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Application of the Fresnel Method to the Calculation of the Radiated Acoustic Field of Rectangular Pistons

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ABSTRACT

Radiation from a uniformly vibrating rectangular piston in an infinite rigid baffle is expressed by an approximate solution in terms of the complex Fresnel integral. Although a similar approximate method has been used by others, the solution given here allows evaluation of a larger region of the field than do previous solutions. Field values for the specific cases of a λ by 2λ piston and a 2λ by 4λ piston are computed and compared for three methods of solution, namely, the present method, an analogous method by Freedman, and the Huygens construction. Favorable comparison is found between the present solution and the less approximate but more complicated solution employing a finite Huygens construction. Also, the present results are in essential agreement with Freedman's near the central axis of the piston. Thus, accurate field values can be derived, using the theory which is given, by the use of the tabulated Fresnel integral or by a graphical method using the Cornu spiral. The accuracy of these derived values vary with field position, but an estimate of the accuracy may be made by comparison with the specific cases which are given.

PROBLEM STATUS

This is an interim report on the problem; work is continuing.

AUTHORIZATION

NRL Problem S01-04
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APPLICATION OF THE FRESNEL METHOD TO THE CALCULATION OF THE RADIATED ACOUSTIC FIELD OF RECTANGULAR PISTONS

INTRODUCTION

The method of the solution to be given in this report was encountered in deriving the solution to the reflected field from a rigid rectangular reflector at any angle of insonification. The radiated field of a rigid piston in an infinite baffle is similar to the special case of reflection from a large plane with normal uniform insonification. The means of computation and specific results for the acoustic radiation of a rigid rectangular piston in a baffle have been given by Stenzel (1) with only limited approximation. A less exact method for dealing with the problem is given by Freedman (2), and although the degree of the approximations are not easily interpreted, the method is somewhat more physically descriptive of a limited region of the field. The solution given here is similar to that of Freedman in its fundamental assumptions, and therefore it is subject to some of the same not-easily-understood limitations. However, it differs in algebraic manipulation, which results in a different and extended domain of validity.

DERIVATION

A rectangular plane piston with dimensions d by d/n is located in a plane, infinite, rigid baffle. Assume a rectangular coordinate system with the origin at the center of the piston, as shown in Fig. 1. The radiation at a point P in half-space in terms of the velocity potential φ resulting from the normal simple-harmonic motion of the piston may be expressed (3) as

$$\varphi = (-\dot{\xi}/2\pi) \exp(i\omega t) \iint (1/r_a) \exp(-ikr_a)\, da \tag{1}$$

where $\dot{\xi}$ is the uniform piston velocity amplitude, r_a is the radial distance to a field point P from an elemental contributing piston element of area da, k is the wave number ($k \equiv 2\pi/\lambda$, where λ is the wavelength), and the integral is taken over the entire radiating area. If the approximation is made that $r_a \approx r$, where r is the distance from the field point P to the center of the piston, the consideration of the differences between the distances to all elements with regard to the magnitude of the velocity potential is eliminated, and Eq. (1) becomes

$$\varphi = (-A/2\pi r) \iint \exp(-i\, 2\pi r_a/\lambda)\, da \tag{2}$$

where $A \equiv \dot{\xi} e^{i\omega t}$.

It can be seen from Fig. 1 that, in general,

$$r_a = \left[(x-\zeta)^2 + (y-\eta)^2 + z^2\right]^{1/2}$$

or

$$r_a = (r^2 + \zeta^2 + \eta^2 - 2x\zeta - 2y\eta)^{1/2}. \tag{3}$$

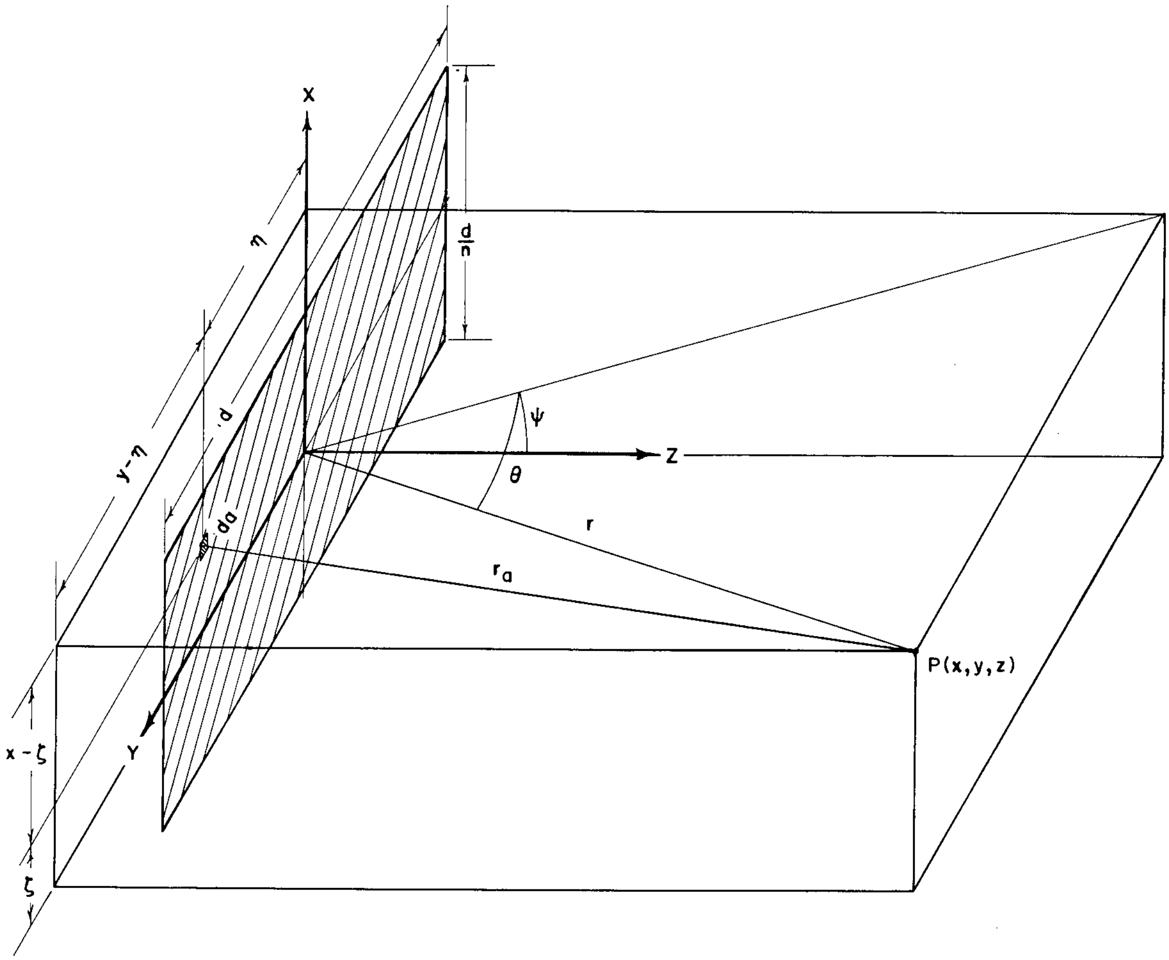


Fig. 1 - The geometrical quantities used to calculate the radiation field from a rectangular piston (large hatched area)

Approximating r_a by the first two terms of the binomial expansion of the right side of Eq. (3) gives

$$r_a \approx r + (\zeta^2 - 2x\zeta + \eta^2 - 2y\eta)/2r,$$

subject to the condition that

$$\left[(\zeta^2 - 2x\zeta + \eta^2 - 2y\eta)/r^2\right]^2 \le 1.$$

The geometrical significance of this limitation is of interest especially if it imposes an angular limitation. In order to avoid the specific evaluation of the region described by the inequality, a region of exclusion only slightly larger than the actual one may be derived as follows.

The solution is not reasonably anticipated to be valid on the surface of the piston itself. Therefore, consider the region on the baffle outside of, and including, the circumscribed circle of the piston where $x \ge \zeta_{max}$, $y \ge \eta_{max}$, and $z = 0$. In this case the above limiting condition becomes $(-x^2 - y^2)^2 \le (x^2 + y^2)^2$ which is identically satisfied for all allowed values of x and y. When $z \neq 0$ the condition becomes

$$(x^2 + y^2)^2 \le (x^2 + y^2 + z^2)^2 = r^4.$$

If only those values of z are allowed such that $r^2 \ge x^2 + y^2$, the inequality will be satisfied and the region of exclusion of the solution may be defined as the interior of the circumscribed hemisphere of the piston.

Equation (2) may now be written

$$\varphi = (-A/2\pi r)\exp(-i\,2\pi r/\lambda) \int_{-d/2n}^{d/2n} d\zeta \int_{-d/2}^{d/2} \exp\left\{(-i\pi/2)(4/\lambda)\right.$$

$$\left.\times \left[(\zeta^2 - 2x\zeta + \eta^2 - 2y\eta)/2r\right]\right\} d\eta. \tag{4}$$

The factors in braces in the exponent of the integral, exclusive of the factor $-i\pi/2$, may be written in the form

$$u'^2(\zeta) + v'^2(\eta)$$

where

$$u'^2(\zeta) \equiv u'^2 \equiv (2/\lambda r)\zeta^2 - (4x/\lambda r)\zeta \tag{5}$$

and

$$v'^2(\eta) \equiv v'^2 \equiv (2/\lambda r)\eta^2 - (4y/\lambda r)\eta. \tag{6}$$

In order to evaluate the limits for the integral in Eq. (4), it is necessary to solve Eqs. (5) and (6) for ζ in terms of u' and for η in terms of v', respectively. From Eq. (5),

$$q\zeta^2 - j\zeta - u'^2 = 0$$

where

$$q = 2/\lambda r \text{ and } j = 4x/\lambda r.$$

Substituting $m = j^2/4q$, it can be shown that

$$\zeta = j/2q \pm (m + u'^2)^{1/2}/\sqrt{q}. \tag{7}$$

The additional transformation $u^2 = m + u'^2$ will greatly simplify the final expression for the integral, so now

$$\zeta = j/2q + u/\sqrt{q} \tag{8}$$

having arbitrarily chosen the plus sign in Eq. (7). (It can be shown that in the end it would not matter which sign had been chosen.) Therefore, solving Eq. (8) for u and substituting for q and j,

$$u = (2/\lambda r)^{1/2}(\zeta - x)$$

and

$$d\zeta = (\lambda r/2)^{1/2}\,du.$$

Since Eq. (6) has a form identical to Eq. (5) it is easily seen that

$$v = (2/\lambda r)^{1/2}(\eta - y)$$

and

$$d\eta = (\lambda r/2)^{1/2}\,dv\,.$$

The complete transformation of Eq. (4) is now

$$\varphi = (-A/2k)\,\exp\left\{(-i\pi/\lambda)\left[2r - (x^2/r) - (y^2/r)\right]\right\}$$

$$\times \int_{u_1}^{u_2} \exp(-i\pi u^2/2)\,du \int_{v_1}^{v_2} \exp(-i\pi v^2/2)\,dv \tag{9}$$

where

$$u_1 = -(2/\lambda r)^{1/2}(x + d/2n) \tag{10}$$

$$u_2 = -(2/\lambda r)^{1/2}(x - d/2n) \tag{11}$$

$$v_1 = -(2/\lambda r)^{1/2}(y + d/2) \tag{12}$$

$$v_2 = -(2/\lambda r)^{1/2}(y - d/2) \tag{13}$$

and k is the wave number, as before.

A mixed coordinate system has been maintained for brevity of notation and ease of manipulation. But transformation may be made to an r, Ψ, θ system since $x = r \sin\Psi\cos\theta$ and $y = r\sin\theta$. Equations (10) through (13) become

$$u_1 = -(2/\lambda r)^{1/2}(r\sin\Psi\cos\theta + d/2n) \tag{14}$$

$$u_2 = -(2/\lambda r)^{1/2}(r\sin\Psi\cos\theta - d/2n) \tag{15}$$

$$v_1 = -(2/\lambda r)^{1/2}(r\sin\theta + d/2) \tag{16}$$

$$v_2 = -(2/\lambda r)^{1/2}(r\sin\theta - d/2)\,. \tag{17}$$

This coordinate system results in radiation patterns in planes tilted about an axis of the piston parallel to an edge. If patterns in rotated and tilted planes are desired, relations between Ψ and θ and the conventional spherical angular parameters α and β may be determined from Fig. 2, and in terms of α and β Eqs. (14) and (16) become

$$u_1 = -(2/\lambda r)^{1/2}(r\sin\alpha\cos\beta + d/2n)$$

$$v_1 = -(2/\lambda r)^{1/2}(r\sin\alpha\sin\beta + d/2)\,.$$

The expressions for u_2 and v_2 are the same as for u_1 and v_1, respectively, with a minus instead of a plus sign before the d inside the parentheses.

The integrals in Eq. (9) can be expressed in terms of the complex Fresnel integral $F(u_1)$ given by

$$F(u_1) = \int_0^{u_1} \exp(-i\pi u^2/2)\,du$$

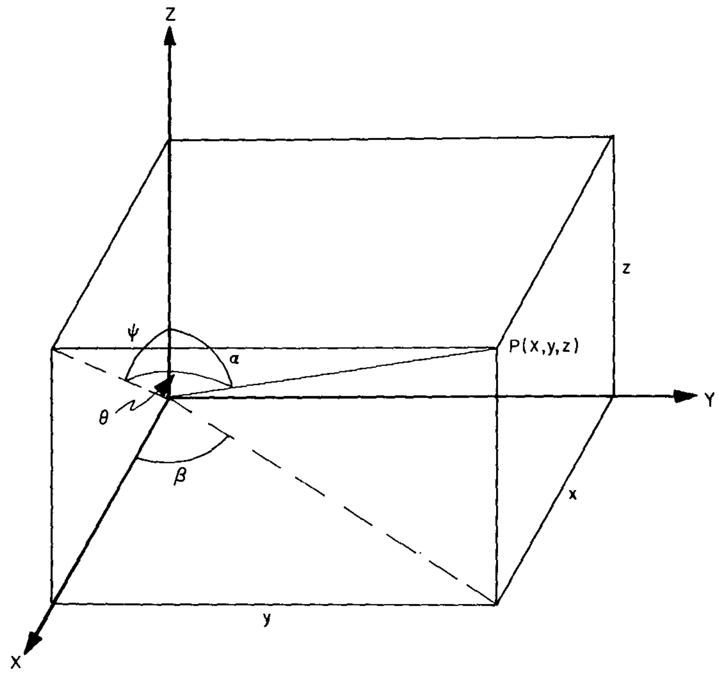


Fig. 2 - The geometry for the transformation from the r, Ψ, θ coordinate system to the conventional spherical r, α, β coordinate system

which, by the application of "Euler's" formula, can be expressed as

$$F(u_1) = \int_0^{u_1} \cos(\pi u^2/2)\, du - i \int_0^{u_1} \sin(\pi u^2/2)\, du.$$

Conventionally,

$$C(u_1) \equiv \int_0^{u_1} \cos(\pi u^2/2)\, du$$

and

$$S(u_1) \equiv \int_0^{u_1} \sin(\pi u^2/2)\, du.$$

The Cornu spiral shown in Fig. 3 is obtained if $C(u_1)$ is plotted versus $S(u_1)$, and $F(u_1)$ may be regarded as a distance from the origin of the plot to the point on the spiral corresponding to u_1. A corresponding integral $F(u_2)$ may be similarly regarded. By manipulation of the integrals it may be shown that

$$F(u_1, u_2) \equiv F(u_2) - F(u_1) = \int_{u_1}^{u_2} \exp(-i\pi u^2/2)\, du$$

$$= \left[C(u_2) - C(u_1)\right] - i\left[S(u_2) - S(u_1)\right]$$

whose magnitude is

$$|F(u_1, u_2)| = \left\{ [C(u_2) - C(u_1)]^2 + [S(u_2) - S(u_1)]^2 \right\}^{1/2}$$

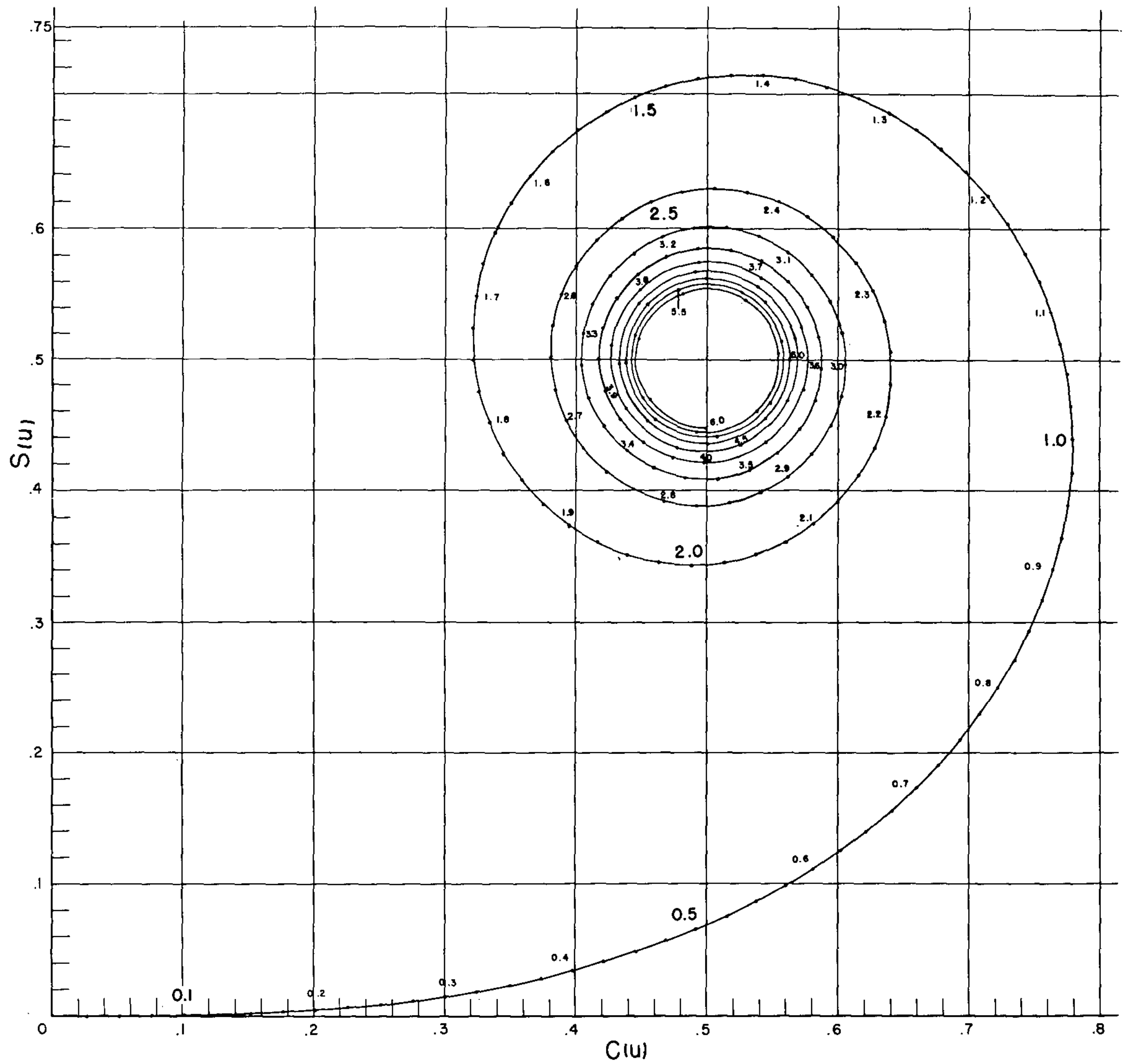


Fig. 3 - The Cornu spiral obtained by plotting $C(u_1) \equiv \int_0^{u_1} \cos(\pi u^2/2)\,du$ vs $S(u_1) = \int_0^{u_1} \sin(\pi u^2/2)\,du$ for the values of u indicated

The line length from u_1 to u_2 in $C(u), S(u)$ space is given by the magnitude of $F(u_1, u_2)$ whose associated phase may be taken as the angle that the directed line u_1, u_2 makes with the positive $C(u)$ axis. The function $S(u)$ and $C(u)$ have been tabulated (4); also, their appropriate expansions (5) may be evaluated with a digital computer.

Equation (9) expresses the field of a square piston when the dimension n (see Fig. 1) is set equal to unity. On a bisecting normal plane of the square piston, say when $\theta = 0$, the evaluation of $|F(u_1, u_2)|$ becomes somewhat simplified since the limits are the same except for sign, i.e., $u_1 = -(d/2)(2/\lambda r)^{1/2}$ and $u_2 = (d/2)(2/\lambda r)^{1/2}$. A characteristic of the Cornu spiral is that $C(u) = -C(-u)$ and $S(u) = -S(-u)$. Therefore, in this case,

$$F(u_1, u_2) = 2F(0, u_1) = 2\left[C(u_1) - i\,S(u_1)\right]$$

and

$$|F(u_1, u_2)| = 2\left[C(u_1)^2 + S(u_1)^2\right]^{1/2}$$

For the axial field of the rectangular piston, since $\Psi = \theta = 0$, a similar simplification may be made so that

$$|F(u_1, u_2)|\,|F(v_1, v_2)| = 4\left\{\left[C(u_1)^2 + S(u_1)^2\right]\left[C(v_1)^2 + S(v_1)^2\right]\right\}^{1/2} \qquad (18)$$

When $n = 1$, i.e., for the square piston, the axial field becomes

$$|F(u_1, u_2)|\,|F(v_1, v_2)| = 4\left[C(u_1)^2 + S(u_1)^2\right]. \qquad (19)$$

For the long-range axial field of a small piston, i.e., when $r \gg (d/n\lambda)$,

$$|F(u_1, u_2)|\,|F(v_1, v_2)| = (2d^2/n\lambda r)$$

since $u_1 = (-d/2n)(2/\lambda r)^{1/2}$ and Eq. (19) may be approximated by $4C(u_1)^2$, and further $C(u_1)$ may be approximated by its upper limit u_1. For such a case Eq. (9) becomes

$$\varphi = -(Ad^2/2\pi n r)\ \exp(-ikr),$$

which displays the direct dependence on area and the inverse dependence on range for the axial far field.

A solution for a specific case can be graphically obtained with fair accuracy by numerically evaluating u and/or v and locating this value on the Cornu spiral in Fig. 3, reading the corresponding values of $S(u)$ and $C(u)$ on the axes, or by scaling the resultant directly from the plot. If u_1 and u_2 are unequal values of opposite sign, the other half of the Cornu spiral is needed for direct plotting of the resultant. However, the application of analytic geometry can produce a resultant with only one half of the spiral.

RESULTS

The radiated field of a rectangular piston does not lend itself to a particularly obvious and characteristic means of description. A plot of the field in terms of r, Ψ, and θ has the advantage of being a convenient, closed, coordinate system, but unfortunately it is somewhat unconventional. Plots which are solutions of Eq. (9) are shown in Fig. 4(a) for the λ by 2λ piston at a constant value of $r = 1.5\lambda$ in which Ψ is taken as the parametric variable. Similar plots at constant values of $r = 10\lambda$ and $r = 20\lambda$ are shown in Figs. 4(b) and 4(c), respectively. Through the progression of graphs in Fig. 4, it may be seen how the null, as well as the secondary peak, predicted by the geometry and size of the piston becomes well formed at $r = 20\lambda$.

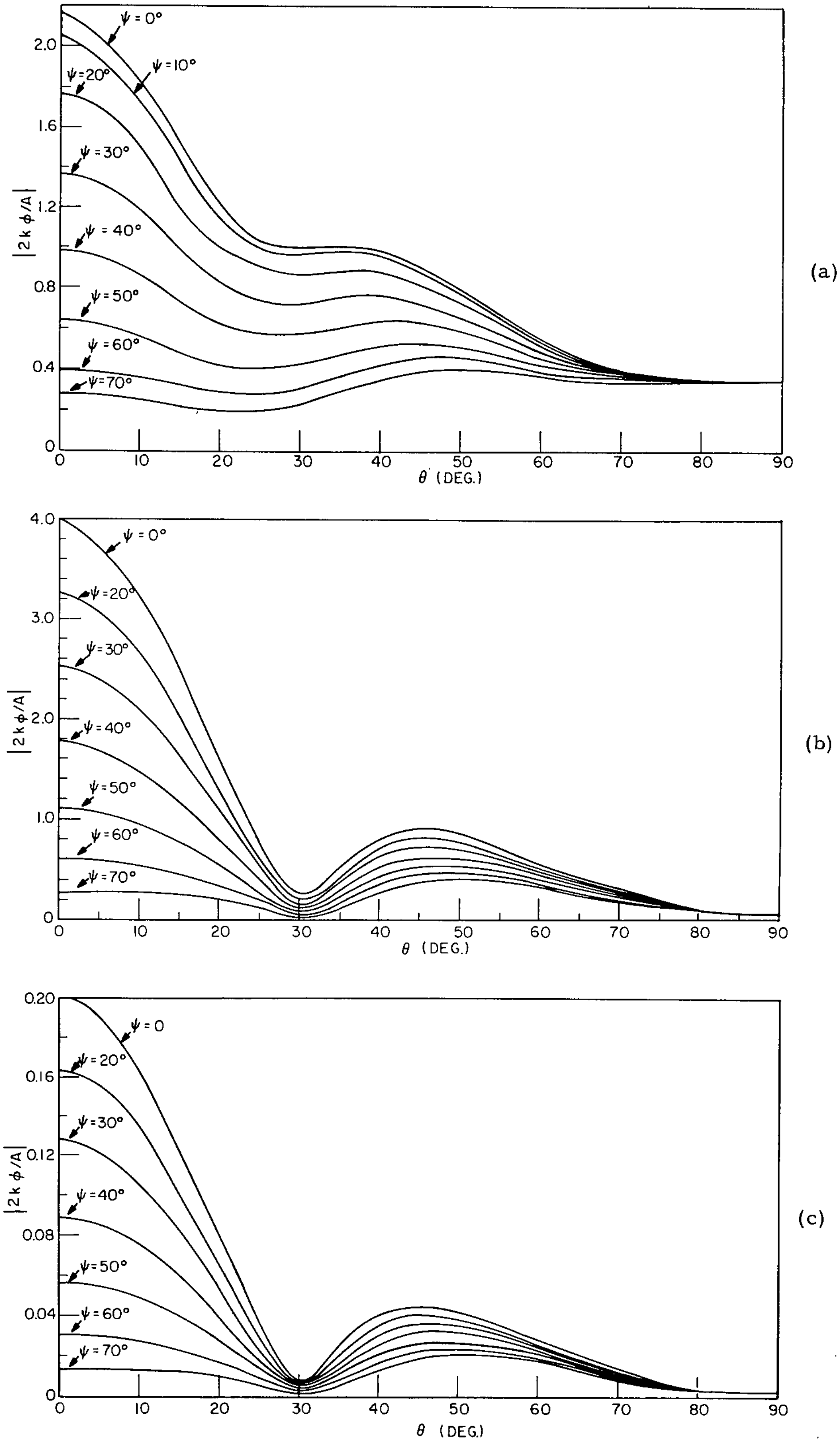


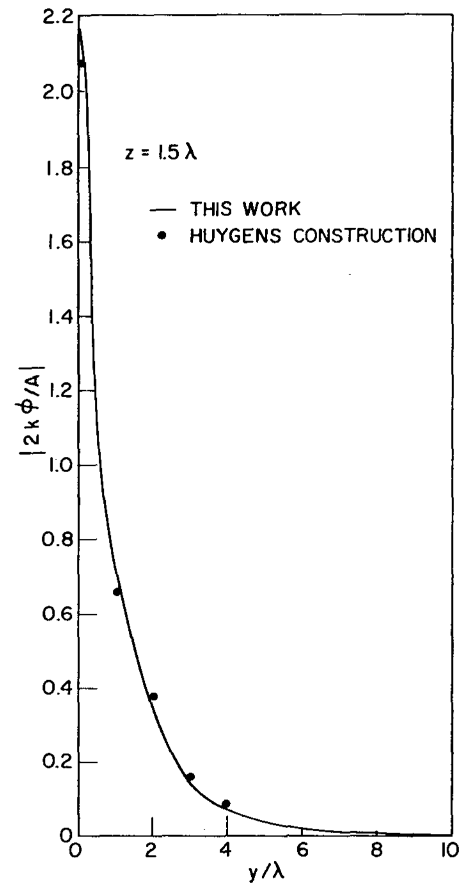
Fig. 4 - The radiated field of a 2λ by λ piston expressed in terms of $|2k\varphi/A|$ vs θ for eight constant values of ψ at (a) $r = 1.5\lambda$, (b) $r = 10\lambda$, and (c) $r = 20\lambda$

As previously stated, the limitations resulting from the approximations of the theory which produce these plots are not easily understood. Since a rectangular piston placed in an infinite baffle is itself a difficult experimental situation to produce with known validity, both with respect to the uniformity of motion of its surface and the rigidity of the baffle, it was felt that a comparison of the theory with experiment would leave one in considerable doubt as to whether the experiment, the theory, or both were unreliable. Fortunately, another group at the Naval Research Laboratory has utilized the basic equation (Eq. (1)) and made a finite Huygens construction on a digital computer. The radiator was divided into a finite number of surface elements, and field values were derived over a range of y for a constant value of z. The same values were then derived again for a further subdivision of the radiating surface into a greater number of smaller surface elements. When the field values did not change significantly as the surface element number increased, the result at that value of z (and larger) was taken as valid. One hundred elements per λ^2 area were used in most calculations. More confidence is felt in this result than in one which could be gotten experimentally.

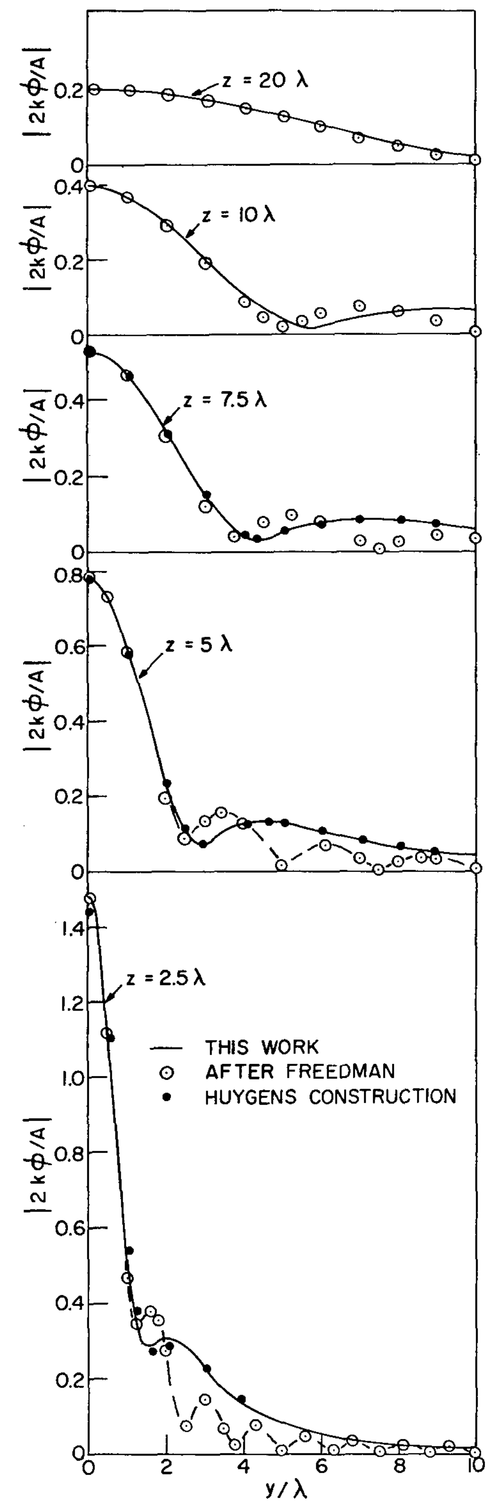
The results of the present work are compared in Fig. 5 with the results of the Huygens construction and with similar results obtained following the method of Freedman (2). Essential agreement is seen to exist between the result of this work and the Huygens construction with respect to the form of the curves, and this agreement exists even at small distances from the piston face in the stated region of validity. Close to the central axis of the piston, agreement is found with Freedman as well. It must be remembered that validity cannot be claimed for the results following Freedman when the field point is both far from the piston face and from the central axis of the piston. This result is inherent in the choice that Freedman makes in his binomial expansion of the field about $r = z$. Moreover, the angular limitation is more severe as the piston is approached. Points beyond this limitation are plotted in some cases since, interestingly, their envelope seems to approach the other plots even in an admittedly invalid region.

The equations describing the axial field in this work actually are equivalent to those given by Freedman. This axial field is plotted in Fig. 6 as a function of the inverse axial distance, in wavelengths, for a λ by 2λ piston and a 2λ by 4λ piston. In the far field the points approach asymptotically the lines which pass through zero. The slopes of these lines are related to the areas of the pistons. At a constant value of the ordinate, the deviation of the radiation from any piston from the inverse distance line is a constant, i.e., at $\lambda/z = 0.2$ (or $z/\lambda = 5$), the λ by 2λ piston field deviation is 2.5 percent, which is equal to the deviation of the 2λ by 4λ piston at $\lambda/z = 0.05$ (or $z/\lambda = 20$). The values of z/λ for constant deviation are also related to each other as the areas of the pistons. It is therefore presumed that one can predict the axial deviation from linear dependence on λ/z for any rectangular piston when given the deviation from linear dependence on λ/z for one piston.

An angular plot similar to the plots in Fig. 4 is shown in Fig. 7 for a 2λ by 4λ piston for field values on the piston bisecting plane, i.e., for $\Psi = 0$, and for parametric values of $r = 2.5\lambda$, 5λ, 10λ, and 20λ. One would expect agreement with a Huygens construction at $r = 20\lambda$ for the 2λ by 4λ piston similar to the agreement found for the λ by 2λ piston at a distance of 5λ. Figure 8(a) shows the comparison of the three methods of solving the problem, again in a rectangular plot similar to Fig. 5, for $z = 5\lambda$. A similar plot is shown in Fig. 8(b) for $z = 20\lambda$. Three-dimensional plots of the field at a constant value of $z = 7.5\lambda$ in a plane parallel to the piston face for both the λ by 2λ and the 2λ by 4λ pistons, for one quadrant where $0 \leq x \leq 11$ and $0 \leq y \leq 10$, are shown in Fig. 9.



(a)



(b)

Fig. 5 - Plot of $|2k\varphi/A|$ computed in a rectangular coordinate system for a range of y/λ values and for $x = 0.05\lambda$. The values of z are (a) $z = 1.5\lambda$, and (b) $z = 2.5\lambda$, 5λ, 7.5λ, 10λ, and 20λ. The results shown are for a piston for which $d = 2\lambda$ and $n = 2$ (i.e., for a λ by 2λ piston). Similar results of a Huygens construction are plotted for comparison in (a) and (b), and the analysis following Freedman is shown in (b).

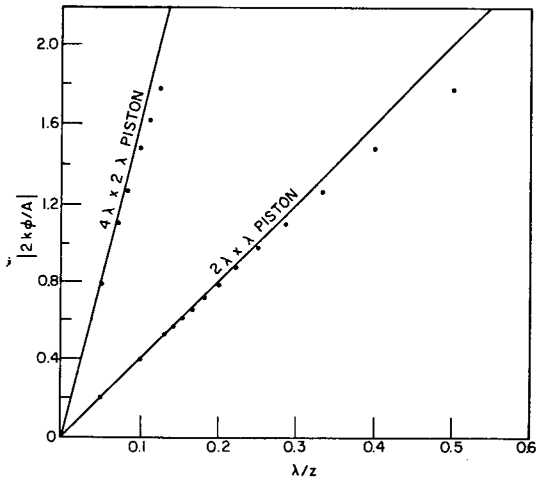


Fig. 6 - Computed values of $|2k\varphi/A|$ plotted vs λ/z showing the deviation from inverse z dependence of the axial field as the piston is approached. The data shown are for two pistons, one with $d = 2\lambda$, $n = 2$ (2λ by λ piston), the other with $d = 4\lambda$, $n = 2$ (4λ by 2λ piston).

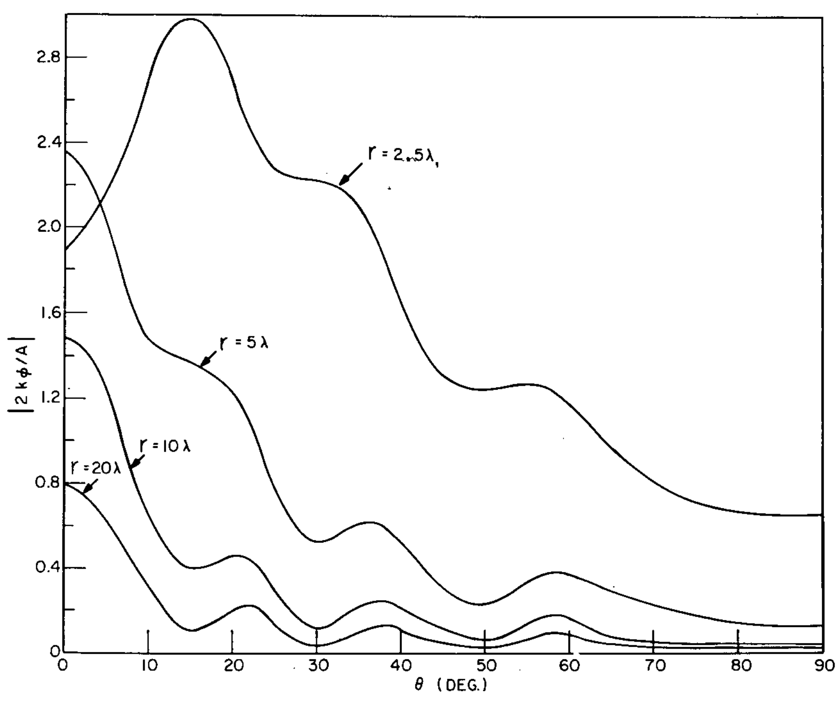
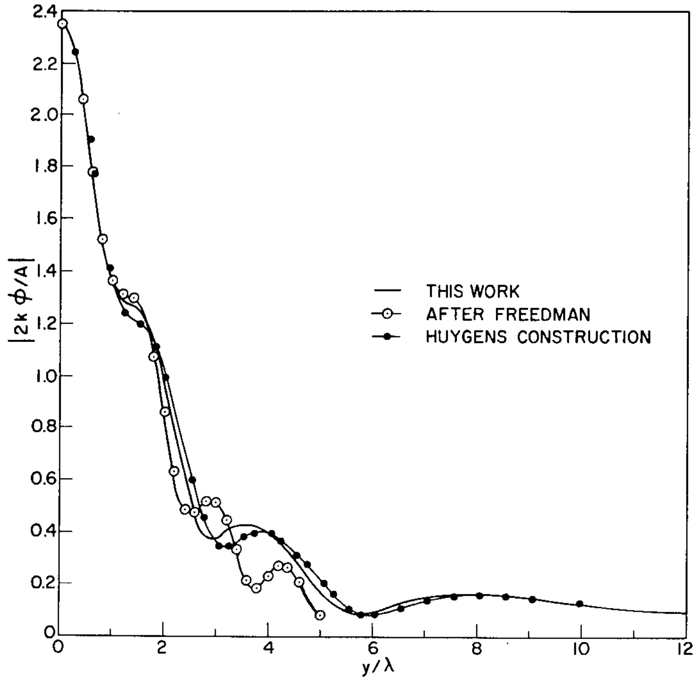
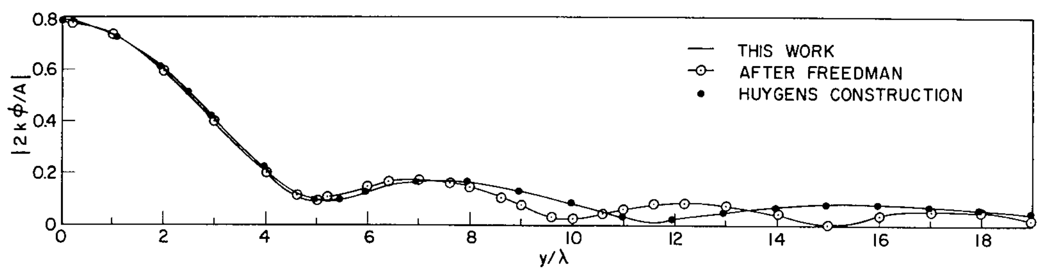


Fig. 7 - The radiated field of a 4λ by 2λ piston expressed in terms of $|2k\varphi/A|$ vs θ for $\Psi = 0$ at $r = 2.5\lambda$, 5λ, 10λ, and 20λ



(a)



(b)

Fig. 8 - Plot of $|2k\varphi/A|$ computed in a rectangular coordinate system for a range of y/λ values and for $x = 0.05\lambda$. The values of z are (a) $z = 5\lambda$ and (b) $z = 20\lambda$. Both plots are for a piston for which $d = 4\lambda$ and $n = 2$ (i.e., a 4λ by 2λ piston). Similar results of an analysis following Freedman and a Huygens construction are plotted for comparison.

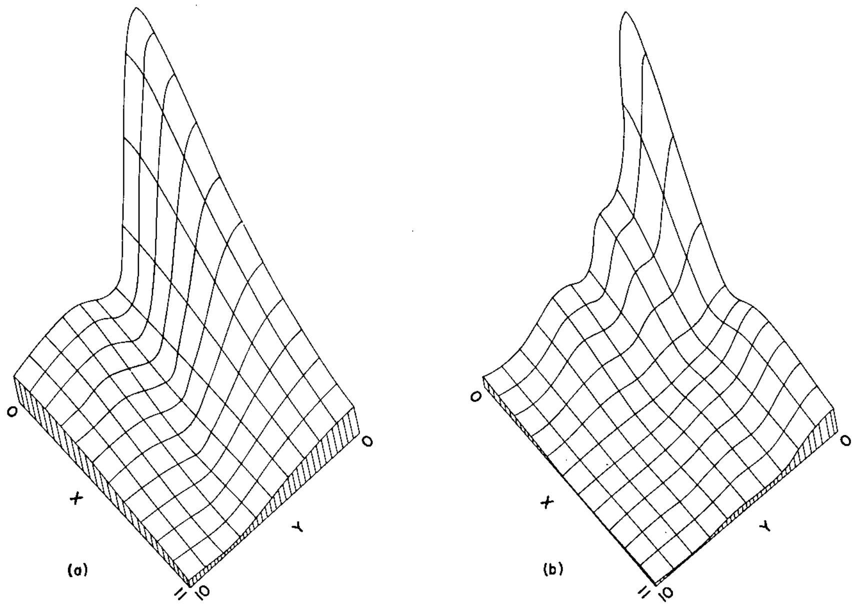


Fig. 9 - Isometric plots of one quadrant of the field in a plane parallel to the piston face at a distance $z = 7.5\lambda$ for (a) a 2λ by λ piston and (b) a 4λ by 2λ piston

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13. ABSTRACT

Radiation from a uniformly vibrating rectangular piston in an infinite rigid baffle is expressed by an approximate solution in terms of the complex Fresnel integral. Although a similar approximate method has been used by others, the solution given here allows evaluation of a larger region of the field than do previous solutions. Field values for the specific cases of a λ by 2λ piston and a 2λ by 4λ piston are computed and compared for three methods of solution, namely, the present method, an analogous method by Freedman, and the Huygens construction. Favorable comparison is found between the present solution and the less approximate but more complicated solution employing a finite Huygens construction. Also, the present results are in essential agreement with Freedman's near the central axis of the piston. Thus, accurate field values can be derived, using the theory which is given, by the use of the tabulated Fresnel integral or by a graphical method using the Cornu spiral. The accuracy of these derived values vary with field position, but an estimate of the accuracy may be made by comparison with the specific cases which are given.

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