Simplified Prediction Equations for the NRL Satellite Position Display

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CONTENTS

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Abstract: A satellite position prediction and display equipment (SPAD) has been conceived and developed at NRL. Details regarding the operation and performance of the equipment have been described previously in NRL Report 6219. The present report concerns itself primarily with the equations utilized in a digital computer for performing the position prediction computations. With periodically updated. orbital elements for each satellite stored in the computer memory, these equations provide position coordinates and height above the earth's surface for any satellite, at any desired time. Restricted by the limited computer speed and memory space available, the equations arrived at, in order to both satisfy the display system requirements and to meet the desired accuracy, are of a degree of complexity which places them between the basic planetary equations of celestial bodies and the much more sophisticated equations of today's space computer centers. Although modifications of the basic equations have previously been developed and reported in NRL Reports 5652 and 5659, they were successful in computing positions of only those satellites whose orbits are relatively stable. The equations of this report, which are extensions of these modified equations, are able to provide geocentric coordinate positions for those unstable satellites whose orbits are decaying rapidly. This extension is accomplished by including the time rate of change of the semimajor axis as an input orbital element and by the manner in which the period is computed.

Results obtained from both the modified and the extended equations are compared to those obtained from NRL's Research Computation Center. When computed for nine days into the future, positions resulting from the extended equations have been obtained with errors ranging from 0.1 degree for stable satellites, to no greater than 1 degree for those satellites whose semimajor axes are decaying at a rate of 10^{-3} earth radii/day. The position coordinates and height of eleven satellites can be computed every 1.1 sec.

In order to minimize the amount of data transmitted via the communication channels, only seven orbital elements are required for each satellite. Certain orbit perturbations, namely, the precession of the node and the rotation of perigee, are computed rather than being obtained from a space computation center. This permits the updating of the raw orbital message with a minimum of data words.

This report contains a derivation of the simplified equations for computing the azimuth and elevation angles of a satellite from a ship's position. Equations for computing the radius of the circle on the earth's surface viewed by a satellite are also included.

INTRODUCTION

The U.S. Naval Research Laboratory has developed a satellite position prediction and display equipment (SPAD) as proposed in Ref. 1. The original proposal envisioned an electronic dynamic display for presenting the predicted positions of satellites at any prediction time on an appropriate map background. A computation device, operating upon the appropriate orbital elements, would provide subsatellite position coordinates lying anywhere on the surface of the earth. A special purpose hybrid computer, *i.e.*, a combination of digital and analog circuits, was initially proposed (Ref. 2) as the prediction computation device. Before actual construction of this hybrid computer had begun, it became possible to make use of a general purpose digital computer, the AN/UYK-1.

This report is concerned with the computer program which provides the desired information to the display, and more specifically, with that portion of the program which performs the actual prediction computation. For detailed discussions pertaining to the equipment performance, operation, and hardware, the reader is referred to Refs. 3 and 4. A simplified treatment of the SPAD equipment operation can be found in Ref. 5.

The SPAD computer program consists of two major sections, the executive and the computational routines. The sequence of instructions comprising the executive routine primarily services the operator-selected requests and also performs

NRL Problem Y01-01; Project SF 019-01-03 (Task 6168). This is an interim report; work on the problem is continuing. Manuscript submitted Feburary 1, 1966.

the necessary data and information transfers and input/output operations. For example, if a request is made to display satellites of a certain category or type, an executive routine will perform a search through the entire store of satellite data. Upon locating satellites of this category, the corresponding data is then moved to a series of memory locations where it can be drawn upon as needed. Further examples of operator requests can be found in a discussion of the operation of the overall equipment (Ref. 3).

The computational routines perform the necessary operations on the data in order to generate the numerical information required, such as latitude, longitude, height, *etc.* The computational routines can be divided into three main parts: (a) position and height, (b) azimuth and elevation, and (c) satellite area of view. The equations and methods comprising these routines are covered in detail in subsequent sections of this report.

The position and height routine computes a subsatellite point and height above the earth's surface on the basis of seven stored orbital elements. A spherical model of the earth is used for this and the other computational portions of the program, although several perturbations due to a nonspherical earth are taken into account.

The azimuth and elevation routines compute as azimuth the angle formed by the North Pole, the ship, and the subsatellite point, and as elevation the angle formed by the line between the satellite and a ship and a plane tangent to the earth at the ship's position.

The satellite area of view routine computes the radius of a circular area on the earth's surface. The radius of this circle is obtained from equations which make use of the satellite's distance above the surface of the earth and its known, or assumed, look-cone angle.

This report discusses the modifications of the simplified equations, derived by members of the Data Processing Branch (Refs. 6 and 7), which permit position predictions of satellites whose orbits are rapidly decaying. These modifications reside chiefly in using $\dot{a}$, the time rate of change of the semimajor axis, and in the particular method chosen for computing the mean anomaly M from the data available. While the equations for computing M are not rigorously derived, they are shown to be plausible and to provide predictions well within the required specifications.

CONSIDERATIONS FOR POSITION COMPUTATION

An earlier study conducted by members of the Data Processing Branch resulted in a set of equations which would provide accurate subsatellite positions by using the parameters of two-body dynamics. Two requirements dictated the degree of sophistication to be employed: The equations had to be complete enough so that the error in the positions computed for one week into the future or past would not exceed 60 naut mi, and yet they had to be simple enough so that their solution time would be compatible with the rate at which position information was to be updated. These equations were tested for accuracy and solution time and were found to adequately satisfy the SPAD requirements. However, since only two perturbation terms, the rotation of perigee and the precession of the node, were taken into account, positions of only those satellites whose orbits are rather stable were accurately computed. At that time, it was felt that only such satellites would be of interest. As the project and the computer program progressed, the decision was made to look into the problem of augmenting the original equations in order to provide accurate positions for any satellite regardless of its orbit stability. As a result, the prediction equations were modified to use another orbital element, the time rate of change of the semimajor axis. Due to the limited memory space available, the addition of this orbital element for each satellite held in store necessarily decreased the total number of satellites which could be made available to the user. The resulting increase in accuracy and capability, however, was believed to provide more than adequate compensation. Furthermore, the total number of satellites held in store was not considered of major concern in demonstrating and evaluating the SPAD concept.

ORBITAL ELEMENTS

The orbit size and orientation of any satellite (see Fig. 1) can be defined by five orbital parameters or elements. A sixth parameter, epoch, is

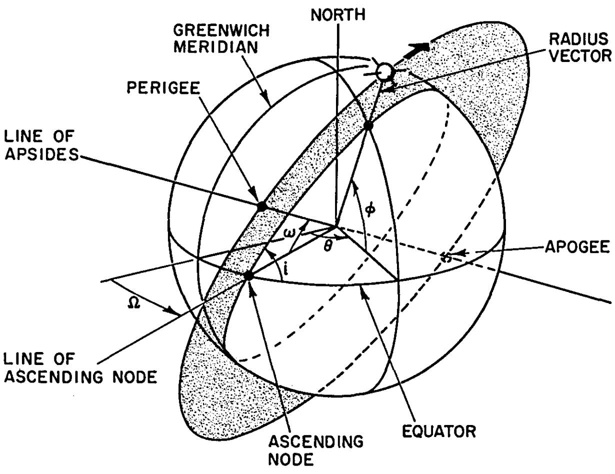

Fig. 1 — Illustration of satellite orbit and some of the geometric quantities used to determine the satellite's position in its orbit: i = angle between earth's equatorial plane and satellite's orbital plane, ω = angle between line of ascending node and line of apsides, Ω = angle between line of ascending node and Greenwich meridian, θ = geocentric longitude, and ϕ = geocentric latitude

the time at which the other five parameters are measured and is necessary in order to locate the position of the satellite in its orbit. In order to simplify the equations used for SPAD, the epoch value is modified to be that time at which the satellite is at its perigee point. Hence, all other time-dependent parameters must be adjusted to this new time. Details of this modification are given in Appendix A.

The two parameters which describe the orbit size and shape are the eccentricity e and the semimajor axis a. As a consequence of the gravitational field of the earth being an inverse-square central force field, any object orbiting this earth model will trace out a closed conic section. Thus, these two parameters are used in their purely geometric sense.

The remaining three parameters are angles which describe the orientation of the orbit with respect to the earth (Fig. 1). The angle of inclination i is the angle measured between the earth's equatorial plane and the satellite's orbital plane. The intersection of these two planes is the line of nodes.

The argument of perigee ω is the angle formed by the line of the ascending node and the line of apsides measured in the direction of the satellite motion. The line of apsides is defined as the line in the orbital plane which passes through the apogee and perigee points. The line of the ascending node is that portion of the line of nodes from the center of the earth to the point at which the satellite crosses the equator in a northerly direction.

The longitude of the ascending node Ω is the angle formed by the line of the ascending node and Greenwich meridian, measured from Greenwich along the equator in an eastward direction.

The foregoing orbital elements and the equations in which they are used are based upon the assumption that the earth has an inverse-square central force field. This assumption arises from the spherically symmetrical model of the earth chosen in the derivations. Due to this central force field simplification, no other forces except a gravitational force directed towards the center of the earth is considered to be affecting the orbiting body. It is known, however, that there are deviations from this spherical symmetry due to the nonhomogeneous nature of the earth's mass distribution as well as that due to the nonspherical shape of the earth. These cause perturbations or disturbances to the regular motion of an orbiting body and should be taken into account in order to obtain accurate subsatellite positions.

A significant source of perturbing forces is the earth's equatorial bulge. This bulge gives rise to an asymmetric gravitational force which is a maximum at the equator. The effect resulting from this additional force is a westerly displacement of the orbital plane for satellites crossing the equator in a northeasterly direction. (An easterly displacement results from a northwesterly satellite motion.) This displacement, which can be found using an available equation, is designated as the precession of the node $\dot{\Omega}$. Another effect attributed to the bulge is the rotation of the line of apsides in the plane of the orbit. The resulting rotation of perigee $\dot{\omega}$ can also be determined by an available equation. Since the equations for computing these two perturbation effects are expressed in terms of the basic orbital elements, these effects were not required as additional input parameters but were included as part of the computational sequence for determining positions. The perturbation due to the nonhomogeneous nature of the earth's mass was assumed to be negligible, and hence no compensations were attempted.

The previous discussion covered only those effects which can be ascribed to the earth's gravitational field. A perturbation effect, which was not accounted for in the previously mentioned NRL studies, arises from aerodynamic drag which causes an orbit degeneration or decay. At perigee, where the effect of this drag is most pronounced, the satellite velocity decreases, resulting in a decrease in the major and minor axes. Apogee decreases more rapidly than perigee, with the subsequent elliptic orbits collapsing into a circle, eventually entering the earth's atmosphere where the satellite burns up. This phenomena can be adequately described by the time rate of change of semimajor axis $\dot{a}$. Since this quantity is not readily computed from the basic elements, it was made part of the set of orbital elements required for each satellite. It should be noted that while this rate is not exactly constant, *i.e.,* there is a rate of change $\ddot{a}$, the latter rate over relatively short prediction times is considered to be of no consequence and is neglected.

THE EFFECT OF ORBIT DEGENERATION ON THE COMPUTATION OF THE MEAN ANOMALY

The mean motion μ per unit of time of a satellite is given (Refs. 8 and 9) as

$$\mu = 2\pi/P \qquad (1)$$

where P is the anomalistic period defined as the time from perigee to perigee. From this relation, μ can be considered as the angle, in radians, described in a unit of time by an imaginary point moving with a constant angular velocity, *i.e.,* as if the orbit was a circle with radius equal to the semimajor axis of the true orbit ellipse. It is thus evident that

$$M = \mu\ (t-\tau) = (2\pi/P)\ (t-\tau) \qquad (2)$$

is the angle described, in radians, by this imaginary point measured from its passage of perigee τ to some arbitrary time t in its orbit. This angle is designated as the mean anomaly and is used in a later section of this report in the solution of Kepler's equation. In practice, the time interval $t-\tau$, or Δt, is the elapsed time from epoch $\tau = T_0$ to prediction time t since these are the only times available. It will be shown later that, for stable orbits, this interpretation of Δt is compatible with the definition of mean anomaly given above. It should be noted that epoch time T_0 as used in these computations is a passage of perigee time, as mentioned in a previous section.

The period P, which is required for the solution of Eq. (2), can be obtained from Kepler's Third Law which states that the square of the period is proportional to the cube of the semimajor axis. In equation form, the period P, in mean solar seconds, is given by

$$P = k\ a^{3/2} \qquad (3)$$

where $k \approx 0.020317541460$* for satellites with a mass small compared to that of the earth (which is obviously true for all artificial earth satellites), and a is expressed in statute miles.

The foregoing equations for finding M are adequate to obtain nine-day subsatellite position predictions with errors no greater than 1 degree, provided a does not change by more than $\approx 10^{-5}$ earth radii (e.r.) in those nine days. This is an emperical rule of thumb arrived at through the experience of the present authors with the equations outlined in this report.

In order to graphically illustrate the effect of changing a, two rather simple cases will be discussed with reference to Fig. 2. One-dimensional time diagrams are shown, with marks on the axis indicating times corresponding to the passage of perigee. Thus, the intervals between marks indicate the length of a period. The diagram of Fig. 2(a) illustrates a situation where a is constant; thus the intervals or periods are constant as is evident from Eq. (3). The prediction time is indicated by t. The subdivision of the time axis into equal P_{const} increments as shown in the figure is equivalent to Eq. (2) where $P = P_{const}$, $\tau = T_0$, and a complete period is equivalent to 2π radians. The result obtained by evaluating Eq. (2) is a number consisting of an integer and a fraction multiplied by 2π. The integer represents the number of complete periods from epoch T_0 to the perigee

*This value of k is taken from p. 4 of Ref. 7.

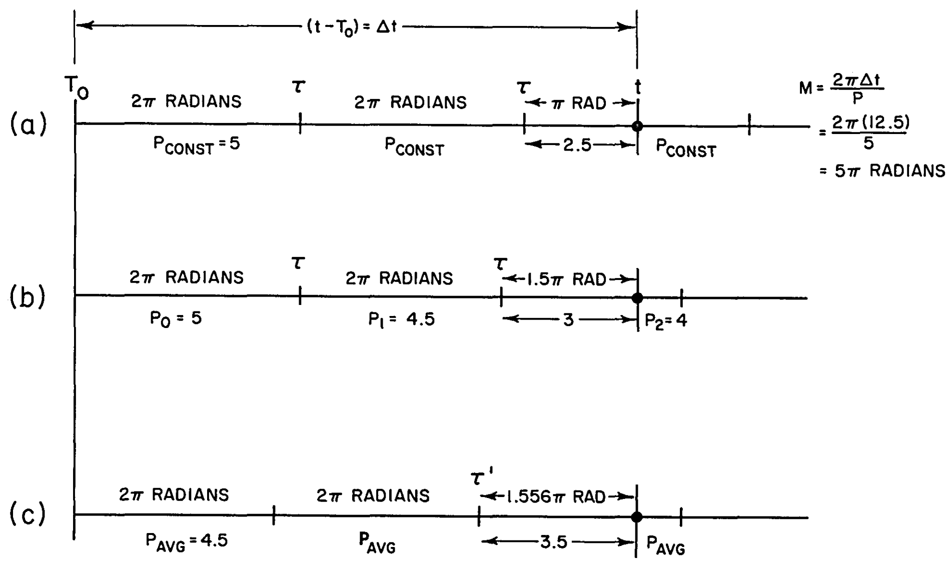

Fig. 2 — One-dimensional time diagrams showing the period P for various satellite orbits: (a) orbit semimajor axis a is constant; (b) satellite orbit is degenerating and a is decreasing; (c) an average value of the period is used to calculate a more accurate value of the mean anomaly $M = 2\pi\ \Delta t/P_{avg}$

immediately prior to the prediction time t; the fraction is the portion of the period that the satellite has traversed since its last perigee. These two quantities can easily be identified from the diagram as two complete periods and one half of a period. Hence, neglecting the integer, the remaining fraction is a value for M which satisfies the definition and thus is used in subsequent computations. It has been pointed out that the mean anomaly M is the angle swept out from the last perigee by a satellite with constant angular velocity; thus it should be clear that the fractional portion of the period which defines the elapsed time since perigee is directly proportional to the angle M. Hence, in the example shown in Fig. 2(a), M is equal to π radians, which is the same result obtained by evaluating Eq. (2) and neglecting the integer. Since the position computation equations depend upon knowing M exactly, it is further clear that the situation just described, which does permit obtaining this exact value, would provide accurate position predictions. This assumes, of course, that all other factors are appropriately accounted for.

In the event, however, that the satellite orbit is degenerating, *i.e.*, the semimajor axis a is decreasing, Eq. (3) shows that the period P is likewise decreasing. This condition is shown graphically in Fig. 2(b) where, for illustrative purposes, the period at epoch is chosen to be the same as the period for the previous case, namely, $P_0 = P_{const}$, and subsequent periods are decreasing linearly. If $P = P_0 = P_{const}$ was used in evaluating Eq. (2), which would be equivalent to subdividing the axis in P_{const} equal increments, the resulting integer and fraction would obviously be the same as obtained in the previous case shown in Fig. 2(a), *i.e.*, $M = \pi$ radians. Now, however, the integer no longer necessarily represents the number of complete periods from epoch to the perigee just prior to prediction time, and of more consequence, the fraction no longer necessarily represents that portion of the period traversed. This is clearly shown in Fig. 2(b), where there are two complete periods and three quarters of a period to time t. Thus, the exact value for M should be $3\pi/2$ radians. So, the resulting error in M is $\pi/2$ radians.

It is at once evident that large values of $\dot{a}$ would cause correspondingly large errors in the period P, as well as the value obtained for M, and the subsequent value for subsatellite positions would increase in error.

In order to provide a capability for predicting subsatellite positions for those satellites with large $\dot{a}$ terms, a method for evaluating the mean anomaly M, which would take into account the variable nature of the period P, is required.

A METHOD FOR COMPUTING THE MEAN ANOMALY M

With the value of $\dot{a}$ available as an orbital element, it is possible to determine the value of the period at any time t in an orbit by using the equation

$$P_t = k\ (a + \dot{a}\Delta t)^{3/2} \tag{4}$$

where Δt is the time elapsed from epoch T_0 to prediction time t.

The purist may argue that since both the period and the semimajor axis are defined in terms of a complete orbit, it is meaningless to speak of these elements as continuous functions of time. However, continuous functions can be defined as follows: At a given moment let the perturbing influence (*e.g.*, atmospheric drag) be suddenly removed, so that thereafter the orbit is that of a pure ellipse. The a and P for this orbit may be thought of as the instantaneous values at the given moment. Furthermore, if the change in the instantaneous value of P is negligible over one orbit, as is most generally the case, the value of period determined by Eq. (4) at time t of an orbit is essentially equal to the period of the complete orbit.

From the preceding discussion it can be seen that the period P of the particular orbit that the satellite is in at time t can be obtained from Eq. (4). However, although this value of period is quite accurate, it in itself, when applied to the example of Fig. 2(b), does not provide an answer for M any more accurately than that obtained when using the value of period obtained at epoch. This is so due to the fact that in order to obtain the mean anomaly M, not only must the length of the period be known, but the relationship of where the prediction time t falls with respect to the beginning of the period, *i.e.*, perigee, must also be known. This latter fact is not provided from Eq. (4).

It appeared, then, that a value of P intermediate between the period at epoch, where $P = P_0$, and the value determined at prediction time, *i.e.*, $P = P_t$, would result in a more useful answer for M. Although it is evident from Eq. (4) that the period does not change in a linear fashion, Appendix B shows that over the relatively short prediction times of interest, the error introduced into the value of P by assuming a linear variation is only 0.001 percent. With this in mind, the best intermediate value of P would be the average value between the two aforementioned values of period. Thus the mean anomaly M is computed from

$$M = \frac{2\pi\ \Delta t}{P_{avg}} \tag{5}$$

where

$$P_{avg} = \frac{P_0 + P_t}{2} = \frac{k\ [(a)^{3/2} + (a + \dot{a}\Delta t)^{3/2}]}{2}. \tag{6}$$

Implementation of these equations in the digital computer resulted in predicted positions up to nine days before and after epoch which satisfied the SPAD specifications for all satellites, including those whose $\dot{a}$ terms were on the order of 10^{-3} e.r./day. The results have been tabulated in Appendix C where the reference position coordinates are compared to the coordinates obtained using the above equations and also to those obtained when the $\dot{a}$ was not taken into account.

For an illustration of this approach, reference is made to Fig. 2(c). The time axis is shown here to be divided into P_{avg} equal increments. This is equivalent to evaluating Eq. (5) above. Although the time interval from the apparent perigee point τ' to prediction time t is longer than the actual elapsed time from true perigee τ, this time interval is taken as a portion of a period which is larger than the actual period. Thus, the resulting fraction can conceivably be quite close to the actual value of the fraction. In the situation illustrated, the fraction obtained is only 0.056 rad different from the actual value. However, in practice, this difference could be greater for some cases, but from the good results obtained in position coordinates, it appears safe to assume that the above method does provide a good approximation to the true value for M.

A tabulation of the reference values for M calculated from the orbital element releases for two satellites are shown in Table 1, along with those

TABLE 1
Comparison of Reference and Calculated Values of Mean Anomaly M

Time	Reference M (radians)	M from P_{avg} (radians)	Error (radians)	M from P_{const} (radians)	Error (radians)	$\dot{a}$ (e.r./day)
Epoch (T_0)	0	0	0	0	0	-10^{-3}
$T_0 +$ 7 Days	0.6872	0.7163	0.0291	0.4196	0.2676	-10^{-3}
$T_0 +$ 10 Days	3.5272	3.6184	0.0912	3.0018	0.5254	-10^{-3}
Epoch (T_0)	0	0	0	0	0	-10^{-5}
$T_0 +$ 7 Days	5.7397	5.7364	0.0033	5.7226	0.0171	-10^{-5}
$T_0 +$ 14 Days	0.5444	0.5373	0.0071	0.4819	0.0625	-10^{-5}

obtained by the above method and those obtained when the variable nature of the period was not taken into account. The computations for M were performed for two separate prediction times, and the differences from the reference values are listed for both methods.

It is not the intent of this discussion to propose that the method used in SPAD is the only one, or even the best one, for finding a good value for the mean anomaly. The intent is rather to illustrate and indicate a method which has been used and which has yielded satisfactory results within the limits of the original specification. Other techniques have been suggested and investigated to a certain extent; however, they either were so lengthy as to not be compatible with the speed and memory available in the computer, or their merit could not be defined conclusively. For other methods, reference can be made to Refs. 10 and 11.

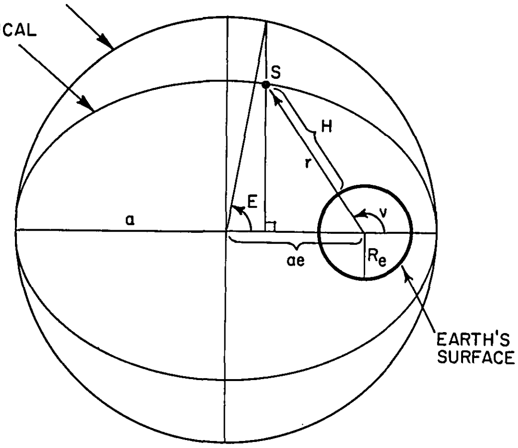

Fig. 3 — Illustration of geometrical quantities pertinent to determination of satellite's position in its orbit: E = eccentric anomaly, v = true anomaly, e = eccentricity, a = semimajor axis, H = satellite's height above a perfectly spherical earth; R_e = earth's mean radius, and r = satellite's distance from the center of the earth

POSITION COMPUTATION

In order to find the position of a satellite in its orbit, it is necessary that the angle designated as the eccentric anomaly E be known. For a geometrical interpretation of this angle, refer to Fig. 3. Since the construction technique of this figure and the reasons thereof are beyond the scope of this report, the reader is referred to Refs. 8 and 9 for this information. The angle E can be determined from the equation

$$E = M - e \sin E \tag{7}$$

where M is the value for mean anomaly found from Eqs. (5) and (6), and e is the eccentricity of the elliptical orbit. This transcendental equation in E is known as Kepler's equation. Although there are many methods for solving this equation (Refs. 8 and 9), an iterative procedure seemed best suited for computer implementation.

As in most iterative processes, a good first approximation of the answer reduces considerably the number of iterations to be performed. Thus, in the SPAD program, the value of E found for subsatellite position computation at time t is stored in the computer memory and is then subsequently used as a first approximation in the iterative process for the next position computation of the same satellite at a new time. The new value of E so obtained is then stored in the memory for use in the next computation, *etc.* Now, since updating of satellite positions in the computer occurs rather rapidly, the values of E for any particular satellite do not change to any large extent from computation to computation. Thus, the iterative process is effectively reduced due to the fact that the first approximation of E is generally quite close to its correct value at the new time.

The desired accuracy for the value of E is obtained when two consecutive iterations of Kepler's equation yield values of E which differ from each other by no more than some small value epsilon. The value of epsilon chosen for SPAD is approximately 1.19×10^{-7} revolution. However, in those cases where the iterations do not converge to the difference value given by epsilon, a maximum of sixty-four iterations are performed. For this condition, the accuracy in the value of E so obtained obviously cannot be defined. From practical experience this latter situation has never arisen; nevertheless, since the possibility for nonconvergence does exist, the computer, upon encountering such a condition, would iterate endlessly in a search to satisfy epsilon unless a limit to the number of iterations which can be performed was imposed.

With the value for E obtained from Kepler's equation, the polar coordinates r and v of a satellite in its orbital plane can be determined. The radius vector r is given as

$$r = a\ (1 - e \cos E) \text{ earth radii} \tag{8}$$

where a is the semimajor axis of the orbit. The true anomaly v can be found from the following relationships:

$$\cos v = \frac{(\cos E) - e}{1 - e \cos E} \tag{9}$$

and

$$\sin v = \frac{(1 - e^2)^{1/2} \sin E}{1 - e \cos E}. \tag{10}$$

Both forms are used in the computer program in order to determine the proper quadrant of v.

The satellite's height above a perfectly spherical earth is determined by the simple relationship

$$H = r - R_e \quad \text{earth radii} \tag{11}$$

where R_e is taken as the earth's mean radius, in earth radii. The quantities H, r, and v are shown in Fig. 3.

The following equations are used to convert the polar coordinates of a satellite's position in its orbit to earth-oriented or geocentric coordinates. Details for the derivations of these equations are contained in an NRL report written by G. Hall (Ref. 6).

The equation for determining the earth latitude ϕ of a subsatellite point can be written as

$$\phi = \arcsin\ [(\sin i) \sin (v + \omega_t)] \tag{12}$$

where ω_t is the argument of perigee at the prediction time t. The quantity ω_t must be adjusted to take into account the perturbation mentioned in a previous section, namely the rotation rate of perigee $\dot{\omega}$. Considering $\dot{\omega}$ to be constant, ω_t can be found from

$$\omega_t = \omega + \dot{\omega}\,(t - T_0) \tag{13}$$

where ω is the argument of perigee at epoch time T_0, and the quantity $t - T_0$ is the time elapsed from epoch time T_0 to prediction time t. An equation found in Ref. 7 gives an approximate value for $\dot{\omega}$ as

$$\dot{\omega} \approx +5\ (R_e/a)^{7/2} \frac{(5 \cos^2 i - 1)}{(1 - e^2)^2} \text{ degrees/day.} \tag{14}$$

The subsatellite geocentric longitude θ can be found from

$$\theta = \beta + \Omega_t \tag{15}$$

where Ω_t is the longitude of the ascending node at time t, and β is an intermediate quantity defined by the equation

$$\beta = \arcsin \left[\frac{(\cos i) \sin (v + \omega_t)}{\cos \phi}\right]. \tag{16}$$

The longitude of the ascending node, being expressed as a geocentric coordinate, is affected by the rotation rate of the earth ω_e as well as the precession of the orbital plane $\dot{\Omega}_R$, which is a perturbation quantity. Considering this precession to be of constant angular velocity and rotating about the same axis as the earth's rotation, Ω_t is found from the equation

$$\Omega_t = \Omega - (\omega_e - \dot{\Omega}_R)\ (t - T_0) \qquad (17)$$

where Ω is the longitude of the ascending node at epoch time T_0, and $(t - T_0)$ is the elapsed time since epoch. From Ref. 7, the precession of the orbital plane $\dot{\Omega}_R$ is given approximately by

$$\dot{\Omega}_R \approx -10\ (R_e/a)^{7/2} \frac{\cos i}{(1-e^2)^2} \text{ degrees/day.} \qquad (18)$$

The three quantities H, θ, and ϕ thus uniquely define a satellite's position in three-dimensional space with reference to the earth's coordinate system. The AN/UYK-1 digital computer utilized with the SPAD equipment performed the foregoing equations for eleven satellites in 1.1 seconds. This time for computation includes the transferring of data, preparation and packing of both the position and the symbol output words, and readout to the display. The accuracy of the results, as shown in Appendix C, indicate that the original specifications were adequately met.

AZIMUTH AND ELEVATION

The azimuth and elevation angles may be used in pointing a shipboard antenna at the satellite. Equations for computing these angles are derived in Appendix D and make use of the following two assumptions: The earth is considered as a perfect sphere, and the point on the earth's surface from which the observation is made is located on a stable platform. These assumptions reduced the problem to one of rather simple spherical trigonometry. The assumptions are further justified since the inclusion of these angles as part of SPAD output was merely to demonstrate a possible feature, and not necessarily to provide the most accurate results for a shipboard environment.

The readout of this information in the SPAD equipment is accomplished via a page printer. The azimuth and elevation angles are computed to several decimal places; however, they are printed out in only whole numbers of degrees in order to conserve space on the printout message. The range of azimuth is from 0 to 359 degrees. Elevation is given as an angle from 0 to 90 degrees. If the satellite is below the horizon, three N's are printed. Obviously, readout in page printer form is not optimum in an operational environment. However, it does not appear too difficult a task to provide the information electronically to appropriate servo systems for automatic antenna pointing when desired.

For shipboard antennas which are platform stabilized, equations of the type implemented here could conceivably serve for satellite acquisition, provided a wide-beam antenna was used. However, the trend towards larger antennas, in some cases, has caused platform stabilization techniques to become prohibitive. The nonstability of the platform, due to the pitch, roll, and yaw of the ship, can be compensated for by so-called data stabilization or by beam steering. In these methods the coordinate transformations due to pitch, roll, and yaw are taken into account in the computation procedures for the pointing angles. Since the transformations of vector rotations are not commutative unless the rotations are infinitesimally small, the individual contributions of pitch, roll, and yaw must be taken in very small increments, with the subsequent transformations performed many times in order to determine the true orientations of the platform. It is evident, then, that a complex set of equations must be solved in a complex high-speed computer for correct pointing angles to be obtained in real-time situations. The memory limit and the speed of the AN/UYK-1 computer is such that a coordinate transformation procedure could not be implemented along with all the other computer tasks for SPAD.

The equations* for determining the azimuth angle α are as follows:

$$\alpha = \arcsin \left[\frac{(\cos \phi_s) \sin (\lambda_s - \lambda_0)}{\sin \beta} \right] \qquad (19)$$

and

$$\alpha = \arccos \left[\frac{\sin \phi_s - (\sin \phi_0)(\cos \beta)}{(\cos \phi_0)(\sin \beta)} \right] \qquad (20)$$

where ϕ_0 is the ship's latitude, λ_0 is the ship's longitude, ϕ_s is the satellite's latitude, and λ_s is

*Both forms of the equations for α are used in the computer in order to determine the correct quadrant for α.

the satellite's longitude. The angle β is obtained from the equation

$$\beta = \text{arc cos } [(\sin \phi_s)(\sin \phi_0) + (\cos \phi_s)(\cos \phi_0) \cos (\lambda_s - \lambda_0)] \tag{21}$$

and is that angle formed by the satellite's radius vector r and a line from the ship to the earth's center. All the foregoing quantities can be easily identified from Fig. D1 of Appendix D.

Since, for SPAD, the azimuth α is defined as an angle measured clockwise from the North Pole ($0 \leq \alpha < 360$ degrees), in certain cases the angle which is read out to the page printer is obtained from

$$360° - \alpha \tag{22}$$

where α is the quantity found from Eqs. (19) and (20). The diagram of Fig. D1 depicts just such a situation.

The elevation angle θ can be obtained from

$$\theta = \psi - 90° \tag{23}$$

where

$$\psi = \text{arc sin } \left(\frac{r \sin \beta}{\rho}\right). \tag{24}$$

The slant range ρ is given as

$$\rho = (r^2 + R^2_e - 2rR_e \cos \beta)^{1/2} \tag{25}$$

where r is the satellite's radius vector, R_e is the earth's mean radius, and β is the angle defined by Eq. (21). The elevation angle and other pertinent quantities are shown in Fig. D2. If the angle ψ is less than 90 degrees, resulting in a negative angle for elevation as is the case shown in Fig. D3, three N's are sent to the page printer indicating that the satellite is below the horizon. More detailed treatments, *e.g.,* those that take the earth's oblateness into account, can be found in Refs. 10 and 11.

SATELLITE AREA OF VIEW

The determination of the area on the earth's surface as viewed by a satellite necessarily requires information concerning the satellite's sensor. In view of the fact that SPAD is a research tool designed for feasibility and desirability studies, the details of how this sensor information would or could be obtained, as well as the significance to be attached to such information, was of no concern to the SPAD project personnel. Thus, given the appropriate information, SPAD personnel were required only to provide the ability for displaying an area, about the subsatellite point, denoting the area viewed by a satellite.

A special purpose analog computer, which provides the necessary voltage waveforms for presenting an area about the subsatellite point on the display CRT, requires only the radius d of the area and the target coordinates (Ref. 12). The equations which follow, and which were implemented in the digital computer for determining this radius d, are based upon a known look-cone angle. This angle is thus required as part of the orbital element message received for each satellite. When this angle is not known, it is assumed to be a maximum of 180 degrees. A spherical earth model is used in the derivation given in Appendix E.

The radius d of the area viewed by the satellite is obtained from

$$d = R_e \left\{\text{arc sin } \left[\frac{r \sin (\gamma/2)}{R_e}\right] - \gamma/2\right\} \tag{26}$$

where r is the satellite's radius vector, R_e is the earth's mean radius and γ is the look-cone angle. When

$$\frac{r \sin (\gamma/2)}{R_e} > 1, \tag{27}$$

the radius d is found from the relation

$$d = R[(\pi/2) - \text{arc sin } (R_e/r)]. \tag{28}$$

The first condition is shown in Fig. E1, Appendix E, and depicts a situation where the look-cone angle subtends an arc on the earth's surface. The second condition, shown in Fig. E2, illustrates the situation where the look-cone angle is such that the satellite's line of sight does not intersect with the earth. Thus, the angle formed by the lines from the satellite and tangent to the earth's surface is found, from which the horizon-to-horizon radius is then computed. It is evident that with unknown look-cone angles, the assumed angle of 180 degrees will always present the latter situation as shown in Fig. E2.

CONCLUSIONS

The mathematical procedure for computing the subsatellite positions as implemented in the SPAD digital computer has been presented. Although the equations are similar to those published in previous NRL reports, the inclusion of the time rate of change of the semimajor axis $\dot{a}$ and the particular method used for computing the mean anomaly M permits the computation of positions accurate to 1 degree for those satellites whose semimajor axis is changing by as much as 10^{-3} earth radii per day. A tabulation of results has been given which demonstrates the position accuracy obtained for four different satellites whose change in semimajor axes ranged from 10^{-3} to 10^{-7} earth radii per day. Position data was computed at epoch and over a period of 3, 6, and 9 days from epoch.

During the several months in which the equipment has been used, forty-eight satellites have been available for display. Whenever ephermeris data was available on satellites other than those tabulated in this report, positions obtained from the SPAD equipment compared favorably and were consistent with the findings reported here. It is, therefore, felt that although a rigorous theoretical treatment of orbital dynamics was not performed, the computational procedure followed by SPAD provides satellite position predictions adequately for display purposes.

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Appendix A
SPAD RAW DATA CONVERSION

Before the NRL Satellite display SPAD can predict satellite positions, the computer store must receive orbital characteristics and other data in a format compatible with the operational program of the SPAD computer (see Table A1). Since the data is not presently available in the desired format, it is necessary to perform a data conversion for purposes of test and evaluation. This conversion process will now be described.

Six quantities are needed to describe the path of a planet through space. One particular set, listed in a book by W. M. Smart,* is: a – semimajor axis, e – eccentricity, T – epoch time, i – angle of inclination of the orbital plane, $\bar{\omega}$ – longitude of perihelion, and ϵ – mean longitude at the epoch. Since an artificial satellite moves in a central force field about the earth in a manner not unlike the motion of a planet around the sun, we would also expect that six quantities should be sufficient to specify its orbit. Unfortunately, this is not entirely true. In dealing with planets, we must realize we are on one planet traveling about the sun trying to specify the position of another planet also traveling about the sun. With an artificial satellite, however, we are trying to specify position about an earth which, for all practical purposes, excluding rotational motion, is a stationary reference system. In addition, we must consider second-order effects such as extraterrestial gravitation, the earth's oblateness, atmospheric drag, and radiation pressure factors which are either nonexistent or negligible in the case of planetary motion. The net effect is that the quantities, or elements as they are sometimes called, will have some slightly different meanings when applied to artificial satellites. We will need at least six elements, possibly more, to specify an orbit, depending upon the accuracy required.

To enable scientific workers to attain the high accuracy necessary in experimental work with artificial satellites, data is available in the form of orbital element lists containing accurate information on the many parameters needed to specify orbits. The particular elements of the present study have been chosen to simplify computational procedures and yet to provide the accuracy required. The elements are:

- a – semimajor axis of the elliptical or circular orbit
- e – eccentricity of the elliptical or circular orbit
- i – inclination of the orbital plane to the earth's equatorial plane
- $\dot{a}$ – time rate of change of semimajor axis
- ω – argument of perigee
- Ω – longitude of the ascending node
- T – epoch time (time at which the foregoing values are true).

The first three elements determine the shape and orientation of the orbit, while the fourth describes the decay associated with atmospheric drag. The last three deal more specifically with positioning of the satellite at some place in the orbit and, for SPAD, must be modified to the passing of perigee. This process will be described later in more detail.

Data for each satellite must be recieved in a format compatible with the SPAD Prediction Computer and Program. This format (Table A1) consists of nine data words, of 30 bits each, containing the seven orbital elements described above, as well as additional quantities which enable SPAD to display ancillary information such as satellite category, SPADAT number, *etc.*, pertaining to each satellite.

The first data word carries two parcels of information: satellite category and look-cone radius angle. For the present, due to the experimental nature of SPAD, responsibility for placing a satellite in one category and not in another is assumed by members of the project. The first data word is coded and the data word bits are arranged in the order shown:

```
30                   15       9       1
 000 000 000 000 000  000 000 000 000 000.
```

Category bits are numbered 15 through 30, while look-cone angle bits are 1 through 9. To designate a particular category, a logical ONE is placed in one, and only one, of the category bit positions in

---

*W.M. Smart, "Textbook on Spherical Astronomy," 4th ed. (rev.), Cambridge:The University Press, 1956.

TABLE A1
Satellite Raw Data

Data Word (30 bits each)	Use
1	Most significant 16 bits—Satellite category indicator Least significant 9 bits—Look-cone angle, scaled 2^7
2	Least significant 16 bits—SPADAT number in binary-coded decimal (BCD)
3	T_0 — Epoch modified to passing of perigee, in seconds from beginning 1960, scaled 2^{29}
4	a — Orbit semimajor axis, in earth radii, scaled 2^4
5	e — Eccentricity, scaled 2^0
6	i — Angle of inclination, in radians $\geqslant 0$, scaled 2^3
7	ω — Argument of perigee, in revolutions $\geqslant 0$, scaled 2^0
8	Ω — Longitude of ascending node, in revolutions $\geqslant 0$, scaled 2^0
9	$\dot{a}$ — Time rate of change of semimajor axis, in earth radii per second, scaled 2^{-25}

accord with the list shown in Table A2. Thus, a ONE in bit position 24 would designate a USA communications satellite, while a ONE in bit position 20 characterizes a Friendly Navigation type. The information contained in the first nine bit positions is used by SPAD to paint a circle around the particular satellite called up by track number. This circle will be of different radius depending upon both the look-cone angle and the height of the satellite above the earth. It is assumed that, at a future date, this data will represent an area which can be viewed or photographed from a particular satellite. One example would be the display of an area or cloud formation photographed by a weather satellite. For the present, however, artificial data is substituted to test the equipment and its capabilities.

The second data word is the SPADAT number in binary-coded decimal (BCD) form, right adjusted to conform to the SPAD Program. The right-most, or low order, 16 bits carry the required information. The high order bits 17 through 30 are logical ZERO. This process may be best illustrated by an example. Consider the decimal number 973. In BCD form this would be

00 0000 0000 0000 0000 1001 0111 0011,

with the binary equivalent word being

000 000 000 000 000 000 100 101 110 011.

As an additional example, if we consider the decimal number 441, in BCD form we would have

00 0000 0000 0000 0000 0100 0100 0001,

with the binary equivalent word being

000 000 000 000 000 000 010 001 000 001.

The third data word is the epoch, modified to passing of perigee and expressed in seconds from

TABLE A2
Bit Positions for Data Words
(First Data Word is Illustrated)

Bit Number	Information Contained	
30	Unknown Category	
29	Other Category	
28	Other Communications	
27	Friendly Other	
26	Friendly Communications	
25	USA Other	
24	USA Communications	
23	Other Weather	
22	Other Navigation	Category
21	Friendly Weather	
20	Friendly Navigation	
19	Friendly Scientific	
18	USA Weather	
17	USA Navigation	
16	USA Scientific	
15	Other Scientific	
14	Logical Zero	
13	Logical Zero	
12	Logical Zero	
11	Logical Zero	
10	Logical Zero	
9	Logical Zero	
8	Integer	
7	Binary fraction	
6	Binary fraction	
5	Binary fraction	Look-Cone Angle
4	Binary fraction	
3	Binary fraction	
2	Binary fraction	
1	Binary fraction	

the beginning of January 1960. This is an arbitrary reference date chosen on the basis of its position in time—early enough to include all satellites of interest, and yet late enough to keep the number of elapsed seconds within a reasonable range for computation. To obtain this number we proceed as follows. From the orbital element list we obtain the epoch date in modified Julian days referenced to midnight. To this number we must add 2,400,000.5 days to obtain the Julian date of the epoch. In "The American Ephemeris and Nautical Almanac," Table I, Julian Day Number, the corresponding Julian date for noon, January 1, 1960, is 2,436,934.0 days, to which date one-half day must be added to advance the time to midnight, January 1. Therefore, the beginning of January 1960 corresponds to a Julian date of 2,436,934.5 days. To find the elapsed time in seconds since 1960, we subtract 2,436,934.5 from the Julian date at epoch, the result being the number of elapsed days. The product of the number of elapsed days and 86,400 sec/day gives the required time in seconds. This quantity is designated by T. It is not necessarily the time of passing of perigee τ. We are interested in τ and must pass through several steps in calculating this quantity. First, using quantities found on the element sheets, the mean anomaly M is calculated by means of the equation

$$M = L - \omega - \begin{bmatrix}\text{algebraic sign} \\ \text{of } \cos i\end{bmatrix} \Omega \qquad \text{(A1)}$$

where

M = mean anomaly, in degrees, at epoch T
L = mean geocentric longitude
ω = argument of perigee
i = angle of inclination
Ω = right ascension of the ascending node.

These and other quantities characterizing an orbit are further defined in many texts on the subject.*†

The mean anomaly found by Eq. (A1) will represent up to one orbit, will be either positive or negative depending on the relative magnitudes of the quantities involved, and will be expressed in units of degrees. The quantity M can also be expressed in revolutions within the orbit plane, with perigee to perigee representing one revolution. Depending upon its sign, M will be used in different ways to determine the passing of perigee τ nearest to epoch time T. The procedure outlined in Figs. A1 and A2 is used to determine the smallest interval of time $\Delta T = (T - \tau)$ required to

*F.R. Moulton, "An Introduction to Celestial Mechanics," 2nd ed., (rev.) New York: MacMillan, 1935.
†W.M. Smart, op. cit.

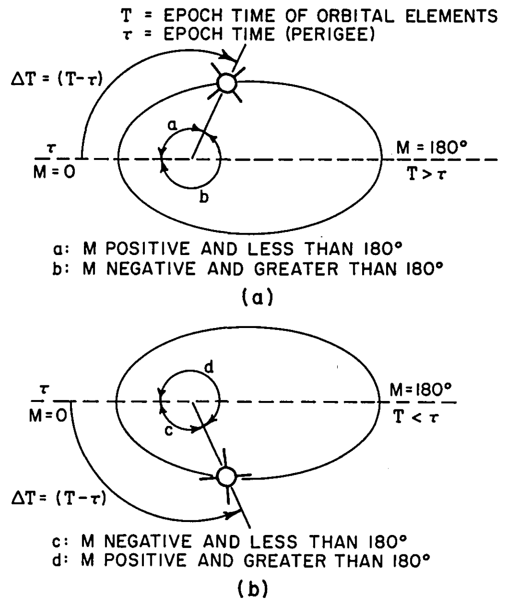

Fig. A1 – Illustration of two cases encountered in bringing the satellite from epoch time T to the epoch τ of the passing of perigee: (a) ΔT is a positive quantity which is subtracted from epoch time T to obtain the nearest passing of perigee; (b) ΔT is a negative quantity which is subtracted from epoch time T to obtain the nearest passing of perigee, and thus the passing of perigee τ occurs later than epoch

bring the satellite from epoch time T to passing of perigee τ, using the anomalistic period P_a, a known element. The value τ so found is then used as SPAD epoch time modified to passing of perigee, *i.e.*, $\tau = T_0$.

Before the third data word can be entered into the machine, it must be scaled and converted to a form the machine can accept.

Although it was not stated explicitly, the first two data words were entered as binary fractions. We must, therefore, convert the third data word, which is a large integral number, into a fractional form of predetermined magnitude capable of fitting into the SPAD operational program. To provide the desired magnitude we must divide by some power of two, in this case 2^{29}, and then convert this fraction to binary. The process of division will henceforth be referred to as scaling. Thus, the third data word was scaled 2^{29} and converted to binary.

The fourth data word is the semimajor axis a given in earth equatorial radii. This information is taken directly from the element sheets, scaled 2^4, and converted to binary.

The fifth data word is the eccentricity e and may be converted to binary with no scaling necessary since the number is a decimal fraction already.

The sixth data word is the inclination angle i of the satellite's plane of motion to the earth's equator. Given in degrees, it must be converted to radians and scaled 2^3. This decimal is then converted to binary for input to the computer.

The seventh data word is the argument of perigee ω, modified to passing of perigee, and stored. The units are in revolutions. The argument of perigee is measured positive from the line of the ascending node γ' to perigee in the direction of satellite motion. The elements give the argument of perigee ω^T measured at epoch. The rate of change of perigee $\dot{\omega}$ is given as a positive or negative quantity. Since we have calculated ΔT (time to nearest perigee) for the second data word, we make use of the equation

$$\omega = \omega^T - \dot{\omega}\,\Delta T \tag{A2}$$

to calculate the argument of perigee at T_0. We have seen that ΔT can be positive or negative, but since we have set our directions by convention, all we need do is combine the signs algebraically to obtain the correct result. This may be seen by referring to Fig. A3, which has been exaggerated for clarity.

From the element sheets we find that $\dot{\omega}$ is positive and therefore advancing in the direction shown in Fig. A3, case I. If we are at epoch time T and this time coincides with passing of perigee τ, or $M = 0$, we have a situation described by case I(b). If we have gone past perigee case I(a), and are at the position shown in the orbit, we have an argument of perigee ω^T greater than ω by an amount $\dot{\omega}\Delta T$. In other words, our line of apsides has rotated through an angle $\dot{\omega}\Delta T$ since perigee. To bring it back we must subtract the amount $\dot{\omega}\Delta T$ from ω^T. If we are at the position given by case I(c), our epoch time has occurred before perigee and we must add an amount $\dot{\omega}\Delta T$ to the argument of perigee to bring us to the situation described by case I(b), which shows passing of perigee.

If we consider negative values of $\dot{\omega}$, we have the situation pictured in Fig. A3, case II. As before, case II(b) represents the satellite at perigee. If the satellite is in the position shown in case II(a), having just gone through perigee, we find ΔT is

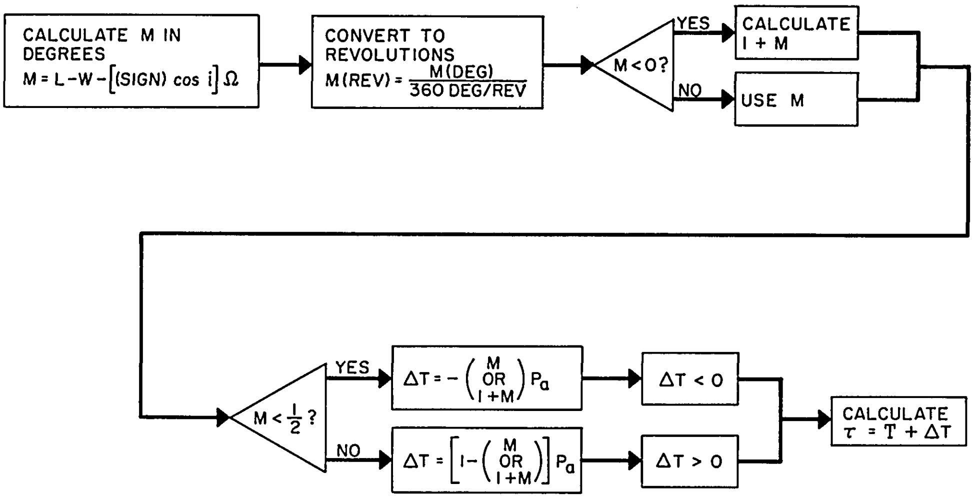

Fig. A2 — Step-by-step procedure used to calculate the epoch τ of the passing of perigee

positive, with the result that $\dot{\omega}\Delta T$ is negative. We see, therefore, that the line of apsides must be rotated by an amount $\dot{\omega}\Delta T$ in the positive direction. In similar manner if we are at a position as described in case II(c), ΔT is negative and $\dot{\omega}\Delta T$ is positive. Thus we see that to arrive at perigee, $\dot{\omega}\Delta T$ must be subtracted from ω^T to find the argument of perigee at perigee. Since this is in decimal fractions of a revolution, no scaling is necessary.

The eighth data word is the longitude of the ascending node in revolutions modified to passing of perigee. The longitude of the ascending node is the angle from Greenwich meridian to the line of the ascending node measured in the equatorial plane. This must be calculated from the right ascension of the ascending node and from the time rate of change of the right ascension of the ascending node.

The right ascension of the ascending node Ω_R is the point in space where the satellite plane intercepts the plane of the earth's equator (the satellite traveling in a northward direction). This point, given in degrees and measured for the epoch time T of the elements, may be converted to Ω^T, the longitude of the ascending node, by the equation

$$\Omega^T = \Omega_R - G_0 \qquad \text{(A3)}$$

where Ω_R is the right ascension of the ascending node for epoch time of the elements, and G_0 is the right ascension of Greenwich. This relationship is graphically illustrated in Fig. A4(a).

The quantity G_0 is found from the table of Universal and Sidereal Times in the "American Ephemeris and Nautical Almanac." The value of G_0 changes with time because of the rotation of the earth. However, due to the fact that the orbital elements are projected to midnight, the tabulated values may be used directly. Looking under the column marked Sidereal Time and finding the value corresponding to epoch in units of hours, minutes, and seconds of arc, we convert these units to degrees using the relations

$$1 \text{ hr} = 15^\circ$$
$$1 \text{ min} = 0.25^\circ$$
$$1 \text{ sec} = 0.004167^\circ.$$

Once we have found G_0, Ω^T may be calculated from Eq. (A3). With reference to Fig. A4(a), we will consider two cases.

Case I: G_0 is greater than Ω_R and both are measured eastward from the vernal equinox γ. In this instance, application of the equation will give a negative value for Ω^T, which is measured in a westerly direction. Since we want the angle measured in a positive or easterly direction, we must take the 360-degree complement as shown.

Case II: G_0 is less than Ω_R and both are measured eastward from γ. In this case Eq. (A3) applies directly.

Now that we have calculated Ω^T, we must compute the longitude of the ascending node Ω at

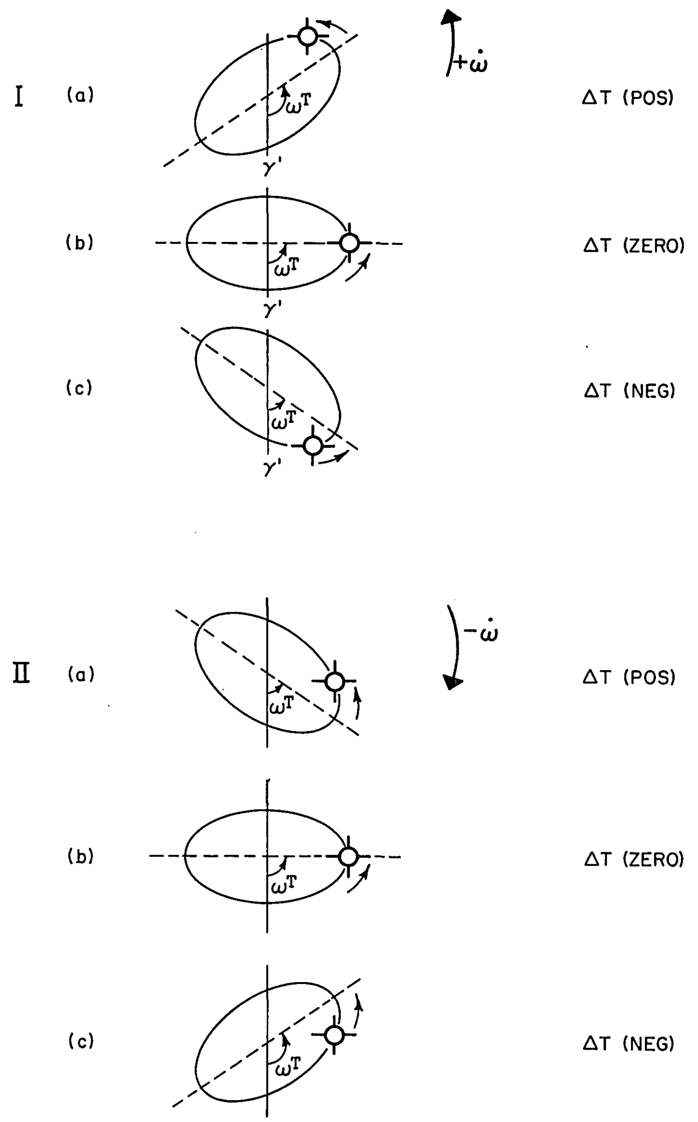

Fig. A3 — Illustration of positive, negative, and zero values for time ΔT to nearest perigee. The argument of perigee at epoch is denoted by ω^T, and γ' is the line of ascending node. Cases I and II illustrate positive and negative rates of change of the argument of perigee, respectively.

the passing of perigee. Since we know the value for the rate of change of the right ascension of the ascending node, $\dot{\Omega}_R$,* the most direct course of action is to convert this rate to the rate of change of the longitude of the ascending node $\dot{\Omega}$. This may be visualized by referring to Fig. A4(b) and to the equation

$$\dot{\Omega} = \dot{\Omega}_R - \omega_e \tag{A4}$$

where ω_e is the rotation of the earth. Consider the two cases where the inclination angle i is greater or less than 90 degrees.

Case I: When i is greater than 90°, Ω_R increases in a positive or easterly direction. Since the earth, and hence the Greenwich meridian, also rotates in an easterly direction, the rotation rate for the longitude of the ascending node $\dot{\Omega}$ is less than

*This quantity, which is available from the orbital element lists and which can be obtained by evaluating Eq. (18) of the text, is also considered as the precession rate of the orbital plane.

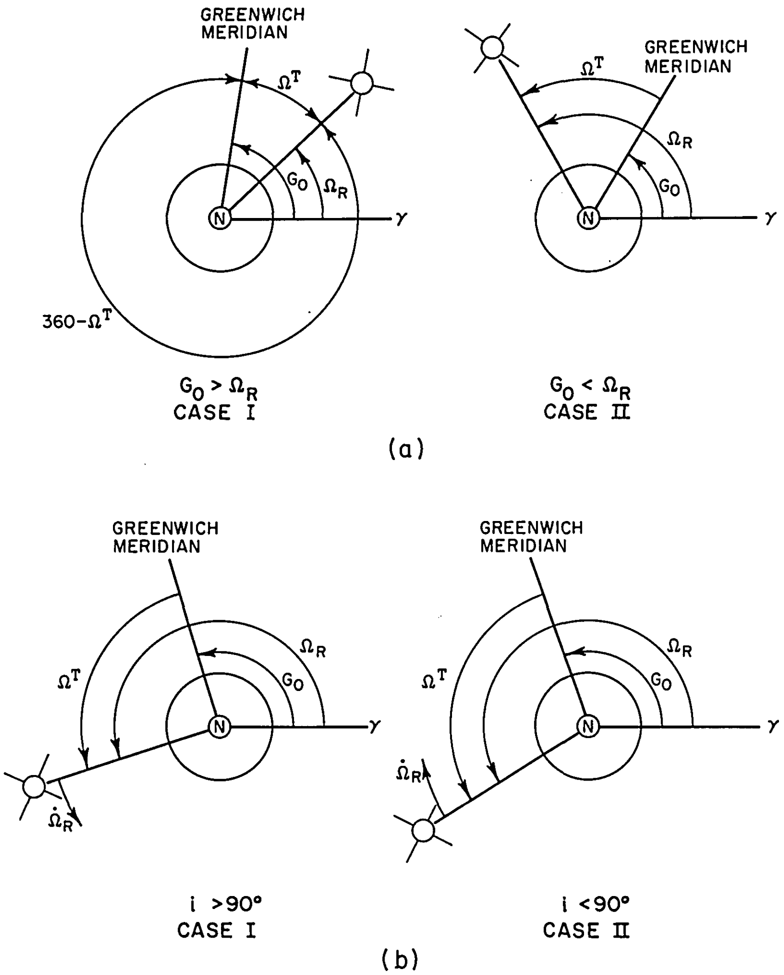

Fig. A4 — Illustration of the relation between the right ascension of the ascending node Ω_R, the longitude of the ascending node Ω^T, and the right ascension G_0 of Greenwich. Ω_R and G_0 are measured counterclockwise (eastward) from the vernal equinox γ when viewed from the north pole N. Equation (A3) and the diagrams in (a) lead to an evaluation of Ω^T; Eq. (A4) and the diagrams in (b) lead to an evaluation of the time rate of change of the longitude $\dot{\Omega}$ of the ascending node at the passing of perigee.

that, $\dot{\Omega}_R$, of the right ascension of the ascending node by an amount ω_e.

Case II: When i is less than, or equal to, 90°, Ω_R decreases in a westerly direction. Since the earth rotates in an easterly direction, the rotation rate of the longitude of the ascending node $\dot{\Omega}$ is greater than that, $\dot{\Omega}_R$, of the right ascension of the ascending node by an amount ω_e.

Now that we have found both Ω^T and $\dot{\Omega}$, and since we already know ΔT, we make use of the equation

$$\Omega = \Omega^T - \dot{\Omega}\Delta T \tag{A5}$$

and find the longitude Ω of the ascending node modified to passing of perigee. As we noted in the seventh data word, both the time-varying quantity and the time interval have an algebraic sign attached to them. Thus, since we have again defined our coordinate system, all we need to do is follow the sign which results upon forming the product $\dot{\Omega}\Delta T$ and evaluate the equations algebraically. This can be seen by the following example.

If $\dot{\Omega}$ from Eq. (A4) is positive, the existing situation is indicated by Fig. A5, case I. When the satellite is at perigee, as indicated in case I(b),

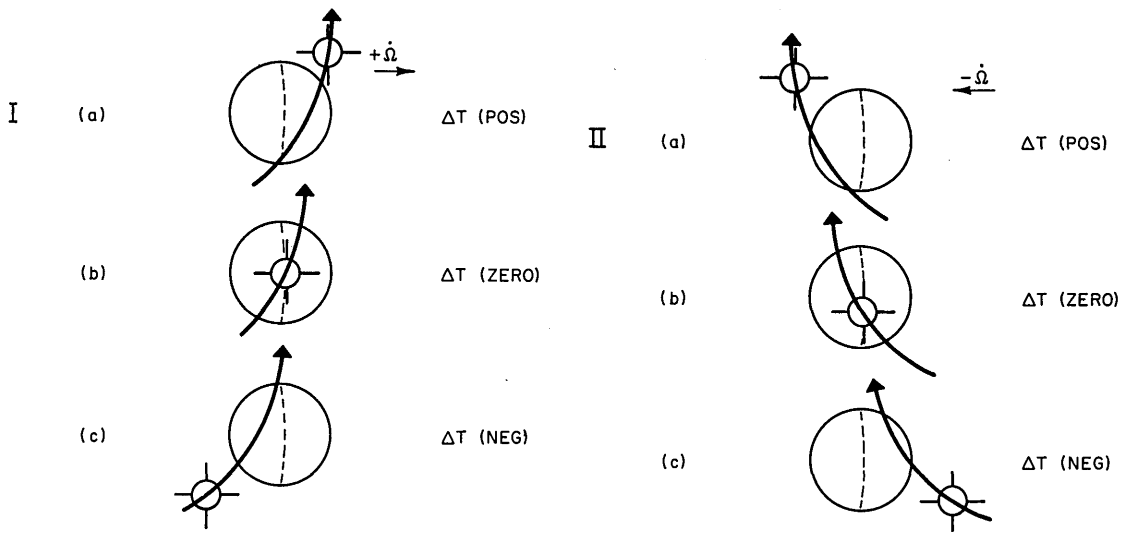

Fig. A5 — Illustration of positive, negative, and zero values for time ΔT to nearest perigee when (a) $\dot{\Omega}$ is positive and (b) $\dot{\Omega}$ is negative

ΔT is zero. When the satellite has gone past perigee, case I(a), ΔT is positive and the plane of the orbit has rotated in the direction shown. Therefore, $\dot{\Omega}\Delta T$ must be subtracted from Ω^T to get Ω. Similarly, when the satellite has not yet reached perigee, case I(c), ΔT is negative and the value $\dot{\Omega}\Delta T$ is in effect added to Ω^T to obtain the correct value for Ω.

On the other hand, should we obtain a negative value for $\dot{\Omega}$ from Eq. (A4), the situation is as indicated in Fig. A5, case II. If the satellite is at perigee, $\Delta T = 0$, case II(b). In the event that the satellite has gone past perigee, as shown in case II(a), ΔT is positive; thus the product $\dot{\Omega}\Delta T$ is negative. Hence, in order to obtain Ω, we in effect add $\dot{\Omega}\Delta T$ to Ω^T. If the satellite is before perigee, the product $\dot{\Omega}\Delta T$ is positive and therefore is subtracted from Ω^T, which brings the plane of motion to that of case II(b), to obtain Ω.

The ninth data word is the time rate of change $\dot{a}$ of the semimajor axis. This may be taken directly from the element sheets and scaled 2^{-25}, whereupon it will be suitable for conversion to binary and for usage in the machine.

Appendix B
LINEAR APPROXIMATION OF PERIOD P

In order to justify using an average value P_{avg} of the period in the computation of subsatellite positions, it was assumed that the actual period varied in a linear fashion over the relatively short time intervals of interest. Since the period actually varies as the 3/2 power of the semimajor axis, as is evident from Eq. 4 of the text, the following procedure was followed in order to determine the maximum deviation between the actual period variation and that of the linear approximation.

In Fig. B1, P_{lin} represents the linear approximation and P_{act} represents the true variation of period. The slope of the straight line P_{lin} is found as

$$m = \frac{P_{t'} - P_{T_0}}{t' - T_0} \tag{B1}$$

where P_{T_0} is the period at epoch time T_0, and $P_{t'}$ is the period at time t'. The equation for the straight line, *i.e.*, the linear period P_{lin} at any time t, is

$$P_{lin} = m\,(t - T_0) + P_{T_0}. \tag{B2}$$

The equation for the actual period P_{act} is given as

$$P_{act} = k\,[a + \dot{a}\,(t - T_0)]^{3/2} \tag{B3}$$

which, by factoring, can be written as

$$P_{act} = ka^{3/2}\left[1 + \frac{\dot{a}}{a}(t - T_0)\right]^{3/2}. \tag{B4}$$

The time t_{max} at which the maximum deviation occurs is determined in the usual manner. Thus, letting $T_0 = 0$, and $P_{T_0} = ka^{3/2}$, ΔP is defined as

$$\Delta P \equiv P_{lin} - P_{act} = P_{T_0} + mt - P_{T_0}\left(1 + \frac{\dot{a}}{a}\,t\right)^{3/2}. \tag{B5}$$

By differentiating Eq. (B5), setting this equal to zero, and solving for t, there results

$$t_{max} = \frac{a}{\dot{a}}\left[\left(\frac{2am}{3P_{T_0}\dot{a}}\right)^2 - 1\right]. \tag{B6}$$

Substituting Eq. (B6) into (B5), in order to find the maximum deviation, yields

$$\Delta P_{max} = \frac{4}{27}\left(\frac{ma}{\dot{a}}\right)^3\left(\frac{1}{P_{T_0}}\right)^2 - \frac{ma}{\dot{a}} + P_{T_0}. \tag{B7}$$

Using the following typical values,

$a = 1.1$ earth radii

$\dot{a} = -10^{-3}$ earth radii per day

$T_0 = 0$

$t' = 9$ days

$k = 0.05867447087$,

from which the following are found,

$P_{T_0} = 0.067\,692\,134$ days

$P_{t'} = 0.066\,863\,069$,

results in the maximum deviation of

$$\Delta P_{max} < 0.001 \text{ percent.} \tag{B8}$$

Evaluating Eq. (B6), it was determined that this maximum deviation occurs approximately 4.7 days from epoch T_0.

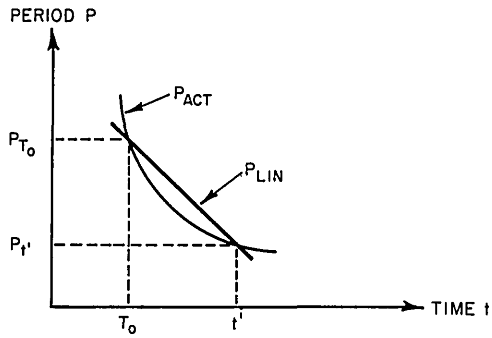

Fig. B1 – Actual and linear-approximation curves for the satellite orbit period P *versus* the time t. T_0 is epoch time.

Appendix C
TABULATION OF SATELLITE POSITION MEASUREMENTS

This Appendix contains tabulations of typical satellite latitude, longitude, and height results obtained from the equations discussed in the text. These quantities have been compiled from orbital data taken over a period of several months. The effect of the modification proposed in this report is illustrated by listing the differences between the sophisticated NAREC predictions, selected as a reference, and the results obtained from the SPAD-implemented equations as reported in this paper. Also included in the tables are results obtained without the addition of the time rate of change $\dot{a}$ of the semimajor axis and the associated mean anomaly M and period P computations.

Table C1 lists the results, taken at the beginning of each day, for the SPADAT satellite No. 564 over a period of 15 days after epoch. It is to be noted that after one week for this satellite, whose $\dot{a}=-10^{-4}$ earth radii (e.r.) per day, position errors are greater than the 1 degree specified if the $\dot{a}$ term is not accounted for in the equations. On the other hand, when the $\dot{a}$ term is used as proposed in the text, errors in position do not exceed 1 degree even after 2 weeks of prediction.

Tables C2, C3, and C4 list the average and maximum differences between the ephemeris of NAREC and SPAD position predictions at epoch and 3, 6, and 9 days after epoch. Three satellites were chosen for these listings in order to provide a reasonable spread of $\dot{a}$ values ranging from -10^{-7} to -10^{-3} e.r./day. The average difference value given in these tables is obtained using position readings taken at 10-minute intervals over one complete orbit. The maximum difference value is the maximum deviation, from the

TABLE C1

Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Latitude, Longitude, and Height. The Effect of Not Including $\dot{a}$ in the SPAD Prediction Equations is Apparent Several Days After Epoch. Data are for SPADAT Satellite No. 564 with $\dot{a} \approx -10^{-4}$ Earth Radii/Day.

Prediction Time (days)	Differences When SPAD Includes $\dot{a}$			Differences When SPAD Does Not Include $\dot{a}$		
	Δ Lat (degree)	Δ Long (degree)	Δ Hgt (stat mi)	Δ Lat (degree)	Δ Long (degree)	Δ Hgt (stat mi)
Epoch (T_0)	0.0	0.1	1	0.0	0.1	1
T_0 +1	0.1	0.1	0	0.0	0.0	0
T_0 +2	0.0	0.1	1	0.3	0.2	0
T_0 +3	0.0	0.1	0	0.5	1.0	1
T_0 +4	0.1	0.2	0	0.6	2.1	3
T_0 +5	0.2	0.0	0	1.5	1.6	7
T_0 +6	0.2	0.1	0	1.8	1.7	8
T_0 +7	0.1	0.0	1	3.1	2.5	5
T_0 +8	0.1	0.2	1	2.6	7.1	3
T_0 +9	0.2	0.2	1	2.1	10.9	14
T_0 +10	0.3	0.1	1	6.3	6.8	26
T_0 +11	0.3	0.1	1	6.7	6.0	31
T_0 +12	0.4	0.1	1	10.5	9.5	20
T_0 +13	0.0	0.6	0	5.9	25.4	4
T_0 +14	0.4	0.3	0	9.8	23.4	36
T_0 +15	0.6	0.0	1	16.0	12.6	58

reference position, observed during that orbit. It should be noted in Tables C2 and C3 that it would not have been necessary to include the $\dot{a}$ term in the position computation in order to meet the 1-degree accuracy requirement. However, the necessity of including this term is clearly shown in Table C4, where the error after 9 days reached a maximum of 101 degrees in longitude for this satellite.

Tables C5, C6, and C7 list the differences in height from the reference value, above the earth's surface for the same three satellites. The readings were obtained for the same times as was the position information of the preceding tables.

TABLE C2

Average and Maximum Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Latitude and Longitude. Including the Parameter $\dot{a}$ in the SPAD Prediction Equations Does Not Appear to be Necessary in This Case. Data are for SPADAT Satellite No. 205 with $\dot{a} = -10^{-7}$ Earth Radii/Day.

Prediction Time (days)	Differences When SPAD Includes $\dot{a}$				Differences When SPAD Does Not Include $\dot{a}$			
	Latitude (degree)		Longitude (degree)		Latitude (degree)		Longitude (degree)	
	Avg	Max	Avg	Max	Avg	Max	Avg	Max
Epoch (T_0)	<0.1	0.1	<0.1	0.1	0.1	0.1	0.1	0.1
T_0+3	<0.1	0.1	0.1	0.1	<0.1	0.1	0.1	0.2
T_0+6	0.1	0.8	0.1	0.5	0.1	0.8	0.1	0.5
T_0+9	0.1	0.6	0.3	0.3	0.1	0.6	0.3	0.3

TABLE C3

Average and Maximum Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Latitude and Longitude. Including the Parameter $\dot{a}$ in the SPAD Prediction Equation Does Not Appear to be Necessary in This Case. Data are for SPADAT Satellite No. 285 with $\dot{a} = -10^{-5}$ Earth Radii/Day.

Prediction Time (days)	Differences When SPAD Includes $\dot{a}$				Differences When SPAD Does Not Include $\dot{a}$			
	Latitude (degree)		Longitude (degree)		Latitude (degree)		Longitude (degree)	
	Avg	Max	Avg	Max	Avg	Max	Avg	Max
Epoch (T_0)	<0.1	0.1	<0.1	0.1	<0.1	0.1	<0.1	0.1
T_0+3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
T_0+6	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.3
T_0+9	0.2	0.3	0.2	0.3	0.2	0.4	0.5	0.9

TABLE C4
Average and Maximum Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Latitude and Longitude. The Effect of Not Including $\dot{a}$ in the SPAD Prediction Equations is Apparent Three Days After Epoch. Data are for SPADAT Satellite No. 632 with $\dot{a} = -10^{-3}$ Earth Radii/Day.

Prediction Time (days)	Differences When SPAD Includes $\dot{a}$				Differences When SPAD Does Not Include $\dot{a}$			
	Latitude (degree)		Longitude (degree)		Latitude (degree)		Longitude (degree)	
	Avg	Max	Avg	Max	Avg	Max	Avg	Max
Epoch (T_0)	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
T_0+3	0.2	0.4	0.3	0.5	5.0	7.3	8.6	15.0
T_0+6	0.4	0.7	0.5	1.1	18.7	29.9	35.4	53.6
T_0+9	0.6	1.1	0.8	1.9	28.9	48.5	61.3	101.1

TABLE C5
Average and Maximum Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Height. Including the Parameter $\dot{a}$ in the SPAD Prediction Equations Does Not Appear to be Necessary in This Case. Data are for SPADAT Satellite No. 205 with $\dot{a} = -10^{-7}$ Earth Radii/Day.

Prediction Time (days)	Height Above Spherical Earth (stat mi)			
	Differences When SPAD Includes $\dot{a}$		Differences When SPAD Does Not Include $\dot{a}$	
	Avg	Max	Avg	Max
Epoch (T_0)	1	1	1	1
T_0+3	0	1	0	1
T_0+6	1	1	1	1
T_0+9	0	1	1	1

TABLE C6

Average and Maximum Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Height. The Effect of Not Including $\dot{a}$ in the SPAD Prediction Equations is Apparent Nine Days After Epoch. Data are for SPADAT Satellite No. 285 with $\dot{a} = -10^{-5}$ Earth Radii/Day.

Prediction Time (days)	Height Above Spherical Earth (stat mi)			
	Differences When SPAD Includes $\dot{a}$		Differences When SPAD Does Not Include $\dot{a}$	
	Avg	Max	Avg	Max
Epoch (T_0)	1	1	1	1
T_0+3	1	1	1	1
T_0+6	1	1	1	2
T_0+9	1	1	2	3

TABLE C7

Average and Maximum Differences Between the Ephemeris of NAREC Predicted and SPAD Predicted Values for Satellite Height. The Effect of Not Including $\dot{a}$ in the SPAD Prediction Equations is Apparent Six Days After Epoch. Data are for SPADAT Satellite No. 632 with $\dot{a} = -10^{-3}$ Earth Radii/Day.

Prediction Time (days)	Height Above Spherical Earth (stat mi)			
	Differences When SPAD Includes $\dot{a}$		Differences When SPAD Does Not Include $\dot{a}$	
	Avg	Max	Avg	Max
Epoch (T_0)	0	1	1	1
T_0+3	3	6	4	8
T_0+6	6	13	13	24
T_0+9	10	19	22	41

Appendix D
AZIMUTH AND ELEVATION DETERMINATIONS

AZIMUTH

The computation of the azimuth angle α is basically one of solving the spherical triangle OPN of Fig. D1. The arc QPN represents the longitude λ_s of the subsatellite position, and the arc MON represents the ship's longitude λ_0. The ship is located at point 0, with its latitude shown as ϕ_0. The line CPS is the radius vector of the satellite from the center of the earth, with the subsatellite point located at point P. The latitude of P is shown as ϕ_s. The great-circle arc shown passing through the points 0 and P subtends an angle β, and is thus designated as β radians.

From a relationship of spherical triangles,

$$\cos \beta = (\cos \phi_s')(\cos \phi_0') + (\sin \phi_s')(\sin \phi_0')(\cos \Delta\lambda). \tag{D1}$$

It can be seen in the figure that

$$NP \doteq \phi_s' = \frac{\pi}{2} - \phi_s \tag{D2}$$

$$NO \doteq \phi_0' = \frac{\pi}{2} - \phi_0 \tag{D3}$$

$$\Delta\lambda = \lambda_s - \lambda_0. \tag{D4}$$

Thus, using basic trigonometric identities,

$$\cos \phi_s' = \cos\left(\frac{\pi}{2} - \phi_s\right) = \sin \phi_s \tag{D5}$$

$$\sin \phi_s' = \sin\left(\frac{\pi}{2} - \phi_s\right) = \cos \phi_s, \tag{D6}$$

and similarly

$$\cos \phi_0' = \sin \phi_0 \tag{D7}$$

and

$$\sin \phi_0' = \cos \phi_0. \tag{D8}$$

Hence, Eq. (D1) can be rewritten in terms of the ship's and the satellite's latitude and longitude as

$$\beta = \text{arc cos } [(\sin \phi_s)(\sin \phi_0) + (\cos \phi_s)(\cos \phi_0)(\cos \Delta\lambda)]. \tag{D9}$$

The Law of Sines, as applied to the spherical triangle OPN, states that

$$\frac{\sin \alpha}{\sin \phi_s'} = \frac{\sin \Delta\lambda}{\sin \beta}. \tag{D10}$$

Now, substituting the relation of Eq. (D6) into (D10), the azimuth angle α, expressed in terms of the ship and satellite coordinates, can be found from

$$\alpha = \text{arc sin}\left[\frac{(\sin \Delta\lambda)(\cos \phi_s)}{\sin \beta}\right], \tag{D11}$$

which is Eq. (19) of the text.

The method used for locating the proper quadrant of α required a determination of the cos α. Thus, applying the same basic spherical triangle relationship as was used for Eq. (D1),

$$\cos \phi_s' = (\cos \phi_0')(\cos \beta) + (\sin \phi_0')(\sin \beta)(\cos \alpha). \tag{D12}$$

Using Eqs. (D5), (D7), and (D8), cos α can be found from

$$\cos \alpha = \frac{\sin \phi_s - (\sin \phi_0)(\cos \beta)}{(\cos \phi_0)(\sin \beta)}, \tag{D13}$$

which is Eq. (20) of the text.

ELEVATION

The elevation angle θ is easily found from the plane triangle of Fig. D2. Figure D3 merely serves to relate the elevation triangle to Fig. D1, which was used in the azimuth computation. The line CPS is the radius vector of the satellite and is designated as r. The line OC is the line from the center of the earth to the ship and is obviously

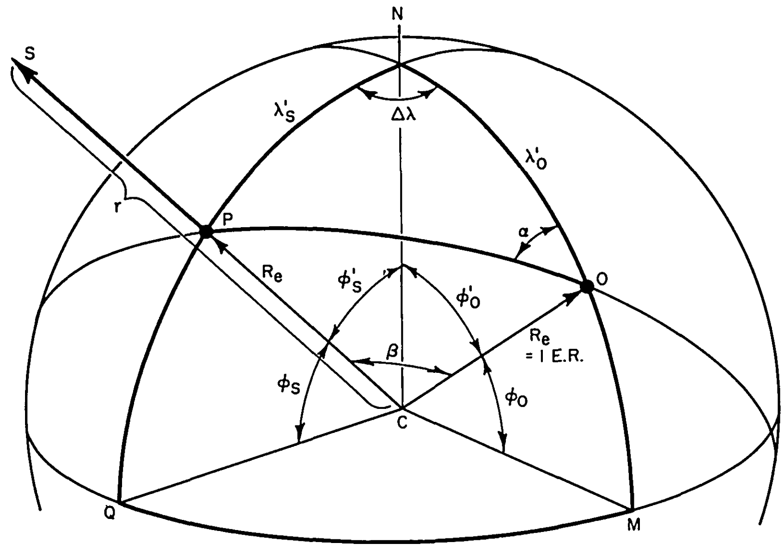

Fig. D1 – Geometric quantities used to calculate the azimuth angle α of the subsatellite point P from the ship's position at 0

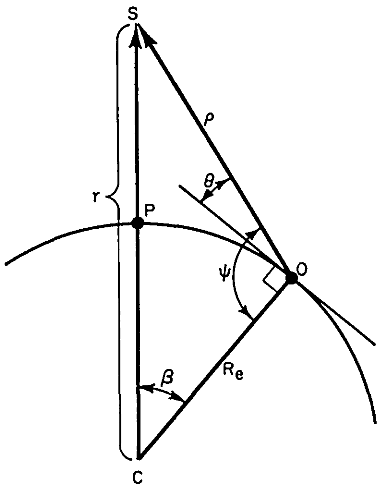

Fig. D2 – Geometric quantities used to calculate the satellite elevation angle θ in terms of the angle ψ

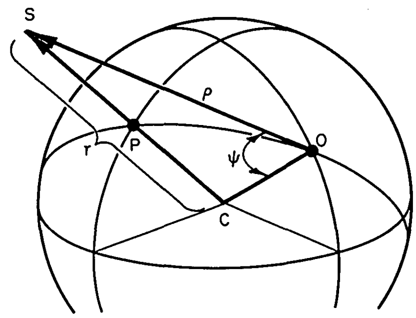

Fig. D3 – Illustration of the geometric relationship between Figs. D2 and D1 the radius of the earth R_e. The line OS is the slant range ρ which is found from the law of cosines as

$$\rho = (R_e^2 + r^2 - 2rR_e \cos \beta)^{1/2}, \qquad \text{(D14)}$$

which is Eq. (25) of the text, and β is the angle formed by the lines CPS and CO which can be obtained from Eq. (D9).

Applying the Law of Sines to the plane triangle CSO yields

$$\frac{\sin \psi}{r} \frac{\sin \beta}{\rho}. \qquad \text{(D15)}$$

Equation (D15) can be solved for ψ to give

$$\psi = \text{arc sin} \left[\frac{r \sin \beta}{\rho}\right], \qquad \text{(D16)}$$

which is Eq. (24) of the text.

It is clear from Fig. D2, then, that the elevation angle θ is simply

$$\theta = \psi - \pi/2 \qquad \text{(D17)}$$

which is Eq. (23) of the text. Figure D4 depicts the special case when ψ is less than $\pi/2$ radians, *i.e.*, the satellite position S is below the horizon. Upon encountering this condition, the SPAD program does not compute the resulting negative angle for elevation, but merely sends out a code word which prints all N's on the page printer.

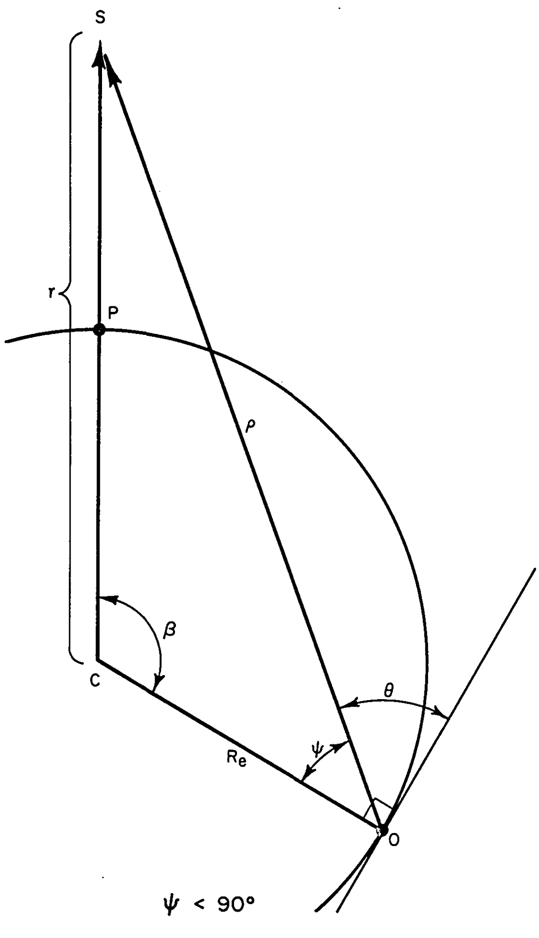

Fig. D4 — Illustration of the elevation angle θ when ψ is less than $\pi/2$

Appendix E
DERIVATION OF THE SATELLITE AREA OF VIEW RADIUS

The radius d of a circle on a perfectly spherical earth is shown in cross section in Fig. E1. In this figure, the line CM is designated as x, which in this case is equal to the earth's radius R_e. The line CPS is the radius vector r of the satellite located at S and can be obtained from Eq. (8) of the text. The angle γ is the satellite's look-cone angle and is a quantity which is required as input data for each satellite. The satellite sensor is also assumed to be pointing along a line perpendicular to the earth's surface.

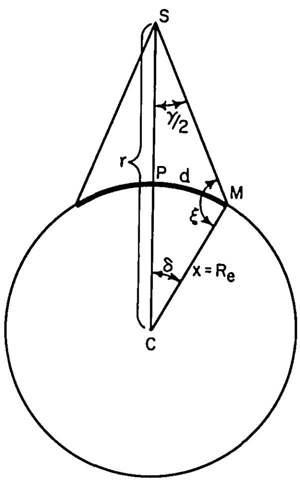

Fig. E1 — Cross-sectional view of the circular area (heavy line) of radius d subtended at the earth by the satellite look-cone angle γ

Applying the Law of Sines to the plane triangle CMS,

$$\frac{r}{\sin \xi} = \frac{x}{\sin (\gamma/2)}. \tag{E1}$$

Solving for $\sin \xi$ gives

$$\sin \xi = \frac{r \sin (\gamma/2)}{x}. \tag{E2}$$

Now since

$$\delta + \gamma/2 = \pi - \xi, \tag{E3}$$

then

$$\sin [\delta + (\gamma/2)] = \sin (\pi - \xi) = \sin \xi. \tag{E4}$$

Substituting Eq. (E2) into Eq. (E4),

$$\sin [\delta + (\gamma/2)] = \frac{r \sin (\gamma/2)}{x}, \tag{E5}$$

and solving for δ gives

$$\delta = \text{arc sin} \left[\frac{r \sin (\gamma/2)}{x}\right] - \gamma/2. \tag{E6}$$

Finally, since

$$d = x\delta \tag{E7}$$

and $x = R_e$, the radius d is found from

$$d = R_e \left\{\text{arc sin} \left[\frac{r \sin (\gamma/2)}{R_e}\right] - \gamma/2\right\}, \tag{E8}$$

which is Eq. (26) of the text.

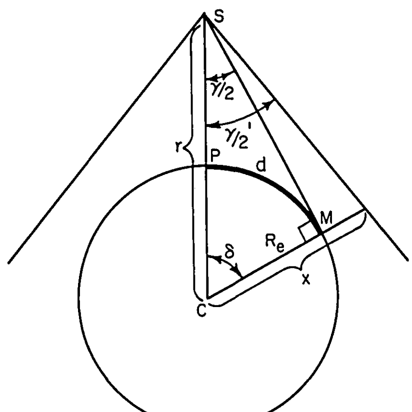

Fig. E2 — Illustration of the case where the earth's surface subtends an angle less than γ. The area (of radius d) shown is the maximum possible amount of the earth's surface that can be viewed from the satellite.

The situation where, due to particular values for r and γ, the earth's surface does not subtend the angle γ is depicted in Fig. E2. (This would also be the case, regardless of the value of r, when γ is unknown and a maximum angle of π is assumed.) The computer tests for the existence of this condition by the relationship

$$\frac{r \sin(\gamma/2)}{R_e} > 1, \qquad \text{(E9)}$$

that is, when $x > R_e$. When this inequality is satisfied, the only meaningful radius is that which corresponds to the line of sight to the horizon. Hence, the line SM is constructed from the satellite point S and tangent to the earth's surface. This line and the line CPS form a new angle, $\gamma'/2$.

Thus, from the right triangle CSM,

$$\gamma'/2 = \text{arc sin } (R_e/r) \qquad \text{(E10)}$$

From the figure it is seen that

$$\delta = (\pi/2) - (\gamma'/2). \qquad \text{(E11)}$$

Hence, by substituting Eq. (E10) into (E11), there results

$$\delta = (\pi/2) - \text{arc sin } (R_e/r). \qquad \text{(E12)}$$

Therefore, since

$$d = R_e\ \delta, \qquad \text{(E13)}$$

the radius d is obtained from

$$d = R_e\ [(\pi/2) - \text{arc sin } (R_e/r)], \qquad \text{(E14)}$$

which is Eq. (28) of the text.

It can be noted that as the satellite point S approaches infinity, $R_e/r \rightarrow 0$. Thus Eq. (E14) becomes

$$d = R_e\pi/2 = 1/4\ (2\pi\ R_e)$$

$$= 1/4 \text{ (earth's circumference).}$$

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13. ABSTRACT

A satellite position prediction and display equipment (SPAD) has been conceived and developed at NRL. Details regarding the operation and performance of the equipment have been described previously in NRL Report 6219. The present report concerns itself primarily with the equations utilized in a digital computer for performing the position prediction computations. With periodically updated orbital elements for each satellite stored in the computer memory, these equations provide position coordinates and height above the earth's surface for any satellite, at any desired time. Restricted by the limited computer speed and memory space available, the equations arrived at, in order to both satisfy the display system requirements and to meet the desired accuracy, are of a degree of complexity which places them between the basic planetary equations of celestial bodies and the much more sophisticated equations of today's space computer centers. Although modifications of the basic equations have previously been developed and reported in NRL Reports 5652 and 5659, they were successful in computing positions of only those satellites whose orbits are relatively stable. The equations of the report, which are extensions of these modified equations, are able to provide geocentric coordinate positions for those unstable satellites whose orbits are decaying rapidly. This extension is accomplished by including the time rate of change of the semimajor axis as an input orbital element and by the manner in which the period is computed.

Results obtained from both the modified and the extended equations are compared to those obtained from NRL's Research Computation Center. When computed for nine days into the future, positions resulting from the extended equations have been obtained with errors ranging from 0.1 degree for stable satellites, to no greater than 1 degree for those satellites whose semimajor axes are decaying at a rate of 10^{-3} earth radii/day. The position coordinates and height of eleven satellites can be computed every 1.1 sec.

In order to minimize the amount of data transmitted via the communication channels, only seven orbital elements are required for each satellite. Certain orbit perturbations, namely, the precession of the node and the rotation of perigee, are computed rather than being obtained from a space computation center. This permits the updating of the raw orbital message with a minimum of data words.

This report contains a derivation of the simplified equations for computing the azimuth and elevation angles of a satellite from a ship's position. Equations for computing the radius of the circle on the earth's surface viewed by a satellite are also included.

DD FORM 1 JAN 64 1473

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