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Study of Waves in a Small Tank

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With the use of relatively simple apparatus, precise measurements were made of wavelength and attenuation versus frequency of surface waves in the transition region between gravity waves and capillary waves. Wave velocities were determined by measuring the wavelengths in photographs of the water surface. The effects of contamination on velocity and attenuation were determined.

The effect of a current flowing perpendicular to a capillary wave system was studied photographically. Below a surface velocity of 6 centimeters per second, the cross current has little effect on wave structure. In the velocity range from 6 to 10 centimeters per second severe distortion of wave shape, and even destruction of the entire wave train, may result.

The attenuation with distance of the wave amplitudes was measured for wavelengths from 0.75 to 4 centimeters. The measured values of attenuation in this wavelength range, obtained with distilled water and a clean surface, are only 6 percent more than the theoretical values. Tap water gave values as much as a factor of five larger than the theoretical values.

INTRODUCTION

Oceanographic studies made with microwave radar have indicated that the radar return is principally from the capillary wave structure on the ocean surface. This report describes a series of experiments, conducted in a small wave tank, where the length of the generated waves fell in the range 0.5 to 4 centimeters and the wave amplitude was from 1 to 2 millimeters. Experimental data are presented which are in conformance with the wave velocities postulated by hydrodynamic theory (1). A photographic study was made of the distortion of the wave structure produced by a current flowing across the generated wave train. The attenuation of wave amplitudes (for wavelengths from 0.75 to 4 centimeters) with distance in both distilled and tap water was measured. In all experiments described, the depth of the water is several half wavelengths and the waves will not "feel" the bottom.

APPARATUS USED TO MEASURE WAVE VELOCITY

The plexiglass wave tank (Fig. 1) is 100 by 60 centimeters inside dimensions. Two partitions run the length of the tank dividing it into three compartments: a wave generating area 100 by 30 centimeters and a reservoir along each side. The partitions are 6 centimeters high and are provided with removable gates so that a cross current may be made to flow across the wave compartment by pumping water from one reservoir into the other. The wave energy is absrobed at either end of the tank by a series of thin triangular pieces of plexiglass whose surfaces have been roughened to increase frictional losses.

The wave generator was constructed using a 6-inch permanent-magnet loudspeaker. The cone and metal support were cut away leaving only the magnet, voice coil, and its support. A "T" made of brass rod was soldered to a metal disk and cemented to the voice coil support. The whole assembly is mounted in a rigid aluminum bar, which is supported at each end by a metal rod. A piece of polyfoam approximately 1 centimeter square and 25 centimeters long was attached to the metal T. The polyfoam bar was so positioned that one of the edges rested in the water. Several materials were tried, but it was found that the low inertial mass of the polyfoam produced the best results, particularly above 15 cycles per second.

The wave generator is driven by a Hewlett-Packard audio oscillator (Model 200 J), whose frequency was monitored by a Beckman (Model 8360 R) Universal Eput and Timer. Impedance matching between oscillator and wave generator is provided by a U.T.C. transformer (Model LS55).

A sheet of Teflon is located on the bottom of the wave compartment to provide a light-reflecting

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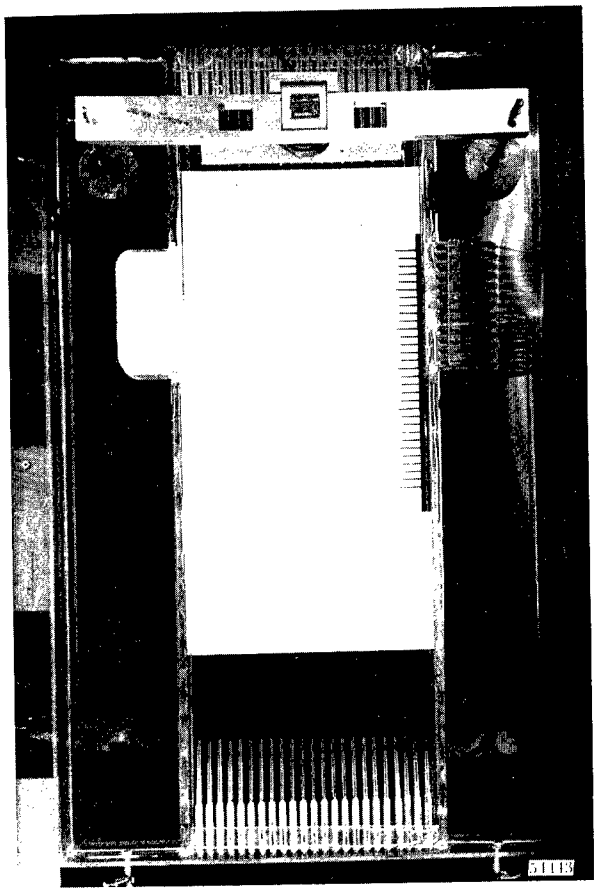


Fig. 1 — Wave tank

surface. A flood lamp is located on one side of the tank approximately 2 feet above the water surface to illuminate the water at an angle of 45 degrees. A Polaroid camera is located above the wave compartment to photograph the wave structure.

Calibration markers are visible along the right side of Fig. 1 in the wave compartment. These were constructed of short pieces of wire inserted in holes spaced 1 centimeter apart in a brass bar. This assembly is laid on the Teflon sheet, and thus appears in all photographs, providing a means for measuring wavelength.

RESULTS OF WAVE VELOCITY MEASUREMENTS

The classical theory of hydrodynamics predicts (1) the velocity of waves in a fluid by the mathematical expression

$$V = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}}$$

where λ is the wavelength and T is the surface tension of the fluid. The value of ρ (density of the fluid) may be taken as unity for water at room temperature. Using the nominal value given (2) for water of 72 dynes per centimeter for T (surface tension), the theoretical curve relating velocity and wavelength is plotted in Fig. 2, along with measured values to be referred to below. The correlation of values measured in the wave tank and those predicted by classical theory was the object of this experiment.

Tap water with a surface tension of 71.5 dynes per centimeter (measured using the Du Nouy ring method) was used for the velocity measurements. Figure 3 is a photograph of the water surface in the wave compartment, with the wave generator operating at 16.5 cycles per second. The calibration markers are visible along the right side of the photograph. The distance between adjacent wave crests, or one wavelength, is indicated on the photograph. The average wavelength is found by comparing, using a pair of dividers, the photographed length of ten wave crests to the photographed calibration scale. The velocity of the tank waves was then computed

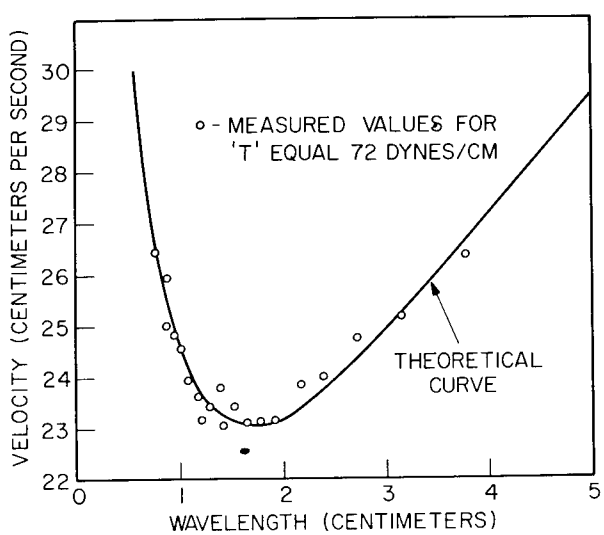


Fig. 2 — Velocity of water waves in a small tank as a function of wavelength

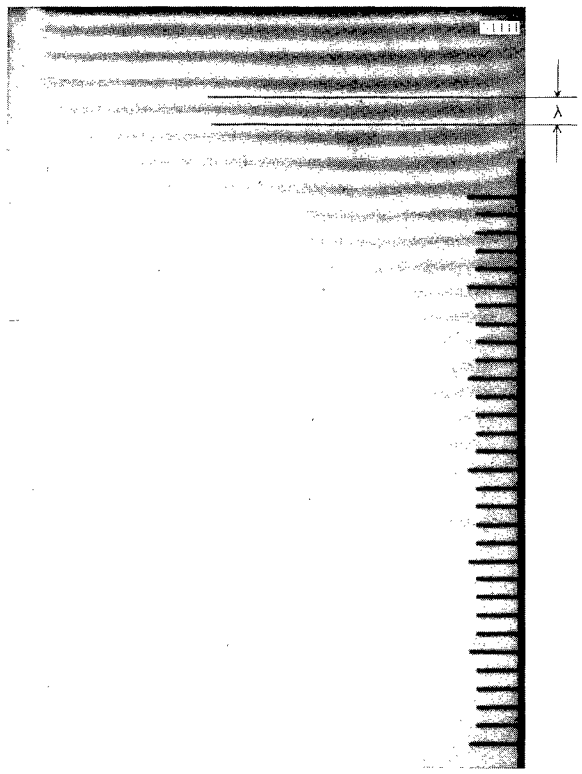


Fig. 3 — Waves photographed with the camera view normal to the water surface

using the relationship $V = f\lambda$. A series of photographs was made with the wave generator operating at frequencies from 6 to 35 cycles per second, increasing the frequency in 1-cycle-per-second steps. The velocities found using this technique are shown in Fig. 2 as the small circles.

The effect of reduced surface tension on wave velocity was investigated in a second part of the experiment. Cod-liver oil (0.001 cubic centimeter) was placed on the surface of the wave compartment. The surface area of the compartment being 3000 square centimeters, the oil film is calculated to be about 33 angstroms thick. The measured surface tension for this condition was 52 dynes per centimeter. The techniques described in the first part of the experiment were used over the same frequency range to measure the wavelength. The complete results for these experiments are shown in Fig. 4.

The low points of these curves indicate the wavelengths at which the transition from capillary waves to gravity waves occur. For tap water, this

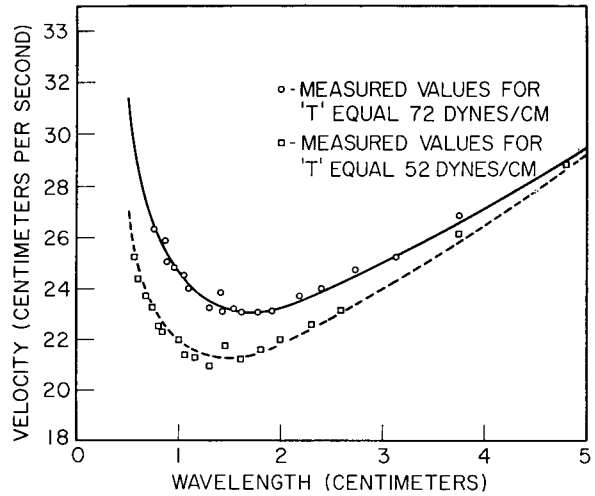


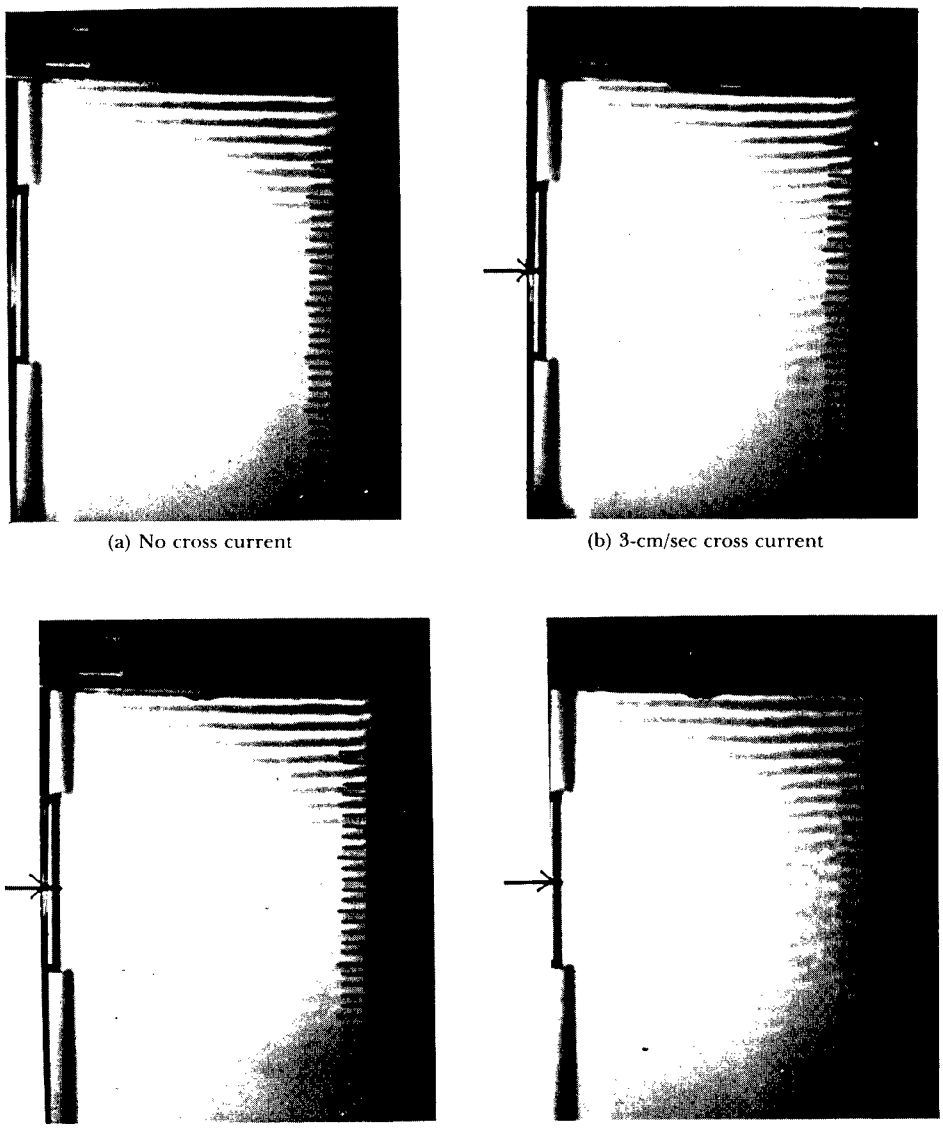
Fig. 4 — Comparison of measured velocities of water waves in a small tank and the theoretical velocities

transition occurs at a wavelength of approximately 1.7 centimeters. When the surface tension is reduced the curves show that the wavelength of the transition point is reduced to approximately 1.5 centimeters.

EFFECT OF A CROSS CURRENT ON WAVE STRUCTURE

Experiments designed to study the effects of surface currents on capillary wave structure were performed next. A current perpendicular to the wave direction was established with the aid of two water troughs, one on either side of the wave area, and a circulating water pump (see Fig. 1). Gates on the sides of the wave area admit water from the trough on one side and exhaust it from the other side. By pumping the water into a 50-centimeter-deep reservoir, connected to the bottom on one trough, a reasonably uniform cross current 15 centimeters wide was produced. The reservoir serves to damp out turbulence produced by the pump, which results in a more uniform flow.

Figure 5(a), shows the capillary waves (wavelength 1.56 centimeters) produced in the wave compartment with no current through the gate. Figure 5(b), shows the same condition with a 3-centimeter-per-second surface velocity. It should be observed that this current (approximately 0.06 knot) has little visible affect on the capillary



(a) No cross current

(b) 3-cm/sec cross current

(c) 6-cm/sec cross current

(d) 10-cm/sec cross current

Fig. 5 — Distortion of water waves by a cross current

waves. In Figs. 5(c) and (d), where the surface velocity is increased to 6 and 10 centimeters respectively, severe distortion of the wave shape is clearly evident.

Additional experiments conducted in the wave tank have shown that a current flowing in opposition to the direction of capillary wave propagation will result in a reduction in the wavelength when compared to that predicted by classical theory. In this instance, there was no visible distortion or breaking up of the wave structure.

EFFECT OF CROSS CURRENTS ON RADAR BACKSCATTER FROM A WATER SURFACE

As was stated in the Introduction, there is strong evidence that radar backscatter from a water surface depends principally on the capillary wave structure which rides on the gross wave structures. Our experiments indicate that when the current is at right angles to the wave direction, even small currents, perhaps less than 6 to 10

centimeters per second, can severely modify or even destroy this capillary wave structure. Radar should therefore be a sensitive detector of such currents. If, however, the currents are either in the same or the opposite direction to the capillary waves, our experiments indicate that their effect is a change in wavelength. This leads to a complicated situation. Radar measurements have been made at a site on the east coast of Florida, where the Gulf Stream is roughly 10 miles wide and flows at approximately 3 knots. Here the interaction of currents, winds, and waves coming from various directions produces very intricate wave patterns. Figure 6 is a photographic record of radar return from an area 7000 yards off shore, and shows in much detail the complex wave structure existing. We will not attempt to analyze this photograph here; we wish merely to point out the potential of high-resolution radar as an oceanographic measuring device.

STUDY OF WAVE STRUCTURE BY PHOTOGRAPHY

When a photograph is made looking normal to the water surface, as in Fig. 3, the capillary waves are clearly visible in only about two-thirds of the area photographed, which corresponds to one-fourth of the tank length. Since a detailed radar picture (Fig. 6) was obtained with the radar looking at a very shallow angle (less than one degree), it was conjectured that the sensitivity of photographic detection of capillary waves could be enhanced by looking from an angle other than 90 degrees. This possibility was investigated.

The camera was positioned at the end of the tank, and while viewing the image on the ground glass screen, the camera angle was varied. At an angle of approximately 40 degrees, the photograph reproduced in Fig. 7 was made. This photograph shows that the capillary waves cover the length of the tank and would seem to indicate that the slope of the wave fronts are more efficient reflecting surfaces than are the wave crests. This method of photographing the wave structure does not, however, provide a means of measuring wavelength unless some method of providing calibration markers could be devised. The lighting in this photograph was from the end of the tank opposite the camera, and the calibration markers are distorted and obscure.

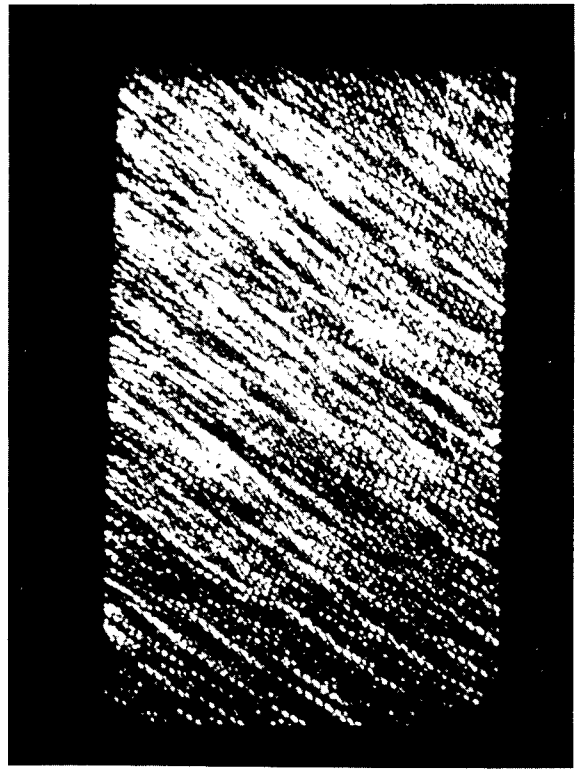


Fig. 6 — Radar picture of waves on an ocean surface as seen from a shallow angle

EXPERIMENTAL SET UP AND THEORETICAL BACKGROUND FOR MEASURING WAVE ATTENUATION

A study of the natural decay of the waves generated in the tank was undertaken. In order to measure the change of wave amplitude with distance, a micrometer wave height measuring assembly was installed above the water surface. Three holes, 10 centimeters apart, were drilled in a metal bar mounted above the water surface in the direction of wave motion so that micrometer heads could be placed above the surface at spacings of either 10 or 20 centimeters. A phonograph needle was secured to each micrometer spindle, so that it was possible to lower the needles until contact was made with the water. A zero reference level was established when no waves were being generated. With the wave generator operating, the needles were raised until they just touched the wave crests. The difference between these two readings gave the wave heights to an accuracy of at least 0.001 inch and permitted estimation

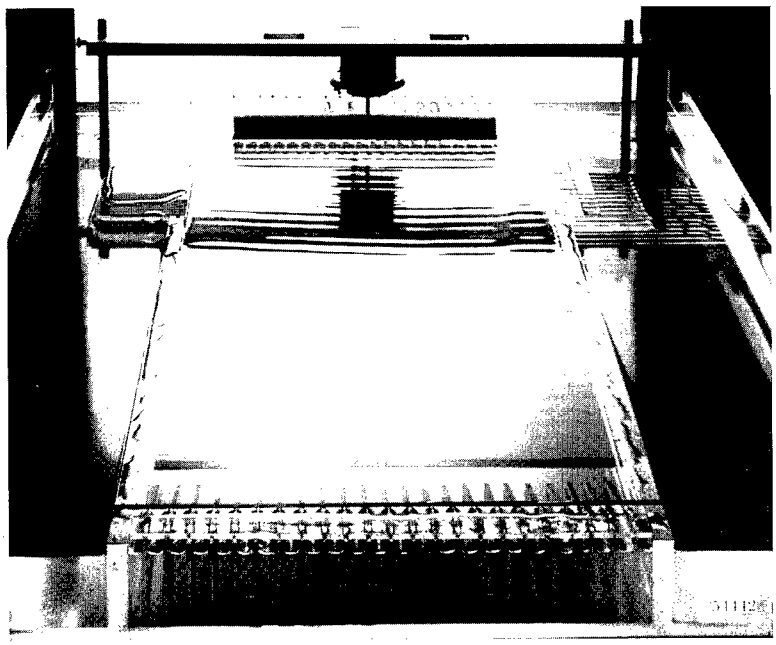


Fig. 7 — Waves photographed with the camera view at 40 degrees to the surface

to 0.0001 inch. Measurements were made for wave frequencies from 3 to 21 cycles per second for a micrometer spacing of 10 centimeters. Additional measurements were made for a 20-centimeter micrometer spacing, and the ratios of wave heights for the two successive 10-centimeter spacings were found to agree within the limits of experimental accuracy. On the basis of this constant ratio, the assumption was made that the waves decay exponentially.

The wave amplitude at the first micrometer position can be represented by the expression $A(X_1) = A_0 e^{-\alpha X_1}$, and for a second micrometer position by $A(X_2) = A_0 e^{-\alpha X_2}$. $A(X_1)$ and $A(X_2)$ are the amplitudes measured at two successive micrometer positions; X_1 was about 10 centimeters in front of the wave generator, and X_2 was exactly 10 centimeters greater than X_1. Solving these two expressions for α, we have

$$\alpha = \frac{1}{10} \ln \frac{A(X_1)}{A(X_2)} \text{ nepers per centimeter.}$$

According to theory, the attenuation constant (α) for a clean water surface is given (3) by $\alpha = 8\pi^2\eta/\rho\lambda^2 V_g$, where V_g is the group velocity of the wave train, and η is the viscosity. The classical expression for phase velocity is

$$V_\theta = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}}.$$

The group velocity is related to the phase velocity by the relation

$$V_g = \frac{V_\theta}{1 - \frac{f}{V_\theta}\frac{\partial V_\theta}{\partial f}}.$$

It is not possible to express V_g by a simple formula in the general case. However, if the waves are predominantly of a capillary wave nature, *i.e.*, the term involving g is negligibly small, then V_g is given by $(3/2)\sqrt{2\pi T/\rho\lambda}$, and the expression for α reduces to $\alpha = 8\pi\eta f/3T$. This expression is quite accurate for frequencies above 25 cycles per second or for wavelengths shorter than 0.75 centimeter. Since the attenuation measurements made in these experiments cover a frequency range from 5 to 25 cycles per second, which corresponds to a wavelength range from 4 to 0.75 centimeter, and since the expression for α given above is not an accurate approximation, the group and phase velocities and attenuation constant have been calculated for a wide range of frequencies using the theoretical formulas previously given and the results are plotted in Figs. 8 and 9.

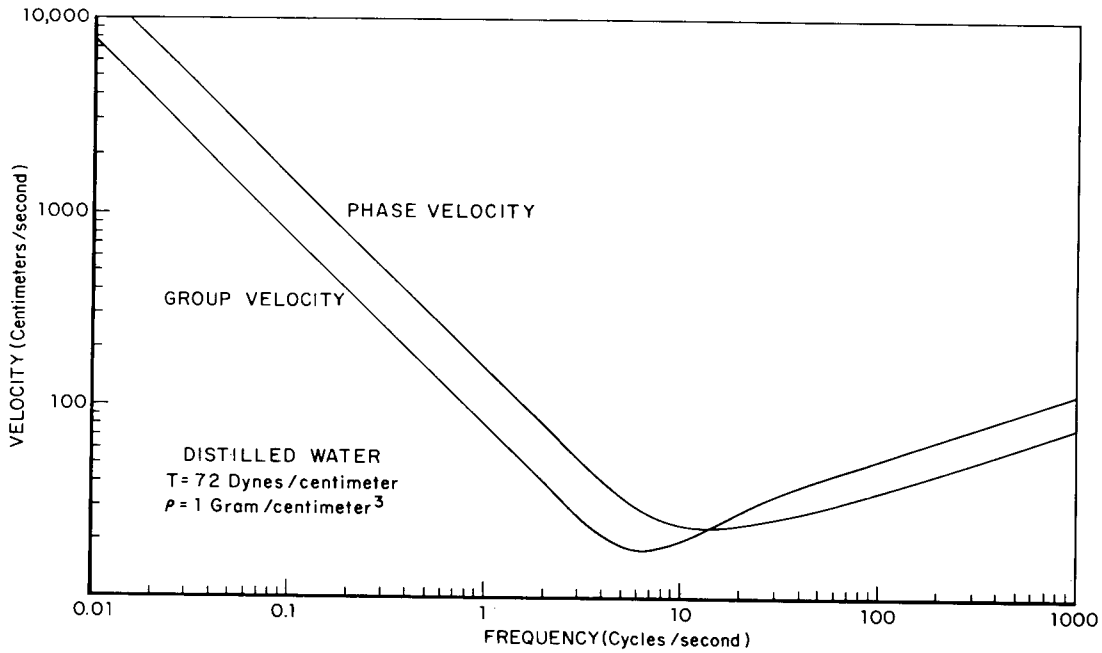


Fig. 8 — Theoretical group and phase velocities of water waves

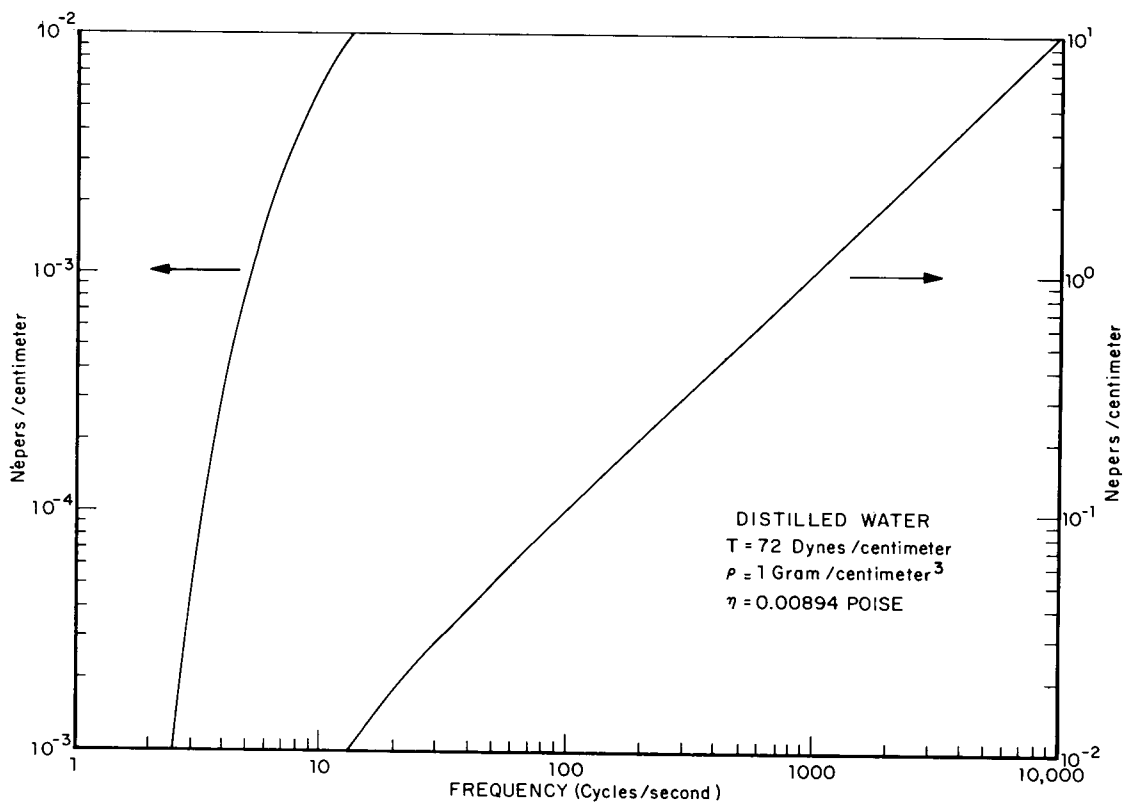


Fig. 9 — Theoretical attenuation of water waves

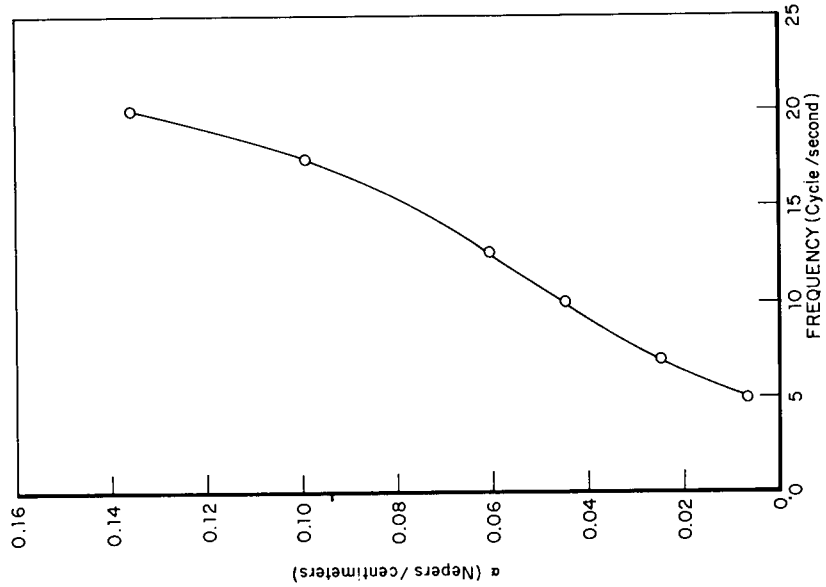


Fig. 11 — Attenuation of water waves in tap water

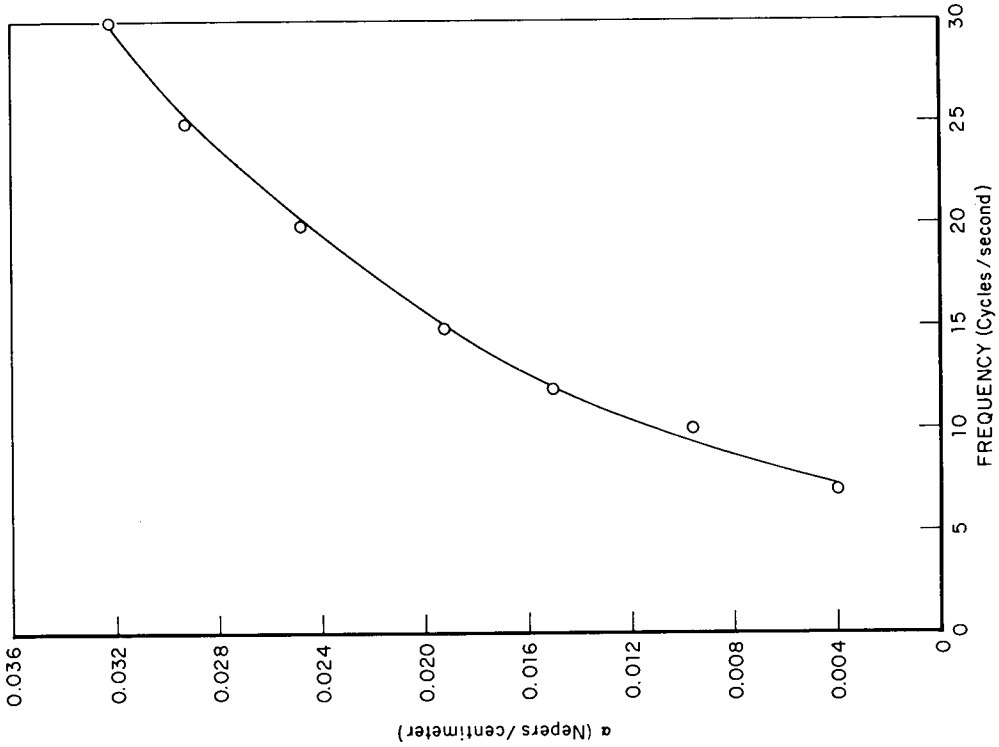


Fig. 10 — Attenuation of water waves in distilled water

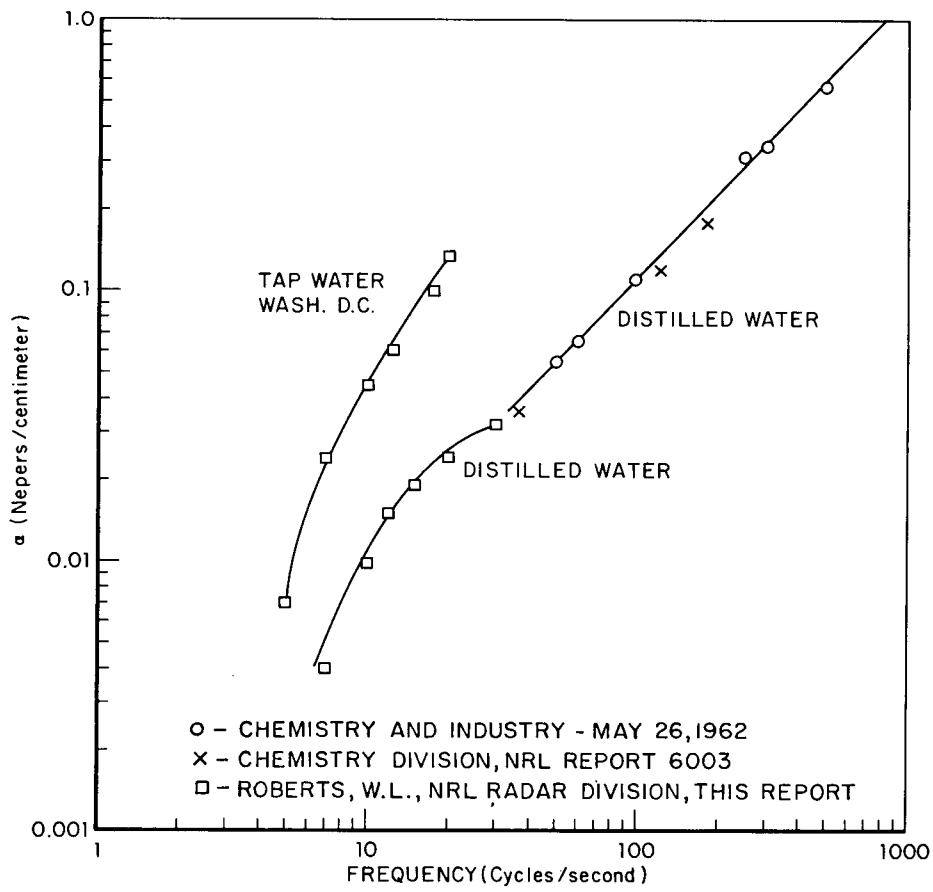


Fig. 12 – Summary of water wave attenuation

Two experiments were performed in the abovementioned frequency range. Distilled water was used first and the measured value of attenuation is shown in Fig. 10. The second experiment used tap water as supplied in Washington, D.C. This tap water contains many impurities, which may be observed by allowing a full tank to set for 24 hours, in which time a siltlike film is seen to cover the bottom of the tank. The values of attenuation for tap water are shown in Fig. 11, and are two to five times greater than those measured for distilled water. This discrepancy occurs in the frequency region where the transition from capillary to gravity waves occurs.

It should be pointed out that low values of attenuation for distilled water are obtained only when the water surface is made very clean. The cleansing is accomplished by sprinkling talc on the water surface and removing it by means of a siphon.

A description of similar experiments at higher frequencies is reported in the literature (3,4) and the values of attenuation found are shown in Fig. 12 for comparison with those of this report.

CONCLUSIONS

The wave velocities in a small wave tank for waves whose wavelengths fall in the region of 0.75 to 4 centimeters can be measured with sufficient accuracy to check closely with classical hydrodynamic theory. The reduction of water surface tension by placing a small amount of cod-liver oil on the surface of tap water is shown experimentally to reduce the wavelength at which the transition from capillary to gravity waves occurs from about 1.7 centimeters to about 1.5 centimeters.

The attenuation of wave amplitude with distance, using distilled water, is found to be about

6 percent higher than the value given by theory. This disparity may be accounted for, at least to some extent, by the fact that losses due to the tank sides are ignored in the expression for attenuation. A clean water surface is required to obtain the results shown. In experiments using tap water the value of attenuation increases by a factor of two to five when compared to distilled water.

Photographic evidence demonstrates that a current normal to the direction of wave propagation will result in severe distortion of the wave pattern when this current approaches 6 centimeters per second (0.12 knot).

ACKNOWLEDGMENT

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