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Digital Computer Studies of Electron Guns

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ABSTRACT

A digital computer program is being used to solve the electron gun problem to a certain degree of approximation. Certain phenomena have been observed in the numerical solution such as the choice of the perveance density influencing the convergence and final results. These phenomena are not completely understood; however, experience has provided guidelines to effective use of the program. The program was used to evaluate two electron guns of current interest to the Sozotron project, namely, the Litton L-3250 and L-232 guns. It was found that both guns appear to be suitable for this application.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing on other phases.

AUTHORIZATION

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DIGITAL COMPUTER STUDIES OF ELECTRON GUNS

INTRODUCTION

The problem of analyzing and designing electrode configurations for electron guns has been attacked with the aid of various analog devices, such as resistance boards and electrolytic tanks (1-3). Except for highly symmetric geometries, numerical calculations have tended to be extremely tedious and, because of the simplifying assumptions generally made, of limited if not dubious validity. Recently, however, the high-capacity digital computer has provided a new technique for this type of problem. Several computer programs have been reported in the literature, some still employing an analog device for part of the calculation (4), others doing the entire problem on a digital machine (5-7). Because of its inherent convenience, flexibility, and adaptability, this latter method appears to offer the greatest promise not only at the present time but also in the future.

The NRL Electron Beams Branch has always been interested in this general problem because of its requirement for high-quality injected beams. An urgent need for a rapid and accurate method of analysis arose when several existing electron gun structures had to be evaluated. Beyond this, it was felt that a design effort might have to be undertaken if these existing structures were found to be unsuitable for the Sozotron project (8). This report describes the first results that have been obtained from a computer investigation of two electron guns of current interest.

THE COMPUTER PROGRAM

The actual computer program employed in these calculations was a slightly modified version of one originally developed at CERN (9). Since the method of computation and most of the input and output are identical, only a brief discussion is given here. The basic problem is to solve Poisson's equation for given boundary values of potential and some distribution of charge density. The procedure adopted is to solve Poisson's equation numerically with an initial guess for the space charge term. From the values of potential found, electron trajectories are traced through the structure, and new space charge terms are evaluated. The program then repeats the solution of Poisson's equation, the entire cycle recurring until the desired accuracy has been obtained.

Poisson's equation in cylindrical coordinates for axially symmetric geometries,

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2} = -\frac{\rho}{\epsilon_0}, \qquad (1)$$

is solved by the successive over-relaxation (SOR) method (10). In its finite difference form for this method, Eq. (1) appears as

$$\varphi_0^{(k+1)} = \varphi_0^{(k)} + \beta\left[C_1\varphi^{(k)} + C_2\varphi_2^{(k)} + C_3\varphi_3^{(k+1)} + C_4\varphi_4^{(k+1)} + C_5 - \varphi_0^{(k)}\right]. \qquad (2)$$

The quantity β is the accelerating factor of the SOR method; C_1, C_2, C_3, and C_4 are functions of the integration mesh employed and C_5 includes the charge density term. From Eq. (2) it is seen that the potential at mesh point zero (0) for the $(k+1)$th iteration is given in terms of the potentials existing at neighboring points 1, 2, 3, and 4 (Fig. 1). Subroutines

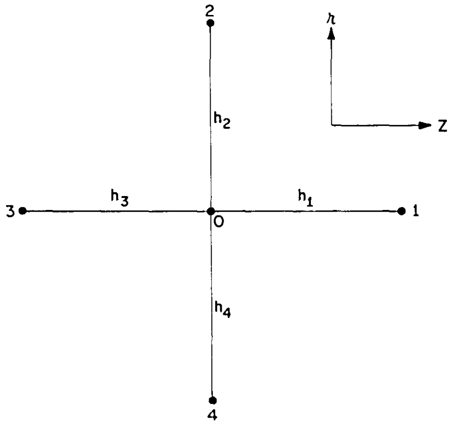

Fig. 1 - Geometry of integration mesh employed for regular points (i.e., not on the boundary)

have been written which automatically compute all the C's for a given mesh point. The accelerating factor β also can be calculated automatically (11), or it can be specified if an optimum value has been found.

Equation (2) is evaluated at each mesh point until the maximum error in the potential is less than a specified amount. The program then switches to its ray-tracing routine. The cathode is divided into a specified number of sections. The current from each section is calculated by assuming that near the cathode one essentially has a number of planar diodes. The solution for the planar diode is given (12) by

$$\varphi(\xi) = K\ \xi^{4/3} \tag{3}$$

where ξ is the distance from anode to cathode in the assumed diode and K is a number depending on the current density j and is given (13) by

$$K = 5.69 \times 10^3\ j^{2/3}. \tag{4}$$

Since φ has been calculated and ξ is set by the computer, the value of K for each section of the cathode can be obtained. From these K's the current from each section of the cathode is determined. These currents are assumed to form rays originating from the center of each section. The rays are traced through the gun, and from the trajectories space charge terms are evaluated. The charge density in the pth cell after the (n + 1)th iteration is given (9) by

$$\rho_p(n+1) = \sum_{i}^{NRAY} \frac{I_i^{(n)}\ \ell_{ip}^{(n)}}{V_{ip}^{(n)}}. \tag{5}$$

Here, NRAY is the number of rays traced, $I_i^{(n)}$ is the current in the ith ray on the nth iteration, $\ell_{ip}^{(n)}$ is the length of the ith ray in the pth cell on the nth iteration, and $V_{ip}^{(n)}$ is the velocity of the ith ray in the pth cell on the nth iteration. Rather than use (5) as it stands, it has been found preferable to apply a relaxation process similar to that used for the potential when calculating the charge density terms. In this case, one uses

$$\bar{\rho}_p^{(n+1)} = \bar{\rho}_p^{(n)} + \alpha\left[\rho_p^{(n+1)} - \bar{\rho}_p^{(n)}\right] \tag{6}$$

where the bar indicated quantities which have been relaxed and $\rho_p^{(n+1)}$ is calculated by Eq. (5). The choice of the accelerating factor α, which is specified by the programmer, has been found to have a great deal of influence on the convergence of the problem. After the new space charge density terms have calculated, the program repeats the solution of Poisson's equation in the form of Eq. (2) at each mesh point. This process of alternately solving Eq. (2) and then tracing trajectories is continued until the solutions meet the accuracy criteria that have been set by the programmer.

Some brief comments regarding the use of the relaxation method for electron gun calculations are now in order. Since the perveance of an electron gun is known to be a geometrical factor, one should be able to determine that the program is working correctly.

This is the case if the perveance computed after each cycle seems to approach a constant value. During actual runs, the perveance usually oscillates with decreasing amplitude about its final value. If the parameters have been chosen properly, the process is very fast, and after just a few iterations the perveance is found to be very close to its final value (Fig. 2). In other cases, however, the problem set for the computer simply will not converge. The perveance, in these cases, does not approach a constant value, but rather continues to oscillate (Fig. 3). Since the potential accelerating factor β is automatically determined (at least initially), one can vary α, the charge density accelerating factor. Typically, α is a number less than unity, with a value of 0.9 generally being used initially. The effect of a slight change in α can be seen in Fig. 4, where α was set at 0.8 and the calculation portrayed in Fig. 3 continued.

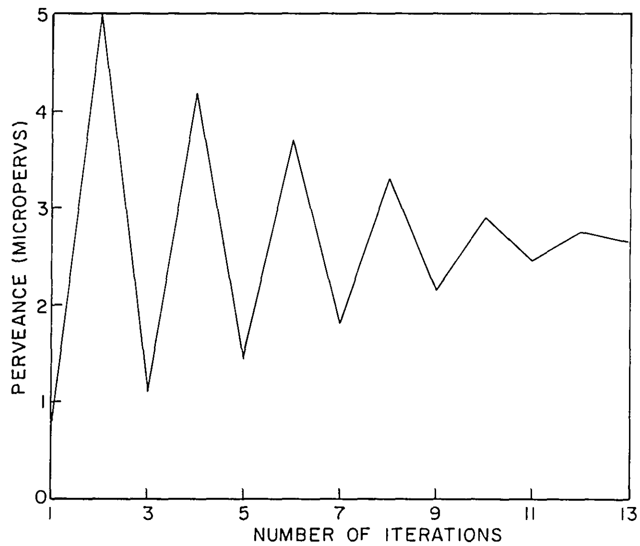

Fig. 2 - Typical convergence of perveance during a computer solution

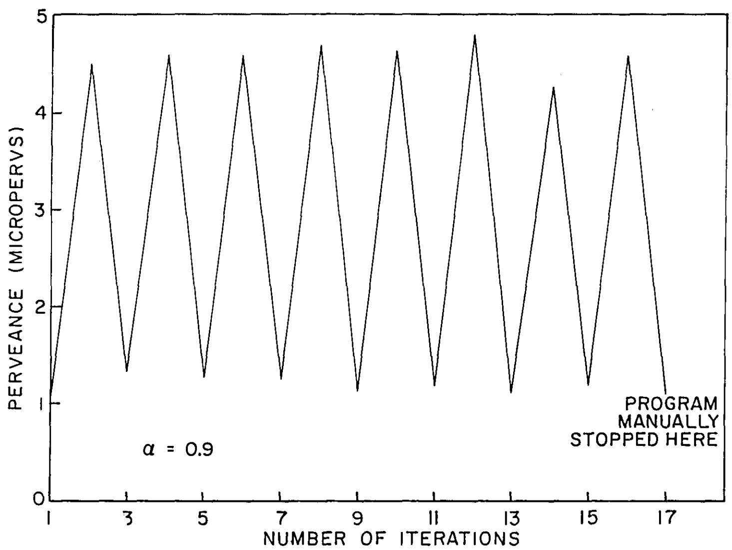

Fig. 3 - Oscillation of perveance during a computer solution. Here $\alpha = 0.9$.

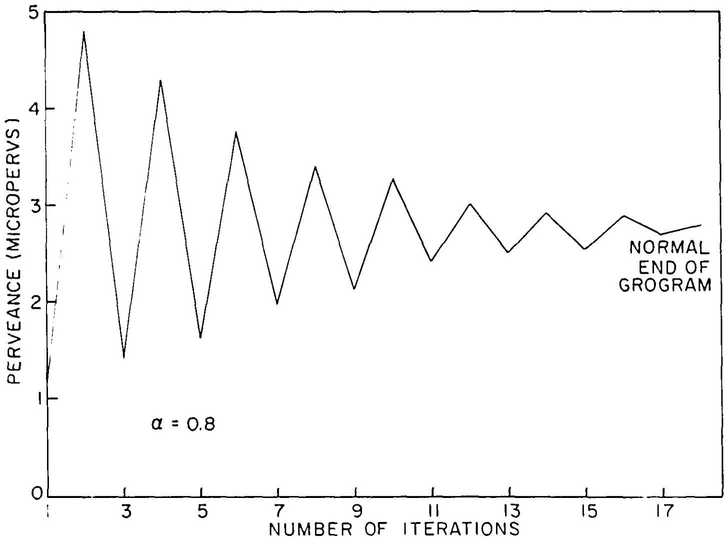

Fig. 4 - Effect of changing α from 0.9 (see Fig. 3) to 0.8

Thus, it has been found that changing α not only influences the speed of convergence, but also in some cases will determine whether the problem will converge at all. Also the initial guess for the charge density has been found to influence the final results somewhat.

Questions certainly can be raised about the behavior described here and about certain other phenomena which have been observed. Unfortunately, some aspects of the SOR process are not well understood, and investigations have not progressed to the point where specific statements can be made. However, experience has provided some empirical guidelines which permit one to use the program quite effectively.

THE SPHERICAL DIODE

The complete spherical diode (14) has proved to be an interesting and useful problem. As a check on the results obtained for axially symmetric guns, this calculation has indicated the areas that are handled very accurately in the program and those which are not. In its own right, the spherical diode forms the basis for most of the high-current electron guns in use today. In studying this problem, several ratios of anode radius to cathode radius were employed, while the size of the integration mesh, the accelerating factors α and β, the initial charge density guess, the number of rays traced, etc., were varied. It was hoped that some general ideas of the effects of these changes might be obtained from this procedure. For rather coarse integration meshes and less stringent accuracy requirements, the problem quickly converged to acceptable solutions almost independently of the values of the various parameters chosen. On the other hand, as the accuracy criteria were made more stringent, the problem became very dependent upon their choice. In fact, when α was improperly chosen, the solution did not converge and in one case, at least, seemed to be oscillating about results that were greatly in error. However, after a few runs the "correct" values became more or less evident, and good results were obtained.

As an indication of these results and the conclusions that have been drawn from them, the solutions for a rather coarse mesh (1.0×1.0 on an area of 10.0×20.0) are exhibited in Figs. 5-7. In this case, α was set equal to 0.9 and fourteen rays were traced, the overall accuracy being specified as 5%.

The current density at the cathode, or the cathode loading, is plotted in Fig. 5. While the theoretical value is a constant, the computed values show a variation of some 10% among themselves and are in all cases higher than the theoretical value. This tendency was also noticed in the original version of this program and has been attributed to "the discrete nature of the way space charge is introduced. . ." (15). Decreasing the mesh size and tracing more rays reduced the magnitude of this variation; the general picture, nevertheless, remained quite the same.

The trajectories of several of the rays traced are shown in Fig. 6. Except along the axis of symmetry used in Fig. 6, the computed rays are in very good agreement with theory. The exact reason for the disagreement of the axial rays with theory is not known at present, although one might blame the round-off error inherent in the calculation. In any case one is led to distrust the trajectory of the axial ray in other calculations.

The average of the computed potential is compared with a theoretical plot in Fig. 7. In no case is the computed value more than 1% in error. This is perhaps better than one can reasonably expect in view of the rather loose accuracy requirements specified for this particular run.

The main conclusions from the spherical diode study may be summarized briefly. The values of potential and the trajectories (off the axis of symmetry) are in excellent agreement with the theoretical values. The charge density terms, on the other hand, show a certain variation which must be considered in their interpretation. In any case, with smaller mesh sizes and more rigorous accuracy requirements, all quantities approach their true values when the problem is correctly set. With these very general considerations in mind, the investigation of two electron guns of current interest may be discussed.

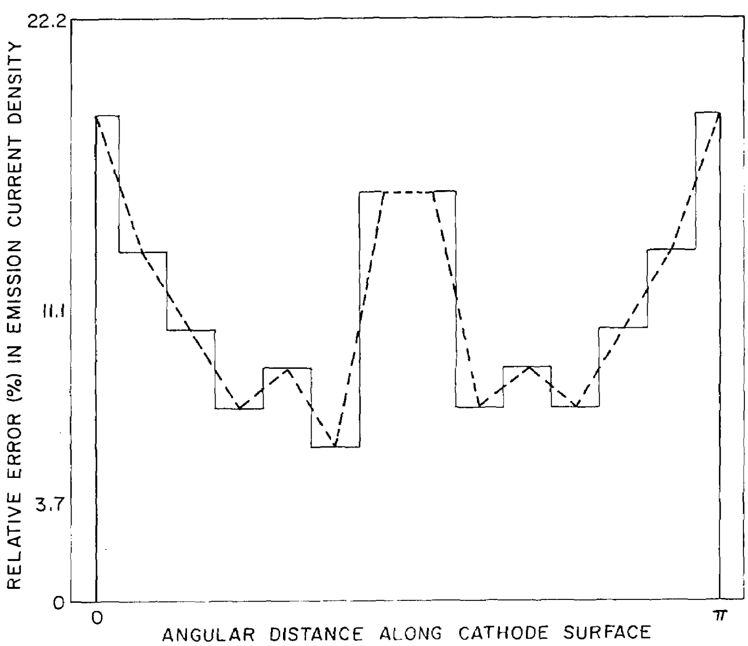

Fig. 5 - Cathode loading computed for a spherical diode with a ratio of anode radius to cathode radius of 1/2 (5.0% accuracy specified in program)

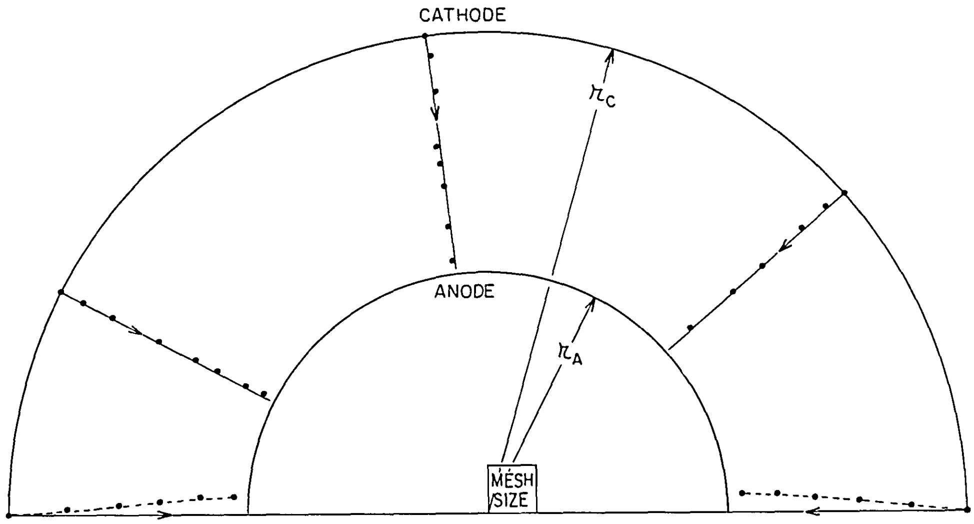

Fig. 6 - Trajectories in a spherical diode with a ratio of anode radius to cathode radius of 1/2 (5.0% accuracy specified in program). The dots are computed values; the solid lines are the theoretical plots.

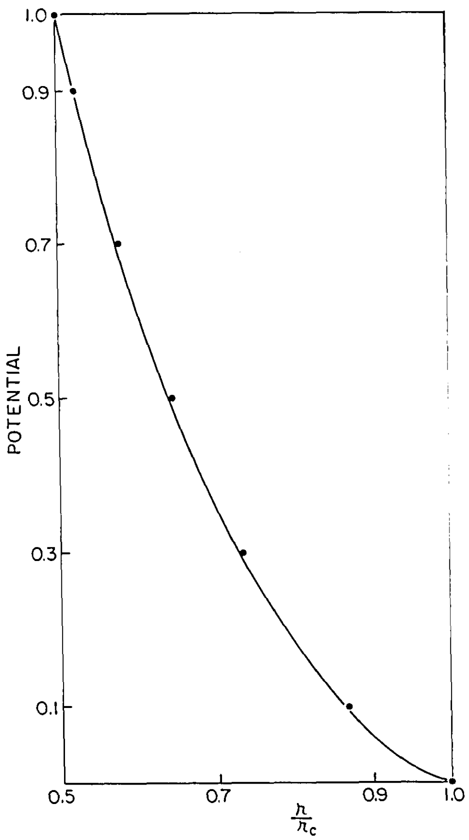

Fig. 7 - Variation of potential in a spherical diode with a ratio of anode radius to cathode radius of 1/2. The dots are computed values; the solid line is the theoretical plot.

THE LITTON L-3250 ELECTRON GUN

The electron gun from the Litton 3250 klystron has been proposed as the original electron source for the Sozotron. Its tentative selection was based on several factors, such as relatively low cost, ready availability, and magnitude of current delivered. However, little was known about the beam quality one might expect from it. For the Sozotron application one needs a space-charge-limited beam, that is, a beam in which the thermal energy is negligible by comparison with the potential energy of the charge distribution (8). Beam quality is usually inferred from the degree of laminarity in the electron flow, although this alone may not always be an accurate criterion. In the computer program, rays are started from the cathode with no initial velocities, that is, with zero temperature. Any marked nonlaminarity of the flow would be enough to give the beam an "effective" temperature, which would be detrimental in the Sozotron application.

The cathode loading for the L-3250 is shown in Fig. 8. The variation of nearly 43% from center to edge is indeed significant if not very surprising. Spherical cathode guns designed by the Pierce method (13) for high perveance (here 2×10^{-6} amp/volt$^{3/2}$) have long been known to exhibit such nonuniform emission current densities.

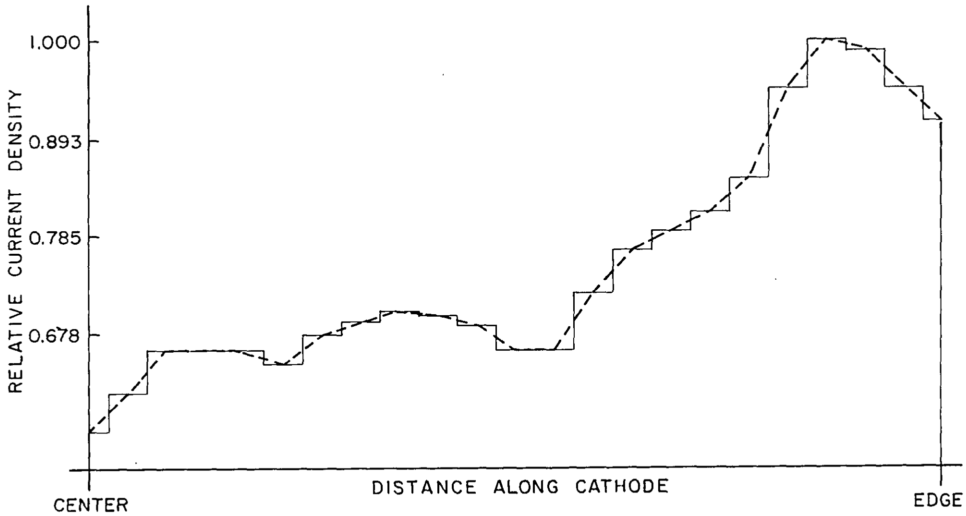

Fig. 8 - Cathode loading for the Litton L-3250 electron gun (0.5% accuracy prescribed)

Several trajectories from a particular L-3250 run are shown in Fig. 9. The same general picture was found with finer mesh sizes and various numbers of rays. The outermost ray was found to pass closer than one mesh distance to the anode structure. Since this is the zero temperature case, it appears that, if no other focusing is used, a considerable amount of current will be intercepted by the anode in an actual setup. Also, quite a few crossovers are in evidence, a condition that is even more apparent when more rays are traced. By plotting beam profiles at various cross sections within the gun one can get an idea of changes in the beam after its formation. Initially, the charge distribution is very definitely depressed at the center. However, the hole in the beam quickly fills only to reappear later. The calculation was not carried far enough to get an indication of the "frequency" of this oscillation; hence no mechanism can be proposed at this time.

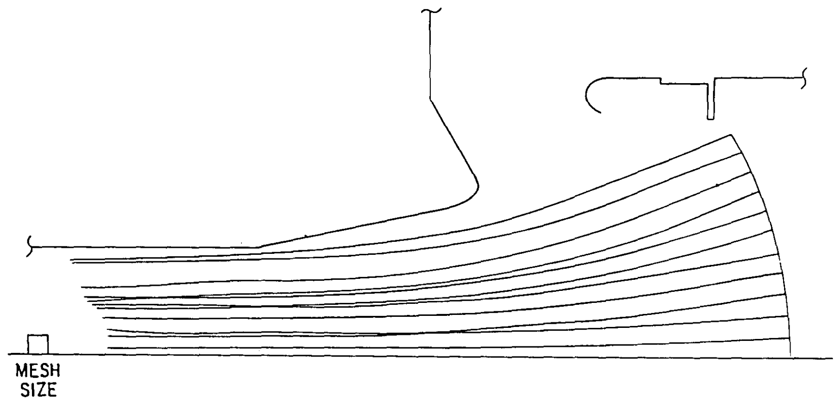

Fig. 9 - Trajectories computed for a Litton L-3250 electron gun (0.5% accuracy prescribed; $\alpha = 0.85$, $\beta = 1.717818$)

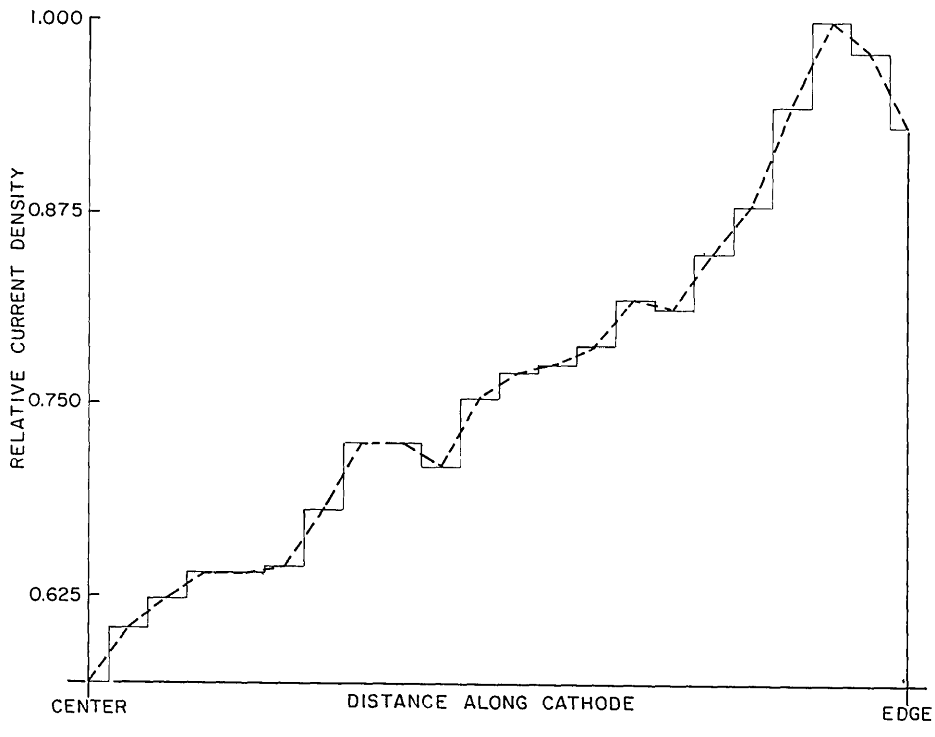

Fig. 10 - Cathode loading for a Litton L-232 electron gun (1.0% accuracy prescribed)

THE LITTON L-232 ELECTRON GUN

The Litton L-232 is a specially designed electron gun reported capable of producing a beam current of 1000 amperes at a perveance of 2.5×10^{-6} amp/volt$^{3/2}$. The cathode is spherical, and again one finds that the cathode loading is definitely nonuniform (Fig. 10). The same general remarks can be made here as were made for the L-3250.

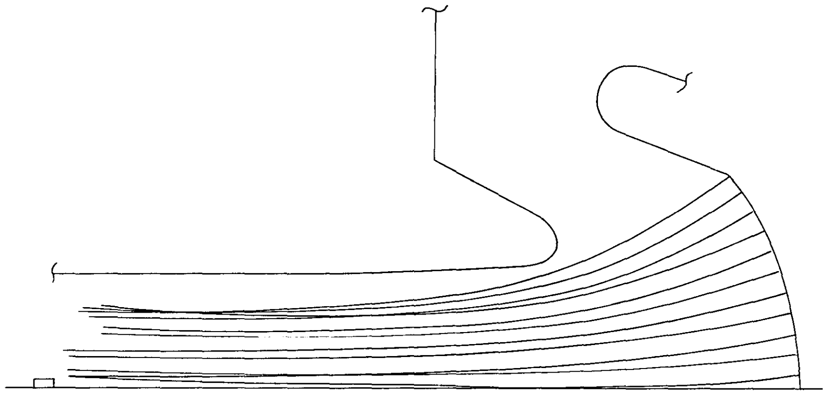

Fig. 11 - Trajectories computed for a Litton L-232 electron gun (1.0% accuracy prescribed; $\alpha = 0.80$, $\beta = 1.766836$)

The trajectory plot (Fig. 11) shows several crossovers, but the flow is quite laminar up to and a little beyond the anode hole. The outermost ray clears the anode structure by a good margin and hence good transmission is expected from this gun. The computed perveance of 2.62×10^{-6} amp/volt$^{3/2}$ is very close to the design value as is the minimum beam diameter (1.5 in.). The beam exhibits qualitatively the same oscillatory behavior at its center as the L-3250 but in general appears to be much smoother in its variations.

DISCUSSION OF RESULTS

Both the L-3250 and L-232 calculations were made using a program in which relativistic phenomena were completely ignored. Since the ratio of kinetic to rest energy is about 0.4 for the L-3250 and 1.0 for the L-232, a complete analysis would demand a full relativistic treatment. Further, the calculations did not account for initial electron velocities, Since the Sozotron application is very dependent on such thermal effects, these should not be ignored in a complete analysis.

On the other hand, one can make some reasonable estimates of the magnitudes of the relativistic and thermal effects and hope to have an accurate picture of the gun performance. In this respect, the present calculations are extremely valuable in that they indicate the salient features of the electron flow. Both guns tested appear to be well designed and cannot be disqualified for possible Sozotron application. The L-232 did appear to possess better flow characteristics than the klystron gun (L-3250), but the latter was not grossly inferior. Detailed examination of the changes that are wrought by including the relativistic and thermal phenomena must await changes in the computer program. It is contemplated that future investigations will be carried out which will include all of these effects and, if possible, reduce any inaccuracies which have been found in the present program.

ACKNOWLEDGMENTS

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13. ABSTRACT

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