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FREE-FLIGHT AIR-DRAG MEASUREMENT TECHNIQUES

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with an Appendix by
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ABSTRACT

The air-drag coefficient, C_D, has been determined for cal .50 spheres in the region of Mach numbers 1.4 to 5.4, using the ballistic pendulum air-drag technique with pendulums of several different lengths. A conventionally suspended pendulum with a period of 3.2 seconds was used to obtain most of the data. The values of C_D so obtained were consistently about 6 percent low. It was suspected that the time of momentum transfer covered too great a fraction of the pendulum period. For this reason, pendulums having seismographic suspension with periods from 17.5 to 30 seconds were installed. After correction for end effects, the values of C_D then obtained agreed closely with those arrived at by other methods. The influence of baffle-hole size and correction for end effects are discussed.

PROBLEM STATUS

This is an interim report; work is continuing.

AUTHORIZATION

NRL Problem F06-01R
NR 436-010

FREE-FLIGHT AIR-DRAG MEASUREMENT TECHNIQUES

INTRODUCTION

Current interest in experimental supersonic aerodynamics has arisen both from the need for guidance in developing appropriate theoretical methods for this field and from practical necessities in design problems of missiles and aircraft. Major emphasis has been placed upon wind tunnels, which are quite expensive in terms both of funds and of manpower. It is desirable, therefore, to find new approaches by which the experimental cost might be reduced.

This report, dealing with a relatively simple, inexpensive technique based on the ballistic air momentum trap, is related to NRL Report O-2851, dated 1 October 1946. Experimental work initiated in August 1945 dealt mainly with cal .30 spheres. In August 1947, pendulums were installed for work with cal .50 projectiles. All the cal .50 work has been concerned with spheres. Briefly, we desire to call attention to five advantages of the method:

(1) The pendulum method measures momentum rather than velocity loss and, for equal precision, utilizes a much shorter trajectory than any of the known chronograph techniques for determining resistance of gases to penetration by projectiles.

(2) About one-percent precision is obtainable readily enough so that the experiment can remain simple and convenient.

(3) The trans-sonic and sub-sonic ranges are not too low for use of the pendulum air-drag technique. In contrast to chronograph methods, accuracy will improve with increasing velocity of the test missile.

(4) Missiles may be of any shape which can be fired in proper orientation through a 100-caliber[1] path in air.

(5) The pendulum method is adaptable for study of such features as head shape, the effect of yaw upon drag, scale effect, and the effects of various physical conditions of the resisting medium. Adaptations of the method to study of lift are possible but will require a considerable amount of experimental development.

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[1] That is, 100 projectile diameters

MEASUREMENT TECHNIQUE

When a missile is fired through a hollow, partly closed pendulum, the pendulum acts as a trap for the momentum imparted to the air. The swing of the pendulum increases in a known way with its momentum gain. In a way not fully understood (since details of momentum transfer from air to pendulum are still obscure) it increases with momentum loss of the missile. There are three principal trajectory sections: (1)The path of the missile, which creates pressure differential against the entrance face of the pendulum; (2) the interior path; and (3) the path which may cause transfer of momentum through the exit face opening. To measure the air-drag coefficient successfully, it is desirable to design the experiment so that sections (1) and (3) are independent of pendulum length, so that the wall or choking effects along section (2) are negligible, and so that the air motion inside the pendulum becomes random in a time less than 1/30th of a quarter period of the pendulum. When these requirements are satisfied, firings at the same measured velocity with several pendulum lengths will serve to determine the end correction and will, therefore, permit absolute determination of the air-drag coefficient.

The pendulums were constructed so that the entrance and exit faces were similar. All the firings were made with spheres, choice of this shape eliminating the yaw problem and reducing expense since commercial ball bearings were used.

EQUIPMENT

Outside the Pendulum Room

A cal .50 sphere was given a horizontal velocity by firing through a cal .50 gun barrel 60 inches long. The amount of gun powder used provided the desired velocities. After leaving the gun, the sphere passed through a steel cylindrical silencer 3 feet in diameter and 10 inches thick. Attached to the silencer on the side facing the pendulum was a blast deflector $1\frac{1}{2}$ feet long and having 14 square aluminum plates 2 inches apart, each plate 1 foot by 1 foot with a 1-inch-diameter hole for the passage of the sphere.

Inside the Pendulum Room

Pendulum bodies, whether of the conventional (Figure 1) or seismographic-type (Figure 2) suspension, were identical in construction. Aluminum, 0.032 inch thick, was formed into a cylinder 10 inches in diameter and of appropriate length. Art board, 1/8 inch, was cut into discs to fit within the pendulum, each disc having at the center a hole 4 or 5 calibers in diameter. Starting at one end, the discs were placed in the cylinder spaced by aluminum frameworks which served to separate the discs by 4 inches. These discs act as baffles, as shown in Figure 3.

On the outer surface of the cylinder an aluminum projection 1/4 x 1 x 1 inch was fastened so that a normal off the 1 x 1-inch face was parallel to the direction of travel of the pendulum. This projection, called a "pusher," served to displace a free pencil lead which rested horizontally in two V supports on a stationary block. A traveling microscope was mounted on a solid stationary support in such a manner that the distance could be measured between the pusher at the pendulum rest position and the edge of the pencil lead after displacement. The sphere passed through the pendulum without direct contact, the air disturbed by the projectile and its shock wave causing the pendulum displacement.

After the rest position of the pendulum is determined, it is necessary that the pendulum be held there until the instant before firing. Restraint was effected by means of a

damper actuated by a solenoid and controlled by pushbuttons, either from the pendulum room or from the firing point.

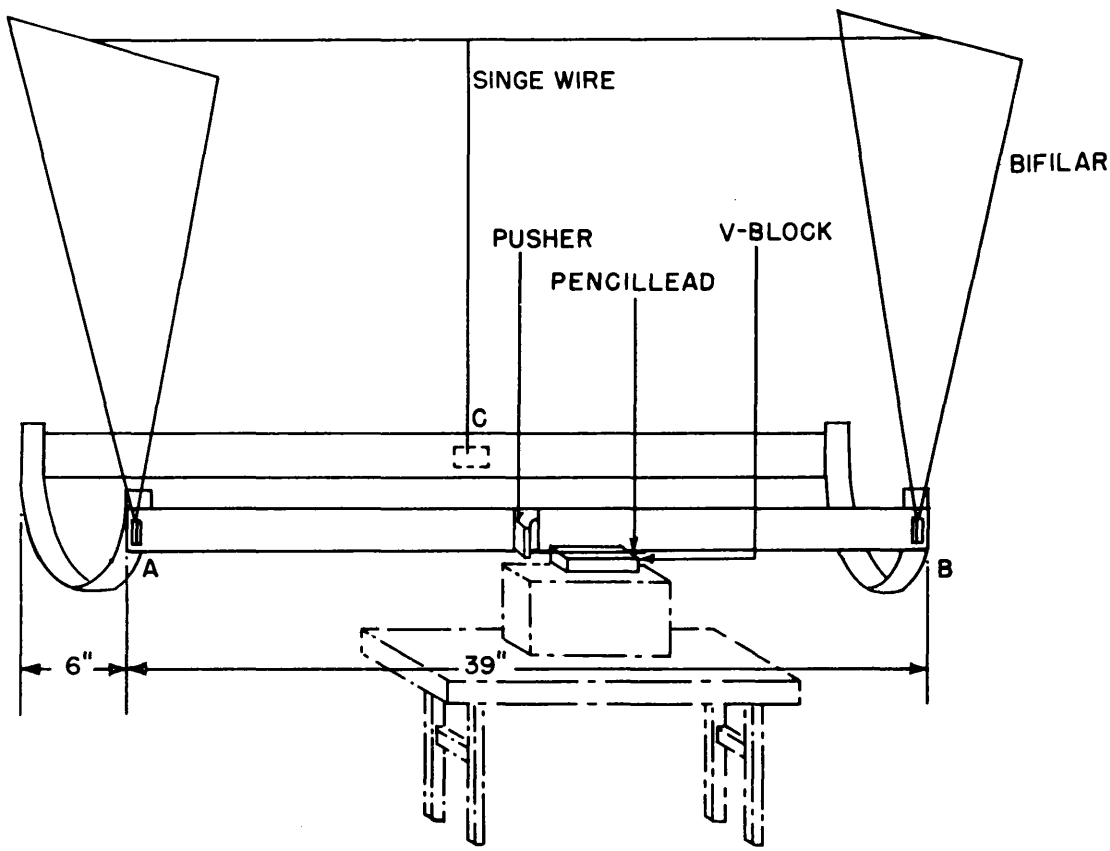

Figure 1 - Conventional cradle pendulum support

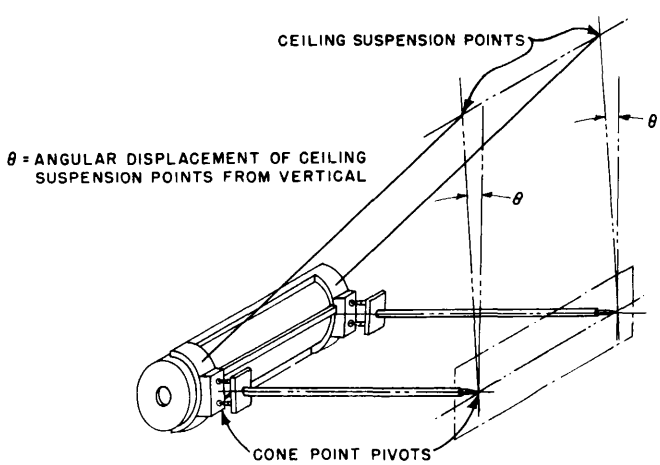

Figure 2 - Seismographic-type pendulum showing the angle θ between the line from ceiling suspension points to cone pivots and the vertical. The period $\Gamma = 2\pi\sqrt{l/g \sin\theta}$, where l is the length of the arms.

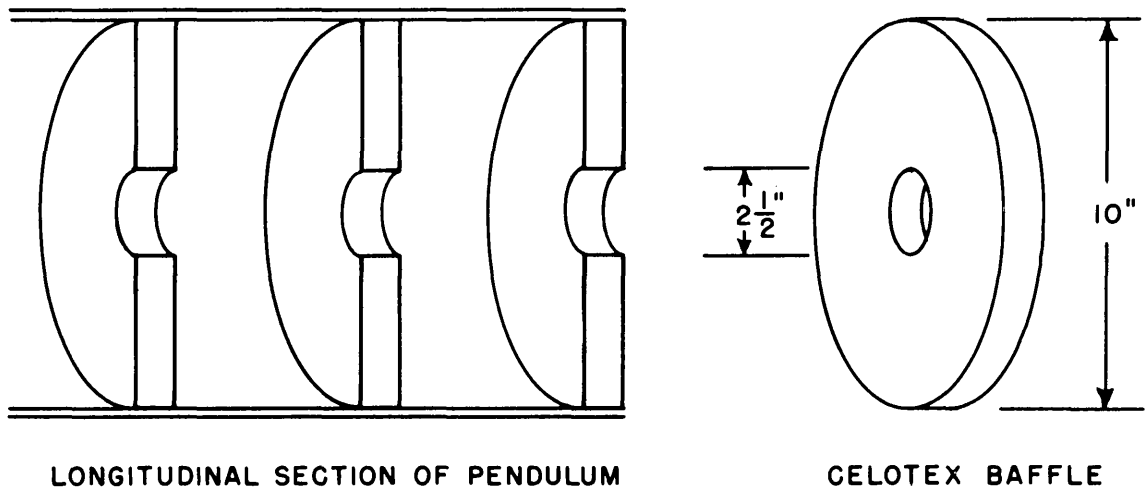

Figure 3 - Arrangement of baffle discs

The velocity of the sphere was determined by measuring the time required for it to traverse two photoelectric screens placed in the trajectory two feet before and two feet behind the pendulum. The screens were approximately 3 feet long, 6 inches wide, and 1 foot high. Each contained a 6- to 8-volt lamp whose light passed through a plane convex lens and two adjustable slits to form a beam of parallel light perpendicular to the trajectory. On the other side of the trajectory line was a convex lens which focused the beam on a 931-A photomultiplier tube. Interruption of the beam caused the photomultiplier tube to trigger a 2050 thyratron which then generated a pulse starting a Potter counter model 78. The second screen, when traversed by the sphere, generated a pulse which stopped the counter. The number of counts furnished an indication of the time of travel between the two screens.

The period of the pendulum was determined with a stopwatch. In the formula $F = -M\omega^2 d$, where $\omega = 2\pi/P$ and P = the period of the pendulum, the force, F, and the displacement, d, were directly measured by holding back the pendulum by a weight over a low-friction pulley. Thus the mass, M, of the pendulum was determined. To support the 176-inch pendulum two cradles were used, and three cradles were needed for the 273-inch pendulum.

The velocity of the projectile at the center of the pendulum was measured by a 1.6-megacycle electronic counter which is operated from pulses fed to it by two photoelectric screens placed about two feet from each end of the pendulum (Figure 4). The exit velocity is then approximately $V_C - \frac{1}{2} V_L$, where V_C is the velocity at the center of the pendulum, computed from the counter observation, and V_L is the velocity lost by the projectile through the length of the pendulum, computed from the observed deflection.

DRAG CALCULATIONS

Let us define the air-drag coefficient, C_D, by the relationship

$$-\frac{dE}{dX} = C_D E \tag{1}$$

whence

$$E = E_0 \exp(-C_D X) \tag{2}$$

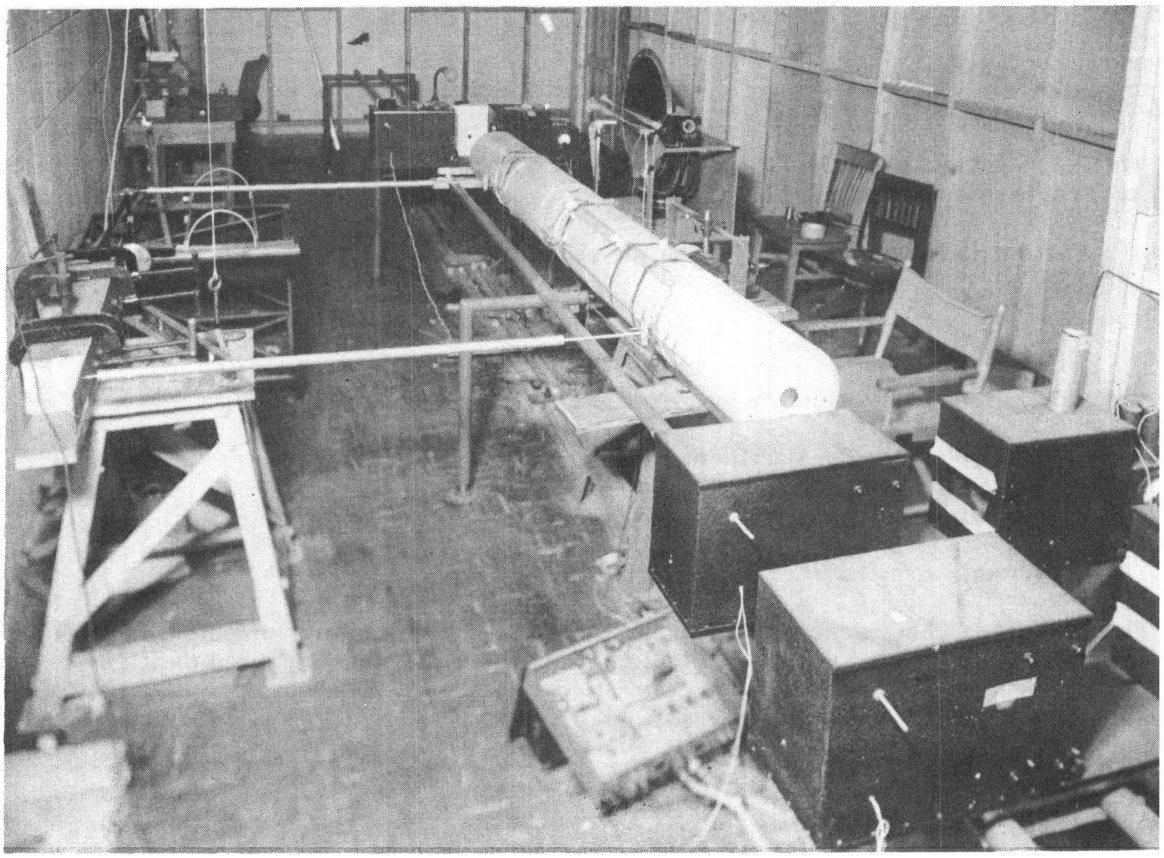

Figure 4 - Arrangement of quiet room. Photoelectric screens at each end of the pendulum

and

$$V = V_0 \exp\frac{(-C_D X)}{2} \tag{3}$$

where $E = \frac{1}{2} m V^2$, kinetic energy of the projectile (no spin),

V = projectile velocity,

m = mass of projectile,

$X = \frac{\rho A S}{m}$,

ρ = air density,

A = area of projectile normal to trajectory, and

S = displacement of projectile in direction of trajectory.

If S = L,

$$X' = \frac{\rho A L}{m} \tag{4}$$

and if S = L + ΔL,

$$X'' = \frac{\rho A (L+\Delta L)}{m} \tag{5}$$

where L = physical length of the pendulum and ΔL = apparent additional length of pendulum, representing the end effect.

For small increments in X and S, the relation may be written

$$1 + \frac{V_L}{V_E} = \exp\frac{(C_D X'')}{2} = \exp\frac{(\alpha X')}{2} \tag{6}$$

where V_L = projectile velocity loss to pendulum over path L + ΔL,

V_E = velocity of projectile at pendulum exit,

α = air-drag coefficient uncorrected for end effect, and

C_D = air-drag coefficient corrected for end effect

Equation (6) is convenient to use since V_L and V_E may be readily computed, X′ is known, and $\alpha X'/2$ can be obtained from a table of the exponential function when the value of

$$1 + \frac{V_L}{V_E}$$

is known. The relation

$$\frac{C_D}{\alpha} = \frac{L}{L + \Delta L} \tag{7}$$

follows directly from Equations (4), (5), and (6).

METHOD OF DETERMINING C_D

If α is plotted against the reciprocal of the pendulum length, one can extrapolate to C_D at 1/L = 0. Figure 5 is plotted in this manner. It is interesting to note that, if this curve is extrapolated so as to intercept the horizontal axis, this intercept will be the reciprocal of the end correction, ΔL.

The equation

$$C_D = \frac{L_1 \alpha_1 - L_2 \alpha_2}{L_1 - L_2} \tag{8}$$

where L_1 and L_2 are any two pendulum lengths and α_1 and α_2 are the corresponding values of α, may be more convenient for the determination of C_D than the graphical method. The validity of this equation may be shown at once by plotting αL vs. L for several values of L. The slope of the resulting curve is C_D, the horizontal intercept is ΔL, and the vertical intercept is $\alpha \Delta L$.

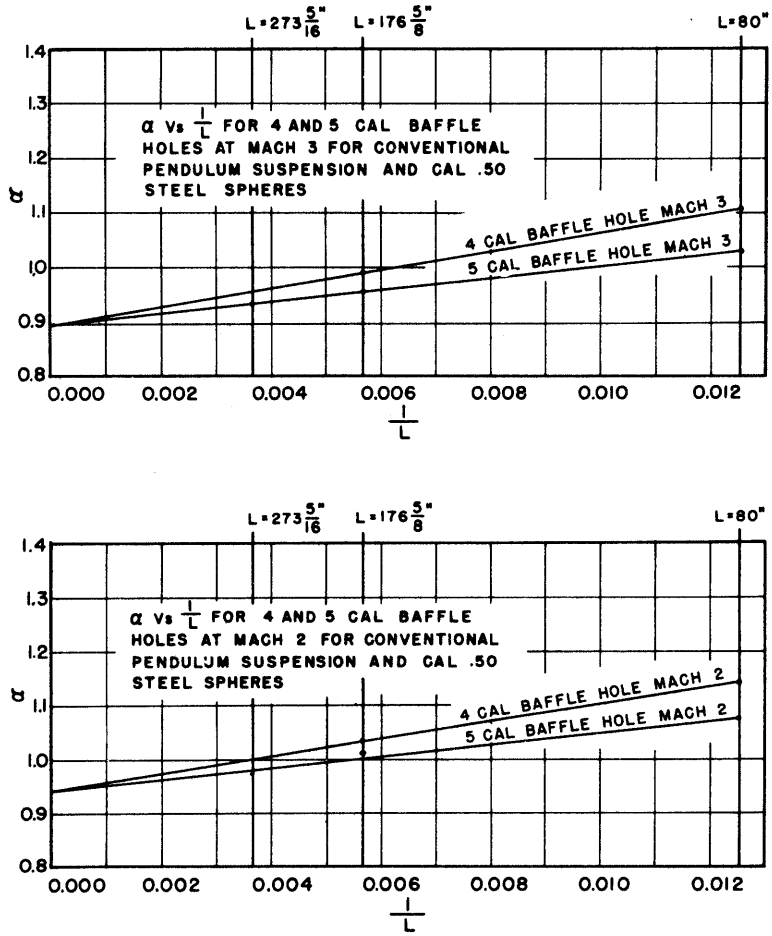

Figure 5

AIR DRAG OF CAL .50 STEEL SPHERES BY A CONVENTIONALLY SUSPENDED BALLISTIC PENDULUM

The values of α were investigated to determine the effect of baffle-hole size and pendulum length. As was expected, the larger the baffle hole the smaller was α for the same Mach number. Curves of α vs. Mach number for the 5-caliber baffle-hole diameter were made for pendulum lengths of 80, 176.6, and 273.3 inches, as shown in Figure 6. Data were taken for a 4-caliber baffle-hole diameter at Mach numbers 2 and 3, using 80- and 176.5-inch pendulums, as shown in Figure 7. Values of C_D, obtained by plotting the values for the 4- and 5-caliber baffle-hole diameters against $1/L$, and extrapolating to $1/L = 0$, were the same (Figure 5).

AIR DRAG OF CAL .50 ALUMINUM SPHERES BY A VELOCITY DIFFERENCE METHOD

In order to have additional data with which to compare results of the ballistic pendulum air-drag technique, C_D was determined by the velocity difference method. Aluminum spheres, cal .50. were used since, because of their low density, their velocity loss over

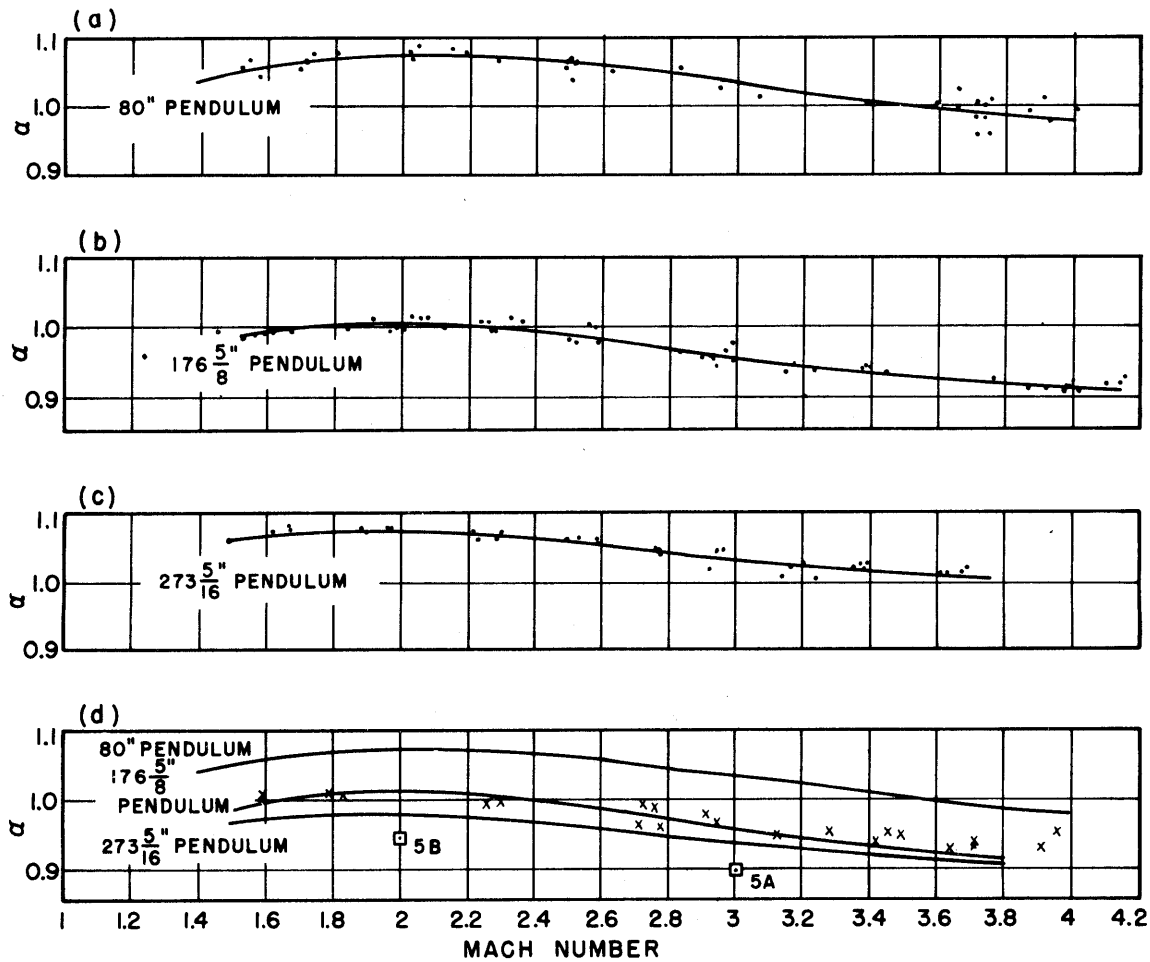

Figure 6 - α vs. Mach number for cal .50 steel spheres, 5-caliber (2-1/2 inch) baffle hole, conventional pendulum suspension

a given distance was almost three times that of the steel spheres. Screens were set up 25 feet apart, giving two nearly equal base lengths.

In the calculation of C_D, no account has been taken of the difference in base lengths between screens. As shown in the Appendix, if the experiment is repeated with the direction of firing reversed, the average C_D calculated for the two directions will be identical with the average C_D calculated with base length differences taken into account. That is, averaged C_D values obtained by this method, ignoring possible errors in the location of the center screen, will not be in error.[2] The values of C_D obtained by this method which are quoted in the report are averages of this kind. The velocity of the projectile

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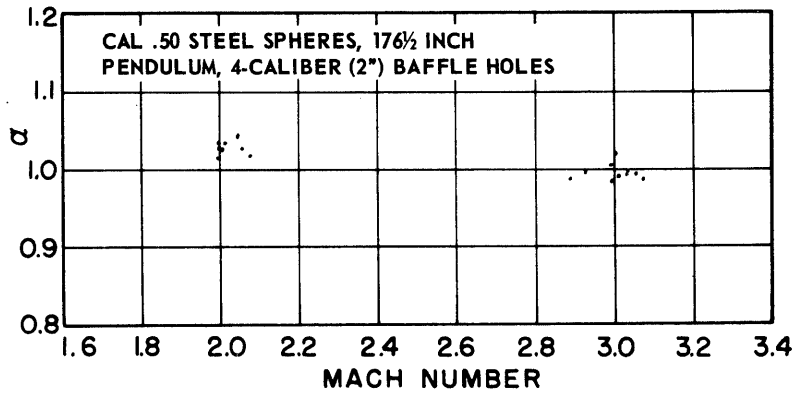
[2] Actually, this statement is correct only insofar as the computation method outlined earlier in the report is used. A more exact treatment shows that this averaged C_D value will be correct within second order terms in $(D_1 - D_2)/(D_1 + D_2)$. See Appendix for further discussion.



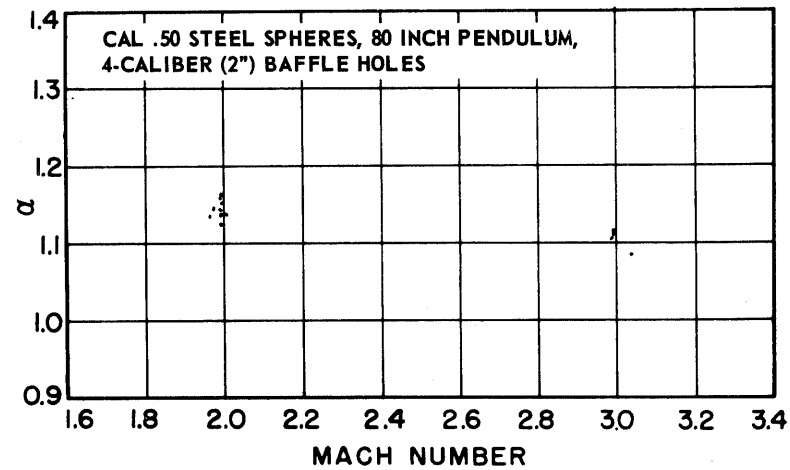


Figure 7 - α values obtained using conventional suspension

was measured over each base length and values of C_D were computed using the equation

$$V_1 / V_2 = \exp\left(\frac{C_D}{2}\right) = T_2 / T_1 \qquad (9)$$

where V_1 and V_2 are the velocities of the projectile over the first base length D_1 and the second base length D_2 respectively, and T_1 and T_2 are the corresponding transit times. Again, time was measured by Potter 1.6-megacycle electronic counters. Photocell light screens were used for triggering. Data obtained in this manner are plotted in Figure 8 together with values obtained by A. C. Charters and R. N. Thomas[3] for 9/16-inch steel spheres. It will be noted that the values of C_D obtained by these two methods agree closely.

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[3] Aberdeen Prov. Gr. Ball. Res. Lab. Rept., 514; Charters, A. C., and Thomas, R. N., J. Aero. Sci., 12, p. 468, Oct. 1945

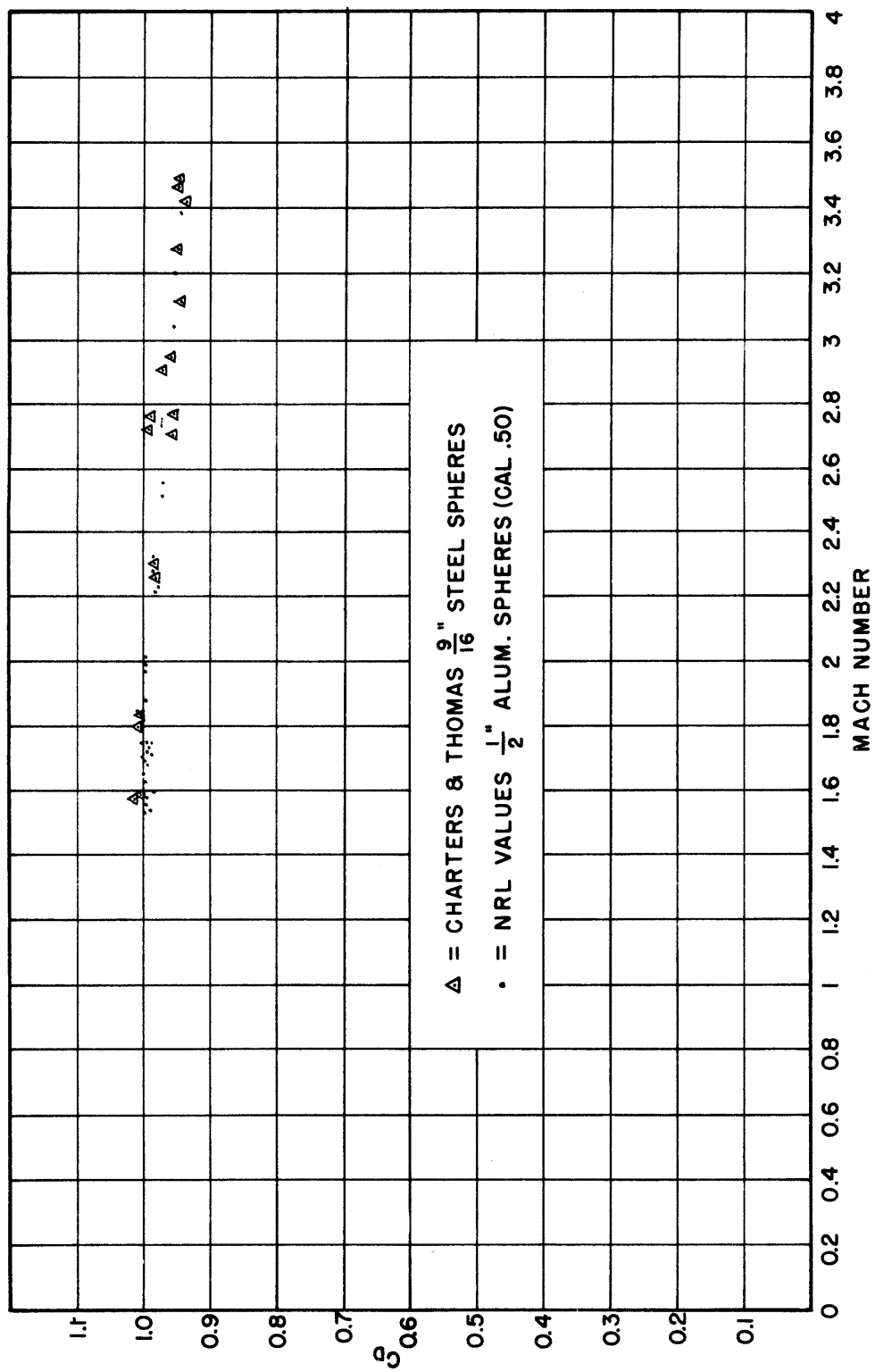

Figure 8 - C_D from NRL free flight velocity difference method compared with Charters and Thomas values

DISCREPANCY IN C_D VALUES OBTAINED BY DIFFERENT METHODS

Values of C_D obtained by extrapolation after plotting α VS 1/L are given in Figure 6d, together with values obtained by Charters and Thomas as well as α curves obtained by the ballistic pendulum method at NRL. It will be noted that the values of C_D corrected in this way are about 6% lower than those reported by Charters and Thomas.

PROBABLE CAUSE OF LOWER C_D VALUES

Since the period of the conventionally suspended pendulum was 3.2 seconds, it seemed probable that the time of momentum transfer was long enough to affect the deflection reading. An analog of the pendulum was set up using a ballistic galvanometer, a charged condenser, and a resistor. The RC product of the condenser and the resistor, plus the resistance of the galvanometer, was used as a measure of the time required for the charge to be transferred through the coil of the galvanometer. The capacity of the condenser and the voltage applied were adjusted to obtain a nearly full-scale reading with no external resistance. The period of the galvanometer was 8.6 seconds. Figure 9 shows the percent of maximum deflection plotted against the time expressed as the percent of the total period of the galvanometer.

From Figure 9, 94% of the maximum deflection corresponds to having an RC product which is 4% of the period of the galvanometer. Analogously, with the period of the conventionally suspended pendulum equal to 3.2 seconds, the momentum reading would be 94% of maximum if the time of momentum transfer were 4% of 3.2 seconds, or 0.128 second. If a 99.5% momentum reading is desired, the time of momentum transfer, judging from Figure 9, should be about 0.7% of the period of the pendulum. This indicates that a pendulum having a period of about 17 seconds should give values of C_D agreeing closely with established ones.

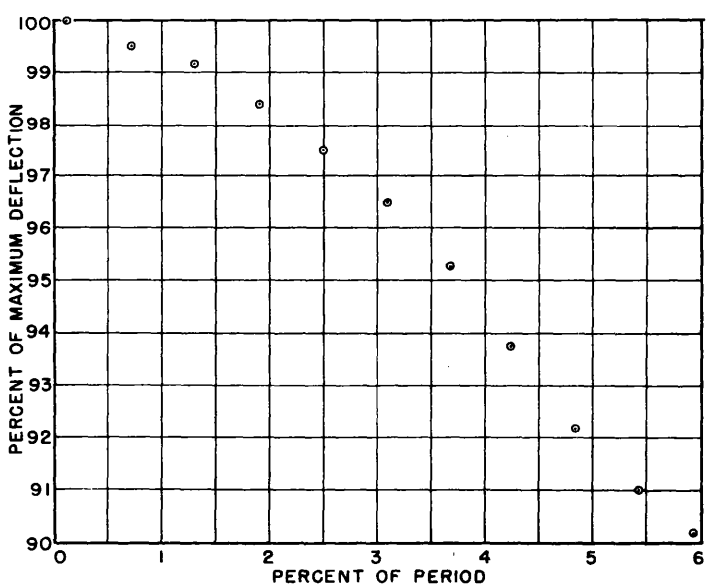

Figure 9 - Percent of maximum deflection vs. percent of period of galvanometer for various RC products

There is considerable data to support the validity of this analog. The first seismographic-type pendulum used had a period of 7.5 seconds, as compared to the 3.2-second period of the conventionally suspended pendulum. The results obtained were disappointing in that the change in values of C_D was only about 1%. However, if it is considered that a specific C_D value may be in error by $\pm\frac{1}{2}$% it can be concluded that, while it is not possible to determine accurately the change in C_D obtained by doubling the pendulum period, the change is in the right direction and is small. It seems to have reduced the previous 6% error to about 5%, or possibly 4%. Since the additional momentum received by the pendulum by doubling the period is shown to be small, it can be inferred that much of the momentum is transferred to the pendulum quickly and that, as shown above from consideration of Figure 9, the period would have to be increased several fold in order to obtain a momentum reading of 99.5%.

Thus, the rate of momentum transfer must be high at first, but after about one second it becomes practically zero. This behavior, though not necessarily exponential, is qualitatively similar to the behavior of the transfer of charge through the coil of the galvanometer. Close correspondence to exponential decay behavior is not essential for the usefulness of the electrical analog. If, as has been demonstrated, one can make the pendulum period long enough so that its ballistic swing represents the momentum transfer accurately, then the time of momentum transfer must obviously be small compared to the quarter-period of such a pendulum. For reasonably accurate representation of the momentum by the pendulum, a short time for momentum transfer is required.

In the case of the air-drag pendulum, a large amount of the momentum is transferred quickly. For this reason, it seems quite safe to assume that, for momentum readings on the order of 95% or higher, the exact shape of the momentum time curve does not affect the deflection of the pendulum very much. Thus the electrical analog agrees with the momentum transfer closely enough to be quite useful in predicting the approximate period needed to obtain data of small systematic error when using the ballistic air-drag pendulum.

USE OF SEISMOGRAPHIC-TYPE SUSPENSIONS TO OBTAIN LONGER-PERIOD PENDULUMS

Seismographic-type suspensions [4] (Figures 2 and 4) were tried using pendulums with periods of 17 to 30 seconds. The values of C_D thus obtained agree closely with those obtained by Charters and Thomas and with values obtained at NRL by taking velocity differences. The values of α for three lengths of seismographic-type pendulums vs. Mach numbers are plotted on Figures 10 and 11. Values of α for this type pendulum are presented in Table 1. For comparison Figure 12 shows these C_D values as well as those obtained by Charters and Thomas. Good agreement will be noted, except at Mach = 1.6 where the pendulum values are about 2.5% low.

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[4] Use of seismographic-type suspensions for this work was suggested to the writers and given initial trials at NOL by Dr. George Shue.

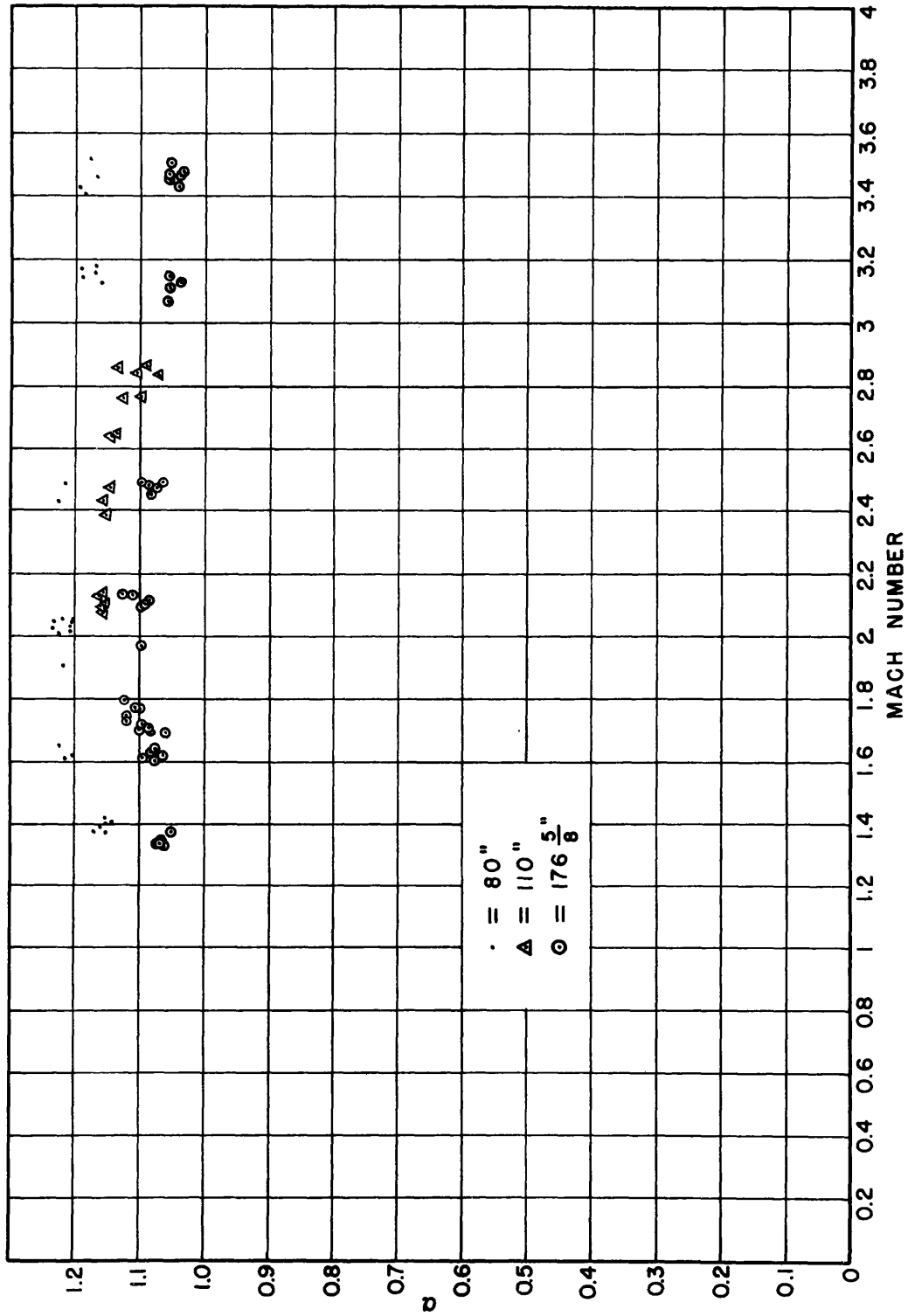

Figure 10 - α vs. Mach number for 3 seismic pendulums

TABLE 1

Data Obtained from Seismographic Pendulums Using Cal .50 (0.50-inch) Steel Spheres

Pendlum Baffle Holes: 2 inches (4-caliber). Values of C_D are computed from the equation:

$$C_D = \frac{L_1\alpha_1 - L_2\alpha_2}{L_1 - L_2}$$

where L_1 and L_2 are any two pendulum lengths and α_1 and α_2 are the corresponding values of α. M = Mach number.

80″ Pend		110.6″ Pend		177.6″ Pend		80″ Pend		110.6″ Pend		177.6″ Pend	
α	M	α	M	α	M	α	M	α	M	α	M
1.147	1.401	No data taken		1.059	1.339	1.162	4.215	No data taken		1.050	4.209
1.142	1.410			1.077	1.339	1.122	4.251			1.046	4.212
1.156	1.381			1.062	1.341	1.168	4.230			1.037	4.191
1.170	1.381			1.049	1.373	1.177	4.172			1.030	4.189
1.159	1.419					1.151	4.213			1.031	4.142
1.169	1.378										
Average α = 1.157				Average α = 1.062		Average α = 1.156		C_D = 0.943 at M = 4.20		Average α = 1.039	
Average M = 1.394				Average M = 1.348		Average M = 4.216				Average M = 4.189	
		C_D = 0.985 at M = 1.37									
1.196	1.626	No data taken		1.079	1.630	1.088	4.522	1.043	4.566	No data taken	
1.205	1.629			1.094	1.611	1.075	4.545	1.085	4.472		
1.197	1.617			1.065	1.619	1.067	4.443	1.052	4.469		
1.210	1.625			1.074	1.599	1.095	4.551	1.048	4.448		
1.212	1.608			1.075	1.623	1.097	4.416	1.010	4.575		
						1.112	4.585	1.059	4.585		
						1.095	4.589	1.069	4.488		
						1.085	4.602	1.049	4.523		
						1.095	4.545	1.046	4.527		
								1.041	4.495		
Average α = 1.204				Average α = 1.077		Average α = 1.090		Average α = 1.050			
Average M = 1.621				Average M = 1.616		Average M = 4.533		Average M = 4.515			
		C_D = 0.973 at M = 1.62				C_D = 0.945 at M = 4.52					
1.217	2.058	1.155	2.074	1.124	2.136	1.083	4.986	1.046	4.852	No data taken	
1.234	2.028	1.157	2.105	1.091	2.113	1.087	5.036	1.066	4.941		
1.234	2.046	1.155	2.135	1.096	2.113	1.096	5.010	1.061	5.077		
1.223	2.010	1.157	2.086	1.089	2.112	1.085	4.973	1.035	5.094		
1.208	2.022	1.178	2.121	1.089	2.109	1.096	5.018	1.033	5.171		
				1.112	2.130						
Average α = 1.223		Average α = 1.160		Average α = 1.100		Average α = 1.089		Average α = 1.048			
Average M = 2.033		Average M = 2.104		Average M = 2.119		Average M = 5.005		Average M = 5.027			
		C_D = 0.998 at M = 2.09				C_D = 0.941 at M = 5.02					
1.187	2.499	1.143	2.638	1.079	2.479	1.082	5.376	1.020	5.456	No data taken	
1.238	2.498	1.137	2.644	1.100	2.500	1.055	5.469	1.044	5.449		
1.224	2.514	1.150	2.384	1.091	2.492	1.103	5.331	1.029	5.405		
1.209	2.489	1.154	2.424	1.086	2.487	1.074	5.292				
1.206	2.479	1.144	2.479	1.071	2.489	1.081	5.337				
				1.083	2.460	1.086	5.352				
Average α = 1.213		Average α = 1.146		Average α = 1.085		Average α = 1.080		Average α = 1.031			
Average M = 2.496		Average M = 2.514		Average M = 2.485		Average M = 5.360		Average M = 5.437			
		C_D = 0.980 at M = 2.50				C_D = 0.903 at M = 5.40					
1.169	3.182	1.096	2.959	1.051	3.098	1.063	5.690	1.037	5.671	No data taken	
1.172	3.157	1.121	2.957	1.054	3.112	1.097	5.631	1.001	5.601		
1.189	3.151	1.107	3.039	1.057	3.068	1.075	5.709	0.999	5.612		
1.160	3.127	1.135	3.058	1.054	3.153	1.079	5.653	1.050	5.671		
1.193	3.173	1.074	3.043	1.036	3.127	1.054	5.641				
		1.087	3.074								
Average α = 1.177		Average α = 1.103		Average α = 1.050		Average α = 1.074		Average α = 1.022			
Average M = 3.158		Average M = 3.022		Average M = 3.112		Average M = 5.665		Average M = 5.639			
		C_D = 0.939 at M = 3.09				C_D = 0.886 at M = 5.65					
1.131	3.499	No data taken		1.044	3.503						
1.176	3.525			1.049	3.616						
1.166	3.465			1.054	3.671						
1.194	3.438			1.057	3.688						
1.185	3.405			1.060	3.554						
				1.051	3.500						
				1.041	3.462						
				1.056	3.459						
				1.036	3.475						
				1.041	3.430						
				1.049	3.461						
Average α = 1.170		C_D = 0.950 at M = 3.50		Average α = 1.049							
Average M = 3.466				Average M = 3.529							

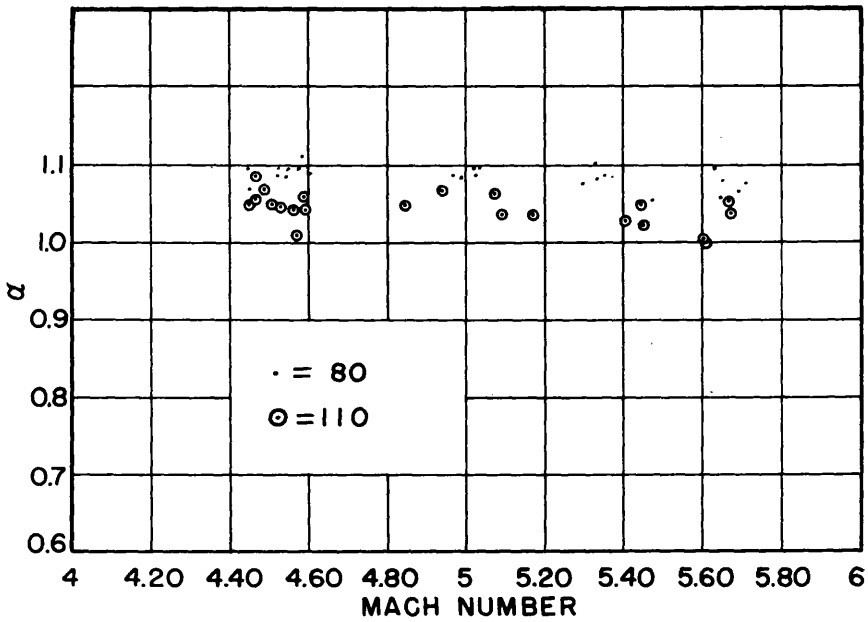

Figure 11 - α vs. Mach number for 2 seismic pendulums above Mach 4

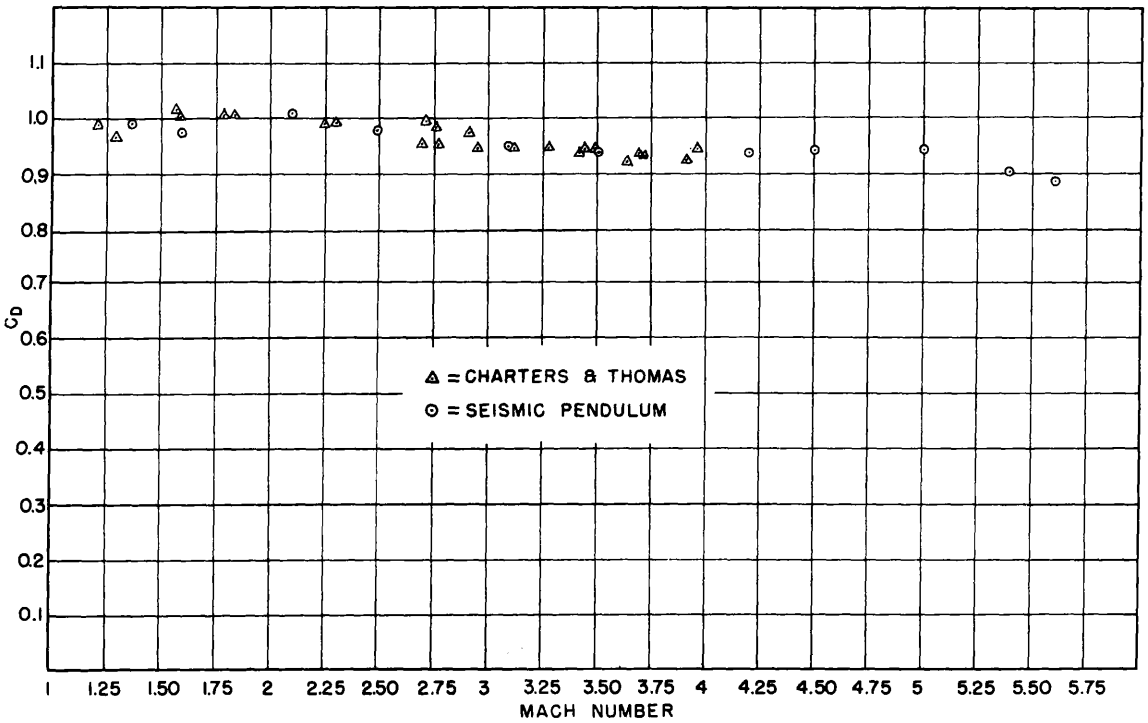

Figure 12 - C_D from seismic pendulum data compared with Charters and Thomas values

CONCLUSION

The ballistic pendulum method of determining the air-drag coefficient is particularly applicable to extremely high Mach numbers, provided the arrangements include suitably reduced baffle-hole sizes and a sufficient reduction of muzzle blast. Since the base length over which the projectile velocity is determined is large, and since the deflection of the pendulum increases with projectile velocity, no decrease of precision in the determination of projectile velocity or momentum loss is noted at high Mach numbers. Since the total length of trajectory need be only 30 feet, somewhat higher Mach numbers can be maintained than are possible with the customary spark chronograph ranges.

This method of determining the air-drag coefficient can be used with missiles of any shape than can be projected through the pendulum, provided the yaw and drift, if present, can be controlled so as to prevent contact between the missile and the pendulum. The convenience of this method makes it adaptable for use in universities, either for the determination of the air-drag coefficient or for instructing students in aerodynamics.

***

SYMBOLS

α = air-drag coefficient uncorrected for end effect

ρ = air density

ω = $2\pi/P$

A = area of projectile normal to trajectory

C_D = air-drag coefficient corrected for end effect

D_1 = base length between 1st and 2nd screen

D_2 = base length between 2nd and 3rd screen

E = $\frac{1}{2}m\ V^2$ kinetic energy of projectile (no spin)

ΔL = apparent additional length of pendulum, representing end effect

L = physical length of the pendulum

m = mass of projectile

P = period of the pendulum

S = displacement of projectile in direction of trajectory

T_1 = transit time over D_1

T_2 = transit time over D_2

V = projectile velocity

V_E = velocity of projectile at pendulum exit

V_L = projectile velocity loss to pendulum over path $L + \Delta L$

V_1 = velocity over 1st base length

V_2 = velocity over 2nd base length

X = $\rho AS/m$

X′ = $\rho AL/m$

X″ = $\rho A(L + \Delta L)/m$

APPENDIX

Robert E. Roberson

Referring to Figure 13 for the experimental arrangement, let D_1 and D_2 denote distances between screens, defining $D = 1/2(D_1 + D_2)$, and let Δt_1 and Δt_2 be the transit times between screens of a projectile fired in the direction indicated by the arrow. It is desired to find the error made by calculating C_D as if $D_1 = D_2$, and to show how it is reduced by reversing the direction of firing and using the average value of C_D calculated for the two directions.

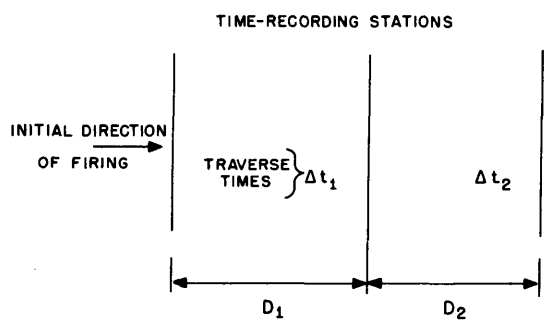

Figure 13 - The experimental arrangement

Let us represent by C_D the value of the drag coefficient calculated recognizing the difference between D_1 and D_2, and by C_D^* the approximation to this value calculated upon the assumption that $D_1 = D_2$. Postulate that (1) the value C_D is calculated from [5]

$$C_D = \frac{2m}{\rho AD} \ln \left(\frac{D_1 \Delta t_2}{D_2 \Delta t_1}\right) \tag{10}$$

and (2) that except for the errors in the approximation $C_D^* \simeq C_D$, the errors of the experiment are the same for the two directions of firing. The first of these postulates makes it unnecessary to examine in this appendix the rationale of the calculation procedure. The second allows the errors in the location of the center screen to be considered aside from the various other experimental errors. As additional notation, let subscripts f and r represent forward and reversed directions of firing and a bar above any quantity represent its average value for the f and r experiments.

From the definition of C_D and Equation (10) there result immediately

$$(C_D)_f = \frac{2m}{\rho AD} \ln \left(\frac{D_1 \Delta t_2}{D_2 \Delta t_1}\right), \tag{11}$$

$$(C_D^*)_f = \frac{2m}{\rho AD} \ln \left(\frac{\Delta t_2}{\Delta t_1}\right), \tag{12}$$

and

$$(C_D)_f - (C_D^*)_f = \frac{2m}{\rho AD} \ln \frac{D_1}{D_2}. \tag{13}$$

---

[5] The listing of the more basic assumptions that lead to this formula is beyond the scope of this appendix.

Clearly, in the reversed firing, the roles of D_1 and D_2 are effectively interchanged. Therefore, by analogy with Equation (13),

$$(C_D)_r - (C_D^*)_r = \frac{2m}{\rho AD} \ln \frac{D_2}{D_1} . \tag{14}$$

Taking one half the sum of Equations (13) and (14), one has

$$\overline{C_D} = \overline{C_D^*} . \tag{15}$$

That is, if the C_D value of the experiment is the average of the C_D values obtained by firing through the screens in two directions, it contains no error by virtue of being calculated on the assumption that $D_1 = D_2$.

A somewhat more sophisticated argument, constructed upon a less restrictive base than postulate 1, shows that actually the error made in $\overline{C_D}$ by considering $D_1 = D_2$ is of second order in $(D_1 - D_2) / (D_1 + D_2)$.

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