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ECLIPSE AREA AND ECLIPSE FUNCTION CALCULATIONS

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ABSTRACT

Tables are calculated for:

(a) the exposed areas of a sun eclipsed by a moon whose diameter is 1.04 of the sun's diameter, Table I,

(b) the exposed areas of a sun eclipsed by a moon of equal size including the corona out to 1.50 of the radius of the sun, Table IV, and

(c) the variation in intensity of the radiation from the sun reaching a distant observer for a 1.04 eclipse for light of wave-length approximately 5600 angstroms, Tables II and III.

ECLIPSE AREA AND ECLIPSE FUNCTION CALCULATIONS

INTRODUCTION

This report presents calculations for the May 20, 1947, eclipse, which were used in the elucidation of the optical and microwave measurements of that eclipse made by parties from this Laboratory. The results are given in Tables and are:

1) Eclipse Areas - The computation of the exposed area of an eclipsed sun, covered by a moon 1.04 times as large. The exposed area of a sun equal in size to the moon was also computed and included the corona out to 1.50 of the radius of the sun. (Table I, page 7, and Table IV, page 11).

2) The Eclipse Function - The variation in intensity of the radiation from the sun reaching a distant observer for various positions of the moon across the sun, computed for light of wavelength of approximately 5600 angstroms for a 1.04 eclipse. (Table II, page 8, and Table III, page 9).

METHODS OF COMPUTATION

The calculation of the eclipse function of the sun was made by means of the Method of Julius[1] with data for the radial distribution of surface brightness across the face of the sun, (called the *brightness function*,) based on the work of Julius, and of Moll, Berger and van der Bilt[2]. Although the calculation during the first half of the eclipse was for a solar eclipse magnitude of 1.00 the results were found to be more than 99 percent valid for an eclipse magnitude of 1.04. Brightness data were for a wavelength of 5500 angstroms.*

The method of Julius consists essentially of dividing the sun into 12 somewhat arbitrary concentric zones and assigning to each an average brightness value. These zones are quite small at the limb of the sun where the brightness function varies rapidly. They are, however, quite large at the center of the sun, where the brightness function changes slowly. Figure 1 shows how the zones were numbered. Zones numbered 5, 6, and -1 through -10 were 0.05R wide; zones 1, 2, 3, and 4 were 0.025R wide; zones 7, 8, 9, and 10 were 0.10R wide; and zones 11 and 12 were 0.20R wide; R was the radius of the sun. Zones -1 through -10 represent the corona of the sun. The moon was moved across the sun in steps of 0.05R except in the first calculation when the last two steps were further divided into two smaller steps of 0.025R each.

*See page 14 for references 1 - 5

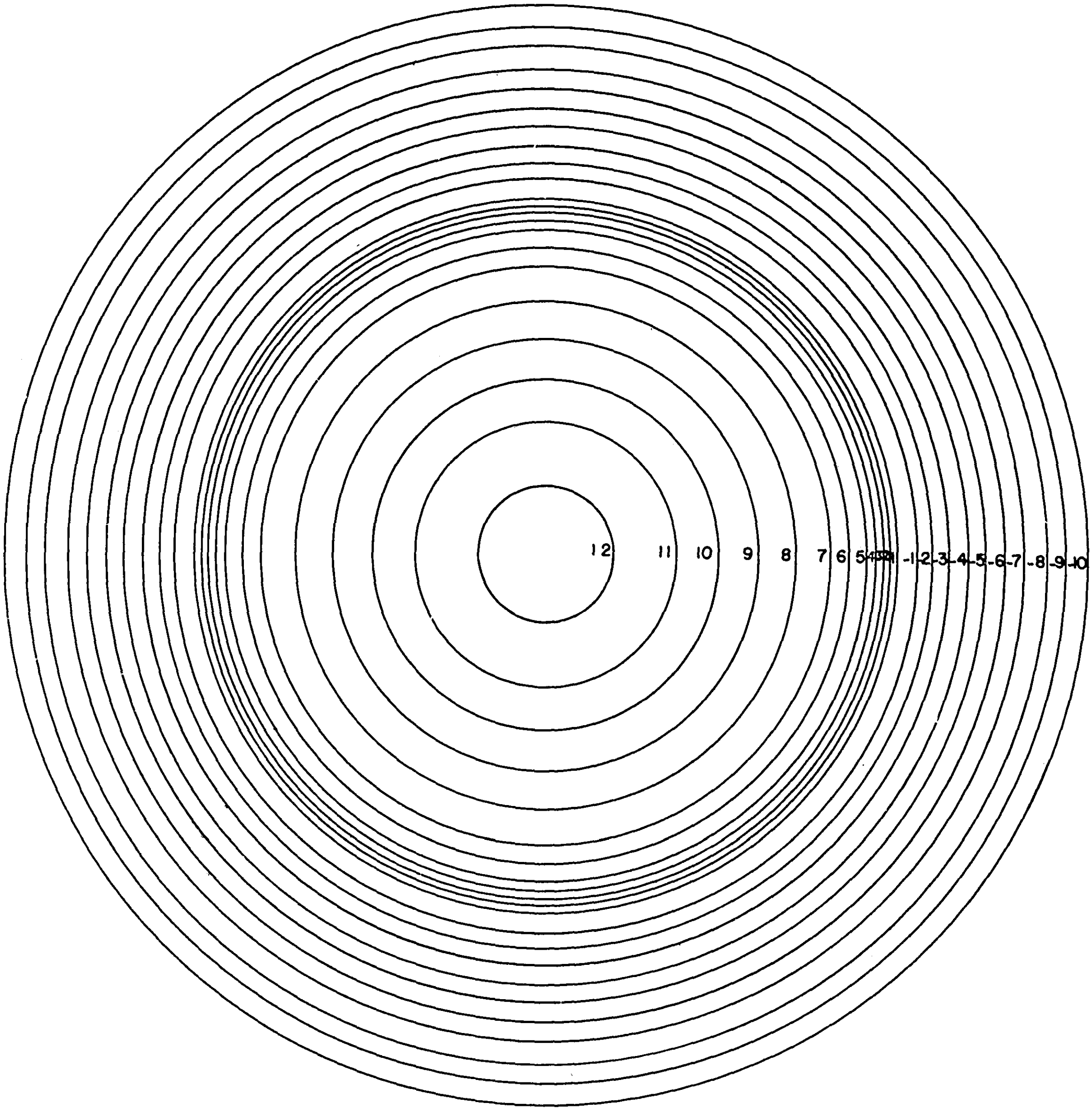

Figure 1. The Sun is Divided into Zones 1 through 12.
The Corona is Divided into Zones -1 through -10.

A value for the radiation intensity of each exposed zone area was obtained by multiplying the exposed portion of the zone area by the brightness function of the zone. The value of the eclipse function for a particular position of the moon was obtained by adding all the resulting zone intensities for this position and normalizing the sum by dividing it by the radiation intensity of the entire sun.

The brightness function values of Julius, obtained by use of a thermopile and given in Table 3 of his paper, were used for all zone areas whose radii were less than 0.85 of the radius of the sun. The values for a wavelength of 5500 Angstroms, given by Moll, Berger, and van der Bilt,

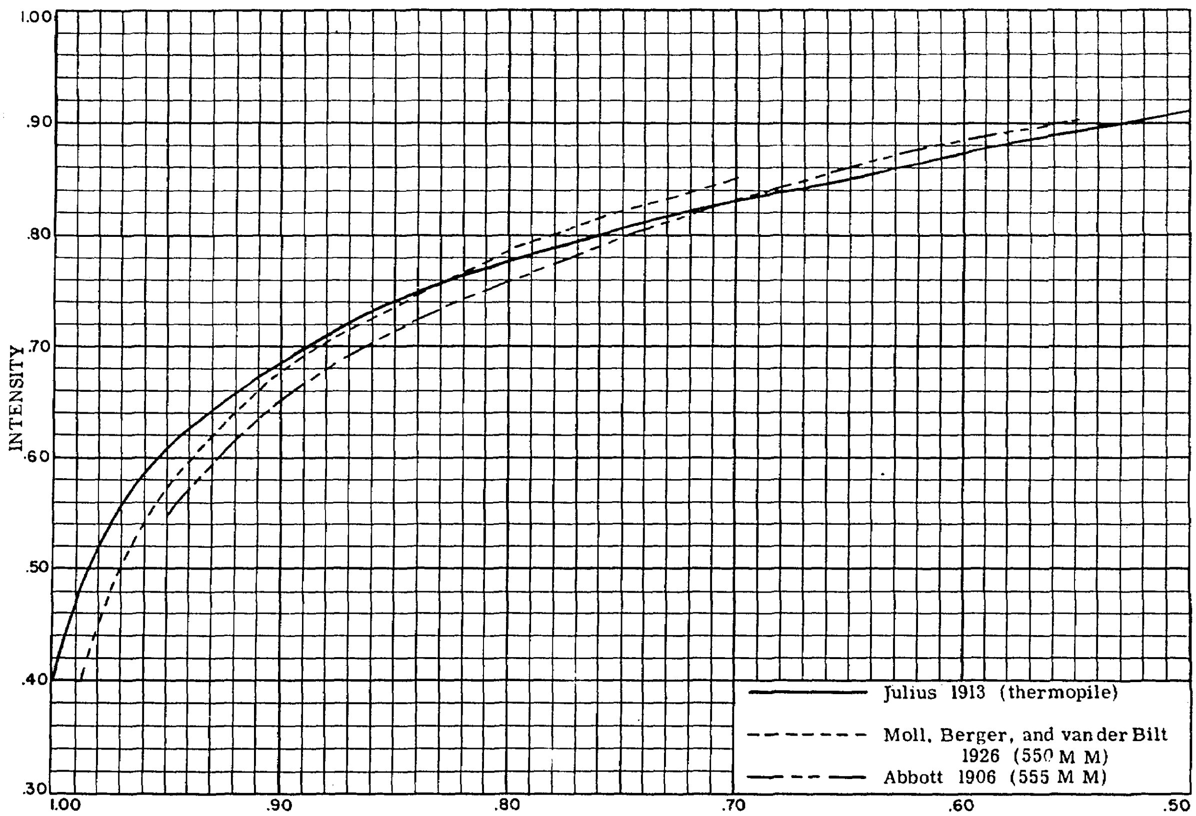

Figure 2. Distance from Center of Sun in terms of Radius of the Sun.

were used for the rest of the sun. For an additional and more detailed calculation, which will be explained later, the brightness function values of Kopal[3] based on the data of Wesselink[4] for the limb of the sun were used. A graph comparing various brightness curves for a wavelength of 5500 Angstroms is shown in Figure 2.

The area of the sun obscured by the moon is formulated in terms of r_1 the angular radius of the sun, r_2 the angular radius of the moon, and $\bar{a}$ the angular amount of the diameter of the sun covered by the moon. As shown in Figure 3, the area of the sun obscured by the moon is the sum of the two areas AEBC and ADBC.

$$\theta = \text{arc cos } x_1/r_1 , \quad (1)$$

$$\theta = \text{arc cos } x_2/r_2 , \quad (2)$$

$$\begin{aligned} AEBC &= 2\,(AO'E - AO'C) \\ &= 2\left(\frac{\theta_1}{2\pi}\,\pi r_1^2 - \tfrac{1}{4}\, r_1^2 \sin 2\theta\right) \quad (3) \end{aligned}$$

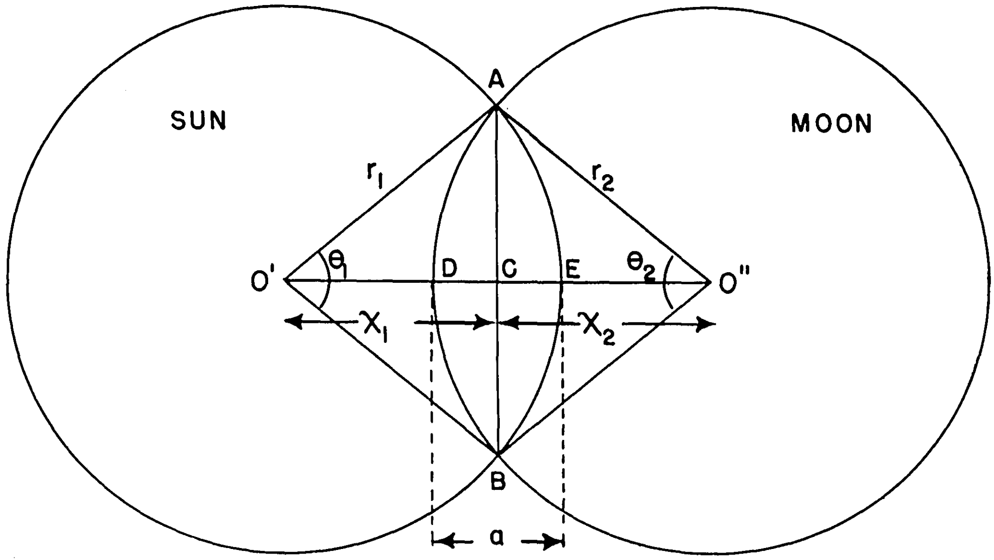

Figure 3. Geometry of Eclipse.

$$\text{ADBC} = 2\left(\frac{\theta_2}{2\pi} \pi r_2^2 - \tfrac{1}{4} r_2^2 \sin 2\,\theta \right) . \tag{4}$$

Since $x_1 + x_2 = r_1 + r_2 - a$,

and $r_1^2 - x_1^2 = r_2^2 - x_2^2$,

$$x_1/r_1 = \frac{(1 - a/r_1)\,(1 + r_2/r_1) + (a/r_1)^2/2}{1 + r_2/r_1 - a/r_1} , \tag{5}$$

$$x_2/r_2 = \frac{(1 - a/r_2)\,(1 + r_1/r_2) + (a/r_2)^2/2}{1 + r_1/r_2 - a/r_2} . \tag{6}$$

Fractional area of the sun obscured by the moon:

$$= \frac{1}{\pi}\left\{ (\theta - \tfrac{1}{2} \sin 2\,\theta) + \left[\frac{r_2}{r_1}\right]^2 (\theta - \tfrac{1}{2} \sin 2\,\theta_2) \right\} . \tag{7}$$

Calculations from Eq. 7 are found in Table 15, page 46 of the Eclipse Supplement to the American Ephemeris for October 1, 1940.

The numerical calculation of the area of each zone by means of Eq. 7 was a lengthy procedure. It was done by computing the exposed area of each zone cut out by two radii which intersected the zone near the spot intersected by the edge of the moon. The radii used were in steps of five degrees. The correction between the nearest computed zone sector and the actual exposed zone area was measured with a planimeter using a paper moon and sun with the sun divided into zones and sectors. This is illustrated in Figure 4. Area 2 was measured with the planimeter and added to area 1 for this particular zone. The total area of all the zones for each moon position was compared with the corresponding area in Table 15, page 46 of the Eclipse Supplement to the American Ephemeris for October 1, 1940. The agreement was exact in most cases and within one percent in all cases.

The paper moon used to cover the zoned sun in the measurements of the first 0.475 of the eclipse for the first area calculation was the same size as the sun, instead of 1.038 as in the 1947 eclipse. The exposed sun area for an eclipse magnitude of 1.04, however, differs by less than one percent when at least 0.50 of the diameter of the sun is exposed (the error is 0.8 percent when 0.50 of the sun's diameter is covered by the moon and less than 0.8 percent for greater exposures of the sun).

In addition, since the greatest zone area differences occur at the limb of the sun where the brightness function is least, the eclipse function for magnitudes of 1.04 and 1.00 correspond even more closely than do the two areas.

More care was exercised in the computation of the eclipse function when 0.50 or more of the sun's diameter was obscured by the moon. The same method was used as for the first half of the eclipse, but the paper moon was 1.04 the size of the paper sun.

The exposed area of each zone was determined for 20 positions of a 1.00-magnitude moon up to 0.475 of the covered portion of the sun's diameter and for 19 positions of a 1.04 magnitude moon during the rest of the eclipse. These calculations are given in Table I. From the zone areas the eclipse function was calculated and is tabulated in Table II. A more precise and detailed table of the exposed area of the sun, given in Table

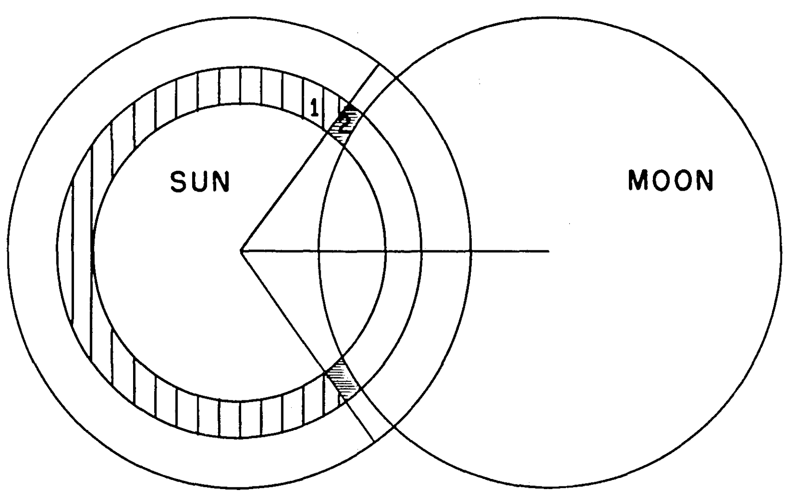

Figure 4. Detail of Zone Calculation.

III, was made for exposures of three percent or less of the sun's diameter. For most of the Table graphs of precisely calculated, exposed-sun-area fractions vs sun's exposed-diameter fractions were used. When only 0.00004 or less of the sun's diameter was exposed, an approximate formula for the area was used:

$$\text{Exposed area} = \frac{1}{\pi r_1^2}\left[2r_1\, r_2/\,(r_2 - r_1)\right]^{1/2} a^{3/2} \qquad (8)$$

$$= 0.03733\ a^{3/2}\ ,$$

where a minutes of arc is the exposed portion of the sun's diameter. The areas were multiplied by the brightness function of Kopal[3] for an effective wavelength of 4540 Angstroms, multiplied by a proper constant to fit the brightness function curve of Moll, Berger, and van der Bilt for a wavelength of 5500 Angstroms. This is also tabulated in Table III.

A second zone area calculation was made to include the corona of the sun out to 1.50 of the radius of the sun. This was done for a sun equal in size to the moon; the zone areas are tabulated in Table IV.

If the brightness function for a given wavelength is not known it may be computed by using the method of Julius from experimental data for the eclipse function for this wavelength and the zone areas given in Table IV. Let A_1 be the measured radiation received when the moon is in position 1, A_2 the measured radiation received when the moon is in position 2, A_3 for position 3, etc.; let a_{1-1} be the exposed area of zone -1 when the moon is in position 1, a_{45} the exposed area of zone 5 when the moon is in position 4, etc.; and let x_1 be the average brightness of zone 1, x_4 the average brightness of zone 4, etc. Then,

$$a_{1-10}\, x_{-10} + a_{1-9}\, x_{-9} + \cdots\ a_{11}\, x_1\ a_{12}\, x_2\ \cdots\ a_{1,12}\, x_{12} = A_1\ ,$$

$$a_{2-10}\, x_{-10} + a_{2-9}\, x_{-9} + \cdots\ a_{21}\, x_1\ a_{22}\, x_2 \cdots a_{2,12}\, x_{12} = A_2\ ,$$

etc.

The set of equations may be written

$$\sum_{\substack{j = -10 \\ j \neq 0}}^{j = 12} a_{ij}\, x_j = A_i \qquad (9)$$

and is solved for x_j by computing the inverse of the matrix by means of the method of Waugh and Dwyer[5]. then x_j = the inverse of a_{ij} times A_i = $a_{ij}^{-1} A_i$. Fifty positions of the moon are tabulated (i.e., there are 50 i's), and any 22, of course, can be used in the computation. Thus, the brightness function for any radiation can be calculated from eclipse function data for this radiation.

TABLE I

Eclipse areas for a 1.04 eclipse

ZONE	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0	42.5	45.0	47.5
1	049375	.04610	.04462	.04344	.04242	.04146	.04068	.04000	.03926	.03860	.03794	.03742	.03685	.03629	.03579	.03514	03483	.03412	.03377	.03356
2	.048125	.04625	.04411	.04278	.04171	.04067	.03987	.03915	.03840	.03770	.03705	.03654	.03592	.03535	.03480	.03418	.03374	.03311	.03293	.03243
3	.046875	.04688	.04376	.04223	.04104	.03996	.03911	.03834	.03753	.03684	.03613	.03561	.03501	.03440	.03386	.03325	03277	.03213	.03188	.03152
4	.045625	.04563	.04397	.04168	.04034	.03923	.03832	.03750	.03672	.03599	.03527	.03469	.03409	.03347	.03291	.03227	.03178	.03108	.03088	.03033
5	.08750	.08750	.08750	.08282	.07894	.07653	.07440	.07270	.07100	.06947	.06800	.06679	.06551	.06425	.06302	.06178	.06077	.05950	.05892	.05761
6	.08250	.08250	.08250	.08250	.07563	.07410	.07167	.06964	.06780	.06614	.06468	.06328	.06200	.06064	.05938	.05806	.05702	.05561	.05500	.05344
7	.15000	.15000	.15000	.15000	.15000	.13930	.13748	.13138	.12685	.12302	.11952	.11654	.11379	.11086	.10823	.10545	.10328	.10034	.09890	.09586
8	.13000	.13000	.13000	.13000	.13000	.13000	.13000	.12582	.11795	.11193	.10752	.10376	.10074	.09748	.09472	.09172	.08923	.08628	.08444	.08127
9	11000	.11000	.11000	.11000	.11000	.11000	.11000	.11000	.11000	.10593	.09851	.09266	.08860	.08503	.08182	.07858	.07594	.07288	.07074	.06753
10	.09000	.09000	.09000	.09000	.09000	.09000	.09000	.09000	.09000	.09000	.09000	.08617	.07902	.07373	.06984	.06631	.06324	.05999	.05765	.05428
11	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.12000	.11631	.10983	.10162	.09259	.08458	.07860	.07177
12	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.03731	.03267	.02681
ZONE	50.0	50.0	55.0	60.0	65.0	67.5	70.0	72.5	75.0	77.5	80.0	82.5	85.0	87.5	90.0	92.5	95.0	96.25	97.5	98.75
1	.03275	.03241	.03183	.03047	.02954	.02882	.02848	.02800	.02751	.02683	.02642	.02562	.02491	.02395	.02320	.02176	.01972	.01827	.01549	.01380
2	.03172	.03139	.03041	.02936	.02846	.02801	.02743	.02682	.02629	.02546	.02489	.02410	.02316	.02217	.02099	.01897	.01604	.01359	.00824	.00000
3	.03067	.03037	.02929	.02828	.02726	.02664	.02606	.02549	.02495	.02407	.02338	.02241	.02157	.02018	.01870	.01622	.01182	.00695	.00000	
4	.02971	.02929	.02830	.02731	.02611	.02549	.02488	.02425	.02357	.02265	.02193	.02096	.01981	.01823	.01640	.01332	.00606	.00000		
5	.05639	.05573	.05344	.05136	.04903	.04782	.04622	.04481	.04328	.0431	.03978	.03737	.03441	.03047	.02565	.01530	.00000			
6	.05239	.05184	.04916	.04714	.04454	.04301	.04146	.04000	.03839	.03615	.03416	.03091	.02741	.02204	.01302	.00000				
7	.09336	.09244	.08761	.08235	.07643	.07319	.06967	.06632	.06244	.05652	.05085	.04246	.03112	.01079	.00000					
8	.07856	.07785	.07277	.06666	.06042	.05676	.05246	.04790	.04254	.03510	.02502	.00948	.00000	.00000						
9	.06432	.0646	.05858	.05232	.04497	.04072	.03570	.02976	.02107	.01484	.00000	.00000								
10	.05157	.05099	.04511	.03851	.03000	.02759	.01693	.00660	.00000	.00000										
11	.06567	.0654	.05288	.03622	.01348	.00531	.00000	.00000												
12	.02081	.02076	.00827	.00000	.00000	.00000	.00000	.00000												

TABLE II

Eclipse function for an eclipse magnitude of 1.00 with 0 to 0.475 of sun's diameter covered, and for an eclipse magnitude of 1.04 with 0.5 to 1 of sun's diameter covered.

Fraction of Sun's Diameter Covered	Fraction of Total Sun Radiation Received	Fraction of Sun's Diameter Covered	Fraction of Total Sun Radiation Received
.000	1.000	.475	.633
.025	.997	.500	.596
.050	.991	.525	.566
.075	.982	.550	.535
.100	.969	.575	.493
.125	.952	.600	.471
.150	.943	.625	.438
.175	.927	.650	.405
.200	.909	.675	.376
.225	.889	.700	.340
.250	.868	.725	.308
.275	.846	.750	.276
.300	.823	.775	.249
.325	.798	.800	.211
.350	.773	.825	.178
.375	.745	.850	.148
.400	.720	.875	.116
.425	.689	.900	.086
.450	.666	.925	.060
		.950	.034

TABLE III

Exposed sun areas and eclipse function for last 0.03 of eclipse.

A	Fraction of Sun's Diameter Covered	Fraction of Area Exposed x10^4	Brightness Function	Eclipse Function x10^4
.002	.999937	.03339	.281	.012
.004	.999874	.09444	.286	.034
.006	.999810	.1735	.291	.064
.008	.999747	.2671	.297	.100
.010	.999684	.3733 .42	.302 .301	.14 .16
.012	.999620	.56	.305	.22
.014	.999557	.70	.307	.27
.016	.999494	.88	.309	.34
.018	.999430	1.03	.311	.40
.020	.999367	1.25	.3135	.495
.025	.999210	1.70	.319	.68
.030	.999052	2.27	.324	.925
.035	.99889	3.0	.327	1.24
.040	.99874	3.7	.329	1.535
.045	.99858	4.5	.333	1.89
.050	.99842	5.385	.334	2.276
.06	.99810	6.93	.337	2.95
.07	.99779	8.6	.339	3.68
.08	.99747	10.4	.341	4.48
.09	.99716	12.3	.342	5.31
.10	.99684	14.5	.345	6.31
.12	.99620	19	.347	8.31
.14	.99557	23	.348	10.1
.16	.99494	28	.350	12.4
.18	.99430	34	.352	15.1
.20	.99367	39	.354	17.4
.25	.99210	53	.357	.00238
.30	.99052	68	.362	.00310
.35	.9889	84	.366	.00387
.40	.9874	100	.37	.00467
.45	.9858	117	.382	.00564
.50	.9842	133	.41	.00688

TABLE III (Continued)

A	Fraction of Sun's Diameter Covered	Fraction of Area Exposed $x10^4$	Brightness Function	Eclipse Function $x10^4$
.55	.9826	150	.42	.00795
.60	.9810	168	.44	.00932
.65	.9795	186	.455	.01067
.70	.9779	204	.47	.0121
.75	.9763	223	.475	.0134
.80	.9747	241	.483	.0147
.85	.9731	259	.491	.0160
.90	.9716	278	.499	.0175
.95	.9700	296	.507	.0189
1.00	.9684	315	.515	.0204

TABLE IV

Eclipse area for a 1.00 eclipse, including the corona out to 1.50 of the sun's radius.

Z \ Δ	-∞	-.225	-.200	-.176	-.150	-.125	-.100	-.075	-.050	-.025	0	.025	.050	.075	.100	.125	.150
-10	.1475	.1425	.1382	.1355	.1332	.1313	.1296	.1281	.1266	.1254	.1243	.1232	.1221	.1211	.1202	.1192	.1183
-9	.1425	.1425	.1375	.1333	.1306	.1282	.1264	.1248	.1232	.1218	.1206	.1194	.1182	.1171	.1162	.1151	.1142
-8	.1375		.1375	.1327	.1283	.1256	.1234	.1215	.1198	.1182	.1169	.1157	.1144	.1133	.1123	.1112	.1101
-7	.1325			.1325	.1275	.1233	.1206	.1186	.1165	.1150	.1134	.1121	.1108	.1095	.1086	.1072	.1062
-6	.1275				.1275	.1226	.1184	.1159	.1136	.1117	.1100	.1084	.1072	.1058	.1046	.1034	.1023
-5	.1225					.1225	.1177	.1127	.1108	.1087	.1069	.1051	.1038	.1022	.1009	.0997	.0985
-4	.1175						.1175	.0045	.1085	.1060	.1038	.1019	.1003	.0988	.0974	.0960	.0948
-3	.1125							.1125	.1077	.1037	.1011	.0988	.0971	.0953	.0935	.0924	.0912
-2	.1075								.1075	.1030	.0988	.0961	.0950	.0922	.0906	.0889	.0877
-1	.1025									.1025	.0979	.0938	.0913	.0892	.0873	.0857	.0842
Z \ Δ	-∞	-.225	-.200	-.175	-.150	-.125	-.100	-.075	-.050	-.025	0	.025	.050	.075	.100	.125	.150
1	.049375										.049375	.04610	.04462	.04344	.04242	.04146	.04068
2	.048125										.048125	.04625	.04411	.04278	.0471	.04067	.03987
3	.046875										.046875	.04688	.04376	.04223	.04104	.03996	.03911
4	.045625										.045625	.04563	.04397	.04168	.04034	.03923	.03832
5	.08750										.08750	.08750	.08750	.08282	.07894	.07653	.07440
6	.08250										.08250	.08250	.08250	.08250	.07563	.07410	.07167
7	.15000										.15000	.15000	.15000	.15000	.15000	.13920	.13748
8	.13000										.13000	.13000	.13000	.13000	.13000	.13000	.13000
9	.11000										.11000	.11000	.11000	.11000	.11000	.11000	.11000
10	.09000										.09000	.09000	.09000	.09000	.09000	.09000	.09000
11	.12000										.12000	.12000	.12000	.12000	.12000	.12000	.12000
12	.04000										.04000	.04000	.04000	.04000	.04000	.04000	.04000

TABLE IV (Continued)

Z \ Δ	.175	.200	.225	.250	.275	.300	.325	.350	.375	.400	.425	.450	.475	.500	.525	.550	.575	.600
-10	.1175	.1168	.1161	.1151	.1149	.1143	.1139	.1135	.1131	.1129	.1127	.1126	.1126	.1128	.1131	.1136	.1143	.1152
-9	.1133	.1125	.1118	.1111	.1104	.1098	.1092	.1088	.1084	.1080	.1077	.1074	.1074	.1072	.1074	.1077	.1081	.1087
-8	.1092	.1084	.1075	.1067	.1061	.1057	.1047	.1042	.1037	.1032	.1027	.1024	.1022	.1020	.1019	.1020	.1022	.1026
-7	.1052	.1043	.1034	.1025	.1018	.1010	.1003	.0997	.0990	.0984	.0979	.0974	.0973	.0970	.0967	.0968	.0966	.0968
-6	.1012	.1008	.0993	.0984	.0975	.0967	.0960	.0952	.0945	.0938	.0934	.0929	.0924	.0919	.0915	.0912	.0911	.0910
-5	.0974	.0963	.0953	.0942	.0936	.0925	.0917	.0909	.0900	.0895	.0889	.0882	.0876	.0870	.0865	.0862	.0859	.0857
-4	.0936	.0924	.0914	.0904	.0894	.0884	.0875	.0866	.0860	.0851	.0844	.0836	.0831	.0824	.0818	.0812	.0808	.0803
-3	.0899	.0887	.0876	.0865	.0855	.0844	.0834	.0827	.0818	.0808	.0800	.0793	.0785	.0777	.0770	.0765	.0759	.0753
-2	.0862	.0850	.0838	.0827	.0815	.0805	.0794	.0786	.0776	.0766	.0758	.0749	.0741	.0732	.0725	.0718	.0723	.0703
-1	.0828	.0813	.0802	.0789	.0778	.0766	.0756	.0747	.0736	.0725	.0717	.0707	.0698	.0690	.0687	.0671	.0665	.0656
Z \ Δ	.175	.200	.225	.250	.275	.300	.325	.350	.375	.400	.425	.450	.475	.500	.525	.550	.575	.600
1	.04000	.03926	.03860	.03794	.03742	.03685	.03629	.03579	.03514	.03483	.03412	.03397	.03356	.03275	.0326	.0322	.0318	.0313
2	.03915	.03840	.03770	.03705	.03654	.03592	.03535	.03480	.03418	.03374	.03311	.03293	.03242	.03172	.0310	.0305	.0301	.0295
3	.03834	.03763	.03684	.03613	.03561	.03501	.03440	.03386	.03325	.03277	.03213	.03188	.03152	.03067	.0293	.0288	.0283	.0278
4	.03750	.03672	.03599	.03527	.03469	.03409	.03347	.03291	.03227	.03178	.03108	.03088	.03033	.02971	.0296	.0291	.0286	.0280
5	.07270	.07100	.06947	.06800	.06679	.06551	.06425	.06302	.06178	.06077	.05950	.05892	.05761	.05639	.0554	.0543	.0532	.0521
6	.06964	.06780	.06614	.06468	.06328	.06200	.06064	.05938	.05805	.05702	.05561	.05500	.05344	.05239	.0509	.0498	.0487	.0475
7	.13138	.12685	.12302	.11952	.11654	.11379	.11086	.10823	.10545	.10328	.10034	.09890	.09586	.09336	.0909	.0885	.0859	.0834
8	.12582	.11795	.11193	.10752	.10376	.10074	.09748	.09472	.09172	.08923	.08628	.08444	.08127	.07856	.0762	.0736	.0708	.0679
9	.11000	.11000	.10593	.09851	.09266	.08860	.08503	.08182	.07858	.07594	.07288	.07074	.06753	.06432	.0617	.0588	.0558	.0527
10	.09000	.09000	.09000	.09000	.08617	.07902	.07373	.06984	.06631	.06324	.05999	.05765	.05428	.05157	.0482	.0451	.0418	.0383
11	.12000	.12000	.12000	.12000	.12000	.12000	.11631	.10983	.10162	.09259	.08458	.07860	.07177	.06567	.0591	.0525	.0451	.0359
12	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.04000	.03731	.03267	.02681	.02081	.0149	.0086	.0032	.0000

TABLE IV (Continued)

Z \ Δ	.625	.650	.675	.700	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.9625	.975	.9875	1.00
-10	.1163	.1181	.1204	.1236	.1279	.1360	.1475											.1475
-9	.1121	.1107	.1123	.1147	.1178	.1222	.1308	.1425										.1425
-8	.1031	.1039	.1051	.1067	.1088	.1097	.1165	.1257	.1375									.1375
-7	.0971	.0973	.0981	.0992	.1007	.1028	.1059	.1106	.1198	.1325								.1325
-6	.0910	.0912	.0917	.0923	.0983	.0946	.0966	.0997	.0981	.1137	.1275							.1275
-5	.0855	.0855	.0856	.0859	.0863	.0871	.0885	.0902	.0931	.0980	.1079	.1225						.1225
-4	.0799	.0798	.0796	.0796	.0798	.0801	.0807	.0819	.0835	.0866	.0910	.1010	.1175					.1175
-3	.0747	.0743	.0739	.0736	.0734	.0734	.0736	.0741	.0748	.0763	.0787	.0820	.0944	.1125				.1125
-2	.0697	.0691	.0685	.0679	.0675	.0671	.0668	.0667	.0668	.0673	.0682	.0699	.0741	.0849	.0948	.1075	.1075	.1075
-1	.0663	.0639	.0632	.0624	.0616	.0610	.0604	.0597	.0591	.0588	.0584	.0584	.0590	.0611	.0713	.0811	.0920	.1025
Z \ Δ	.625	.650	.675	.700	.725	.750	.775	.800	.825	.850	.875	.900	.925	.950	.9625	.975	.9875	1.00
1	.0309	.0305	.0300	.0295	.0291	.0287	.0283	.0278	.0273	.0268	.0263	.0256	.0249	.0239	.0230	.0214	.0161	.0000
2	.0291	.0286	.0281	.0276	.0271	.0266	.0261	.0255	.0249	.0243	.0235	.0224	.0211	.0187	.0164	.0108	.0000	.0000
3	.0273	.0268	.0263	.0258	.0252	.0247	.0241	.0235	.0228	.0220	.0210	.0196	.0176	.0138	.0088	.0000		
4	.0275	.0269	.0263	.0257	.0251	.0245	.0238	.0231	.0222	.0212	.0199	.0180	.0151	.0079	.0000			
5	.0510	.0498	.0486	.0473	.0460	.0445	.0429	.0410	.0389	.0362	.0326	.0273	.0169	.0000				
6	.0462	.0450	.0436	.0422	.0406	.0388	.0369	.0346	.0319	.0283	.0234	.0139	.0000					
7	.0807	.0777	.0746	.0712	.0675	.0632	.0583	.0523	.0443	.0316	.0124	.0000						
8	.0647	.0612	.0576	.0535	.0487	.0432	.0362	.0253	.0094	.0000	.0000							
9	.0492	.0455	.0412	.0362	.0300	.0209	.0077	.0000	.0000									
10	.0344	.0300	.0247	.0170	.0063	.0000	.0000											
11	.0242	.0136	.0050	.0000	.0000													
12	.0000	.0000	.0000	.0000	.0000													

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