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A FURTHER STUDY OF THE CROSS-EYE COUNTERMEASURE SYSTEM

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CONTENTS

ABSTRACT

A continuation of studies of a countermeasure technique, called the Cross-Eye system, has led to improvements in and simplification of earlier models reported in NRL Report 4661. This countermeasure is a technique which utilizes the large errors which may be introduced in a tracking radar by a pair of echo sources of controlled phase to increase target noise and bias the mean tracking point off the target.

The major efforts of the initial study of the system were directed toward a countermeasure to monopulse tracking radar. The system was not of optimum design for lobing radars, because of nonuniform illumination of the target on transmission. The nonuniform illumination of the target may be compensated for by minor modifications of the countermeasure which make it equally effective against all types of tracking radars. Simplifications have been made in the system which improve its effectiveness while increasing the system bandwidth to where it is limited by the amplifying element only.

PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

NRL Problem R02-13
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A FURTHER STUDY OF THE CROSS-EYE COUNTERMEASURE SYSTEM

INTRODUCTION

A microwave system has been developed which provides a countermeasure effective against all available radar tracking techniques, including the monopulse tracking radar (1). This countermeasure, which has been labeled the Cross-Eye System, takes advantage of the large errors that may be introduced into tracking radars by a pair of echo sources with a 180^0 relative phase. In the initial experimental Cross-Eye system, the echo sources were located at the aircraft wingtips. The 180^0 relative phase was maintained by receiving the radar signal on one wingtip, transmitting through the aircraft to the opposite wingtip, and retransmitting to the radar. By this means, identical paths, except for the 180^0 phase shift in one path, were provided for the two echo sources independent of aircraft yaw and pitch. This technique caused increased radar tracking noise and a bias of the means tracking point off the target.

The initial study of the countermeasure, being limited by available time, was concentrated on the monopulse tracking radar, since this radar has less over-all vulnerability to available countermeasure techniques than the lobing radars. The resulting initial system was found to be not optimum for lobing radars. Since it is intended that the system be equally effective against all tracking radars, the study of the Cross-Eye countermeasure has been continued in this report to show the cause of loss of effectiveness of the initial system with lobing radars, and to describe a simple means of regaining this effectiveness.

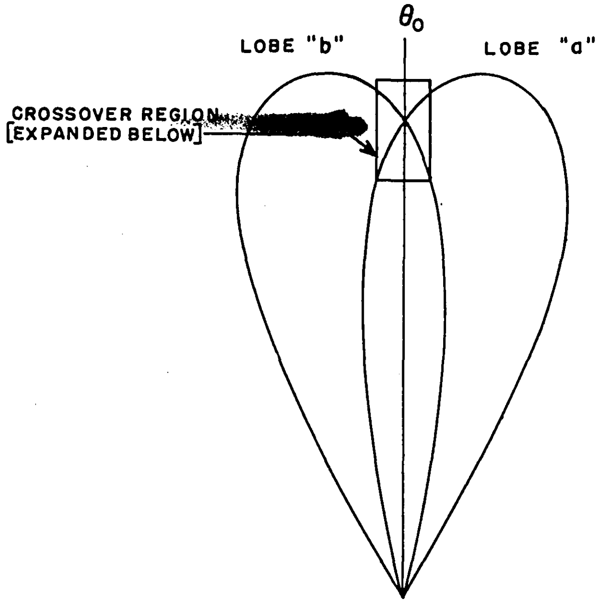
THEORY OF THE CROSS-EYE COUNTERMEASURE WITH MONOPULSE AND LOBING TRACKING RADARS

The effect of the Cross-Eye countermeasure on a lobing radar may be shown to be similar to the theoretical analysis in the initial study (1). The theory is based on the linear portion of the antenna pattern at crossover, as shown in Fig. 1, where only the train angle coordinate is considered. The crossover point (Fig. 1a) is assumed to be a linear pattern (Fig. 1b) with slope k of the output or input of the lobes versus angle θ. The assumptions of linear patterns is not necessary, but the same results are obtained as if some pattern function as $\sin x/x$ were used, and with a minimum of complication.

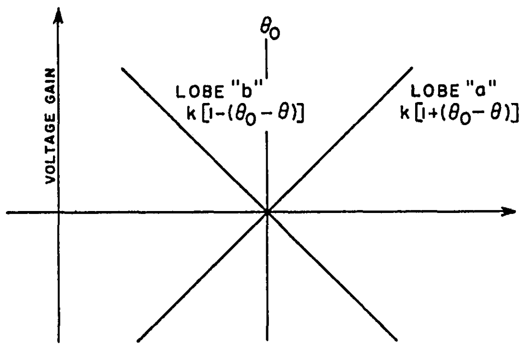
The Cross-Eye system (Fig. 2) provides a signal from two points on a target, θ_1 and θ_2. The relative phase of the two signals is held constant by providing identical triangular paths for each signal, and the phase relation is adjusted to 180^0 by an appropriate phase shift in one path with respect to the other. A third echo, θ_3, is used to represent the aircraft echo in addition to the countermeasure signal.

Figure 3 indicates voltage versus angle for illumination and reception. For the monopulse radar, the target is uniformly illuminated with a signal $2k \cos \omega t$, since power is transmitted simultaneously on both lobes. The signal received at either θ_1 or θ_2 is $2k \cos \omega t$. If path 1 (Fig. 2) retransmits a signal $2k\ A_1 \cos \omega t$ from the angle θ_1 and path 2 retransmits a signal $2k\ A_2 \cos(\omega t+\phi)$ from angle θ_2, the signal received in lobe "a" is

$$e_a = A_1 k[1 + (\theta_0-\theta_1)]\ [2k \cos \omega t] + A_2 k[1 + (\theta_0-\theta_2)]\ [2k \cos(\omega t+\phi)] \qquad (1)$$



(a) True antenna pattern and crossover point



(b) Assumed linear crossover point

Fig. 1 - Antenna pattern

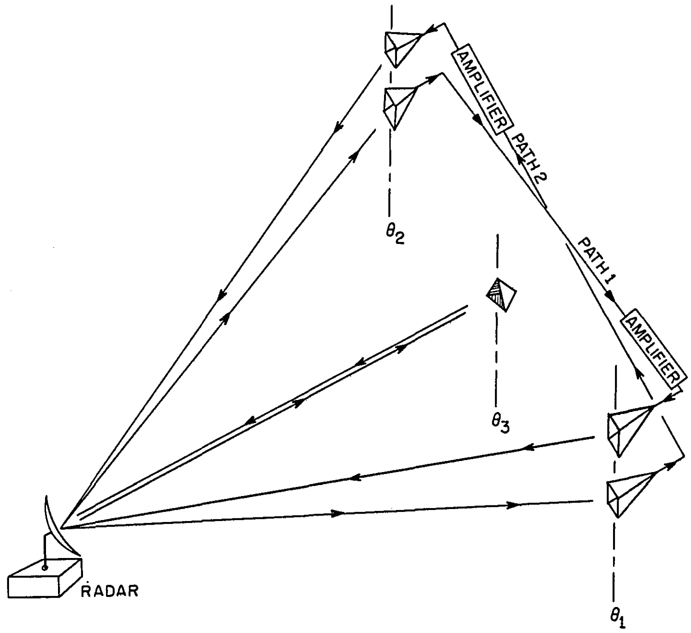
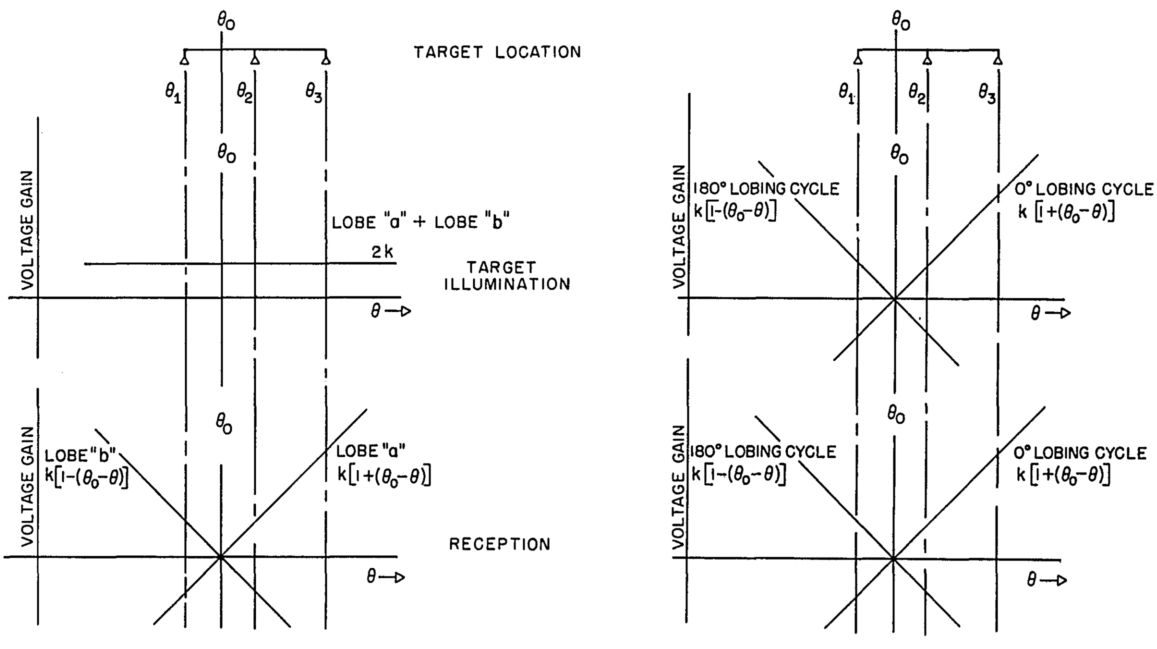


Fig. 2 - Cross-Eye system



(a) Monopulse (b) Lobing

Fig. 3 - Assumed linear crossover for monopulse and lobing radars. (Equations are for one volt received at θ_0)

and the signal received in lobe "b" is

$$e_b = A_1k[1 - (\theta_0-\theta_1)]\ [2k \cos \omega t] + A_2k[1 - (\theta_0-\theta_2)]\ [2k \cos(\omega t+\phi)] \tag{2}$$

where

A_1 = the portion of the received signal in path 1 that is retransmitted

A_2 = the portion of the received signal in path 2 that is retransmitted

ϕ = the relative phase of the two signals from the Cross-Eye system.

If the aircraft return at θ_3 is considered such that the phase of the echo from θ_3 is some relative phase, ψ, with respect to the signal from θ_1, then a term $A_3k[1 + (\theta_0-\theta_3)]\ k \cos(\omega t+\psi)$ is added to lobe "a" and a term $A_3k[1 - (\theta_0-\theta_3)]\ k \cos(\omega t+\psi)$ is added to lobe "b" where A_3 is amplitude of the aircraft echo.

The tracking point for the two and three reflector condition has been solved assuming a perfect servo (1). For the countermeasure signals only (reflector at θ_3 neglected) the tracking point is

$$\theta_0 = \theta_t\left[\frac{1 - a^2}{2(1+a^2+2a \cos\phi)}\right] \tag{3}$$

where

θ_t = span of the two signal sources

a = the ratio A_2/A_1

θ_0 = the tracking point with the midpoint of the countermeasure as the zero reference.

The value of θ_0 has been plotted (1) in a family of curves with a and ϕ as variables. This plot shows how the radar may readily be driven many target spans off the target and cause the radar to break lock.

Adding a reflector at θ_3 to represent the aircraft and setting $\phi = 180^\circ$, the tracking point (1) is

$$\theta_0 = \theta_r \frac{b(1-a) \cos \psi + b^2}{(1-a)^2 + 2b(1-a) \cos \psi + b^2} + \theta_t \frac{1-a^2 + (1+a)\ b \cos \psi}{2[(1-a)^2 + 2b(1-a) \cos \psi + b^2]} \tag{4}$$

where

θ_r = angle from target center to θ_3

$b = A_3/A_1$ (ratio of the reflected signal from θ_3 to the signal from θ_1).

As explained in the initial report on the Cross-Eye system, the first term in Eq. (4) is the normal target noise from the aircraft, and the second term is the noise and tracking error introduced by the countermeasure. The initial report (1) gives plots of tracking point θ_0 versus ψ for selected values of a and b. The maximum tracking error occurs where the target return A_3 is smallest, and maximum tracking noise occurs for values of b approximately equal to the resultant signal amplitude from the Cross-Eye system (1-a).

This review of monopulse tracking of a target with the Cross-Eye system is given to compare with an analysis of lobing radar to show where the two radars differ in the Cross-Eye system analysis. The difference between the monopulse and lobing radars in terms of operation of the Cross-Eye system is in the illumination of the target. A type of lobing radar does exist which lobes on receiving only, while illuminating the radar with a beam on axis, and will give essentially the same result as the monopulse radar. The normal lobing radar, however, transmits on the same lobe with which it receives (Fig. 3). As the antenna beam is lobed to the right, which may be called the zero degree point of the lobing cycle, the target is illuminated by the function $e = k[1 + (\theta_0-\theta)]$, and the echo signal is received on this same antenna pattern. The Cross-Eye system, designed to give identical paths for the two signals returned to the radar, receives on one side of the target and retransmits from the other. Thus (Fig. 2) the signal received at position θ_2 in the lower horn is $k[1 + (\theta_0-\theta_2)] \cos\omega t$ and is retransmitted to the radar at position θ_1, to fall on the $k[1 + (\theta_0-\theta_1)]$ point of the receiving lobe. One may, therefore, set up the equations for the two signals from the Cross-Eye system. For the zero degree point of the lobe cycle,

$$e_a = \{k\ [1 + (\theta_0-\theta_2)]\}\ \{A_1 k\ [1 + (\theta_0-\theta_1)]\} \cos \omega t$$

$$+ \{k\ [1 + (\theta_0-\theta_1)]\}\ \{A_2 k\ [1 + (\theta_0-\theta_2)]\} \cos(\omega t+\phi). \tag{5}$$

Similarly, at the 180° point of the lobing cycle, the antenna beam is lobed to the left and the signal received will be

$$e_b = \{k\ [1 - (\theta_0-\theta_2)]\}\ \{A_1 k\ [1 - (\theta_0-\theta_1)]\} \cos \omega t$$

$$+ \{k\ [1 - (\theta_0-\theta_1)]\}\ \{A_2 k\ [1 - (\theta_0-\theta_2)]\} \cos(\omega t+\phi). \tag{6}$$

These equations may be reduced to give

$$e_a = [1 + (\theta_0-\theta_2)]\ [1 + (\theta_0-\theta_1)]\ [A_1 k^2 \cos \omega t + A_2 k^2 \cos (\omega t+\phi)] \tag{7}$$

and

$$e_b = [1 - (\theta_0-\theta_2)]\ [1 - (\theta_0-\theta_1)]\ [A_1 k^2 \cos \omega t + A_2 k^2 \cos (\omega t+\phi)]. \tag{8}$$

One may observe, in the above equations, that the antenna pattern functions may be factored out of the two received signals from the Cross-Eye system. The relative phase of the signals ϕ and the ratio of their amplitudes will control only the amplitude of the received signal and have no effect on the tracking point. These equations were used in Appendix A to solve for the tracking function at the lobing radar. The tracking point was found to fall at the midpoint of the Cross-Eye system and remains at this midpoint regardless of the values of A_1, A_2, and ϕ.

The lobing radar tracking point has been solved (Appendix B) to show the effect of the reflector at θ_3 representing the target, similarly to the analysis of monopulse. The tracking point was found to be

$$\theta_0 = \theta_r \frac{b^2 + b(1-a)\cos\psi}{(1-a)^2 + 2b(1-a)\cos\psi + b^2}. \tag{9}$$

This function is identical to the normal tracking noise portion of Eq. (4) for monopulse with no additional noise or tracking error.

The above theoretical analysis shows that the initial Cross-Eye technique for use against the monopulse radar is ineffective against the lobing radar. The difference of the two radars, in terms of operation of the Cross-Eye system, was shown to be in illumination of the target. Effectiveness of the Cross-Eye system with lobing radars may be gained by a minor modification of the systems such as the use of a microwave agc to smooth the lobing modulation received in the Cross-Eye system. This operation would cause an effective uniform illumination of the Cross-Eye system, and the same theoretical results will be obtained with the lobing radar as with the monopulse radar.

ANALYSIS OF THE INITIAL EXPERIMENTAL RESULTS OF THE CROSS-EYE SYSTEM WITH LOBING RADARS

During the initial analysis of the Cross-Eye system, the limitation of operation with lobing radar was known. Unfortunately, there was insufficient time to instrument an agc and other desired items. On the S-band lobing radar, however, experimental results with the initial system without instrumentation of an agc did show effective operation against lobing radars, contrary to theory.

It was found that, at close ranges, the amplifiers in the Cross-Eye system began to saturate, which caused a smoothing of the Cross-Eye illumination over the lobing cycle. This action is effectively a poor form of agc. One could observe the increase of effectiveness of Cross-Eye as the range decreased, which verifies the results of the above analysis and shows that some means of agc will regain performance of the Cross-Eye system with lobing radars.

In the initial test, results were obtained with a passive Cross-Eye system where amplifiers were not used, on an X-band Mark 25 lobing radar which indicated that the Cross-Eye system was effective where the theory indicates that it should not be. The theory is based on perfect linear components, which should be a valid assumption for waveguide, horns, and waveguide fittings. The passive X-band system, however, also used a ferrite component (1). The nonreciprocal characteristics of the ferrite component are used to allow the two signals to travel the same guide with a relative phase shift of 180^0 and an adjustable relative amplitude.

The ferrite component has exhibited linear characteristics on the bench, but, with higher than room temperatures, even as low as 60^0 to 70^0 centigrade, significant changes in characteristic take place and nonlinearities are observed. The experimental Cross-Eye system was located in the unventilated wing tanks of an aircraft where such temperatures are likely. Minor nonlinearities in the Cross-Eye system have been taken into consideration (Appendix C) and have been shown to increase the effectiveness of the countermeasure. This result further indicates, as one might expect, that use of normal agc is not the only means of making the Cross-Eye system effective on lobing radars and is not necessarily optimum.

One can observe how a nonlinearity may cause a change in effectiveness of the Cross-Eye system by comparing the new tracking point θ_0 (Appendix C) with the previous analysis.

Where a linear component will give a voltage output directly proportional to the voltage input, an effective nonlinearity is introduced (Appendix C) by a change in slope from a value of unity to some value m. For the lobing radar, the tracking point becomes

$$\theta_0 = \theta_r \left[\frac{b^2 + b(1-a)\cos\psi}{(1-a)^2\left(\frac{1+m}{2}\right) + \left(1 + \frac{1+m}{2}\right)b\,(1-a)\cos\psi + b^2} \right] + \frac{\theta_t}{2}\left(\frac{1-m}{1+m}\right)\left[\frac{1 - a^2 + (1+a)b\cos\psi}{(1-a)^2 + \left(1 + \frac{1}{1-m}\right)b\,(1-a)\cos\psi + \frac{b^2}{1+m}} \right]. \tag{10}$$

One observes that without nonlinearity, or $m = 1$, the countermeasure effectiveness is lost as the second term goes to zero; θ_0 then is the same as Eq. (9), the solution for lobing radar. If $m = 0$, which is equivalent to the use of agc by complete smoothing of the lobing modulation at the input, one obtains the tracking point

$$\theta_0 = \theta_r \left[\frac{b^2 + b(1-a)\cos\psi}{\dfrac{(1-a)^2 + 3b\,(1-a)\cos\psi}{2} + b^2} \right] + \frac{\theta_t}{2}\left[\frac{1 - a^2 + (1+a)b\cos\psi}{(1-a)^2 + 2b(1-a)\cos\psi + b^2} \right]. \tag{11}$$

This tracking point is essentially the same as Eq. (4), the tracking point with monopulse. The second term, containing the countermeasure noise and tracking error, is identical; and the first term, representing normal target noise, is only slightly modified, but will not cause a significant change.

One may even consider a further step in analyzing the effect of the nonlinearity factor m, where m becomes negative. This condition is equivalent to an "excess gain control" where the lobing modulation on the received signal is actually reversed. If m - 0.75, for example, the tracking point is

$$\theta_0 = \theta_r \left[\frac{b^2 + b(1-a)\cos\psi}{\dfrac{(1-a)^2 + 9b(1-a)\cos\psi}{8} + b^2} \right] + 7\,\frac{\theta_t}{2}\left[\frac{1 - a^2 + (1+a)b\cos\psi}{(1-a)^2 + 5b(1-a)\cos\psi + 4b^2} \right]. \tag{12}$$

The changes in the denominators of the terms has little effect on the value of the fractions. One will observe, however, that the second term, containing the noise and tracking error introduced by the Cross-Eye system, is amplified seven times, for this value of m, causing a considerable increase in effectiveness of the countermeasure.

Countermeasure repeater systems are under development which use a technique similar to the "excess gain control," in which the modulation on a signal received for a lobing radar is actually inverted and retransmitted back to the radar. However, this repeater system and other similar types of countermeasures can be detected and dealt with by various radar techniques.

The original Cross-Eye system, with a simple agc to allow it to operate on both lobing or monopulse radars, introduces tracking noise and tracking error which is very difficult to recognize as a countermeasure. In addition, much effort has been given in the initial report and later studies to determine a counter-countermeasure to the Cross-Eye system for our own protection. However, since the Cross-Eye system is essentially an enhancement of the natural angle noise of a target (2), it is almost impossible to counter with known radar techniques (1).

The use of "excess gain control" in the Cross-Eye system merely amplifies the tracking noise introduced by the countermeasure, and is not as readily detected as the repeater system described. Techniques to increase the Cross-Eye system effectiveness are under investigation, but one must take care not to increase its detectibility, give aid to other radars, or make the system susceptible to countermeasures.

A BROADBAND VERSION OF THE CROSS-EYE SYSTEM

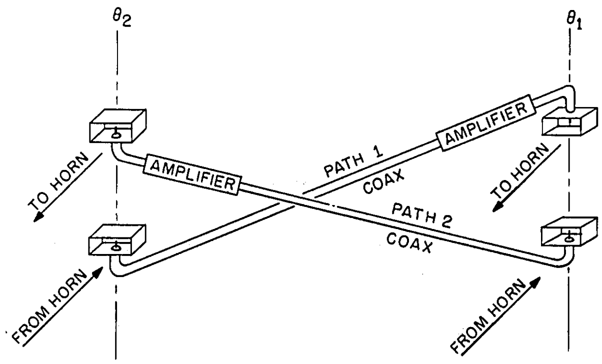
The initial version of Cross-Eye was constructed in a limited period of time for the sole purpose of proving that the system could work. Further consideration has been given to techniques for a practical system.

The question has arisen as to how to make the system broadband when a 180° relative phase shift must be introduced in the two paths. One technique (Fig. 4) requires that the two paths be given identical electrical length, including amplifiers. The amplifiers may be stabilized by a phase comparison circuit and phase control of the amplifiers by feedback. The 180° phase shift is introduced, while maintaining identical electrical length, by feeding one antenna from the top rather than the bottom. Thus, the electric field is set up of reverse polarity, or exactly 180° out of phase, in the one antenna. For waveguide, a 180° twist some where in the long stretch of one path will accomplish the same results.

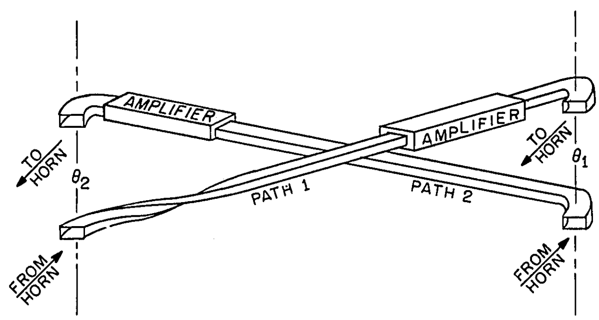
By this means, the only bandwidth limitation will be in the amplifier. Traveling-wave tube amplifiers, even in this early stage of development, are very broadband in comparison to other techniques of microwave amplification, and make possible a very broadband countermeasure system. Available amplifiers can cover, for example, from 4 to 8 kilo-megacycles or 7 to 13 kilo-megacycles in a single amplifier without tuning.

CONCLUSIONS

1. The theoretical Cross-Eye tracking radar countermeasure as initially described (1) is not effective on lobing radars as a result of the nonuniform illumination of the target. (Lobe on receive only radars will be affected the same as monopulse radars by the Cross-Eye system.)



(a) Coax to waveguide feed; identical path electrical lengths, but 180° relative phase.



(b) Waveguide transmission; identical path electrical lengths, but 180° twist in long run of guide.

Fig. 4 - Broadband Cross-Eye system

2. The Cross-Eye system can be made equally effective on all radars by use of nonlinear elements, such as an agc, to obtain an effective uniform illumination of the Cross-Eye antennas.

3. The effectiveness of the Cross-Eye system on lobing radars may be amplified by modulation reversing techniques, although the detectibility of the system may be increased. The application of this technique to the Cross-Eye system is under study.

4. By use of a new technique for obtaining exactly a 180° phase shift, the Cross-Eye system can be made broadband, limited only by the amplifying components which are available with wideband characteristics.

* * *

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* * *

APPENDIX A

DERIVATION OF THE TRACKING POINT WITH A LOBING RADAR TRACKING THE CROSS-EYE SYSTEM

In this Appendix, the tracking point of the lobing radar will be determined for the Cross-Eye system without background signals. It will be assumed that the radar uses essentially a product type detector operating on the signal received from one extreme of the lobing cycle, 0°, and the signal received from the other extreme of the lobing cycle, 180°, for train angle detection. Although other techniques of analysis may be more appropriate for lobing radar, the product detector gives the same results; since the product detection technique was used for the monopulse radar, one may more readily compare the two radars by this analysis. The product detector essentially multiplies the sum and difference of the signals received at the 0° and 180° points of the lobing cycle, e_a in Eq. (7) and e_b in Eq. (8), respectively. This product is low-pass filtered to obtain the error voltage. Equations (7) and (8) may be expanded to give

$$e_a = k^2[1 + (\theta_0-\theta_2) + (\theta_0-\theta_1) + (\theta_0-\theta_2)(\theta_0-\theta_1)]\,[A_1 \cos \omega t + A_2 \cos(\omega t+\phi)] \tag{A1}$$

and

$$e_b = k^2[1 - (\theta_0-\theta_2) - (\theta_0-\theta_1) + (\theta_0-\theta_2)(\theta_0-\theta_1)]\,[A_1 \cos \omega t + A_2 \cos(\omega t+\phi)]. \tag{A2}$$

The second-order term, $(\theta_0-\theta_1)(\theta_0-\theta_2)$, will be neglected, since the previous analyses are based on linear antenna pattern theory. A solution may be obtained retaining the second-order terms which agrees with the solution with linear patterns, but this introduces unnecessary complications.

The sum of the signals is

$$(e_a+e_b) = 2k^2[A_1 \cos \omega t + A_2 \cos(\omega t+\phi)] \tag{A3}$$

and the difference is

$$(e_a-e_b) = 2k^2[(\theta_0-\theta_2) + (\theta_0-\theta_1)]\,[A_1 \cos \omega t + A_2 \cos(\omega t+\phi)]. \tag{A4}$$

The product detector multiplies the sum and difference functions to obtain

$$(e_a+e_b)(e_a-e_b) = 4k^4[(\theta_0-\theta_2) + (\theta_0-\theta_1)]$$
$$[A_1^2 \cos^2 \omega t + 2A_1A_2 \cos \omega t \cos(\omega t+\phi) + A_2^2 \cos^2(\omega t+\phi)]. \tag{A5}$$

Expanding trigonometric terms

$$(e_a+e_b)(e_a-e_b) = 2k^4[(\theta_0-\theta_2) + (\theta_0-\theta_1)]$$
$$[A_1^2 + A_1^2 \cos 2\omega t + 2A_1A_2 \cos \phi + 2A_1A_2 \cos(2\omega t+\phi) + A_2^2 + A_2 \cos 2(\omega t+\phi)]. \tag{A6}$$

The angle error voltage is equal to this product. After low-pass filtering, which removes all functions with rf or the ω term, the error voltage becomes

$$e_r = 2k^4[(\theta_0-\theta_2) + (\theta_0-\theta_1)] \; [A_2 + 2A_1A_2 \cos \phi + A_2^2]. \tag{A7}$$

To determine where the radar will track, the error voltage may be set equal to zero, which is equivalent to closing the loop in the tracking radar. The antenna position θ_0 must, therefore, move to the correct position to maintain zero error voltage. With e_r set equal to zero, an antenna tracking position

$$\theta_0 = \frac{\theta_1 + \theta_2}{2} \tag{A8}$$

will satisfy Eq. (A7). This solution indicates that the lobing radar tracks the mid-point of the two signal sources from the Cross-Eye system as if it were a single reflector regardless of the signal amplitudes or relative phase.

* * *

APPENDIX B

DERIVATION OF THE TRACKING POINT WITH A LOBING RADAR TRACKING THE CROSS-EYE SYSTEM WITH AIRCRAFT RETURN

The derivation of the tracking point of the Cross-Eye system with aircraft return will be made similar to the analysis in Appendix A, except that a signal from position θ_3, to represent the aircraft return, will be added into e_a and e_b. Since θ_3 is both illuminated by the pattern functions $[1 \pm (\theta_0-\theta_3)]$ and received on the same pattern function, one will receive the signals

$$e_a = k^2[1 + (\theta_0-\theta_1) + (\theta_0-\theta_2) + (\theta_0-\theta_1)(\theta_0-\theta_2)] \\ [A_1 \cos \omega t + A_2 \cos(\omega t+\phi)] \\ + A_3[1 + 2(\theta_0-\theta_3) + (\theta_0-\theta_3)^2] \cos(\omega t+\psi) \tag{B1}$$

and

$$e_b = k^2[1 - (\theta_0-\theta_1) - (\theta_0-\theta_2) + (\theta_0-\theta_1)(\theta_0-\theta_2)] \\ [A_1 \cos \omega t + A_2 \cos(\omega t+\phi)] \\ + A_3[1 - 2(\theta_0-\theta_3) + (\theta_0-\theta_3)^2] \cos(\omega t+\psi) \tag{B2}$$

where

A_3 = the echo amplitude from θ_3

ψ = the phase of the echo from θ_3 relative to the signal from θ_1.

Neglecting second order terms, the sum is

$$(e_a+e_b) = 2k[A_1 \cos \omega t + A_2 \cos(\omega t+\phi) + A_3 \cos(\omega t+\psi)] \tag{B3}$$

and the difference is

$$(e_a-e_b) = 2k[(\theta_0-\theta_1) + (\theta_0-\theta_2)] [A_1 \cos \omega t + A_2 \cos(\omega t+\phi)] \\ + 4k(\theta_2-\theta_3) A_3 \cos(\omega t+\psi). \tag{B4}$$

The product of the sum and difference is then

$$(e_a+e_b)(e_a-e_b) = 4k^2\{[(\theta_0-\theta_1) + (\theta_0-\theta_2)] [A_1^2 \cos^2 \omega t + 2A_1A_2 \cos \omega t \cos(\omega t+\phi) \\ + A_2^2 \cos^2(\omega t+\phi)] + 2(\theta_0-\theta_3) [A_1A_3 \cos \omega t \cos(\omega t+\psi) + A_2A_3 \cos(\omega t+\phi) \cos(\omega t+\psi)] \\ + [(\theta_0-\theta_1) + (\theta_0-\theta_2)] [A_1A_3 \cos \omega t \cos(\omega t+\psi) + A_2A_3 \cos(\omega t+\phi) \cos(\omega t+\psi)] \\ + 2(\theta_0-\theta_3) A_3^2 \cos^2(\omega t+\psi)\}. \tag{B5}$$

By expanding trigonometric terms and by low-pass filtering, one obtains the error voltage

$$\begin{aligned} e_r = 2k^2\{&[(\theta_0-\theta_1) + (\theta_0-\theta_2)]\ [A_1^2 + 2A_1A_2 \cos\phi + A_2^2] \\ &+ [(\theta_0-\theta_1) + (\theta_0-\theta_2) + 2(\theta_0-\theta_3)]\ [A_1A_3 \cos\psi + A_2A_3 \cos(\psi-\phi)] \\ &+ 2(\theta_0-\theta_3)\ A_3^2 \cos\psi\}. \end{aligned} \tag{B6}$$

If $A_2 = a\ A_1$ and $A_3 = b\ A_1$, and ϕ is set equal to 180°, which is the normal operation of the Cross-Eye system, then the error voltage reduces to

$$\begin{aligned} e_r = 2k^2\ A_1^2\{&2\theta_0\,[(1-a)^2 + 2b(1-a)\cos\psi + b^2] \\ &- (\theta_1+\theta_2)\ [(1-a)^2 + b(1-a)\cos\psi] \\ &- 2\,\theta_3\,[b^2 + b(1-a)\cos\psi]\}. \end{aligned} \tag{B7}$$

The tracking loop is effectively closed by setting e_r to zero and solving for θ_0, the antenna pointing position, as in Appendix A. If one chooses the mid-point of the Cross-Eye system as zero reference, then $\theta_2 = -\theta_1$ and the tracking point is

$$\theta_0 = \theta_r\,\frac{b^2 + b(1-a)\cos\psi}{(1-a)^2 + 2b(1-a)\cos\psi + b^2}\,. \tag{B8}$$

The tracking point, θ_0 in Eq. (B8) is the normal tracking point for a single reflector located at the mid-point of the Cross-Eye system, in addition to the reflector at θ_3. Thus, as one would expect from Appendix A, the Cross-Eye system with lobing radar appears as just an additional reflector added to the aircraft at the mid-point of the system with an amplitude equal to the difference of its two signal returns to the radar.

* * *

APPENDIX C

DERIVATION OF THE EFFECT OF NONLINEARITIES OF THE CROSS-EYE SYSTEM COMPONENTS ON THE TRACKING POINT OF A LOBING RADAR

This analysis is almost identical to Appendix B, except for the introduction of nonlinearity into the energy transfer within the Cross-Eye system. In a linear circuit, the output is directly proportional to input, and a nonlinearity is introduced by a change of the slope of unity to a slope m about the average power received in the two paths of the Cross-Eye system. There are many other forms of nonlinearity which may be introduced into the Cross-Eye system; however, this form might very possibly exist in the ferrite component of the X-band Cross-Eye system, and it has further significance as described in the text.

Introducing the nonlinearity in Eqs. (B1) and (B2), one obtains

$$\begin{aligned} e_a &= A_1k^2[1 + (\theta_0-\theta_1) + m(\theta_0-\theta_2) + m(\theta_0-\theta_1)(\theta_0-\theta_2)] \cos \omega t \\ &+ A_2k^2[1 + (\theta_0-\theta_2) + m(\theta_0-\theta_1) + m(\theta_0-\theta_1)(\theta_0-\theta_2)] \cos(\omega t+\phi) \\ &+ A_3k^2[1 + 2(\theta-\theta_3) + (\theta_0-\theta_3)^2] \cos(\omega t+\psi) \end{aligned} \tag{C1}$$

and

$$\begin{aligned} e_b &= A_1k^2[1 - (\theta_0-\theta_1) - m(\theta_0-\theta_2) + m(\theta_0-\theta_1)(\theta_0-\theta_2)] \cos \omega t \\ &+ A_2k^2[1 - (\theta_0-\theta_2) - m(\theta_0-\theta_1) + m(\theta_0-\theta_1)(\theta_0-\theta_2)] \cos(\omega t+\phi) \\ &+ A_3k^2[1 - 2(\theta_0-\theta_3) + (\theta_0-\theta_3)^2] \cos(\omega t+\psi). \end{aligned} \tag{C2}$$

By the same assumptions and manipulations as in Appendix B, one finds the tracking point

$$\begin{aligned} \theta_0 &= \theta_r \left[\frac{b^2 + b(1-a)\cos\psi}{\dfrac{(1-a)^2(1+m) + (3+m)b(1-a)\cos\psi}{2} + b^2} \right] \\ &+ \frac{\theta_t}{2}\left(\frac{1-m}{1+m}\right)\left[\frac{1 - a^2 + b(1+a)\cos\psi}{(1-a)^2 + \left(\dfrac{2+m}{1+m}\right) b(1-a)\cos\psi + \dfrac{b^2}{1+m}} \right]. \end{aligned} \tag{C3}$$

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