

THE LOSS FACTOR OF FORMVAR

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ABSTRACT

The loss factor of a dielectric is shown to be applicable to four discrete concepts of phenomena, viz,

1. the measure of the power loss in the dielectric,
2. a component of a complex dielectric constant,
3. the effect arising from mixtures of materials with different time constants,
4. a measure of activation in energy.

Each of these is studied in turn and the relationships applied to Formvar insulation.

PROBLEM STATUS

This is an interim report; further work is in progress.

AUTHORIZATION

NRL Problem No. C06-11 (Bureau of Ships Letter S62-2 (335) SRD-346/46 dated 3/11/46).

THE LOSS FACTOR OF FORMVAR

STATEMENT OF THE PROBLEM

In attempting to develop a method of insuring the operation of the Navy's electrical equipment, a thermal evaluation study was undertaken on Formvar wire insulation.¹ In the series of studies, use was made of the coefficient called the dielectric loss factor, ϵ'' , and a relationship was derived between this variable and the quality of an electrical insulator.

Depending on the viewpoint taken, the dielectric loss factor may indicate one of four physical phenomena. These are tabulated below.

<u>View</u>	<u>Meaning of Loss Factor, ϵ''</u>	<u>Ultimate Units</u>
Electrical Engineering	Power Loss	Watts
Physics	Component of Complex Dielectric Constant	Dimensionless
Mathematics	Time Constants	Seconds
Chemistry	Activation Energy	Calories/mole.

The purpose of this report is to correlate these four concepts and apply the results to the characteristics of Formvar.

THE MEANINGS OF LOSS FACTORS

Loss Factor ϵ'' as a Coefficient of Power

The most direct approach to the concept of dielectric loss factor is that of the electrical engineer who treats it simply as a measure of power loss in a capacitive assembly.²

Classic Approach—Consider a circuit containing a parallel plate condenser which employs Formvar as an insulant. Measurement of the current,

¹ T. D. Callinan, Second Report on the Thermal Evaluation of Formvar Resin, NRL Report C-3191, 8 October 1947.

² A. E. Knowlton, Standard Handbook for Electrical Engineers, McGraw-Hill, 7th ed. (1941) Section 3-252.

voltage, and wattage of the system yields the phase angle, ϕ . The total losses, W^1 , in the condenser are then given by the product of the voltage, current, and $\sin(1-\phi)$ or $\sin \delta$. Since the effective capacitance, C , of this condenser is by definition

$$\frac{I \cdot \cos \delta}{V \cdot \omega} \quad (1)$$

where I is the current; V , the voltage $\omega = 2\pi$ (frequency); and δ is the loss angle; W^1 , the losses, must then be $V^2 \omega C \tan \delta$.

The total capacitance, C , of the parallel plate condenser is a function of the dielectric constant ϵ^1 at the test frequency, according to the relationship $C = \epsilon^1 C_0$, where C_0 is the geometric capacitance of the assembly with vacuum as the dielectric. The total losses W^1 in the condenser may now be written as

$$W^1 = \epsilon^1 \tan \delta (\omega C_0 V^2). \quad (2)$$

It becomes apparent that under a given voltage, frequency, and dimensional array, the losses in the condenser are directly proportional to the product of the dielectric constant ϵ^1 and the tangent of the loss angle δ . For later convenience this may be written as:

$$\epsilon'' = \epsilon^1 \cdot \tan \delta \quad (3)$$

where ϵ'' is called the Loss Factor. With this simplification, the power loss, in watts is:

$$W^1 = \epsilon'' (\omega C_0 V^2). \quad (3a)$$

This brings out clearly the truism that the introduction of a material, the dielectric constant of which is greater than 1, increases the losses of the system even if $\tan \delta$ does not decrease. Hence the value of developing thermally stable synthetics of lower rather than higher dielectric constants, other things being equal.

Application to Formvar

In the test cell (diam. = 2 cm; thickness = $\theta = .0075$ cm) employed for studying the electrical properties of Formvar at 10,000 cycles, an alternating potential of 110 V (0.33 esu) was maintained across a condenser having a geometric capacitance of 5 mmf at 25°C. From Figure 1 it appears that the dielectric constant under this condition initially was 3.3 and the loss factor 0.08. The power loss in the circuit was then:

$$\begin{aligned} W^1 &= \epsilon'' (2\pi \cdot f \cdot C_0 \cdot V^2) = 0.08 (2\pi \cdot 10,000 \cdot 5 \cdot 10^{-12} \cdot 0.3^2) \\ &= 2.10 \cdot 10^{-8} \text{ watts,} \end{aligned}$$

or for the sake of the chemist (where 1 watt = 1 joule/sec = $(4.18)^{-1}$ calories/sec),

$$W^1 = 4.8 \cdot 10^{-9} \text{ calories/sec.}$$

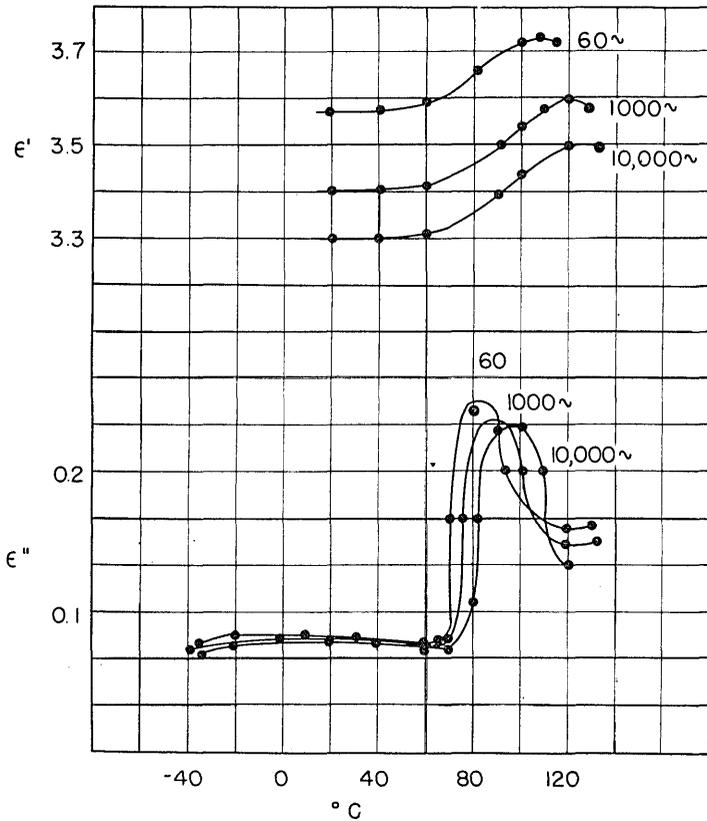


Figure 1 - Electrical properties of polyvinyl formal

no way for the losses; it reasons simply that most materials employed as dielectrics tend to change the phase angle, that these losses are proportional to the dielectric constant of the dielectric array and to the power factor, when the geometry, the frequency, and the voltage gradient of the system are constant.

LOSS FACTOR AS THE IMAGINARY PART OF A COMPLEX DIELECTRIC CONSTANT

In determining the amount of losses which arise in a dielectric or condenser assembly, the bridge network (Figure 2) developed by Wien is well suited.³ It comprises a network of resistance-shunted air capacitors arranged for judicious balancing. By considering the conditions required for matching the several impedances, the physicist has found that in the simple proposition $C = \epsilon C_0$, the dielectric constant is a complex number $(A + j B)$ possessing both a real and an imaginary part.

The volume of Formvar found between the plates of the test cell was

$$\begin{aligned} cc &= \pi \cdot r^2 \cdot \theta \\ &= \pi (1 \text{ cm}^2)(.0075 \text{ cm}) \\ &= (0.0235) \text{ cc.} \end{aligned}$$

Consequently the losses in a cc. of Formvar under these conditions are $4.8 \cdot 10^{-9} / .0235 = 2 \cdot 10^{-7}$ calories/cc/sec, or since the molecular weight is of the order of 20,000 and the density is 1.21 gms/cc, the loss is $3.3 \cdot 10^{-8}$ calories/mole/sec or 3300μ calories/mole/sec. It is interesting to note that the increase in power factor which Formvar undergoes on being heated increases the losses by only 200μ calories/mole/sec, but the increase in dielectric constant, ϵ' , increases the losses by 2000μ calories/mole/sec.

Conclusion

This concept of dielectric loss factor accounts in

³ B. Hague, Alternating Current Bridge Methods, Pitman Pub. Co, New York, 4th ed. (1938)

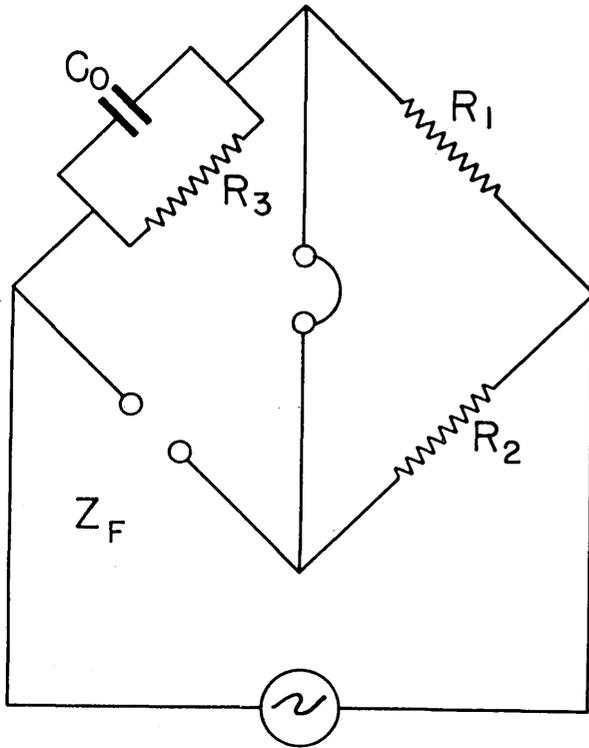


Figure 2 - Wien bridge network

Derivation

Two arms of the Wien bridge network are nonreactive resistances R_1 and R_2 . The third position in the bridge is a resistance R_3 shunted by a variable air condenser of capacity C_0 . The total impedance Z_3 of this assembly is:

$$Z_3 = \frac{1}{\frac{1}{R_3} + j \omega C_0} \quad (4)$$

Now the condenser containing the dielectric whose properties are required constitutes the fourth arm and is considered to correspond to a capacitor whose dielectric has unlimited resistance but shunted by a resistor which permits some in-phase current to flow. In this way some of the molecules are thought to be perfect dielectrics and others perfect conductors and that each type acts independently of the other. Equiva-

lently then, the impedance of the cell containing Formvar film is given by:

$$Z_F = \frac{1}{\frac{1}{R_F} + j \omega C_F} \quad (5)$$

Since $\tan \delta = \frac{1}{\omega C R}$, (6)

or $R_F = \frac{1}{\omega C_F \tan \delta}$, (7)

the impedance of the Formvar is given by:

$$Z_F = \frac{1}{\frac{1}{1/\omega C_F \tan \delta} + j \omega C_F} \quad (8)$$

which on simplification yields

$$Z_F = \frac{1}{\omega C_F \tan \delta + j \omega C_F} \quad (9)$$

$$= \frac{1}{\omega C_F (\tan \delta + j)} \quad (10)$$

But $C_F = \epsilon'_F C_o$, and consequently substitution eventuates in

$$Z_F = \frac{1}{\omega C_o (\epsilon'_F \tan \delta + j \epsilon'_F)} \quad (11)$$

or by Equation (3),

$$Z_F = \frac{1}{\omega C_o (\epsilon''_F + j \epsilon'_F)} \quad (12)$$

which is equivalent to

$$Z_F = \frac{1}{j \omega C_o (\epsilon'_F - j \epsilon''_F)} \quad (13)$$

If the product $\epsilon'_F \tan \delta$ is designated as ϵ''_F , the loss factor is obtained once again, this time entering as a part of a complex dielectric constant. This complex dielectric constant is the measured value of the formula

$$Z_F = \frac{1}{j \omega \epsilon C_o} \quad (14)$$

and hence

$$\epsilon = \epsilon'_F - j \epsilon''_F \quad (15)$$

Interpretation

This is another way of saying that the complex dielectric constant of Formvar in an alternating field is a function of the total polarization of the molecules and also of the losses. In other words, under the conditions of measurement, ϵ'_F is the measured value of the dielectric constant and ϵ''_F is the loss factor; these are components of a complex coefficient, mathematically necessary for balancing the bridge network.

If the arms be arranged as shown in Figure 2, balance obtains when

$$\frac{R_1}{R_2} = \frac{\frac{1}{\frac{1}{R_3} + j \omega C_3}}{j \omega C_o \left(\epsilon'_F - j \epsilon''_F \right)}, \quad (16)$$

or rearranging,

$$\frac{R_1}{R_2} = \frac{\omega C_o \epsilon''_F - j \omega C_o \epsilon'_F}{\frac{1}{R_3} + j \omega C_3}, \quad (16a)$$

and equating like parts,

$$\frac{R_1}{R_2} = \frac{\omega C_o \epsilon''_F}{\frac{1}{R_3}} = \frac{-j \omega C_o \epsilon'_F}{j \omega C_3} \quad (16b)$$

which on simplification becomes

$$\frac{R_1}{R_2} = \omega C_o R_3 \epsilon''_F = \frac{-C_o}{C_3} \epsilon'_F \quad (17)$$

The contribution of the loss factor ϵ'' to the complex dielectric constant ϵ , of which it forms an imaginary part, is quite small for Formvar. Thus the material at 10,000 cycles and 20°C, has an observed dielectric constant of 3.3 and a loss factor of 0.08. Its complex constant is then

$$\epsilon = 3.3 - j (0.08), \quad (18)$$

which may be considered as the equivalent of vector subtraction⁴ and has the solution

$$\epsilon = 3.299. \quad (19)$$

⁴ F. S. Woods, Advanced Calculus, Ginn and Co., New York, New Ed., (1934) p. 203.

Evaluation

It appears from this that there arises in an alternating field a total capacitance the value of which is the product of its geometric capacitance (capacitance in a vacuum) and a complex dielectric constant $\epsilon' - j \epsilon''$ instead of a simple dielectric constant ϵ . It becomes apparent also that ϵ'' is the same factor which arose by evaluating the electrical engineers view, but that whereas the previous meaning was a measure of the power loss in the system, it here has the meaning that the losses in a dielectric not only affect the overall capacitance of a circuit but that they actually correspond to an imaginary part (an in-phase component) of the complex dielectric constant.

The interpretation of how the losses in a dielectric can affect the electrical properties of dielectric constant and loss factor has led to the two hypotheses to be described later. Suffice to say, the deduced equation does not postulate a molecular mechanism. In this way the physicists' viewpoint resembles the engineers' since neither specify the substance's character.

$$W' = \epsilon'' (\omega C_0 V^2) \quad (3a)$$

$$\epsilon = \epsilon' - j \epsilon'' \quad (15)$$

LOSS FACTOR ϵ'' AS A MEASURE OF UNMATCHED DIELECTRIC TIME CONSTANTS

Both equations (3a and 15) developed so far have indicated that losses are inherent in dielectric systems but neither have postulated a mechanism for explaining these losses. In fact the equations fail to account for a phenomenon that is observed in many dielectrics, viz., anomalous dispersion. This term indicates that the dielectric constant decreases with increasing frequency. Simultaneously, the loss factor goes through a maximum.⁵

In terms of the two previously derived meanings of loss factor, the energy dissipation increases as the frequency increases until a certain maximum frequency f_{\max} after which it decreases and the complex dielectric constant changes appreciably because, while the real part is decreasing, the imaginary part passes through a maximum.

Some investigators believed that this was an absorption phenomenon similar to that of specific heat. In an attempt to show that this was not true, and in fact, that even materials which did not undergo anomalous dispersion individually, could be assembled to give a condenser possessing an "absorption," J. Clerk Maxwell in a classic study developed an elegant mathematical array.⁶

⁵ P. Debye, Polar Molecules, New York Chemical Catalog Co., (1929) pp. 77-108.

⁶ J. C. Maxwell, A Treatise on Electricity and Magnetism, Clarendon Press, Oxford, 3rd ed. (1904) Vol. 1. p. 456.

Classical Derivation

Consider an imperfect condenser to be composed of an air condenser in series with a perfect resistance. The time it takes this assembly to discharge 67 percent of its original voltage is called its time constant, \mathcal{T} . Suppose now that two such assemblies possessing different time constants were placed in series. It is apparent that if one discharged in a matter of seconds and the other in a matter of hours, that the total time of discharge is in terms of hours. Now consider the effects which would arise if an alternating potential were impressed across the system. If the frequency were anything less than once an hour one of the units would never reach its full potential and therefore the capacity of the system would be less than the calculated geometric capacitance. Similar reasoning leads to the conclusion that the discharging current forms a counter-flow to the charging current and the losses increase. The losses which arise are then related to: (a) the time-constants of the array and, (b) to the frequency. If these losses can be expressed in terms of loss factor, ϵ'' , then a relationship $\epsilon''(\mathcal{T}, \omega)$ explaining the change in capacitance will have been secured.

Since most electrical insulators are not chemically pure and consequently may contain discrete particles imbedded in a secondary matrix, the possibility of Maxwell's postulate arising in nature is highly probable. Consider the mechanism by which Formvar films are cast from solution: as the solvent evaporates, the concentration approaches saturation eventuating in the precipitation of the less soluble components. Thus the molecules of larger molecular weight and greater chain length congeal as a pellicle first, followed by the molecules and chains of lower molecular weight. Thus on going from bottom to top of a film the percentage of larger molecular weight aggregates diminishes. Electrically the heterogeneity of this system may be represented as two condensers shunted by two resistances as shown in Figure 3. When such a condition obtains, the impedance of the system is given by

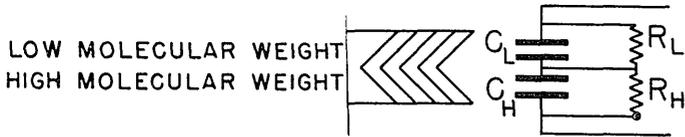
$$\frac{1}{Z} = \sum \frac{1}{R} + \sum \frac{1}{\frac{1}{\omega C}} \quad , \quad (20)$$

the solution of which is

$$Z = \frac{R}{1 + j \omega C R} = \frac{R_1 + R_h}{1 + j \omega \frac{(C_1 C_h)(R_1 + R_h)}{(C_1 + C_h)}} \quad (20 a)$$

where the product $CR = \mathcal{T}$, the time constant of the assembly and is a composite of the time constants of the individual components

$$\mathcal{T}_1 = C_1 \cdot R_1 \quad \text{and} \quad \mathcal{T}_h = C_h \cdot R_h.$$



Equation (20a) may also be written as

$$\frac{1}{Z} = \omega C_o (j \epsilon'' + \epsilon'), \quad (21)$$

Figure 3 - Electrical equivalent of layer dielectrics

as shown in Appendix A. This is actually another way of stating that the overall capacitance of two imperfect condensers in series is a function of the frequency and the losses intrinsic to such an array, for on rearranging Equation (21),

$$\frac{1}{\omega Z} = C = j C_o \epsilon'' + C_o \epsilon'$$

In this case the loss factor ϵ'' is the observed value of a composite dielectric. In actuality the real part of the complex dielectric constant is given by

$$\epsilon' C_o = \frac{C_l C_h}{C_l + C_h} \quad (22)$$

In order to make equation (21) suitable for examining an actual case, substitution and transformation yields

$$\frac{C_l C_h}{C_l + C_h} = - \left(\epsilon'' j C_o + \frac{10^{12}}{\omega Z} \right), \quad (23)$$

where all capacitances are given in micromicrofarads and the loss factor ϵ'' is the maximum over the dispersion range.

Application

In the case of Formvar, this maximum has been observed⁷ at 25°C at 100,000 cycles, and the loss factor reached a value of 0.020. The apparent dielectric constant was 3.0 under these conditions. Since the power factor of the system was given by

$$\text{Power Factor} = \tan \delta = \frac{\epsilon''}{\epsilon'} = \frac{1}{\omega CR} = 0.007, \quad (24)$$

$R = 96$ megohms, where R is the experimentally determined sum of the parallel resistances R_1 and R_2 .

⁷ A. von Hippel, et al, Tables of Dielectric Materials, Report V., NDRC Dn. 14 Report 237, February 1944, p. 111

The impedance Z is then

$$\frac{1}{Z} = \frac{1}{R} + j \omega C,$$

or

$$Z = 9 \cdot 10^4 \text{ ohms.}$$

Substituting the known quantities in equation (23),

$$\frac{C_1 C_h}{C_1 + C_h} = \frac{10^{12}}{2\pi (10^5)(9 \cdot 10^4)} - (0.02)(5) j \quad (25)$$

$$= -(18 + j 0.1) = 18.01, \quad (26)$$

or

$$C_1 C_h = 18.01 C_1 + 18.01 C_h.$$

Whence

$$C_1 (C_h - 18.01) = 18.01 C_h. \quad (27)$$

A few of the possible combinations of capacitances in series which might arise in these tests, the combined capacitance of which is 15 mmf, are tabulated below.

TABLE I

Capacitance Combinations of Strata in Formvar Films

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_h}$					
C		C _h		C ₁	
MMF	Thickness Mils	MMF	Thickness Mils	MMF	Thickness Mils
15	4	15	4.00		0.00
15	4	16	3.75	240	0.25
15	4	17	3.48	128	0.52
15	4	18	3.25	90	0.75
15	4	19	3.17	71	0.83
15	4	20	3.00	60	1.00
15	4	30	2.00	30	2.00
15	4	60	1.00	20	3.00

By substituting values of 15 to 30 mmf in Equation (27) the results listed in Table II were obtained.

TABLE II
Values of C_1 Obtained by Substituting
Real Values of C_h in the Equation

$$C_1 \cdot (C_h - 18.01) = 18.01 C_h$$

C_h	$C_h - 18.01$	$18.01 C_h$	C_1
18.01	0	324	00
20.00	1.99	356	182
25.00	6.99	450	64
30.00	11.99	540	45

Two conclusions may be drawn: first that if a definite laminated structure is present in Formvar the capacitance of the high molecular weight fraction is greater than 18.01 mmf and hence less than 3.25 mils in thickness, and secondly that the low molecular weight fraction or layer may occupy as much as 3.25 mils thickness.

Limitations

From this it becomes evident that there are many combinations of dielectric assemblies which result in the same anomalous dispersion. Formvar is actually a mixture of polymeric chains of unequal lengths and hence its properties are described in terms of unmatched time constants only in the broadest terms.

It is because of the lack of more accurate correlation that the Maxwell theory of anomalous dispersion has not enjoyed such wide approval in interpreting dielectrics as the more celebrated Debye theory, to be discussed in the next section.

LOSS FACTOR ϵ'' AS A MEASURE OF ACTIVATION ENERGY

Thus far, the concept of dielectric loss factor ϵ'' , has been limited to changes which occur in a circuit, due either to changes in the nature of the substances or in the arrangement of them. Both of these are gross changes and are based on the supposition of mathematical homogeneity in matter.

The overall or gross losses in a dielectric indicate that all the molecules in the field do not act similarly for they are distributed randomly at any given instant.

Boltzmann discussed this phenomenon when he indicated that in general the probability p , of a molecule undergoing substantial change was given by

$$p = s e^{-q/kt}$$

This may be said to indicate that each molecule must be energized to a value q in order to suffer changes and that the possibility of doing this was improved by raising the temperature T or by changing the molecule's properties s .

Activation Energy

Actually the value q has importance to chemists because it is a measure of the kind of reaction the molecule undergoes. Thus if the energies are of the order of 30 to 75 kilocalories/mole the reaction involves atoms, and if 1 to 10 cal/mole adsorption or van der Waals' forces are involved.

Consider an aggregate of atoms constituting a dielectric molecule. When a potential is impressed the molecule seeks the state of lowest potential energy. This is most easily obtained by atomic polarization according to the law which states that the magnitude of the displacement current D is directly proportional to the dielectric constant ϵ and the voltage gradient E , viz, $D = \epsilon E$; secondly by molecular movement eventuating in kinetic energy; thirdly by bond rotation; and fourthly by chemical reaction. Now the first type is the classical Newtonian body of absolute elasticity. It corresponds to the distortion of the outer shell electrons and is independent of such gross effects as temperature. The movement of these electrons results in a current D of magnitude proportional to the electrical gradient or to the dielectric constant.

The second phenomenon takes into account the surface chemistry of the system. Since each segment of a molecule obeys the law of Newtonian attraction $F = m'm/\epsilon' \cdot d^2$, an adhesiveness between molecules exists corresponding to the term a/v^2 in van der Waals' law.

$$(P + a/v^2)(V - b) = n R T.$$

This is an energy term; it eventuates in interfacial tension, in boiling point and vapor pressures and the colligative properties of matter; it is related to atoms through Sudgen's Parachor, and finally it is related to viscosity.

By using a relationship between the temperature of matter and its viscosity, Fuoss⁸ has been able to determine the type of physical effect from the loss factor ϵ'' . This is the liaison between the Physical Chemist's concept of dielectrics and the loss factor ϵ'' which thus far has been found to relate to: (a) power losses, (b) an imaginary part of a dielectric constant, and (c) to the time constant of a series of dielectrics.

Mechanical Model

These four concepts may be examined in terms of the mechanical model shown in Figure 4. Part A of the Figure represents the configuration of the

⁸ R. M. Fuoss, J. Am. Chem. Soc. 63, pp. 369-378 (1941)

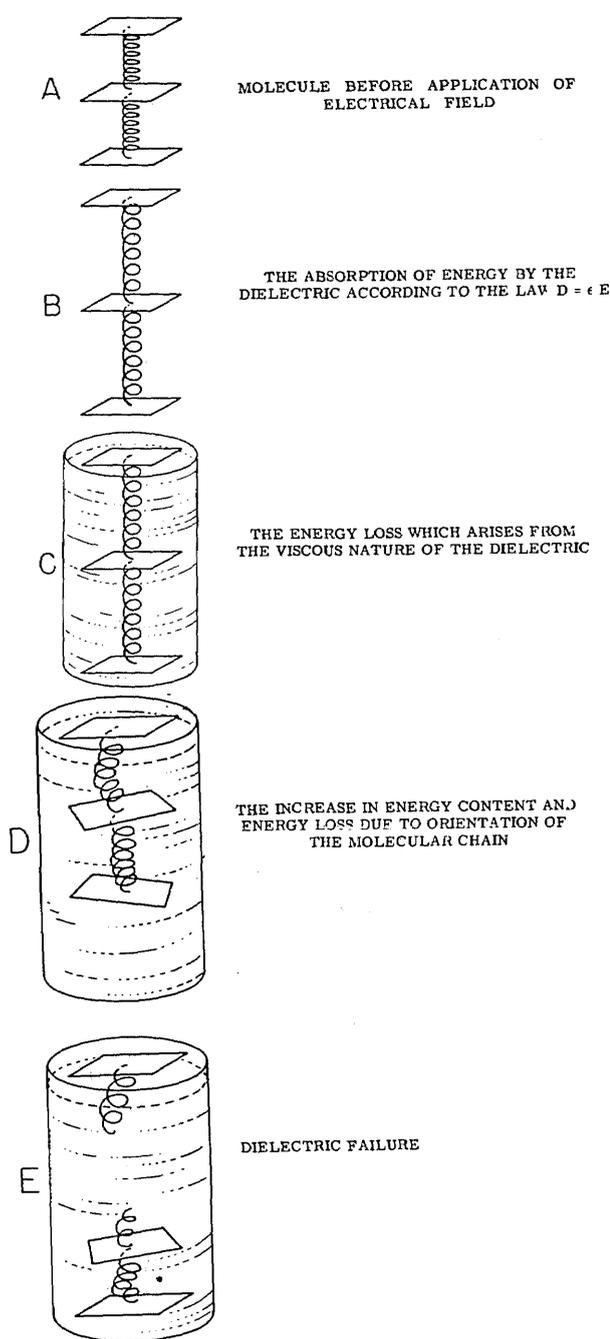


Figure 4 - Mechanical representation of dielectric processes

molecule before an electrical stress has been applied. The electrical field is applied resulting in a stretching of the springs (Part B) according to Hookes Law. Now since the molecules actually drag the atmosphere when they move or are subjected to strain, Part C indicates that some energy is dissipated by the gross movement of the molecule under electrical stress, as it acts against its viscosity η . Let this method of dissipating energy be called Fuossian.

It is apparent that all the energy in the condenser (or in this case the system of springs) would be dissipated if one of the springs broke (Part E). This is mechanistic view of dielectric breakdown; it must be understood that this energy is quite large and only one of the springs needs be ruptured.

One possibility remains as a mechanism for transforming the electrical energy into latent energy, and that is by twisting the springs. Such a possibility is highly probable if the system does not possess an electrical center of gravity but is actually an electrical doublet, or constitutes a dipole. This is Debye⁹ concept of molecules (Part D); the energy dissipated in the form of heat in rotating or twisting the system of springs in a medium is of course dependent on the viscosity of the medium and consequently on the surface energy of the particles. It is apparent then that the loss mechanism of Fuoss (Part C) and that of Debye (Part D) are similar in that they both relate to viscosity.

⁹ P. Debye "Polar Molecules", New York Chemical Catalog, 1929, pp. 77-108.

They may be distinguished however, if it be possible to determine whether any other changes should occur in the system simultaneously.

The energy which the system could acquire by the Debye mechanism is considerably higher than that which would arise from the phenomenon postulated by Fuoss. The electrical analog would be that the capacity of the condenser (and hence its dielectric constant and loss factor) would be considerably greater by the Debye concept than by Fuossian. In actuality both phenomena are found in different regions of the spectrum and each therefore serves as a measure of the quantity of adhesiveness of the molecular aggregates.

Application

Thus in the case of polyvinyl formal, the energy dissipated at 10,000 cycles and 25°C is 0.05 watts-secs/mole at a dielectric constant of 3.0.

Since this is a maximum or peak over a temperature range -60 to 70°C without a noticeable change in dielectric constant, it is considered to indicate the Fuossian phenomenon or van der Waals' forces. On the other hand at temperatures above 70°C, the plastic suffers viscosity change and both the dielectric constant and loss factor change appreciably. The change in ϵ' is attributed by Debye to the ability of the dipoles (in Figure 4 the springs) to twist and follow the field. Consequently the loss factor ϵ'' now is a measure of the energy dissipated as the molecular segment rotates. Its order of magnitude is about five times that of Fuoss' phenomenon which substantiates the idea that it is atomic, relating to bond energies, rather than to gross viscosities.

This may be shown more conclusively by the following considerations. The rapidity with which the springs in the mechanical analogy return to their original position is dependent on the viscosity of the medium.

Thus if the medium is cooled to raise the viscosity, the time it takes the spring to relax and reestablish equilibrium will be greater. This is another way of saying that the relaxation time τ of the system is directly proportional to the viscosity or inversely proportional to the absolute temperature.

$$\tau = \frac{c' \eta}{T} \quad (35)$$

Now if a material has a viscosity coefficient (or an energy of adhesion) η having the dimensions erg secs/cc,¹⁰ its molecules must possess the energy postulated by Boltzman:

$$\frac{1}{p} = \eta = s e^{q/kt} \quad (36)$$

¹⁰ H. S. Taylor, ed. A treatise on physical chemistry, D. Van Nostrand Co., (1931), Vol. 1, p. 153.

This law is more easily recognized in the form

$$\ln \eta = \frac{q}{k T} + s, \quad (37)$$

which states that the viscosity of a liquid falls off logarithmically with increasing temperature.¹¹

Equation (35) on rearranging becomes

$$\eta = \frac{\mathcal{T} T}{c'} \quad (38)$$

$$\ln \mathcal{T} T = \frac{q'}{k t} + \frac{c'}{s} \quad (39)$$

If, instead of using the generalized energy relation q'/k , the substitution be made for Molar quantities, then

$$\frac{q'}{k T} + \frac{c'}{s} = \frac{Q}{R T} + B$$

whence,

$$\ln \mathcal{T} T = \frac{Q}{R T} + B, \quad (40)$$

If then the natural logarithm of the product of the relaxation time of the dielectric \mathcal{T} and the absolute temperature be plotted against the reciprocal of the temperature, the activation energy per mole may be determined. Now the relaxation time of the dielectric is derived directly from the loss factor:

$$\frac{\epsilon''}{\epsilon''_{\max} \pm \sqrt{\epsilon''_{\max}^2 - \epsilon''^2}} = \frac{1}{\omega \tau} \cdot \frac{\epsilon_{\infty} + 2}{\epsilon_0 + 2} \quad (41)$$

In the case of Formvar, this reduces to:

$$\frac{\epsilon''}{0.25 \pm \sqrt{(0.25)^2 - \epsilon''^2}} = \frac{10^{-8}}{\tau} \quad (42)$$

¹¹ J. M. Davies, et al., J. Am. Chem. Soc. 63, pp. 361-369, (1941).

Substituting the value of τ in Equation (40),

$$\ln \frac{0.25 \pm \sqrt{(0.25)^2 - \epsilon''^2}}{10^6 \epsilon''} T = \frac{Q}{R T} + B, \quad (43)$$

or

$$\frac{10^{-6} (0.25 \pm \sqrt{(0.25)^2 - \epsilon''^2}) T}{\epsilon''} = B e^{Q/R T} \quad (44)$$

This may be simplified to yield

$$\frac{10^6 \epsilon''}{0.25 \pm \sqrt{(0.25)^2 - \epsilon''^2}} = B' T e^{-Q/R T} \quad (45)$$

Hence the loss factor ϵ'' bears a definite relationship to the activation energy and to the mechanism of the losses of a system. It is also apparent that the absolute value of ϵ'' is not the governing factor but the height of the maximum. This eliminates satisfactorily an error which might arise due to dc conductivity in the sample.

Effect of Temperature on the Loss Factor

If instead of maintaining T constant, ω had been made a constant, equation 41 indicates the fact that the loss factor at a given temperature should change over a temperature spectrum if various types of phenomena are present. Thus if the loss factor ϵ'' in the expression,

$$\frac{\epsilon''}{\epsilon'' \pm \sqrt{\epsilon''^2 - \epsilon''_{max}^2}} = B \omega T T e^{-Q/R T}$$

passes through a peak while the temperature is increasing, the value of Q must have changed appreciably and gone through a definite rise in energy level as shown in Figure 5 to offset the maxima. These are the distinguishing characteristics of the loss factor.

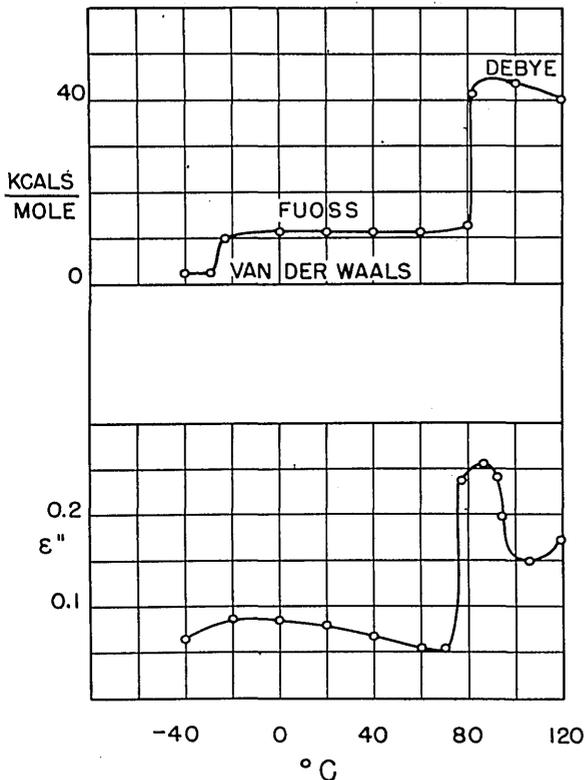


Figure 5 - Energy relations of polyvinyl formal

CONCLUSIONS

The loss factor ϵ'' is related to four phenomena, two of which arise by the nature of electrical circuits, one of which depends on the arrangement of circuit components and the last depending on the fundamental constitution of the material. It is precisely because the loss factor unites these four concepts that it serves so adequately as a measure of the quality of insulants. The determination of the changes which the loss factor undergoes under various service conditions offers itself as a method for evaluating electrical insulants. Work is therefore continuing in this field.

APPENDIX A

Loss Factor of Two Condensers Having Unmatched Dielectric Time Constants

The total shunt resistance R (Figure 3) is equal to the sum of the resistances R_1 and R_h . The total capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_h}$$

or on rearranging,

$$C = \frac{C_1 C_h}{C_1 + C_h}$$

Substitution in Equation 20 yields

$$\frac{1}{Z} = \frac{1}{R_1 + R_h} + j \omega \frac{C_1 C_h}{C_1 + C_h}$$

Now equation 20 may be rearranged in terms of the usually measured values of power factor according to the following procedure.

$$\begin{aligned} \frac{1}{Z} &= \frac{1}{R} + j \omega C \\ &= \frac{1 + j \omega CR}{R} \\ &= \frac{1 + 1/\tan \delta}{R} \\ &= \frac{\tan \delta + 1}{\tan \delta (R)} \end{aligned}$$

$$= (\tan \delta + 1) \omega C$$

$$= (\tan \delta + 1) \omega \epsilon' C_o$$

$$= \omega C_o \epsilon'' j + \omega \epsilon' C_o$$

$$= \omega C_o (j \epsilon'' + \epsilon')$$

$$= \omega(C_o \epsilon'' + C).$$

$$\frac{1}{\omega Z} = C_{\text{observed}} = C_o \epsilon'' + C_{\text{calculated}}$$

but,

$$C = \frac{C_1 C_h}{C_1 + C_h}$$

$$C = \frac{1}{\omega Z} = \epsilon'' C_o + \frac{C_1 C_h}{C_1 + C_h},$$

or,

$$\frac{C_1 C_h}{C_1 + C_h} = C - C_o \epsilon'',$$

or

$$C_1(C_h - C + C_o \epsilon'') = (C - C_o \epsilon'') C_h.$$

APPENDIX B

List of Symbols Used in This Report

- A Constant comprising the real part of a complex number.
- a Constant for each material to which the perfect gas law is applicable.
- B Constant comprising the imaginary part of a complex number.
- B' Steric constant arising in the energy of activation formula.
- b Constant for each material to which the perfect gas law is applicable.
- C The electrical capacitance of a system.
- C₀ The calculated capacitance of a geometric arrangement of electrical conductors.
- C' Constant of proportionality in the Boltzman equation.
- D Total displacement current; a term introduced by Maxwell to indicate the current arising when a condenser is charged.
- d Distance; centimeters.
- δ Loss angle; the complimentary angle to the phase angle, ϕ , (q.v.) hence equal to 1-d.
- E Voltage gradient; electrical stress; volts/cm.
- e The base of the natural logarithm defined as $u = \log_e e^u$.
- ε The complex dielectric constant.
- ε' The measured dielectric constant; the real part of the complex dielectric constant.
- ε₀ The dielectric constant at constant voltage (zero frequency).
- ε_∞ The dielectric constant at optical frequencies.
- ε'' The loss factor defined by $\epsilon', \tan \delta$.
- η The viscosity coefficient.
- F Mechanical force in dynes.
- f Frequency or cycles per second
- I Current in amperes.

j	The imaginary operator, $\sqrt{-1}$.
k	Boltzman's constant, $1.4 \cdot 10^{-16}$ ergs/deg.
μ	One-one millionth.
n	Number of gram-molecular weights.
P	Pressure in grams/cm ² .
p	Probability constant.
π	Universal constant, 3.1416.
Q	Molar energy of activation.
q	Energy of activation expressed in ergs.
R	Electrical resistance expressed in ohms.
R ₀	Universal gas constant, 1.9864 calories per °C per mole.
S	Steric constant in the Boltzman derivation.
S'	Constant of proportionality.
Σ	Algebraic summation.
T	Temperature in degrees absolute.
τ	Relaxation time of the dielectric
τ_c	Time constant expressed in seconds.
V	Voltage
v	Volume in cubic centimeters.
W	Total electrical energy.
W'	Total electrical losses.
ϕ	Phase angle.
θ	Thickness in cms of test cell.
