NRL Report 6190

Effective Lengths of Antennas in Magneto-Ionic Media

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February 12, 1965

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ABSTRACT

The voltages induced in dipoles and loop antennas immersed in a magnetoactive plasma are calculated using the Rayleigh-Carson theorem for magnetoactive media as derived in NRL Report 6163. The voltage induced is given in terms of an effective length times the applied electric field, and in the present case the tensor dielectric character of the magneto-ionic medium complicates the effective length calculations. A first approximation for the effective length of a linear dipole in a magnetoactive plasma is derived which can be used if the antenna is short. Because loop antennas are also used in ionospheric propagation experiments, the ratio of the voltage induced in a small loop antenna to that induced in a dipole antenna is derived.

PROBLEM STATUS

This is an interim report on one phase of the problem; work continues on other portions.

AUTHORIZATION

NRL Problem R01-36
Project SF 019-02-02-7447

Manuscript submitted September 28, 1964.

EFFECTIVE LENGTHS OF ANTENNAS IN MAGNETO-IONIC MEDIA

INTRODUCTION

This report summarizes part of the theoretical effort relative to the Lofti program and is directed toward an understanding of certain phenomena associated with antennas in the ionosphere.

The voltage induced in an antenna is ordinarily given in terms of an effective length times the applied electric field. In free isotropic space the effective length is easy to define and calculate. Unfortunately the problem is considerably more difficult in a magneto-ionic medium (the ionosphere) due to the tensor dielectric character of the plasma. In order to aid experimental work relative to radio propagation in the ionosphere a first approximation to the effective length of a dipole antenna is presented herein. Unfortunately the result as will be given by Eq. (10) depends upon a parameter ($n(\theta)$) which is not precisely defined at present. However, in the limit of a very short antenna the effective length is independent of the precise form of $n(\theta)$.

EFFECTIVE LENGTH OF A LINEAR DIPOLE IN A MAGNETOACTIVE PLASMA

In order to calculate the effective antenna length, consider two dipoles immersed in a magneto-ionic medium. Both antennas are presumed to have the same orientation in space as given by the polar angles θ_a and φ_a and are a very great distance apart. This situation is represented schematically in Fig. 1, which shows the antennas, each of length 2ℓ, orientated in a direction given by θ_a and φ_a (i.e., the unit vector λ). The static magnetic field is represented by $\mathbf{H}_0$, which is presumed to be in the z-y plane and is orientated at an angle α relative to the z axis. Since the antennas are assumed to be a great distance apart, the wave due to antenna 1 can be assumed to be essentially a plane wave in the neighborhood of the second antenna. Thus, in a restricted region near the second antenna, the electric field can be written as

$$\mathbf{E}_\varkappa = \mathbf{E}_{0_\varkappa} e^{-(i\omega/c)n_\varkappa z} \tag{1}$$

where $n_\varkappa$ is the complex index of refraction for a plane wave propagating along the z axis at an angle α with respect to the magnetic field. According to Ginzburg (1a), the index of refraction for a lossless plasma is

$$(n_\varkappa)^2 = 1 - \frac{2\frac{\omega_0^2}{\omega^2}\left(1-\frac{\omega_0^2}{\omega^2}\right)}{2\left(1-\frac{\omega_0^2}{\omega^2}\right) - \frac{\omega_g^2}{\omega^2}\sin^2\alpha + \sqrt{\frac{\omega_g^4}{\omega^4}\sin\alpha + 4\frac{\omega_g^2}{\omega^2}\left(1-\frac{\omega_0^2}{\omega^2}\right)\cos^2\alpha}} \tag{2}$$

where ω_0 and ω_g are the plasma and cyclotron frequencies respectively. There are two modes of propagation, namely, the ordinary mode corresponding to the positive sign on the radical and the "extraordinary mode" corresponding to the negative sign. These two

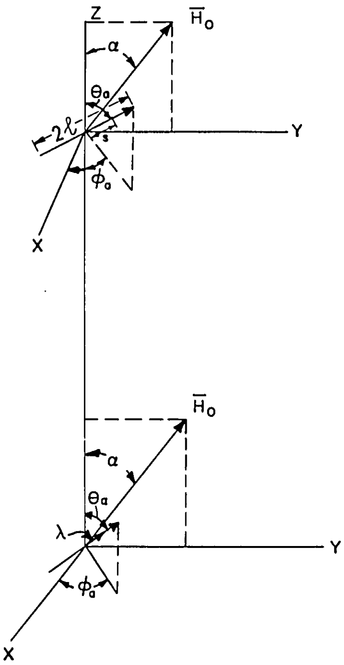

Fig. 1 - Arrangement of coordinate systems and antennas

modes are indicated by the subscript $\varkappa$ on the appropriate quantity. In general, for the ionosphere it is found that only one of the two possible modes survive after propagating some distance, due to the large attenuation of one of the modes of transmission.

Let the directional orientation of the antennas be described by a unit vector λ, i.e.,

$$\boldsymbol{\lambda} = \mathbf{i} \cos \varphi_a \sqrt{1 - \mu_a^2} + \mathbf{j} \sin \varphi_0 \sqrt{1 - \mu_a^2} + \mathbf{k}\mu_a \tag{3}$$

where $\mu_a = \cos \theta_a$.

Now according to the reciprocity theorem discussed (2) in NRL Report 6163,

$$\int \mathbf{E}^1 \cdot \hat{\mathbf{J}}^2 \, d\tau = \int \hat{\mathbf{E}}^2 \cdot \mathbf{J}^1 \, d\tau = V I_2 .$$

Therefore the short-circuit current $dI(s)$ in an element ds at a distance s from the center of antenna 2 is (cf. Ref. 3)

$$dI_{sc} = \frac{\boldsymbol{\lambda} \cdot \hat{\mathbf{E}}_\varkappa(z) \, I(s)}{V} \, ds \tag{4}$$

where $\hat{\mathbf{E}}_\varkappa$ represents the $\varkappa$ mode of propagation (either ordinary or extraordinary) in the reciprocal medium defined by the transpose of the dielectric tensor $\bar{\epsilon}$. $I(s)$ is the current at a distance s from the center of antenna 1 and V is the voltage applied to the terminals of antenna 1. If z_0 denotes the distance between the centers of the two antennas, then at any point on antenna 2

$$z = z_0 + \mu_a s . \tag{5}$$

Use of Eq. (4) with Eq. (5) leads to

$$dI_{sc} = \frac{\boldsymbol{\lambda} \cdot \hat{\mathbf{E}}_\varkappa(z_0) \, e^{-(i\omega/c) n_\varkappa \mu_a s} \, I(s)}{V} \tag{6}$$

where $\hat{\mathbf{E}}_\varkappa(z_0)$ is the electric field in the transposed medium due to antenna 1 at the center of antenna 2. Now over distances of the order of the wavelength, which is assumed to be small compared to distances between the antennas, it is permissible to neglect the effect of collisions so that $n_\varkappa$ may be calculated from Eq. (1).

Let θ represent the angle between the direction of the static magnetic field and the antenna, so that

$$\mu_0 \equiv \cos \theta = \frac{\mathbf{H}_0 \cdot \boldsymbol{\lambda}}{|H_0|} = \left(\mathbf{i} \sqrt{1 - \mu_a^2} \cos \varphi_a + \mathbf{j} \sqrt{1 - \mu_a^2} \sin \varphi_a + \mathbf{k}\mu_a \right) \cdot \left(\mathbf{j} \sqrt{1 - \mu^2} + \mathbf{k}\mu \right)$$

$$= \sqrt{(1 - \mu_a^2)(1 - \mu^2)} + \mu\mu_a \tag{7}$$

where $\mu \equiv \cos \alpha$. It is assumed in the following that the index of refraction appropriate to waves traveling along the antennas depends only upon the angle θ. Then in order to apply Eq. (7) it will be assumed that the current can be represented by

$$I = I(0) \frac{\sin \frac{\omega}{c} n(\theta)(\ell - |z|)}{\sin \frac{\omega}{c} n(\theta)\ell} . \tag{8}$$

Here $n(\theta)$ is the appropriate value of the index of refraction required to describe the propagation along the antenna. There are two choices that can be made: $n(\theta)$ can be taken as the index of refraction in the direction of the antenna and calculated by inserting the angle θ for α in Eq. (2), or $n(\theta)$ can be chosen as the optimum index of refraction as given by Ament (4), namely, $n(\theta) = \{\epsilon_{11}[\epsilon_{11}\mu_0^2 + \epsilon_{33}(1-\mu_0^2)]\}^{1/4}$.

The total short circuit current in antenna 2 is obtained from Eq. (8) by integrating over the entire antenna:

$$I_{sc} = \frac{I(0)\ \hat{\mathbf{E}}_\varkappa(z_0)\cdot\boldsymbol{\lambda}}{V \sin \frac{\omega}{c} n(\mu_0)\ell} \int_{-\ell}^{+\ell} e^{-(i\omega/c)n_\varkappa \mu_a s} \sin\left[\frac{\omega}{c} n(\mu_0)(\ell - |s|)\right] ds$$

or

$$I_{sc} = \frac{2I(0)\ \hat{\mathbf{E}}_\varkappa(z_0)\cdot\boldsymbol{\lambda}}{V \sin \frac{\omega}{c} n(\mu_0)\ell} \int_0^{\ell} \sin\left[\frac{\omega}{c} n(\mu_0)(\ell - |s|)\right] \cos\left(\frac{\omega}{c} n_\varkappa \mu_a s\right) ds$$

$$= \frac{2I(0)\ \hat{\mathbf{E}}_\varkappa(z_0)\cdot\boldsymbol{\lambda}}{V \sin \frac{\omega}{c} n(\mu_0)\ell} \frac{\left[\cos \frac{\omega}{c} n_\varkappa \mu_a \ell - \cos \frac{\omega}{c} n(\mu_0)\ell\right]}{\frac{\omega}{c} n(\mu_0) \left\{1 - \frac{\left(\frac{\omega}{c} n_\varkappa \mu_a\right)^2}{\left[\frac{\omega}{c} n(\mu_0)\right]^2}\right\}} . \tag{9}$$

According to Thevenin's theorem, the open circuit voltage is $V_{oc} = ZI_{sc}$ where Z is the antenna impedance. The open circuit voltage can therefore be obtained from Eq. (9) by multiplying by Z, and since the driving voltage is $V = ZI(0)$,

$$V_{oc} = \frac{2\ \hat{\mathbf{E}}_\varkappa \cdot\boldsymbol{\lambda}}{\sin \frac{\omega}{c} n(\mu_0)\ell} \frac{\left[\cos \frac{\omega}{c} n_\varkappa \mu_a \ell - \cos \frac{\omega}{c} n(\mu_0)\ell\right]}{\frac{\omega}{c} n(\mu_0) \left\{1 - \left[\frac{n_\varkappa \mu_a}{n(\mu_0)}\right]^2\right\}} . \tag{10}$$

Ginzburg (1b) has shown that for propagation of waves of the type under consideration the ratio (for no collisions) of the y component of the electric field to the x component is

$$\frac{E_{\varkappa_y}}{E_{\varkappa_x}} = -i\gamma = \frac{-i2 \frac{\omega_g}{\omega}\left(1 - \frac{\omega_0^2}{\omega^2}\right)\mu}{\frac{\omega_g^2}{\omega^2}(1-\mu^2) \mp \sqrt{\frac{\omega_g^4}{\omega^4}(1-\mu^2)^2 + 4\frac{\omega_g^2}{\omega^2}\left(1 - \frac{\omega_0^2}{\omega^2}\right)\mu^2}} \tag{11}$$

where $\mu = \cos\alpha$, and the upper sign on the radical corresponds to the ordinary mode while the lower sign corresponds to the extraordinary mode. Furthermore,

$$\frac{E_{\varkappa_z}}{E_{\varkappa_x}} = -i\delta = \frac{-i\,(1-\mu^2)^{1/2}\left(\frac{\omega_g\,\omega_0^2}{\omega^3} + \gamma_\varkappa\,\frac{\omega_g^2\,\omega_0^2}{\omega^4}\,\mu\right)}{\frac{\omega_g^2}{\omega^2} - \left(1 - \frac{\omega_0^2}{\omega^2}\right) - \frac{\omega_H^2\,\omega_0^2}{\omega^4}\,\mu^2} . \tag{12}$$

Therefore, the electric field can be represented by

$$\mathbf{E}_\varkappa = E_{\varkappa_x}[\mathbf{i} + \mathbf{j}(-i\gamma) + \mathbf{k}(-i\delta)] . \tag{13}$$

An examination of Eqs. (11) and (12) indicates that for the reciprocal medium, $\hat{\bar{\epsilon}}$, the signs of γ and δ are reversed. This is due to the fact that the medium characterized by $\hat{\bar{\epsilon}}$ corresponds to a static magnetic field opposite to that in the original medium, which is specified by $\bar{\epsilon}$. Therefore

$$\hat{\mathbf{E}} = E_{\varkappa_x}[\mathbf{i} + \mathbf{j}(i\gamma) + \mathbf{k}(i\delta)] \tag{13'}$$

and

$$|\hat{\mathbf{E}}| = |\mathbf{E}| = E_{\varkappa_x}(1-\gamma^2-\delta^2)^{1/2} . \tag{14}$$

Furthermore, it immediately follows that

$$|\hat{\mathbf{E}}\cdot\boldsymbol{\lambda}| = |\mathbf{E}\cdot\boldsymbol{\lambda}| = E_{\varkappa_x}\left[\lambda_x^2 - (\gamma\lambda_y)^2 - (\delta\lambda_z)^2\right]^{1/2} \tag{15}$$

where λ_x, λ_y, and λ_z are the Cartesian coordinates of $\boldsymbol{\lambda}$ as given by Eq. (3). The ratio of the absolute value of the open circuit voltage induced in the antenna to the absolute value of the component of the electric field along the antenna is thus

$$\frac{|V_{oc}|}{|\mathbf{E}\cdot\boldsymbol{\lambda}|} = \left| \frac{\left[\cos\frac{\omega}{c}\,n_\varkappa\,\mu_a\,\ell - \cos\frac{\omega}{c}\,n(\mu_0)\ell\right]}{\frac{\omega}{c}\,n(\mu_0)\,\sin\frac{\omega}{c}\,n(\mu_0)\ell\left\{1 - \left[\frac{n_\varkappa\,\mu_0}{n(\mu_0)}\right]^2\right\}} \right| \approx \ell \tag{16}$$

which is defined as the effective length and is approximately equal to ℓ if $\omega n(\mu_0)\ell/c \ll 1$ as is generally the case. It should be noted that for this approximation the antenna has the same effective length for both modes of propagation.

VOLTAGE INDUCED IN A LOOP ANTENNA IMMERSED IN A MAGNETOACTIVE PLASMA

Satellites have been used in ionospheric propagation experiments that have utilized both dipole and loop antennas. Therefore in order to complete the discussion of "effective length" of antennas, the voltage induced in a small loop antenna will be considered and compared with the voltage induced in a dipole. The discussion will be based on the assumption that the propagation of the signal, at least in the neighborhood of the loop, can be considered to be plane parallel.

In Eq. (13) it was pointed out that the electric field for a plane parallel wave propagating along the z axis in a magnetoactive medium is given by

$$\mathbf{E}_\varkappa = E_{\varkappa_x}(\mathbf{i} - i\gamma\mathbf{j} - i\delta\mathbf{k}) \tag{13}$$

where

$$E_{\varkappa_x} = E_{\varkappa_0} e^{-(i\omega/c)n_\varkappa z}$$

and $n_\varkappa$, $\gamma_\varkappa$ and $\delta_\varkappa$ are given by Eqs. (2), (11), and (12), respectively. The magnetic field associated with the electric field $\mathbf{E}_\varkappa$ can be obtained from the field equation $\nabla \times \mathbf{H} = -i\omega\mathbf{H}/c$:

$$\frac{-i\omega}{c}\mathbf{H}_\varkappa = n_\varkappa E_{\varkappa_x}[(i\gamma_\varkappa)\mathbf{i} + \mathbf{j}] = n_\varkappa E_{\varkappa_0}(i\gamma_\varkappa\mathbf{i} + \mathbf{j})\, e^{-(i\omega/c)n_\varkappa z} . \tag{17}$$

Consider now an antenna made of a single loop whose diameter is small relative to a wavelength, so that the magnetic intensity will be the same at all points within the loop and have (approximately) the value appropriate to the center of the loop. Let the loop have an area $\mathbf{A}$ and let the normal to the plane of the loop have the direction of the unit vector $\boldsymbol{\sigma}$,

$$\boldsymbol{\sigma} = \left(\mathbf{i}\sqrt{1-\mu_b^2}\cos\varphi_b + \mathbf{j}\sqrt{1-\mu_b^2}\sin\varphi_b + \mathbf{k}\mu_b\right) \tag{18}$$

where θ_b and φ_b are the polar colatitude and azimuthal angles of the normal to the loop and $\mu_b \equiv \cos\theta_b$.

The voltage induced in the loop by the magnetic field is

$$V_m = \frac{-i\omega}{c}\int \mathbf{H}_\varkappa \cdot d\mathbf{A} = \frac{-i\omega}{c} A\mathbf{H}_\varkappa(z_0)\cdot\boldsymbol{\sigma}$$

$$= \frac{-i\omega}{c} n_\varkappa A E_{\varkappa_x}(z_0)\left(i\gamma_\varkappa\sqrt{1-\mu_b^2}\cos\varphi_b + \sqrt{1-\mu_b^2}\sin\varphi_b\right) \tag{19}$$

where z_0 is the z coordinate of the center of the loop. The ratio of the absolute value of loop to dipole voltage can be found from the approximate form of Eq. (16) and Eqs. (15) and (19):

$$\frac{|V_m|}{|V_{oc}|} = \frac{\omega}{\ell c} n_\varkappa A \frac{(\sin^2\varphi_b + \gamma_\varkappa^2\cos^2\varphi_b)}{\left[\cos^2\varphi_a - \gamma_\varkappa^2\sin^2\varphi_a - \delta^2\left(\frac{\mu_a^2}{1-\mu_0^2}\right)\right]}\sqrt{\frac{1-\mu_b^2}{1-\mu_a^2}} . \tag{20}$$

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Security Classification

DOCUMENT CONTROL DATA - R&D

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1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
U.S. Naval Research Laboratory Washington, D.C. 20390	Unclassified 2b. GROUP

3. REPORT TITLE

Effective Lengths of Antennas in Magneto-Ionic Media

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

An interim report on one phase of the problem.

5. AUTHOR(S) *(Last name, first name, initial)*

Faust, William R.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
February 12, 1965	9	4

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
NRL Problem R01-36 b. PROJECT NO. SF 019-02-02-7447 c. d.	NRL Report 6190 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Department of the Navy (Bureau of Ships)

13. ABSTRACT

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DD FORM 1 JAN 64 1473

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Theory Antennas Magneto-ionic media Ionospheric propagation Dipole effective length Loop antenna						

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