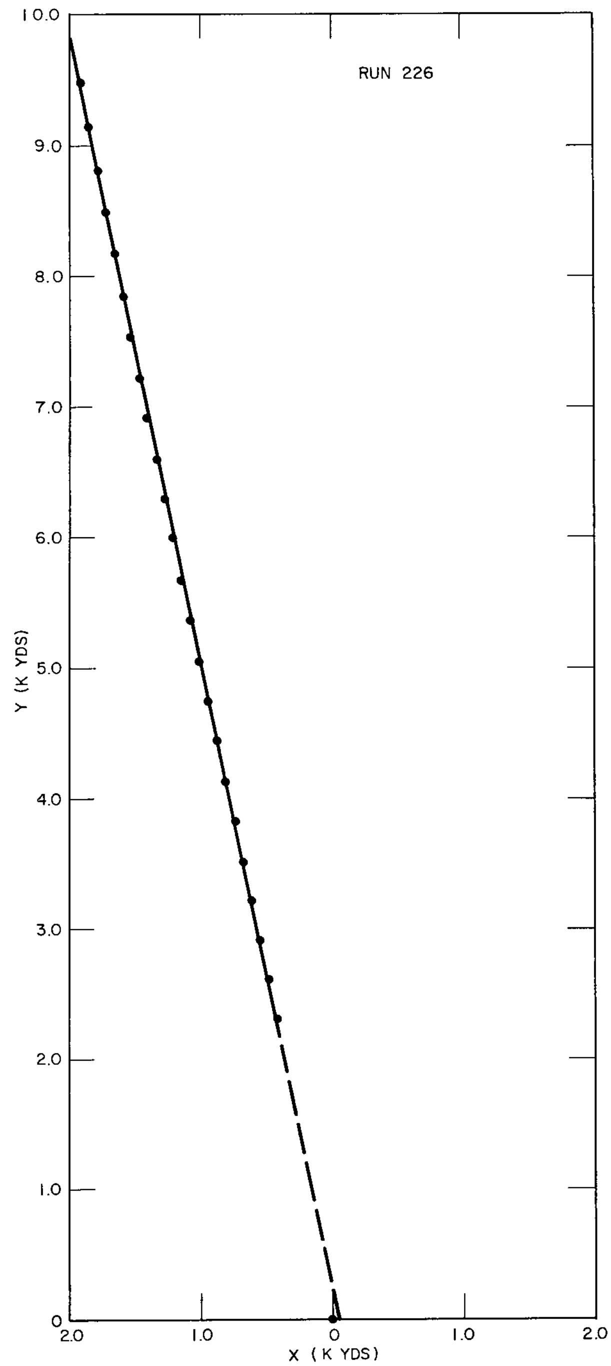


Fig. 3 - Plotter printout from the program-generated tape

In order to fulfill the straight-course requirement and to minimize the number of runs necessary to obtain good overall angular coverage, a TACAN system has been installed near the radar site. Figure 4a and 4b plots the tracks of the ground and elevation coordinates taken from the printout of a data run. The objective of this course was to obtain a nose-on aspect ($A = E = 0°$), for the entire data run. Through use of the TACAN, the pilot approached the beacon on a radial passing over the site. At the specified range, which the pilot read from the aircraft indicator, a dive was started and held at a predetermined angle for the run.

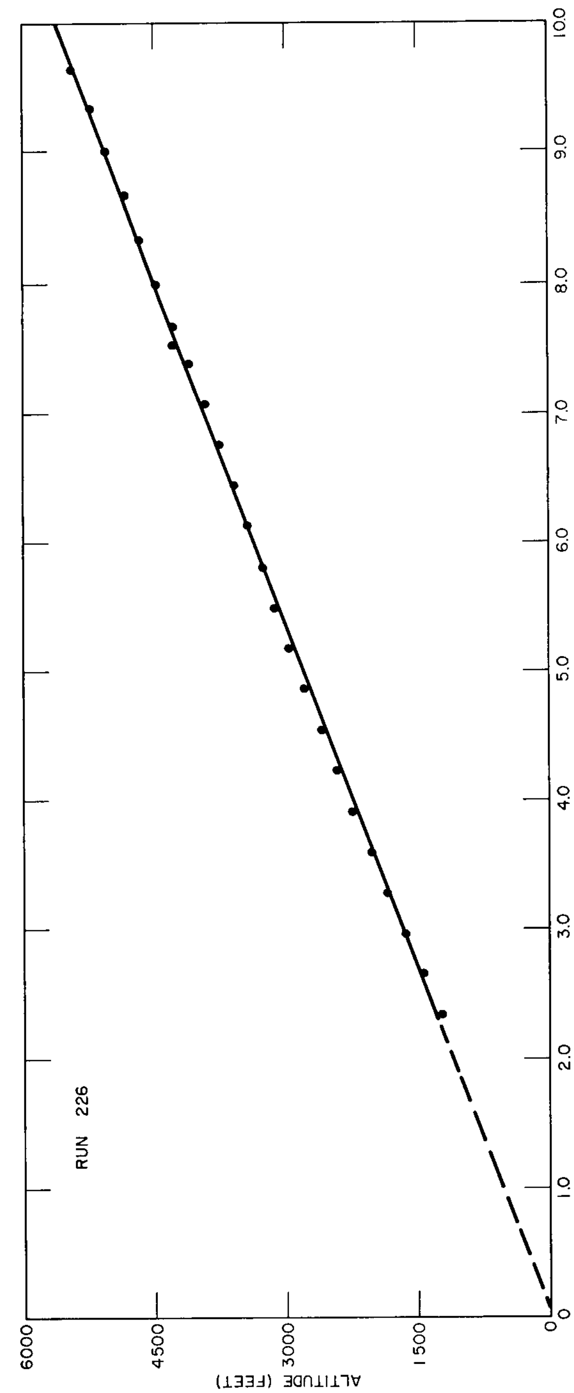
CONCLUSIONS

The program has been used for a few hundred aircraft runs with satisfactory results. Accuracy has proved sufficient, since the overall program objective has been to average radar cross-section data over a 10 by 10 degree aspect angle cell. Aircraft crab angle has been the greatest problem, although corrections based on the pilots' measurements have been applied to the computation, when the angle became significant.



(a) Ground coordinates

Fig. 4 - Projected coordinates from a dive run



(b) Altitude profile

Fig. 4 - Projected coordinates from a dive run

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The radar cross section of a target varies with the aspect angle of the radar beam relative to the target. Among the problems in radar cross-section measurement of full-size aircraft in flight is the difficulty associated with correlating the actual measured values with the aircraft aspect angles. Since the only aspect information available from the radar consists of target locations in spherical coordinates, a solution is desired from these data alone. By imposing restrictive conditions on the aircraft flight courses, target-oriented coordinates may be specified. When these are then combined with the radar beam position, the desired solution is established.

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