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DETERMINATION OF THE J-VALUES OF s-TYPE NEUTRON RESONANCES BY MULTIPLE SCATTERING, PART 2

S. Podgor

Nuclear Reactions Branch
Radiation Division

and

A. W. Sáenz

Analysis and Theory Branch
Radiation Division

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U. S. NAVAL RESEARCH LABORATORY
Washington, D.C.

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ABSTRACT

This report is a continuation of an earlier Monte Carlo study (NRL Report 5592, February 17, 1961) on the relevance of multiple scattering measurements in determining the J-values of s-type neutron resonances of a predominantly scattering variety. In the present work, Monte Carlo calculations were carried out for unpolarized neutrons incident on unpolarized targets with plane-parallel slab geometry. The major part of these computations deal with "general" values of the pertinent nuclear parameters, rather than with values corresponding to specific nuclei. These values bracket the ones previously used. We find for the resonances of interest here that the ratio B/F of neutrons scattered in the backward direction to those (scattered and unscattered) traveling in the forward direction is strikingly insensitive with respect to changes of the relevant parameters. These results and the ones obtained in the earlier report mentioned above furnish strong indications that only a rough knowledge of Γ_γ/Γ and Γ/E_o (E_o is the resonance energy) are necessary in principle to determine the J-values of suitable scattering resonances by the measurement of B/F or of similar ratios. The 132-ev resonance of Co^{59} is probably an ideal one for applying the present method. In carrying out the measurement of B/F for this and similar large resonances, our Monte Carlo work indicates that one could employ a nonmonochromatic beam of incident neutrons with an energy spread of about Γ, essentially centered at E_o.

PROBLEM STATUS

This is a final report on this phase of the problem.

AUTHORIZATION

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INTRODUCTION

The present investigation is a continuation of work previously reported.* That work was motivated by the idea that multiple scattering measurements could perhaps be exploited to determine J-values of s-type neutron resonances. A Monte Carlo program was developed to investigate the validity of this idea and allowed one to treat the multiple scattering of neutrons of any initial polarization by nuclei polarized to any given degree. The neutrons were assumed to be normally incident on a plane parallel target and three specific nuclei were considered: Co^{59}, Mn^{55}, and Ga^{71}.

In that investigation, the incident neutron energy, E_{inc}, was taken equal to the resonance energy, E_o. The most interesting results from an experimental viewpoint were obtained for the case of unpolarized neutrons incident on unpolarized nuclei. The important parameters for this case are as follows: the atomic weight A; the resonance energy E_o; the total width Γ; the neutron width Γ_n; the gamma width Γ_γ; the thickness t of the slab measured in mean free paths corresponding to the nominal cross section $(2\pi/k_o)(\Gamma_n/\Gamma)^2$, where $k_o \equiv (2m E_o/\hbar^2)^{1/2}$, m being the neutron mass; and the thickness τ of the slab in mean free paths corresponding to the peak total cross section for purely resonant scattering for $J = I \pm 1/2$. The relation between the thicknesses t and τ is given by

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NOTE: A summary of the main results of this report is given in Bull. Am. Phys. Soc. 7:22 (1962).

*"Determination of the J-values of s-type Neutron Resonances by Multiple Scattering," A. W. Sáenz and S. Podgor, NRL Report 5592, February 17, 1961. In that report, there are some incorrect formulas, which occurred because of copying errors and are absent from the Monte Carlo code, and therefore from the numerical results in that report. The following corrections should be made in the formulas of the report in question: (a) on page 2, the factor

$$\left\{1 + \frac{2kR\Gamma}{\Gamma_n} + \frac{4k^2R^2}{\Gamma_n^2}\frac{(2I+1)}{(I+[\eta])}\left[(E-E_o)^2 + (\Gamma/2)^2\right]\right\}^{-1}$$

should be inserted in the right-hand side of the Eq. (1) for $\sigma_{\alpha\beta}^{(s)}(\eta)$, and (b) the factor

$$\left\{1 + \frac{2kR\Gamma}{\Gamma_n} + \frac{4k^2R^2}{\Gamma_n^2}\left[(E-E_o)^2 + (\Gamma/2)^2\right]\right\}^{-1}$$

occurring in two of the Eqs. (2) on page 2 and in Eqs. (4) on page 3 should be replaced by the factor given in (a) above; (c) the factor

$$\left\{1 + \frac{(2I+1)}{(I+[\eta])}\frac{2kR}{\Gamma_n}\left[(E-E_o)^2 + (\Gamma/2)^2\right]\right\}$$

in the definition of r in Eqs. (2) should be replaced by

$$\left\{1 + \frac{kR\Gamma}{\Gamma_n}\frac{(2I+1)}{(I+[\eta])}\right\};$$

(d) the factor Γ_γ/Γ in front of the curly brackets in the first Eq. (4) should read Γ_γ/Γ_n. In addition, some minor errors which crept into this earlier report are given in the errata sheet issued June 28, 1961, and can be corrected as follows: on page 4, replace $(2mE_o/\hbar)^{1/2}$ by $(2mE_o/\hbar^2)^{1/2}$ and replace "cross section" in the line above by "maximum total cross section for unpolarized incident neutrons"; in Fig. 1 replace (3.1) by (5); on page 11 delete "but that B/F appears to be somewhat more sensitive with regard to changes in Γ_γ/Γ than in Γ".

$$\tau = \frac{2(I + [\eta])}{(2I + 1)} t, \tag{1}$$

where $\eta = \pm 1$ for $J = I \pm \frac{1}{2}$ and where $[\eta] = 1\ (0)$ for $\eta = 1\ (-1)$.

The following result is typical of those which were obtained for the phase of the work covering unpolarized neutrons and unpolarized targets. For the 132-ev resonance of Co^{59}, taking $\Gamma = 5.0$ ev, $\Gamma_\gamma/\Gamma = 0.10$, $E_{inc} = 132$ ev, and $\tau = 2$, the ratio B/F of reflected neutrons emerging from the back face of the slab to those (scattered and unscattered) coming out of the front face was found to be 1.16 ± 0.02 for $J = 3$ and 1.54 ± 0.03 for $J = 4$, for 10,000 histories. It is seen that the estimated standard deviations, indicated by the $\pm$ symbols, are much smaller than the difference between the values of B/F for $J = I - 1/2$ and $J = I + 1/2$. This affords a very strong indication that the J-value of this resonance could be determined by the measurement of B/F or of similar ratios.

PRESENT RESULTS

After the previous results were obtained for Co^{59}, Mn^{55}, and Ga^{71}, it was thought desirable to study how sensitively these results depended on changes in the relevant parameters. For unpolarized neutrons and unpolarized targets, neglecting Doppler broadening, the numbers B and F are completely specified in the limit of an infinite number of histories by the quantities

$$\frac{\Gamma}{E_o}, \frac{\Gamma_\gamma}{\Gamma}, A, \tau, \frac{E_{inc}}{E_o}, \tag{2}$$

and k_oR, where k_o is the initial neutron wave number and R is the nuclear radius, computed here from the formula $R = 1.4 \times 10^{-13} A^{1/3}$ cm. The dependence on k_oR arises from potential scattering effects and is completely negligible for the cases considered in this report. In contrast with the previous work, in which we only considered specific nuclei, the present calculations deal for the most part with "general" values of the parameters (2) above. In the majority of the examples considered here, the values of these parameters bracket the ones previously used. But in the present calculations, we also changed each of these parameters independently and are thus able to ascertain the effect of this change on B/F. Our overall conclusion is that, for a given τ, the B/F ratio is remarkably insensitive with respect to variations of the remaining parameters (2). We must, however, stress the point that this insensitivity has only been established for a certain class, albeit a wide one, of scattering resonances.

We now summarize the results obtained in this investigation. Each of the Monte Carlo calculations on which they are based involves 10,000 histories.

Variation of B/F with Γ/E_o, Γ_γ/Γ, and A

Table 1 refers to the case $E_{inc} = E_o$. This table gives the values of B/F for all combinations of the following values of the parameters (2): $\Gamma/E_o = 0.01, 0.05, 0.10$; $\Gamma_\gamma/\Gamma = 0.05, 0.10, 0.15$; $A = 50, 70$; and $\tau = 1, 2, 3$. In these computations, we took $E_o = 100$ ev for numerical convenience. The values in Table 1 should be almost completely independent of this choice of E_o, because of our previous remark that the parameters (2) essentially determine B and F for the resonances of interest here.

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An inspection of Table 1 reveals that, for a given τ, the B/F ratio is surprisingly insensitive with respect to changes of Γ/E_o, Γ_γ/Γ, and A, except for the narrowest resonance, i.e., for $\Gamma/E_o = 0.01$. The obvious reason for the larger variation of B/F with A for this last resonance is that neutrons are very quickly knocked out of the resonance region for such narrow resonance. The specific nuclei considered in our work had ratios $\Gamma/E_o > 0.025$.

Table 1
Variation of B/F with Γ/E_o, Γ_γ/Γ, and A

τ	Γ/E_o	B/F					
		$\Gamma_\gamma/\Gamma = 0.05$		$\Gamma_\gamma/\Gamma = 0.10$		$\Gamma_\gamma/\Gamma = 0.15$	
		A = 50	A = 70	A = 50	A = 70	A = 50	A = 70
1	0.01	0.51±0.01*	0.54±0.01*	0.52±0.01*	0.55±0.01*	0.54±0.01*	0.57±0.01*
	0.05	0.61±0.01	0.59±0.01	0.63±0.01	0.60±0.01	0.64±0.01	0.62±0.01
	0.10	0.57±0.01	0.56±0.01	0.58±0.01	0.56±0.01	0.58±0.01	0.57±0.01
2	0.01	0.95±0.02*	1.05±0.02*	0.97±0.02*	1.09±0.02*	1.03±0.02*	1.13±0.03*
	0.05	1.28±0.03	1.29±0.03	1.31±0.03	1.31±0.03	1.39±0.03	1.40±0.03
	0.10	1.23±0.03	1.20±0.03	1.30±0.03	1.27±0.03	1.31±0.03	1.33±0.03
3	0.01	1.30±0.03*	1.53±0.03*	1.33±0.03*	1.59±0.04*	1.34±0.03*	1.70±0.04*
	0.05	1.98±0.04	1.90±0.04	2.06±0.05	2.03±0.05	2.19±0.06	2.27±0.06
	0.10	1.90±0.04	1.88±0.04	2.06±0.05	1.99±0.05	2.21±0.06	2.17±0.06

*The Monte Carlo routine eliminated a neutron when its energy in the center of mass frame was such that $|E_o - E| \leq 10\Gamma$. The asterisks in Table 1 identify those cases in which some neutrons reached this cutoff energy. The number of such neutrons was greatest for A = 50 and $\tau = 3$. But in no case did the number of neutrons removed reach 2% of the total number of histories. The standard deviations in Table 1 for these starred cases do not take into account this elimination, which in any event would not be expected to alter them appreciably.

Figure 1 exhibits graphically one aspect of the results in Table 1: the variation of B/F with Γ/E_o when A = 50 and $\Gamma_\gamma/\Gamma = 0.10$. The thin target (one-collision) approximation for $A \to \infty$ is shown as a dotted line in this figure. The curve $\Gamma/E_o = 0.01$ is observed to be closer to this dotted line than are the curves $\Gamma/E_o = 0.05$, 0.10, and similar results are seen to hold if one plots the corresponding curves for the other pairs of values of A and Γ_γ/Γ in Table 1. This is precisely what one would expect in view of a remark in the preceding paragraph.

Figure 2 illustrates another facet of the results in Table 1: the variation of B/F with Γ_γ/Γ for the case A = 50 and $\Gamma/E_o = 0.05$. In accordance with one's expectations, B/F is seen to increase monotonically with Γ_γ/Γ, and this is also true for the other cases in Table 1. No values of Γ_γ/Γ greater than 0.15 were considered in our work because of the well-known fact that the intensity of multiple scattered neutrons decreases very rapidly with increasing Γ_γ/Γ.

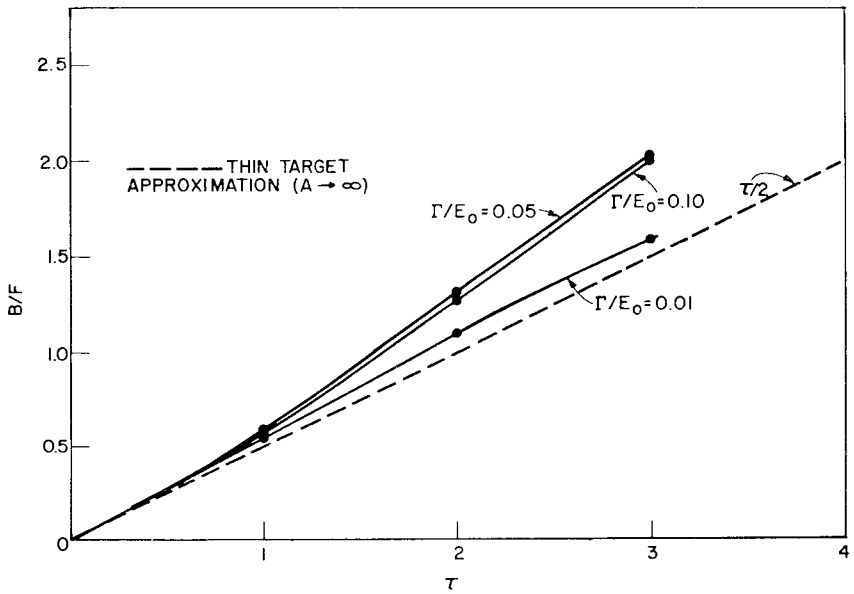

Fig. 1 - Variation of B/F with Γ/E_o for A = 50, $E_{inc} = E_o$ = 100 ev, and $\Gamma_\gamma/\Gamma = 0.10$

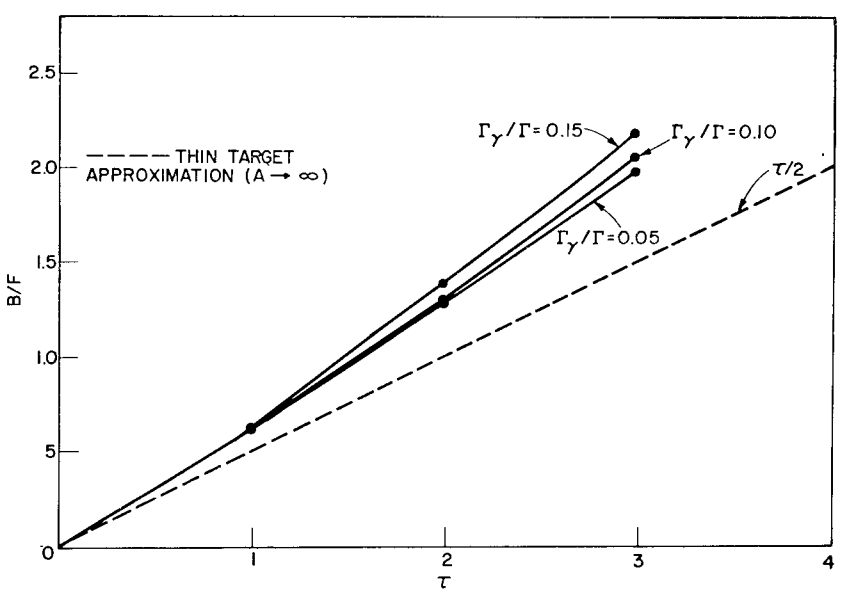

Fig. 2 - Variation of B/F with Γ_γ/Γ for A = 50, $E_{inc} = E_o$ = 100 ev, and $\Gamma/E_o = 0.05$

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Variation of B/F with E_{inc}

Table 2 exhibits the variation of B/F with the initial neutron energy for Co^{59}, taking E_o = 132 ev, Γ_n = 4.9 ev, and Γ_γ/Γ = 0.10.* These calculations were done for all combinations of E_{inc}/E_o = 1, 1 ± $\Gamma/2E_o$; t = 1, 2, 3; and η = ± 1.† The B/F values in Table 2 for given t and η are the same for these three values of E_{inc}/E_o, to within the estimated standard deviations, i.e., to within 2 or 3 percent. Similar results, not given in this report, have been obtained for other large scattering resonances of the type studied in our previous Monte Carlo work. We therefore, conclude that the B/F ratio for the 132 ev resonance of Co^{59} and for other large scattering resonances are independent of the initial neutron energy distribution, if this distribution has a width of about Γ and is roughly centered at E_o. This Co^{59} resonance furnishes an ideal case for applying the present method of determining J.

Table 2
Variation of B/F with E_{inc} for
Co^{59}, E_o = 132 ev, Γ_n = 4.9 ev, and Γ_γ/Γ = 0.10

τ	η	B/F		
		$E_{inc} = E_o - \Gamma/2$	$E_{inc} = E_o$	$E_{inc} = E_o + \Gamma/2$
1	+1	0.70±0.02	0.71±0.02	0.70±0.02
	-1	0.54±0.01	0.55±0.01	0.52±0.01
2	+1	1.52±0.03	1.56±0.04	1.55±0.04
	-1	1.14±0.02	1.16±0.02	1.16±0.03
3	+1	2.24±0.05	2.28±0.06	2.28±0.06
	-1	1.77±0.04	1.82±0.04	1.82±0.04

DISCUSSION

The present investigation of unpolarized neutrons incident on unpolarized nuclei furnishes strong evidence that the B/F ratio is extremely insensitive with respect to changes of the pertinent nuclear parameters and of the initial neutron energy. This in turn indicates that a fairly rough knowledge of the values of these parameters and a moderate monochromatization of the incident neutrons are sufficient, in principle, to determine the J-value of a scattering neutron resonance of the type considered here by multiple scattering measurements.

*These values of E_o and Γ_n are from B.N.L. 325 (Supplement No. 1), January 1, 1960. To the authors' best knowledge, no definitive measurements of Γ_γ/Γ have been reported for this resonance.

†The calculations in Table 2 were carried out in terms of the variables t and η, instead of τ, because this way of presenting our results is more useful from an experimental viewpoint.