

**THE RESONANT FREQUENCY OF A CAVITY-TYPE FILTER  
AS A FUNCTION OF THE SIZE OF THE COUPLING IRIS**

W. W. Balwanz

April 3, 1949

Approved by:

Mr. F. M. Gager, Head, Special Research Branch  
Dr. R. M. Page, Superintendent, Radio Division III



**NAVAL RESEARCH LABORATORY**

CAPTAIN F. R. FURTH, USN, DIRECTOR

**WASHINGTON, D.C.**

**APPROVED FOR PUBLIC  
RELEASE - DISTRIBUTION  
UNLIMITED**

## CONTENTS

<b>Abstract</b>	iv
<b>Problem Status</b>	iv
<b>Authorization</b>	iv
<b>Acknowledgment</b>	iv
<b>INTRODUCTION</b>	1
<b>METHOD OF INVESTIGATION</b>	2
Experiments Conducted	2
Instrumentation	5
Accuracy of Measurements	5
<b>CALCULATION OF THE RESONANT FREQUENCIES OF THE IDEAL UNLOADED CAVITY</b>	5
<b>DISCUSSION OF RESULTS</b>	8
Modes Observed	9
Explanation of Unexcited Modes	9
Percentage of Incident Power Transmitted	10
Deviations Between Theoretical and Actual Cavity Dimensions	11
Band-Pass Characteristics	14
<b>CONCLUSIONS</b>	14
<b>SELECTED BIBLIOGRAPHY</b>	31

## ABSTRACT

Experiments were conducted to determine the degree to which dependance can be placed upon simple cavity theory for determining the required cavity dimensions of a cylindrical cavity-type filter for use at microwave frequencies, and to establish some general rules for determining the size and location of irises to be used in coupling to the cavity. The results show that calculations based upon simple cavity theory are accurate to within two percent, provided certain precautions are taken in determining the size and position of the coupling iris. Primary considerations in iris location are optimum coupling to the desired mode and minimum distortion of the field within the cavity. An iris diameter equal to the narrow dimension of the rectangular wave guide used gave ninety-eight percent power transmission in the  $TE_{0,1,1}$ - $TM_{1,1,1}$  mode for the cavity used.

## PROBLEM STATUS

This is a final report, summarizing the work done under this problem.

## AUTHORIZATION

NRL Problem No. R07-23 - Closed 12 May 1948.

## ACKNOWLEDGMENT

Acknowledgement is made of the assistance rendered by G. C. Schleiter, who directed the work on the system integration problem, and to that given by E. B. McIntyre who assisted in conducting the measurements.

## THE RESONANT FREQUENCY OF A CAVITY-TYPE FILTER AS A FUNCTION OF THE SIZE OF THE COUPLING IRIS

### INTRODUCTION

In the integration of aircraft electronic systems, the need of a high-frequency, low-loss, tunable, band-pass filter was recognized. A study of the cylindrical cavity-type filter as a possible solution was undertaken.

Band-pass filters at frequencies above 5000 megacycles generally require the use of cavity elements which are coupled by windows or irises to the wave guide used for transmission. In this report phases of engineering design not adequately explained in the literature are studied.

Simple theory exists for the design of unloaded cavities of limited shapes. This theory requires no discontinuities in the cavity and no disturbing influence on the field within the cavity. Theory is also advanced for deviations from the ideal case, but in general, it is too complicated for practical engineering use. Theory on coupling through irises is available, but this also is rather complex. Practical considerations for use in designing the size of the cavity and the size and location of the coupling irises without recourse to this complex theory are discussed in this report.

The degree to which dependence can be placed upon simple cavity theory for the design of a tunable, cylindrical, cavity-type filter is determined experimentally. The more important causes of the indicated deviation from theoretical values are discussed, particularly with respect to keeping such deviations within the limitations of engineering design. The mechanism of iris coupling and its effect upon the cavity dimensions and the power transmitted is discussed. General guides for determining the size and location of the irises are developed.

In order to restrict the magnitude of the problem, little attention is given to the actual band-pass characteristics at frequencies other than the resonant frequency. However one curve is shown to give an indication of the band-pass range to be expected from a simple cavity-type filter of the type discussed here.

Reference to the fore-mentioned theoretical work and other background material is given in the bibliography.

#### METHOD OF INVESTIGATION

##### Experiments Conducted

A cylindrical cavity with inside dimensions as shown in Figure 1, was used as the filter element. The size and location of the coupling irises is shown in Figure 2. A functional diagram of the experimental system is shown in Figure 3.

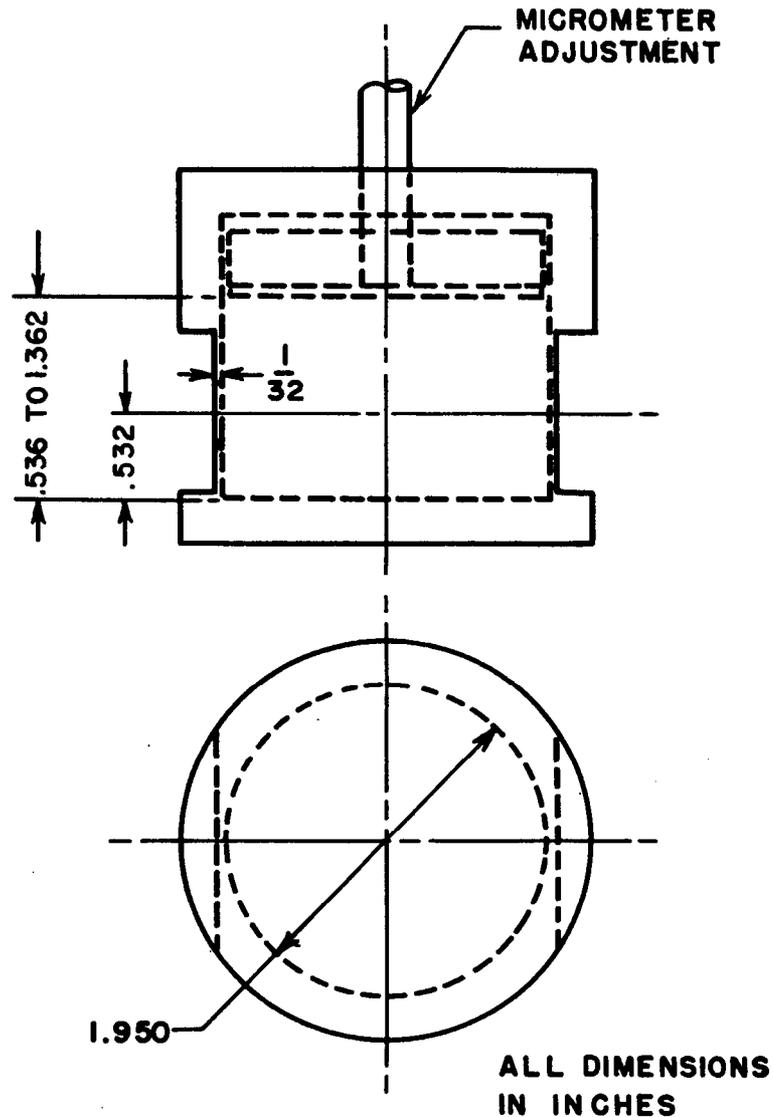


Figure 1 - Cavity dimensions

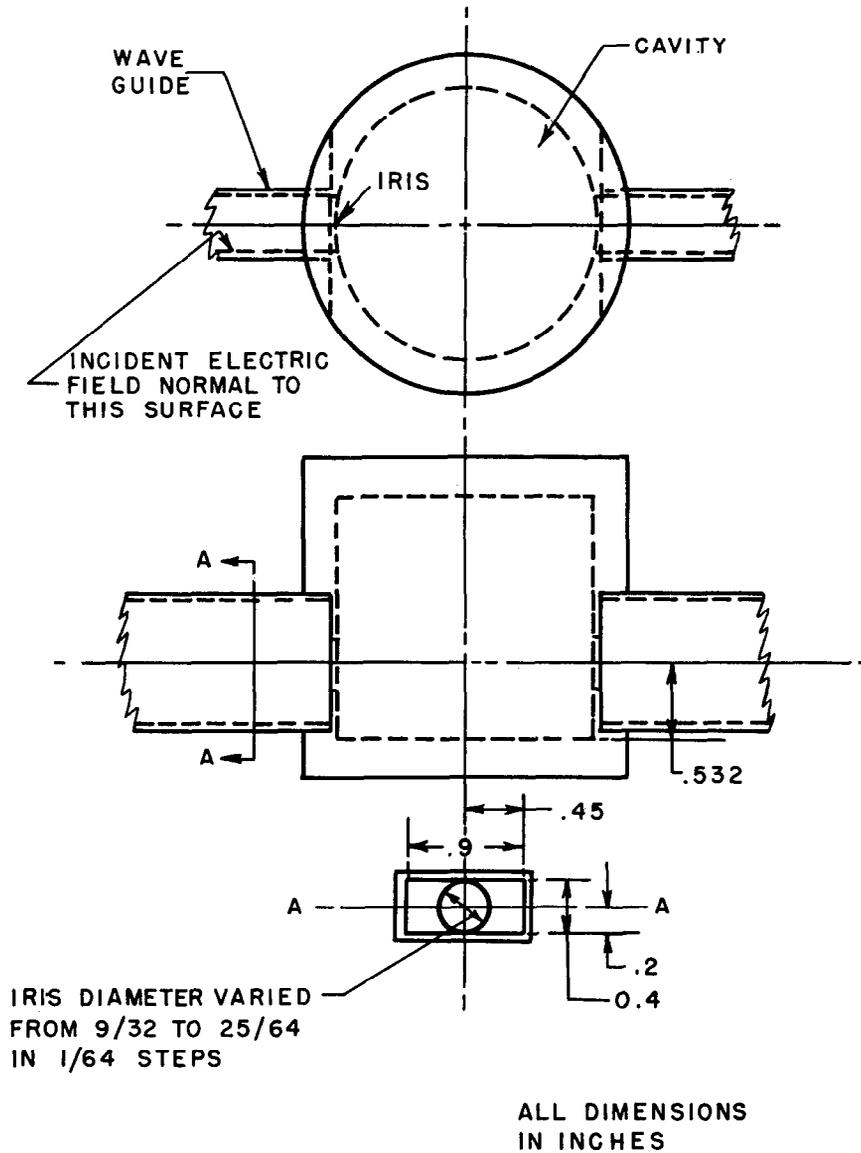


Figure 2 - Location and sizes of coupling irises

Measurements at resonance to obtain the length of the cavity and the power transmitted, were made for each mode of oscillation of the cavity and for each of the eight sizes of iris used. They were taken at approximately 50-Mc intervals over the frequency range from 8500 to 9500 Mc. Several measurements of power transmission were also made as the frequency was varied slightly from resonance.

A micrometer screw was used for adjusting and measuring changes in the length of the cavity. Resonance was indicated and the relative power transmitted was measured by means of a voltage pick-up probe in a slotted-line section ahead of the cavity. This method was used for two reasons: a) it gave a more sensitive indication of resonance than any available method

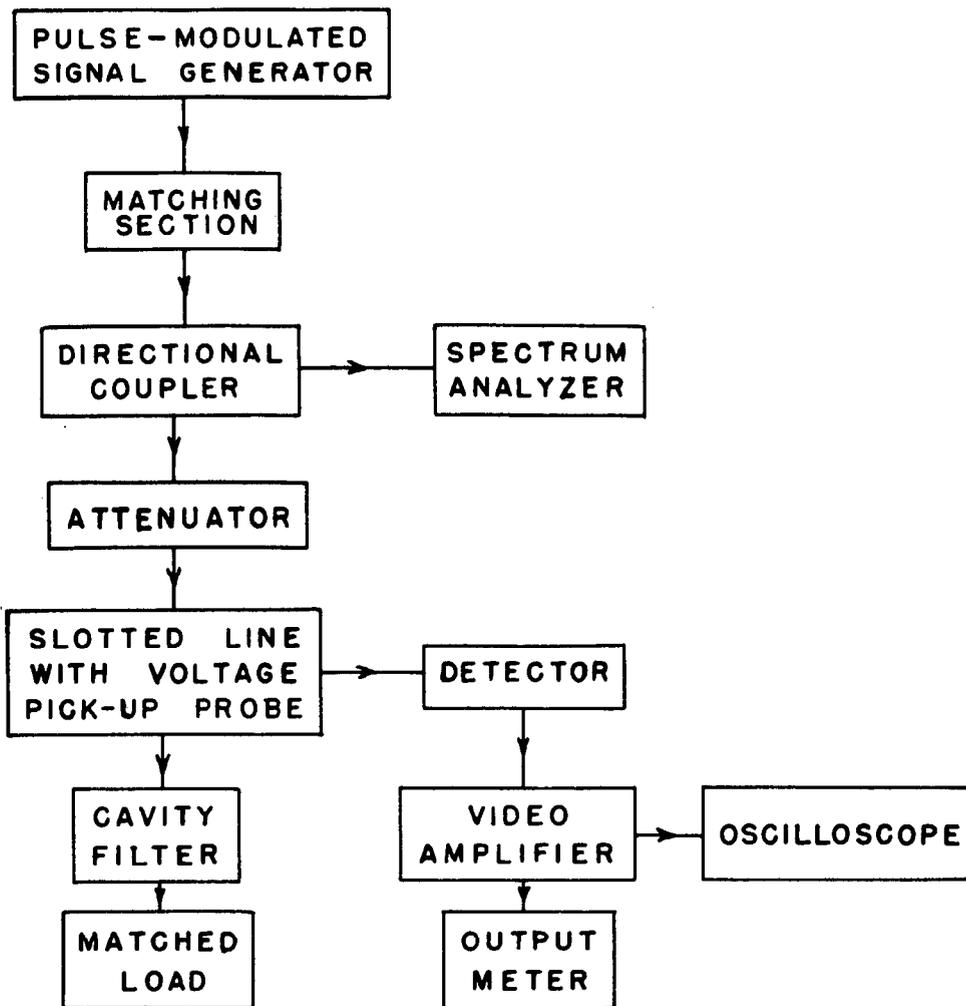


Figure 3 - Functional diagram of equipment used

operating on the output side of the cavity and b) it was a simple means of determining the relative power transmitted through the cavity, absolute measurements of power at these frequencies being rather difficult to make.

When completely detuned, the cavity transmits negligible power, since essentially all the incident energy is reflected. Thus, a standing wave appears in the slotted line section. The ratio of the signal strength at maximum and minimum points along this standing wave, as measured by the relative output from the pick-up probe, is extremely large. As the cavity is tuned to resonance for any particular mode, power is transmitted, reducing the magnitude of the reflected wave and giving a standing-wave ratio closer to unity. Thus when the standing-wave ratio is a minimum, maximum power is transmitted and resonance is indicated. The value of the ratio gives a measure of the relative power reflected. Since the losses within a cavity have been determined to be negligible for engineering purposes, this ratio is also a measure of the relative power transmitted. An attempt to measure the loss within the cavity showed it to be smaller than the limits of accuracy of the measuring equipment.

### Instrumentation

Two details of the instrumentation appear to be worthy of further mention, the stability of the signal source and the tuning of the cavity.

A klystron oscillator was the only available signal source which could be tuned easily over a wide range of frequencies in the band desired. With air cooling, the frequency of this oscillator was not sufficiently stable to permit accurate measurements. After an investigation had been made of a number of different methods, the difficulty was overcome by immersing the oscillator in an oil cooling bath.

During the adjustment of the length of the cavity at positions near resonance, the position of minimum voltage in the wave guide changed slightly with cavity length. Thus for manual manipulation of the probe position, considerable juggling back and forth between probe position and cavity length was required before the minimum standing-wave ratio could be determined. This detail of the work was materially expedited by the use of a motor to sweep the probe repetitively over a length of the slotted-line section. The output of the detector was amplified and viewed directly on a synchronized oscilloscope, thus permitting visual observation of a plot of the standing-wave ratio as the cavity length was tuned. This method also permitted direct observation of noise in the detector system. Such noise frequently came in from external sources and until eliminated caused errors in the measurements.

### Accuracy of Measurements

The equipment used was accurate to within the following tolerances:

Frequency	0.06%
Cavity Dimensions	0.2%
Standing-wave ratio	3.0%

### CALCULATION OF THE RESONANT FREQUENCIES OF THE IDEAL UNLOADED CAVITY

By an ideal cavity is meant one within which the dielectric constant and the permeability are the same as that of free space, and whose surfaces are unbroken and have infinite conductivity. Hansen<sup>6,4</sup> has shown that the resonant wavelength of an ideal cylindrical cavity is given by

$$L = 4(A^2 + B^2)^{1/2}$$

where

L	= wavelength for resonance
A	= $\rho/Z_0$
B	= $2u/a\pi$
$Z_0$	= half the length of the cavity

- a = radius of the cylinder
- u =  $x_{n,m}$  for transverse magnetic ( $TM_{n,m,p}$ ) modes of oscillation or  $x'_{n,m}$  for transverse electric ( $TE_{n,m,p}$ ) modes of oscillation
- $x_{n,m}$  = the value of the argument of the nth order Bessel function of the first kind at the mth zero
- $x'_{n,m}$  = the value of the argument of the 1st derivative of the nth order Bessel function of the first kind at the mth zero
- p = zero or a positive integer equal to the number of half-wave variations of electric field along the length of the cavity (zero is excluded for the TE wave)
- n = zero or a positive integer equal to the number of full-wave variations of electric field encountered along  $360^\circ$  of the angular or  $\theta$  co-ordinate
- m = a positive integer equal to the number of half-wave variations of electric field along the radial or r co-ordinate.

Resonant frequencies are given by the expression

$$f = 7.494 (A^2 + B^2)^{\frac{1}{2}}$$

where  $Z_0$  and a are in centimeters and f is in kilomegacycles. Substituting the value of a for the cavity studied, the expression becomes

$$f = 7.494 (A^2 + 0.0661u^2)^{\frac{1}{2}}.$$

Now if frequencies are confined to the range 8.5 to 9.5 kilomegacycles, the following inequalities must hold.

$$8.5 < 7.494 (A^2 + 0.0661u^2)^{\frac{1}{2}} < 9.5 \text{ and}$$

$$1.287 < A^2 + 0.0661u^2 < 1.608.$$

Next, if the length of the cavity is confined to the range used in these studies (1.30 to 3.48 centimeters), limitations on  $Z_0$  are given by

$$0.65 < Z_0 < 1.74.$$

If  $u = 0$ , its minimum value, the maximum possible value of  $A^2$  is found to be 1.608. Now since

$$p = AZ_0$$

the maximum value that  $p$  can have is

$$p = (1.608)^{\frac{1}{2}} \times 1.74 = 2.21.$$

Since  $p$  is restricted to zero or positive integers, the permitted values are

$$p = 0, 1 \text{ or } 2.$$

With these limitations on  $p$  and  $Z_0$ , restrictions are imposed upon the value of  $A$  in the equation above. Within these restrictions  $u$  is limited to the ranges of values,

$$\begin{array}{ll} 4.41 < u < 4.93 & \text{for } p = 0 \\ 0 < u < 4.40 & \text{for } p = 1 \\ 0 < u < 2.09 & \text{for } p = 2. \end{array}$$

However,  $u$  is further restricted to values\* giving zeros of the Bessel functions. (Values are given in Tables 1 and 2). This requirement and the above limitations on  $u$  restrict  $u$  and the modes of oscillation to the following:

MODE	u
TE <sub>0,1,1</sub>	3.832
TE <sub>1,1,1</sub>	1.841
TE <sub>1,1,2</sub>	1.841
TE <sub>2,1,1</sub>	3.054
TE <sub>3,1,1</sub>	4.201
TM <sub>0,1,1</sub>	2.405
TM <sub>1,1,1</sub>	3.832

From these values, the resonant lengths for the different modes of oscillation have been determined as a function of the frequency. Curves of these quantities are shown on Figure 4. The calculated figures are shown with greater accuracy in Table 3.

\* Tables are shown at end of report.

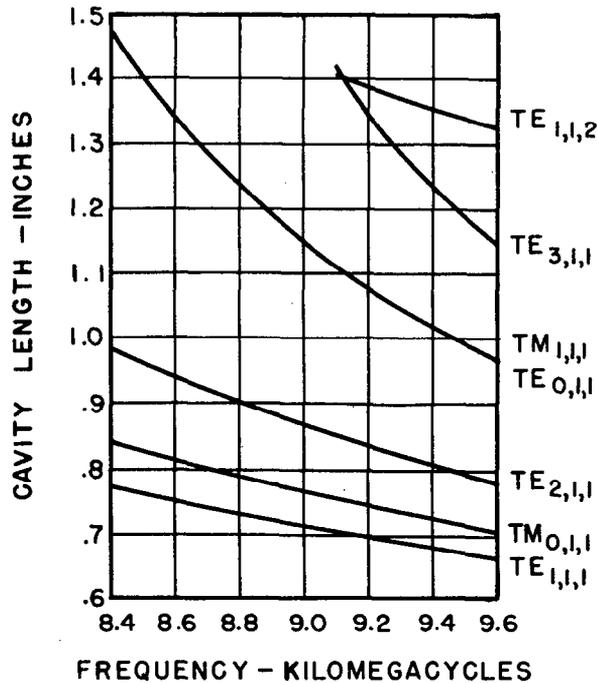


Figure 4 - Calculated cavity length for each mode of an ideal unloaded cavity at resonance

#### DISCUSSION OF RESULTS

The experimental data, given in Tables 4 through 11 of the appendix, shows that the cavity length at resonance for the excited modes agrees very closely with the calculated values. Since the differences are small, the data can be discussed more easily if it is expressed in terms of this deviation. The difference, expressed as a percentage of the calculated length, is shown in Tables 12 through 15 of the appendix. When examined in this form it is seen that the deviation for each mode of oscillation is relatively independent of the frequency. Thus an average can be taken for each mode over all frequencies used, resulting in a further condensation of the data.

In general, caution must be exercised in averaging of this sort. When tuning the cavity to resonance as the frequency is changed, the length of the cavity is changed. Since only one end of the cavity used is movable, changing the length actually moves the position of the coupling iris with respect to field configuration within the cavity. Calculations by Bethe and Schwinger<sup>18</sup> have shown that an iris may either increase or decrease the resonant length of a cavity depending upon the location of the iris with respect to the mode being excited. Thus, in general, averaging over a frequency range may not be permissible. In this case, however, the data indicates averaging to be feasible for the generalizations to be made.

The experimental data indicates that the percentage of incident power transmitted is relatively independent of frequency, allowing an average here also. All the data, therefore, is condensed to average values, and used in that form.

Before examining this data, one more fact should be mentioned. The parameters, variation in cavity length, relative power transmitted, iris size,

iris location, and coupling through the iris are all interrelated and are difficult to consider separately. However, for clarity, they must be introduced in some orderly fashion. Therefore the observed modes are first compared with those calculated theoretically.

#### Modes Observed

The experimental results show that indications of resonance are obtained at only four different modes, whereas the calculations for an ideal cavity indicate seven. Actually only six of these should be observed since two ( $TE_{0,1,1}$  and  $TM_{1,1,1}$ ) occur at the same frequency. A comparison of the calculated values with the measured ones shows close agreement with the  $TE_{1,1,1}$ ,  $TE_{2,1,1}$ ,  $TE_{3,1,1}$ , and the combined  $TE_{0,1,1} - TM_{1,1,1}$  modes. There is no indication of excitation of the  $TE_{1,1,2}$  and the  $TM_{0,1,1}$  modes. To understand this the expressions for the field configurations within the cavity for these two modes are now examined.

#### Explanation of Unexcited Modes

Starting with the wave equations, Hansen<sup>34</sup> has shown that the electric field for these two modes are as follows:

For the  $TE_{1,1,2}$  mode,

$$E_r = \frac{J_1(1.547r)}{1.547r} \cos\theta \sin\pi Z/Z_0$$

$$E_\theta = J'_1(1.547r) \sin\theta \sin\pi Z/Z_0$$

$$E_z = 0.$$

For the  $TM_{0,1,1}$  mode,

$$E_r = (-k_3/k) J'_0(0.974r) \sin k_3 Z$$

$$E_\theta = 0$$

$$E_z = (0.974/k) J_0(0.974r) \cos k_3 Z.$$

Where

$E_r$ ,  $E_\theta$  and  $E_z$  = r,  $\theta$ , and Z components respectively of the electric field.

- $r$  = radial distance from axis of symmetry,  
 $Z$  = axial distance from one end of the cavity,  
 $k_3 = \pi/2Z_0$ ,  
 $k = (k_3^2 + 0.948)^{1/2}$ ,  
 $J_n(k_1 r) =$  Bessel function,  
 $J'_n(k_1 r) =$  First derivative of the Bessel function.

From the expression for the  $TE_{1,1,2}$  mode, the field is seen to be zero on a plane passing through the center of the cavity and normal to the axis of symmetry. Thus the excitation, which is applied essentially at the plane, is feeding into a nodal point of the mode. Consequently negligible excitation of this mode is to be expected.

Next consider the  $TM_{0,1,1}$  mode. The wave guide used to feed energy to the cavity transmits in such a manner that the electric component is normal to the broad side of the wave guide. The orientation of the wave guide as shown by Figure 2 is such that the exciting electric field is parallel to the  $\theta$  component of the field within the cavity, and normal to the  $r$  and  $Z$  components. But as shown by the equations above, the  $\theta$  component of the field within the cavity for this mode is zero. Thus the mode is not appreciably excited.

In those cases where modes are excited, similar reasoning indicates that identical coupling to all modes should not be expected.

#### Percentage of Incident Power Transmitted

Figure 5 shows the average power transmitted as a function of the iris size for three of the modes. The fourth transmits less than 64 percent of the incident power. Here it is seen that there is an appreciable difference in the relative power transmitted by the different modes. As should be expected, it is observed that the power transmitted increases with the size of the coupling iris.

To get a clearer picture of this variation in coupling for the different modes, one more case is examined. Figure 6 shows a section of the unperturbed electric field for the  $TE_{2,1,1}$  and the  $TE_{3,1,1}$  modes, in a plane normal to the axis of the cylinder at its mid-point. The relative size of a 25/64-inch iris is shown for each case. A comparison of the field of the two modes at the iris, shows that the tangential component over the area of the iris is much less for the  $TE_{3,1,1}$  mode than for the  $TE_{2,1,1}$  mode. Since there is coupling only to the tangential component, the  $TE_{3,1,1}$  mode should be excited less than the  $TE_{2,1,1}$  with a consequent smaller percentage of transmitted power. This is confirmed by Figure 5.

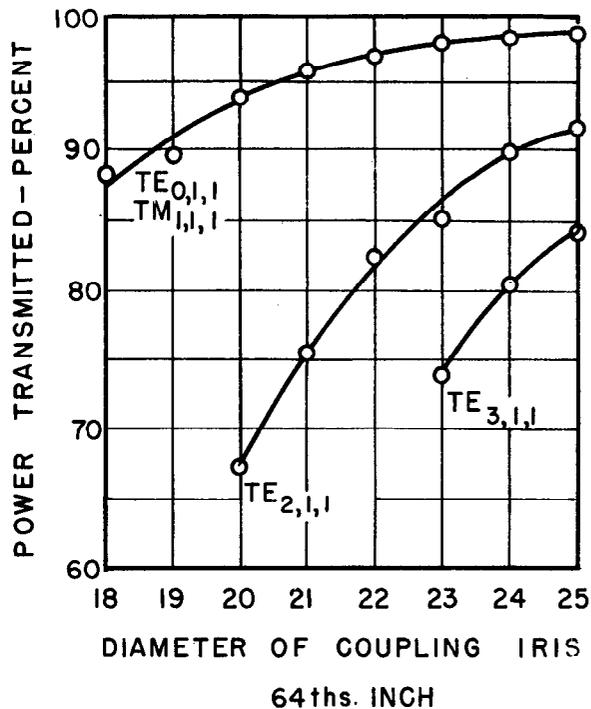


Figure 5 - Average power transmitted versus size of coupling iris

For most modes, it is obvious that the field within the cavity near the iris will be distorted since the excitation is in conflict with the theoretical field configuration. It is to be expected that this distortion will modify the actual cavity dimensions required to establish resonance.

#### Deviations Between Theoretical and Actual Cavity Dimensions

As previously stated, the resonant length of the cavity for the modes which are excited differs somewhat from the calculated resonant length of the ideal cavity. The average percent deviation is shown in Figure 7. A number of reasons may be advanced for this deviation. The dielectric constant and the permeability of the cavity differ slightly from the values for free space. The conductivity of the cavity surface is finite rather than infinite.<sup>10</sup> The adjustable end of the cavity does not make direct contact with the sides of the cavity. The coupling holes and the energy fed into and extracted from the cavity distort the field of the cavity. These last interrelated items are of major importance.

An inspection of the curves of Figure 7, shows that large deviation exists only for the larger holes, indicating that the size of the coupling hole may be the major contributing factor. By observing the curve for the TE<sub>1,1,1</sub> mode it is seen that the deviation is not appreciably affected by increasing the iris diameter. However, this mode transmits negligible power, indicating that the effect of the power transmitted through the cavity may also have to be considered.

Now examination of the curves for the TE<sub>2,1,1</sub> and the TE<sub>3,1,1</sub> modes in Figures 5 and 7, shows that the deviation in length is greater for the TE<sub>3,1,1</sub>

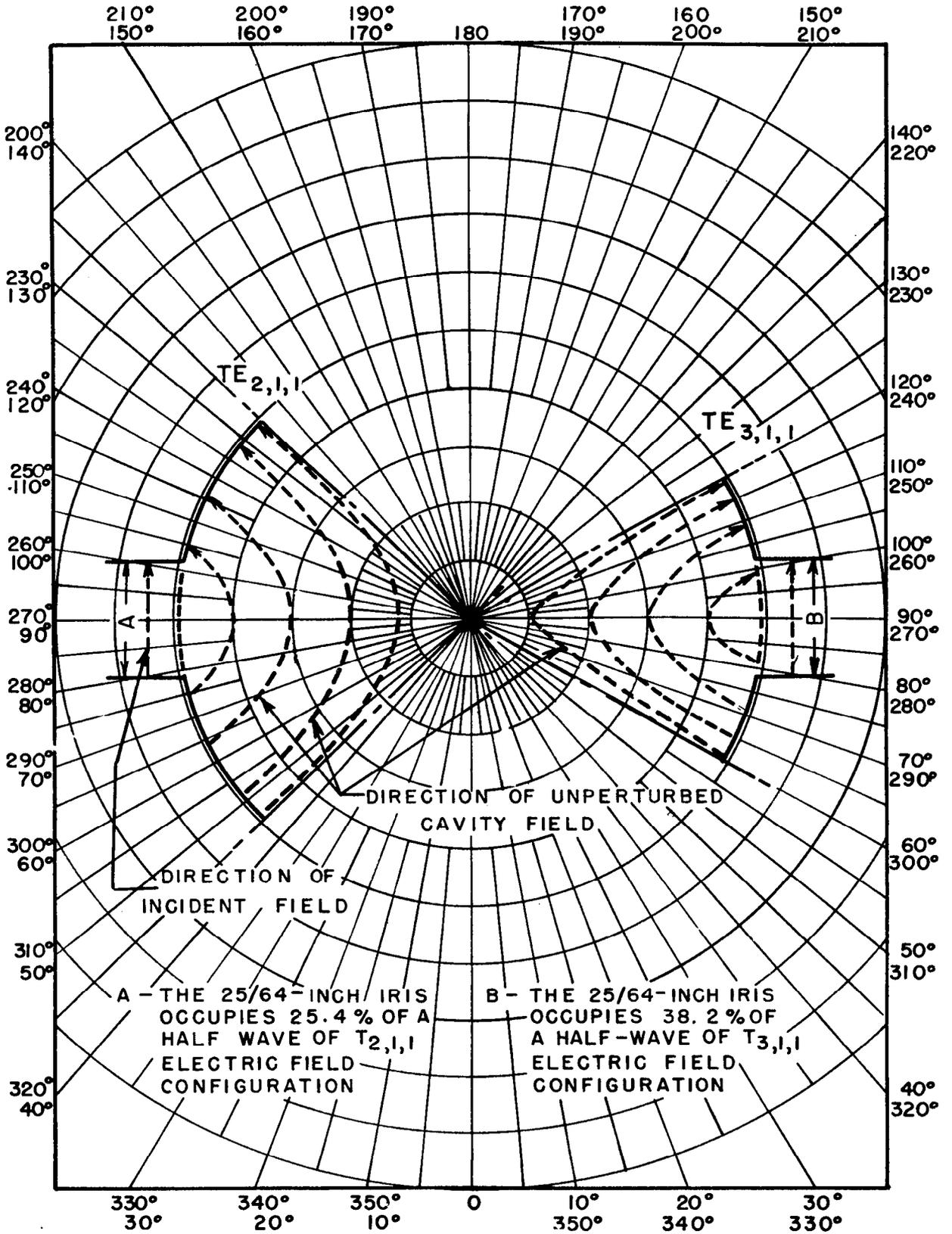


Figure 6 - Comparison of electric field configuration for TE<sub>2,1,1</sub> and TE<sub>3,1,1</sub> modes

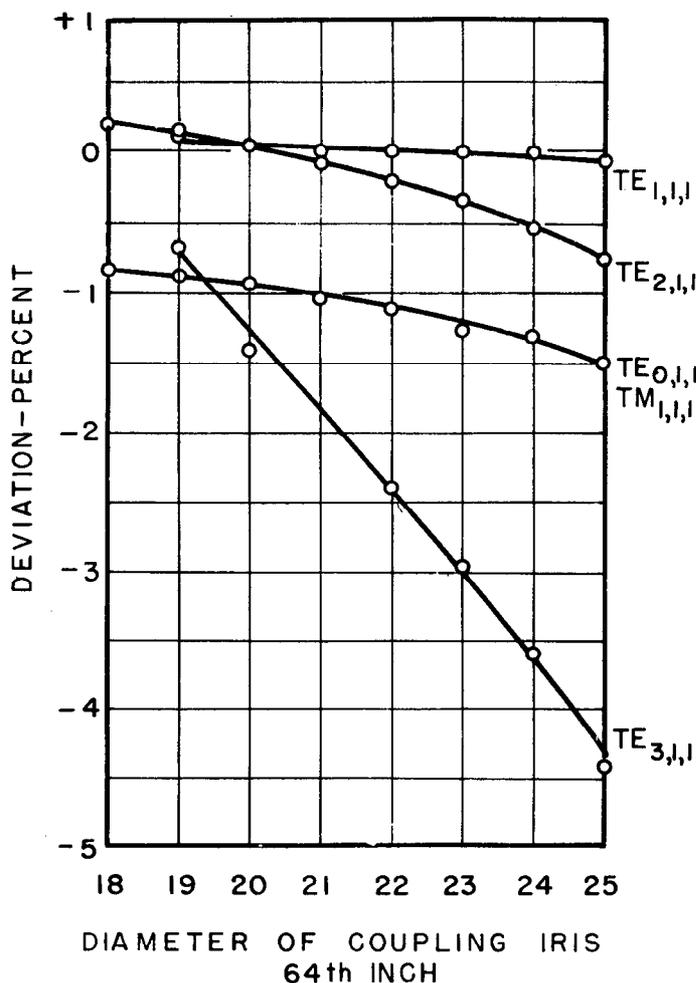


Figure 7 - Average deviation of resonant length versus size of coupling iris

mode, even though this mode transmits less power. Thus it is seen that other contributing factors must be examined.

A glance at the field configuration for these two modes gives a lead to the answer. From Figure 6, it is seen that both modes have radial components over the area of the coupling iris, the radial component being greater for the TE<sub>3,1,1</sub> mode than for the TE<sub>2,1,1</sub> mode. Now the exciting field, which is essentially tangential over the iris, does not excite the radial component at the iris. Thus greater distortion of the field within the cavity should be expected for the TE<sub>3,1,1</sub> mode. If distortion of the field is of primary importance, a greater deviation between the actual and calculated cavity lengths should be expected for this mode. This agrees with the experimental results and ties in with the effects of variation of the iris diameter, and of the power transmitted.

Thus the conclusion is reached that the essential factor determining the variation between actual and calculated cavity dimensions is the distortion of the field within the cavity. Further, it appears, at least for

the low power levels used in these measurements that the more important factors determining this distortion are the size and position of the coupling iris and the orientation of the exciting field.

One other result requires some comment. In Figure 7, it is seen that for  $TE_{1,1,1}$  and the  $TE_{2,1,1}$  modes, with a small coupling iris, the measured resonant cavity length is longer than the calculated value; but as the diameter of the iris is increased, the resonant length decreases relative to the calculated value, passing through zero and becoming shorter than the calculated length. This may possibly be a function of the change in the coupling. However, the limitations imposed by the accuracy of the measuring equipment are of the same magnitude. Since the effect of this shift is negligible insofar as the conclusions to be drawn are concerned, it is not proposed to investigate the phenomena further.

The discussions to this point lead to the conclusion that the various parameters are all interrelated through a common factor. This factor is the direction of the incident field just outside the coupling iris and the direction of the unperturbed field just inside the iris. If they are parallel, appreciable coupling exists with negligible distortion; if they depart from this condition, the coupling through the iris is reduced and the energy which is transmitted distorts the field within the cavity. A similar phenomena exists for the power leaving the cavity on the output side.

Since the cavity-type filter discussed in this report passes a narrow band of frequencies for any particular mode of oscillation and cavity size, some comments should be made on its performance off resonance.

#### Band-Pass Characteristics

In this study little attention has been given to the band-pass characteristics of the cavity at frequencies other than at resonance. The pass range observed was rather narrow, varying somewhat with the mode of oscillation. In fact, if the cavity were to be used as a filter with coupling no greater than used here, more than usual care would have to be used to stabilize the frequency of the oscillator, since normal short-time oscillator variation is greater than the pass band observed.

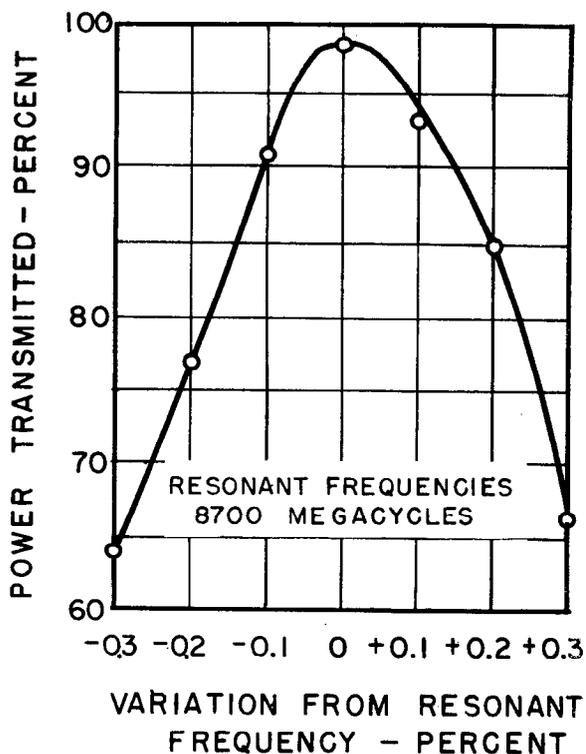
To give some indication of the characteristics, the power transmitted vs frequency for the  $TE_{0,1,1}$  -  $TM_{1,1,1}$  mode is shown in Figure 8. Because of limitations of the test equipment used, the curve shown does not extend to cutoff. Indications were that the curve drops rapidly beyond the frequency deviation of plus or minus three percent shown.

#### CONCLUSIONS

From these measurements and considerations, certain conclusions may be drawn:

- 1) Energy can be transmitted with negligible loss over a narrow band of frequencies and with a relatively sharp cutoff, through a cylindrical cavity-type filter with iris coupling.

Figure 8 - Band-pass characteristics of filter operating on the  $TE_{0,1,1}$ - $TM_{1,1,1}$  mode with 25/64-inch diameter coupling iris



2) With a tunable cavity the calculations can be made on the basis of an ideal unloaded cavity with reasonable expectation that the tuning range will be within two percent of the calculated value provided certain precautions are taken when determining the size and location of the coupling iris. These precautions are:

- (a) The iris should be located in a position where the unperturbed electric field for the desired mode has negligible normal component over the area of the iris.
- (b) The iris should be located adjacent to a maximum of the electric field of the desired mode.
- (c) The wave guide coupled to the cavity should be oriented properly so that the incident electric field is parallel to the unperturbed electric field of the desired mode just within the cavity.
- (d) The iris diameter should be no larger than necessary to obtain the desired coupling. The measurements made here indicate an iris diameter of approximately three-tenths of a wave length (i.e., the narrow dimension of the usual rectangular wave guide), is suitable for transmission with negligible loss when using the  $TE_{0,1,1}$  -  $TM_{1,1,1}$  mode.

\* \* \*

**TABLES**  
**Experimental and Calculated Data**

**TABLE 1**  
**Values  $x_{n,m}$  Giving the  $m$ th Zero of the  $n$ th Order Bessel Function of the First Kind,  $J_n(x_{n,m}) = 0$ .**

n	$x_{n,m}$		
	m = 1	m = 2	m = 3
0	2.405	5.520	8.654
1	3.832	7.016	10.173
2	5.135	8.417	11.620

**TABLE 2**  
**Values of  $x'_{n,m}$  Giving the  $m$ th Zero of the First Derivative of the  $n$ th Order Bessel Function of the First Kind,  $J'_n(x'_{n,m}) = 0$**

n	$x'_{n,m}$		
	m = 1	m = 2	m = 3
0	3.832	7.016	-----
1	1.841	5.33	8.55
2	3.054	6.71	9.97
3	4.201	8.02	11.36
4	5.30	9.30	12.68

TABLE 3

Calculated Lengths of An Ideal Unloaded Cavity at Resonance

f*	Length at resonance - inches					
	TM <sub>0,1,1</sub>	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>	TE <sub>1,1,1</sub>	TE <sub>1,1,2</sub>	TE <sub>2,1,1</sub>	TE <sub>3,1,1</sub>
8400	0.842	1.472	0.775		0.984	
8500	.828	1.400	.764		.962	
8600	.814	1.338	.753		.941	
8700	.801	1.282	.743		.921	
8800	.789	1.232	.733		.902	
8900	.776	1.187	.723		.884	
9000	.765	1.146	.713		.866	
9100	.753	1.109	.704	1.408	.850	1.419
9200	.742	1.075	.695	1.390	.834	1.349
9300	.732	1.043	.686	1.373	.819	1.288
9400	.722	1.014	.678	1.356	.805	1.234
9500	.712	0.987	.670	1.339	.791	1.186
9600	.702	.962	.662	1.323	.778	1.143

\*f - frequency in megacycles

UNCLASSIFIED

TABLE 4  
Filter Characteristics at Resonance  
For 9/32-Inch Coupling Iris

f	Mode of Oscillation					
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR
8500			0.973	*		
8600	1.334		.950	*		
8800			.911	*		
8900	1.183	1.8				
8950	1.163	1.8	.894	*		
9000	1.141	1.9				
9050	1.124	2.0				
9100	1.106	1.85				
9150	1.089	2.1				
9197	1.074	2.1	.843	*	1.210	*
9200	1.072	2.4				
9250	1.056	2.0				
9300	1.041	2.1				
9350	1.028	2.2				
9375	1.019	2.4				
9450	0.997	2.5	.804	*		
9600	.959	1.8	.784	*	1.147	*

\* - Larger than 4. No attempt was made to evaluate these larger values

L - Cavity length in inches

SWR - Input voltage standing wave ratio.

TABLE 5  
Filter Characteristics at Resonance  
For 19/64-Inch Coupling Iris

f	Mode of Oscillation							
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR	L	SWR
8495			0.771	*	0.970	*		
8580	1.347	2.3	.762	*	.953	*		
8630	1.316	2.2	.751	*				
8680	1.290	2.0			.933	*		
8730	1.257	1.9						
8780	1.238	1.85			.914	*		
8850	1.205	1.8						
8900	1.182	1.8			.891	*		
8950	1.162	1.8						
9000	1.143	2.0			.874	*		
9050	1.132	2.0						
9100	1.106	2.1			.857	*		
9150	1.088	2.1						
9200	1.073	2.1			.841	*		
9250	1.056	1.9						
9300	1.041	2.3			.826	*	1.284	*
9350	1.026	2.0						
9400	1.011	1.6			.810	*	1.227	*
9450	0.997	2.0			.807	*	1.215	*

TABLE 6  
Filter Characteristics at Resonance  
For 5/16-Inch Coupling Iris

f	Mode of Oscillation					
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR
8495			0.971	3.8		
8530			.962	3.8		
8580	1.343	1.76	.952	3.8		
8630	1.314	1.70	.942	3.8		
8680	1.288	1.66	.932	3.6		
8730	1.262	1.65	.922	3.5		
8780	1.237	1.61	.912	3.4		
8850	1.203	1.60	.900	3.4		
8900	1.181	1.54	.890	3.5		
8950	1.161	1.49	.881	3.5		
9000	1.143	1.62	.873	3.6		
9050	1.123	1.60	.864	3.8		
9100	1.106	1.58	.857	3.9		
9150	1.087	1.58	.848	4.0	1.362	*
9200	1.072	1.70	.841	*	1.332	*
9250	1.055	1.62	.833	*	1.305	*
9300	1.040	1.68	.824	4.0	1.278	*
9350	1.025	1.69	.818	4.0	1.253	*
9400	1.011	1.58	.811	4.0	1.226	4.0
9450	0.996	1.60	.803	3.8	1.201	3.0

TABLE 7  
 Filter Characteristics at Resonance  
 For 21/64-Inch Coupling Iris

f	Mode of Oscillation					
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>1,1,1</sub>		TE <sub>2,1,1</sub>	
	L	SWR	L	SWR	L	SWR
8495			0.771	*	0.969	3.0
8530			.767	*	.961	3.0
8580	1.336	1.60	.762	*	.951	2.9
8630	1.313	1.53	.756	*	.940	2.8
8680	1.287	1.53	.751	*	.931	2.8
8730	1.260	1.56	.745	*	.920	2.7
8780	1.235	1.48	.741	*	.911	2.7
8850	1.203	1.48	.734	*	.898	2.7
8900	1.180	1.42	.729	*	.888	2.7
8950	1.160	1.40	.724	*	.880	2.9
9000	1.141	1.46	.719	*	.872	2.8
9050	1.122	1.55	.713	*	.863	2.9
9100	1.104	1.45	.709	*	.856	3.0
9150	1.086	1.47	.705	*	.847	3.0
9200	1.070	1.58	.701	*	.839	3.2
9250	1.054	1.50	.698	*	.832	3.2
9300	1.039	1.49	.694	*	.824	3.4
9350	1.024	1.52	.687	*	.817	3.5
9400	1.010	1.48	.683	*	.810	3.6
9450	0.996	1.58	.681	*	.802	3.6

TABLE 8  
Filter Characteristics at Resonance  
For 11/32-Inch Coupling Iris

f	Mode of Oscillation							
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR	L	SWR
8495			0.771	*	0.967	2.5		
8530			.767	*	.959	2.5		
8580	1.341	1.50	.762	*	.949	2.5		
8630	1.312	1.50	.856	*	.939	2.5		
8680	1.285	1.42	.751	*	.929	2.5		
8730	1.259	1.41	.746	*	.919	2.5		
8780	1.235	1.40	.741	*	.910	2.4		
8850	1.202	1.36	.734	*	.897	2.3		
8900	1.179	1.35	.728	*	.887	2.3		
8950	1.160	1.32	.724	*	.880	2.4		
9000	1.140	1.36	.719	*	.871	2.4		
9050	1.120	1.42	.714	*	.862	2.4		
9100	1.104	1.36	.709	*	.855	2.5		
9150	1.085	1.40	.706	*	.846	2.6	1.344	*
9200	1.066	1.53	.702	*	.838	2.7	1.321	*
9250	1.053	1.46	.697	*	.831	2.7	1.291	*
9300	1.040	1.40	.693	*	.823	2.8	1.264	4.0
9350	1.023	1.45	.689	*	.817	2.8	1.240	3.2
9400	1.009	1.40	.685	*	.809	2.8	1.215	2.8
9450	0.995	1.42	.679	*	.798	2.8	1.190	2.4

TABLE 9  
Filter Characteristics at Resonance  
For 23/64-Inch Coupling Iris

f	Mode of Oscillation							
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR	L	SWR
8495			0.771	*	0.965	2.3		
8530			.767	*	.959	2.3		
8580	1.341	1.38	.762	*	.948	2.2		
8630	1.309	1.35	.756	*	.937	2.2		
8680	1.285	1.32	.751	*	.928	2.2		
8730	1.257	1.31	.746	*	.918	2.2		
8780	1.233	1.31	.741	*	.909	2.2		
8850	1.200	1.31	.733	*	.895	2.1		
8900	1.179	1.31	.728	*	.887	2.2		
8950	1.158	1.34	.723	*	.878	2.2		
9000	1.138	1.30	.718	*	.869	2.1		
9050	1.119	1.34	.714	*	.861	2.1		
9100	1.102	1.30	.710	*	.853	2.3		
9150	1.084	1.28	.706	*	.845	2.3	1.342	4.0
9200	1.068	1.36	.702	*	.837	2.4	1.313	3.9
9250	1.052	1.38	.696	*	.830	2.6	1.286	3.8
9300	1.037	1.28	.692	*	.822	2.5	1.258	2.9
9350	1.022	1.38	.690	*	.815	2.7	1.233	2.5
9450	0.994	1.35	.681	*	.801	2.4	1.183	2.1

TABLE 10  
Filter Characteristics at Resonance  
For 3/8-Inch Coupling Iris

f	Mode of Oscillation							
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR	L	SWR
8495			0.770	*	0.963	1.95		
8530			.767	*	.955	2.00		
8580	1.339	1.34	.761	*	.946	1.95		
8630	1.308	1.34	.756	*	.935	1.95		
8680	1.283	1.34	.751	*	.926	1.85		
8730	1.256	1.35	.746	*	.916	1.85		
8780	1.232	1.30	.740	*	.907	1.90		
8850	1.206	1.26	.736	*	.893	1.81		
8900	1.177	1.25	.729	*	.885	1.85		
8950	1.156	1.25	.724	*	.876	1.90		
9000	1.137	1.22	.719	*	.868	1.86		
9050	1.117	1.30	.714	*	.859	1.90		
9100	1.101	1.30	.710	*	.852	1.95		
9150	1.083	1.30	.705	*	.844	2.00	1.334	3.2
9200	1.067	1.36	.701	*	.836	2.00	1.305	3.1
9250	1.051	1.30	.696	*	.829	2.00	1.279	2.4
9300	1.036	1.28	.692	*	.821	2.00	1.249	2.5
9350	1.021	1.28	.689	*	.814	2.00	1.224	2.2
9450	0.993	1.21	.681	*	.800	2.10	1.175	1.9

TABLE 11  
Filter Characteristics at Resonance  
For 25/64-Inch Coupling Iris

f	Mode of Oscillation							
	TE <sub>0,1,1</sub> TM <sub>1,1,1</sub>		TE <sub>1,1,1</sub>		TE <sub>2,1,1</sub>		TE <sub>3,1,1</sub>	
	L	SWR	L	SWR	L	SWR	L	SWR
8495			0.770	*	0.961	2.00		
8530	1.365	1.25	.766	*	.953	1.85		
8580	1.335	1.27	.761	*	.943	1.75		
8630	1.306	1.24	.756	*	.933	1.78		
8680	1.279	1.29	.750	*	.923	1.70		
8730	1.253	1.27	.745	*	.914	1.70		
8780	1.229	1.24	.740	*	.905	1.75		
8850	1.196	1.27	.733	*	.886	1.60		
8900	1.174	1.30	.728	*	.883	1.70		
8950	1.155	1.18	.723	*	.875	1.82		
9000	1.136	1.25	.718	*	.866	1.65		
9050	1.116	1.22	.714	*	.858	1.78		
9100	1.099	1.22	.710	*	.850	1.85	1.354	2.6
9150	1.082	1.25	.704	*	.842	1.80	1.324	2.8
9200	1.065	1.30	.701	*	.834	1.90	1.296	2.7
9250	1.049	1.26	.696	*	.828	1.88	1.268	2.5
9300	1.034	1.30	.693	*	.820	1.85	1.241	2.3
9350	1.020	1.20	.688	*	.813	1.85	1.216	1.95
9450	0.991	1.17	.680	*	.798	1.90	1.166	1.70

TABLE 12  
 Variation from the Calculated Cavity Length  
 at Resonance for the  $TE_{0,1,1}$ - $TM_{1,1,1}$  mode  
 as a Function of Frequency and Iris Size

f	L'	Decrease in Length (%)							
		18*	19	20	21	22	23	24	25
8580	1.350	0.7	0.7	1.0	1.5	1.1	1.1	1.3	1.4
8630	1.321	-	0.7	1.0	1.1	1.1	1.4	1.4	1.6
8680	1.293	-	0.7	0.8	0.9	1.1	1.1	1.2	1.5
8730	1.267	-	1.3	0.9	1.0	1.1	1.3	1.3	1.6
8780	1.242	-	0.8	0.9	1.0	1.0	1.2	1.3	1.5
8850	1.209	-	0.8	1.0	1.0	1.1	1.2	1.2	1.6
8900	1.187	0.8	0.9	1.0	1.1	1.2	1.2	1.3	1.6
8950	1.166	0.8	0.9	1.0	1.0	1.0	1.2	1.4	1.5
9000	1.146	1.0	0.8	0.8	1.0	1.0	1.2	1.3	1.4
9050	1.127	0.8	1.0	.9	0.9	1.2	1.2	1.4	1.5
9100	1.109	0.8	0.8	.8	1.0	1.0	1.7	1.3	1.4
9150	1.092	0.8	0.9	1.0	1.1	1.2	1.1	1.4	1.5
9200	1.075	0.8	0.7	0.8	1.0	1.4	1.2	1.3	1.5
9250	1.059	0.9	0.9	0.9	1.0	1.1	1.2	1.3	1.5
9300	1.043	0.8	0.8	0.9	1.0	0.9	1.2	1.2	1.4
9350	1.028	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5
9400	1.014	--	0.9	0.9	1.0	1.1	-	-	-
9450	1.000	0.9	0.9	1.0	1.0	1.1	1.2	1.3	1.5
9600	0.962	0.9	-	-	-	-	-	-	-

L' - Calculated cavity length in inches

\* - Iris diameter in 64ths of an inch.

TABLE 13  
 Variations from the Calculated Cavity Length  
 at Resonance for the  $TE_{1,1,1}$  Mode  
 as a Function of Frequency and Iris Size

f	L'	Decrease in Length (%)					
		19	21	22	23	24	25
8495	0.764	*-0.1	-0.1	-0.1	-0.1	0.0	0.0
8530	.761		0	0	0	0	0.1
8580	.755	-0.1	-0.1	-0.1	-0.1	0	0
8630	.750		0	0	0	0	0
8680	.745		0	0	0	0	.1
8730	.740		0.1	0	0	0	.1
8780	.735		0	0	0	-0.1	.1
8850	.728		0	0	.1	-0.3	.1
8900	.723		0	.1	.1	0	.1
8950	.718		0	0	.1	0	.1
9000	.713		0	0	.1	0	.1
9050	.709		0	.1	.1	.1	.1
9100	.704		.1	.1	0	0	0
9150	.700		.1	0	0	.1	.3
9200	.695		0	-0.1	-0.1	0	0
9250	.691		-0.1	0	.1	.1	.1
9300	.686		-0.3	-0.1	0	0	-0.1
9350	.682		0.1	-0.1	-0.3	-0.3	0
9400	.678		.1	-0.1			
9450	.674		-0.1	.1	-0.1	-0.1	0

\* The negative numbers indicate an increase in length, the unmarked a decrease.

TABLE 14  
 Variations from the Calculated Cavity Length  
 at Resonance for the TE<sub>2,1,1</sub> Mode  
 as a Function of Frequency and Iris Size

f	L'	Decrease in Length (%)							
		18*	19	20	21	22	23	24	25
8495	0.963	-0.3	0.0	-0.2	0.0	0.2	0.4	0.6	0.8
8530	.955			-0.1	0	.2	.2	.2	.8
8580	.945	-0.3	-0.2	-0.1	0	.2	.3	.5	.8
8630	.935			-0.1	0.1	.2	.4	.6	.9
8680	.925		-0.2	-0.1	0	.2	.3	.5	.9
8730	.915			-0.1	.1	.2	.3	.5	.8
8780	.905	-0.3	-0.3	-0.1	0	.1	.2	.4	.7
8850	.892			-0.2	0	.1	.3	.6	1.3
8900	.884		-0.1	0	.2	.3	.3	.6	0.8
8950	.875			0	.1	.1	.3	.6	.7
9000	.866		-0.2	-0.1	0	.1	.3	.5	.7
9050	.858			0	.1	.2	.3	.6	.7
9100	.850		-0.1	-0.1	0	.1	.4	.5	.7
9150	.842			0	.1	.2	.4	.5	.7
9200	.834	-0.4	-0.1	-0.1	.1	.2	.4	.5	.7
9250	.827			0	.1	.2	.4	.5	.6
9300	.819		-0.1	0.1	.1	.2	.4	.5	.6
9350	.812			0	.1	.1	.4	.5	.6
9400	.805		0.1	0	.1	.2			
9450	.798	0	-0.4	0.1	.2	.8	.4	.5	.8
9600	.779	0.1							

**TABLE 15**  
**Variations from the Calculated Cavity Length**  
**at Resonance for the  $TE_{3,1,1}$  Mode**  
**As a Function of Frequency and Iris Size**

f	L'	Decrease in Length (%)					
		19	20	22	23	24	25
9100	1.419						5.0
9150	1.384		2.0	3.3	3.5	4.0	4.8
9200	1.349		1.7	2.5	3.1	3.8	4.6
9250	1.319		1.5	2.6	3.0	3.5	4.3
9300	1.288	0.8	1.2	2.3	2.8	3.5	4.1
9350	1.261		1.1	2.0	2.7	3.4	4.0
9400	1.234	1.1	1.1	2.0			
9450	1.210	0.1	1.2	2.1	2.7	3.4	4.1

**TABLE 16**  
**Average Variation from the Calculated Cavity Length**  
**at Resonance for Each Excited Mode**  
**as a Function of the Iris Diameter**

Diameter of coupling iris (64ths inch)	Average decrease in length (%)			
	$TE_{0,1,1}$ $TM_{1,1,1}$	$TE_{1,1,1}$	$TE_{2,1,1}$	$TE_{3,1,1}$
18	0.82		-0.20	
19	.86	-0.10	-0.15	0.67
20	.92		-0.05	1.40
21	1.03	-0.01	0.07	
22	1.11	-0.01	.20	2.40
23	1.21	0	.34	2.97
24	1.31	0	.53	3.60
25	1.50	0.06	.75	4.41

TABLE 17  
 Average Percent of Incident Power  
 Transmitted for Each Excited Mode as a Function of  
 the Diameter of the Coupling Iris

Diameter of coupling iris (64ths inch)	Average power transmitted (%)			
	$TE_{0,1,1}$	$TE_{1,1,1}$	$TE_{2,1,1}$	$TE_{3,1,1}$
18	88.2	*	*	*
19	89.5	*	*	*
20	93.8	*	67.2	*
21	95.6	*	75.4	*
22	96.8	*	82.4	*
23	97.9	*	85.1	73.9
24	98.2	*	89.8	80.5
25	98.6	*	91.5	84.2

\* Less than 64%

## SELECTED BIBLIOGRAPHY

### Periodicals

1. Andrews, C.L. "Diffraction pattern of a circular aperture at short distances." Phys. Rev., 71: 777-786, June 1947.
2. Barrow, W.L. and Mieher, W.W. "Natural oscillations of electrical cavity resonators." Proc. I.R.E., 28: 184-191, April 1940.
3. Bernier, J. "Sur les cavities electromagnetiques." L'onde Electrique, 26: 305-317, Aug.-Sept. 1946.
4. Bethe, H.A. "Theory of diffraction by small holes." Phys. Rev. 66: 163-182, Oct. 1-15, 1944.
5. Bleaney, B., Loubser, J.H.N., and Penrose, R.P. "Cavity resonators for measurements with centimetre electromagnetic Waves." Proc. Phys. Soc., 59: 185-199, March 1, 1947.
6. Fano, R.M. "Microwave filters using quater wave couplings." Proc. I.R.E. 35: 1318-1323, Nov. 1947.
7. Hahn, W.C. "A new method for the calculation of cavity resonators." J. App. Phys. 12: 62-68, Jan. 1941.
8. Hansen, W.W. "A type of electrical resonator." J. App. Phys. 9: 654-663, Oct. 1, 1938.
9. Harries, J.H.O. "Apertures in cavities." Electronics 19: 132-135, Dec. 1941.
10. Kinzer, J.P. and Wilson, I.G. "End plate and side wall currents in circular cylindrical cavity resonator." Bell System Tech. J. 26: 31-79, Jan. 1947.
11. Korman, N.I. "The theory and design of several types of wave selectors." Natl. Electronic Conf. 2: 404-423, Oct. 3-5, 1946.
12. Mayer, E. "Resonant frequencies of the nosed-in cavity," J. App. Phys. 17: 1046-1055, Dec. 1946.
13. Mota, H. "Calculation of the electromagnetic field, frequency and circuit parameters of high-frequency resonator, cavities." J. Inst. Elec. Engrs. (London), Pt. III, 93: 335-343, Sept. 1946.
14. Ortusi, J., "Filtrage des ondes guidees." Bull. Soc. Franc Elect., 6: 589-596, Nov. 1946.
15. Spangenberg, K.R. "The universal characteristics of triple-resonant circuit band-pass filters." Proc. I.R.E., 34: 629-634, Sept. 1946.
16. Sutro, P.J. "Theory of mode separation in a coaxial oscillator." Proc. I.R.E., 34: 960-962, Dec. 1946.

## Unpublished Documents

17. Bethe, H.A. "Lumped constants for small irises." Rad. Lab. MIT Report 43-22, Mar. 24, 1943.
18. Bethe, H.A. and Schwinger, J. "Perturbation theory for cavities." NDRC Div. 14 Report 117: Cornell U. Mar. 4, 1943.
19. Bethe, J.A., Schwinger, J., Carlson, J.F. and Chu, L. J. "Transmission of irises in wave guides." NDRC Div. 14 Report 111: Cornell U. Nov. 3, 1942.
20. Bethe, H.A., Marshak, R.E., and Schwinger, J. "Theoretical results on the t-r box." NDRC Div. 14 Report 116: Cornell U. Jan. 20, 1943.
21. Colby, N.C. and Zabel, C.W. "Various 3cm t-r box characteristics." Rad. Lab. MIT Report 41-7, Jan. 6, 1943.
22. Cork, B. "Transmission of higher harmonics through a t-r cavity." Rad. Lab MIT Report 72-3, Jan. 11, 1943.
23. Group 43, "Wave guide handbook." Rad. Lab. MIT Report 43, First Revised Edition, Feb. 7, 1944, (Several supplements were issued at later dates).
24. Jaynes, E.T. "Theory of microwave coupling systems." N.R.L. Report CRG-84, Aug. 1945.
25. Lawson, J.L. and McCreary, R.L. "Measurement of the Q value of a t-r box." Rad. Lab. MIT Report 64-6, July 13, 1942.
26. Leiter, H.A. "A microwave band-pass filter in wave-guide." Rad. Lab. MIT Report 814, Nov. 16, 1945.
27. McCreary, R.L. "Direct coupling in the t-r box." Rad. Lab. MIT Report 64-9, November 3, 1942.
28. Pickering, W.H. and Hagelbarger, D.W. "A new method for the precision measurement of wave guide discontinuities." NDRC Div. 14 Report 317 CIT Oct. 1944.
29. Richards, P.I. "A note on uhf coupled circuits." NDRC Div. 15. Harvard U. Report RRL 411-115c. Mar. 20, 1945.
30. Richards, P.I. and Cohn, S.B. "Resumé of uhf tuned circuit preselectors." NDRC Div. 15 Harvard U. Report RRL 411-259, Jan. 11, 1946.
31. Richards, P.I. and Cohn, S.B. "Resumé of uhf filter development." NDRC Div. 15, Harvard U. Report RRL 411-234. Sept. 10, 1945.
32. Schwinger, J.S. "Theory of obstacles in resonant cavities and wave guides." Rad. Lab. MIT Report 43-34, May 21, 1943.
33. Seeley, S. "Cavity resonators." Rad. Lab. MIT Report 64. June 7, 1944.

34. Seeley, S. and Pollard, E.C. "Notes on microwaves." Rad. Lab. MIT, unnumbered, undated. (These notes are based on a series of lectures by W. W. Hansen, and treat at considerable length the theory underlying the development, transmission, and propagation of microwaves. They are contained in 5 volumes and total in excess of 1000 typewritten pages).
35. Wallman, H. "Realizability of filters." Rad. Lab. MIT Report 637. (Dec. 8, 1944).

\*\*\*