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THE MICROTHERMAL STRUCTURE OF THE OCEAN NEAR KEY WEST, FLORIDA

PART II - ANALYSIS

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ABSTRACT

The microthermal variations described in Part I (NRL Report No. S-3392) are analyzed, and the average variations, both thermal and spatial, are plotted against depth. Autocorrelation curves are presented for typical recordings.

The extent to which the observed variations are capable of bending sound waves or producing reverberation is estimated.

PROBLEM STATUS

This is an interim report, the second of two on one phase of this problem; work on other phases of the problem is continuing.

AUTHORIZATION

NRL Problem S02-04R

THE MICROTHERMAL STRUCTURE OF THE OCEAN NEAR KEY WEST, FLORIDA PART II - ANALYSIS

INTRODUCTION

Part I of this report has given a qualitative description of the types of microthermal variations at constant depth obtained with a sensitive thermopile mounted on a submarine underway at constant speed in the waters adjacent to Key West, Florida.* Part II is an analysis of some of the typical records obtained and an estimate of some of the effects of the thermal microstructure on the propagation of sound in the ocean.

It was found in Part I that the "isothermal" water off Key West in summer is characterized by thermal variations at constant depth that are small, both spatially and thermally. The thermocline on the other hand is characterized by large fluctuations at constant depth that sometimes have a distinct periodicity.

THERMAL VARIATIONS

Figure 1 is a plot for the microthermal cross sections (Part I) of "root-mean-square maximum (RMSM) temperature variation" against depth. For each record used for the cross section, a figure which may be called the RMSM temperature variation was obtained in the following way. The maximum and minimum of the irregular record were read off, subtracted from the mean, and squared. When converted to temperature and averaged, the mean square maximum variation is obtained, the square root of which is the RMSM variation. Only maxima and minima greater than 20 percent of the total variation of the record were used. Thus the RMSM value is an artificial, though convenient, estimate of the magnitude of the microthermal variation of a record; it is in fact an average of those peaks and troughs that are greater than 20 percent of the total deviation.

It is seen from Figure 1 that the above qualitative difference between the isothermal and gradient layers shows up strongly in the RMSM plots. The connecting lines in Figure 1 serve to join points computed from records obtained in the same area on the same day. Since it is likely that a real microthermal discontinuity exists at the top of the thermocline, no line is drawn so as to cross this boundary, except for the cases of Area D (Gulf Stream) and Area E (Cuba Coast water), where the bathythermogram showed no sharp boundary between the mixed and gradient layers, and where, similarly, none seems to exist in the RMSM plot. The geometric mean of the plotted points in the isothermal layer is 0.013°C and in the gradient layer 0.45°C. In the gradient layer the greatest temperature

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* Urick, R. J. and Searfoss, C. W., "Microthermal Structure of the Ocean Near Key West, Florida, Part I - Description," NRL Report S-3392, December 1948 (Unclassified)

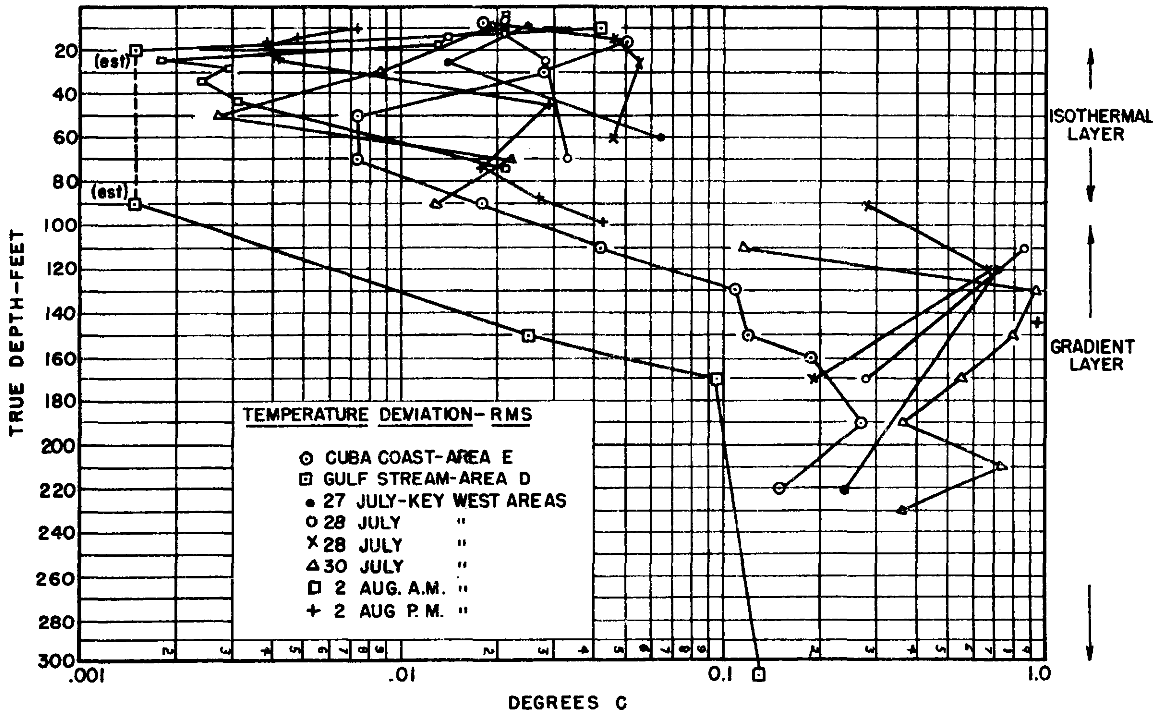

Fig. 1 - Root-mean-square maximum temperature deviation vs depth

variation is found just below the top of the layer, with decreasing variation below. The isothermal water is characterized by an increasing variation with depth, except for a near-surface layer in which greater microthermal variation is found.

SPATIAL VARIATIONS

A somewhat similar plot is shown as Figure 2a, where "patch size" in yards is plotted against depth. The "patch size" for a particular record is determined by counting the number of peaks and troughs greater in amplitude than 20 percent of the largest swing of the record, and dividing that number into the over-all length of the record in yards. The result is the average spatial size of those temperature fluctuations that are greater than the arbitrary value of 20 percent of the over-all deviation of the record. Thus the "patch size" defined in this way is the average half period of the larger microthermal variations at constant depth.

It is seen from Figure 2a that in the isothermal water there is an increase of patch size with depth. A larger patch size is observed in the gradient layer, where there is a slight tendency for the size to decrease with depth. Thus except for the near-surface layer, the similarity between Figures 1 and 2a indicates that water bodies of large patch size are of large thermal size as well. This is of course necessary for thermal stability.

Figure 2b shows typical bathythermograms for Key West areas A, B, and C. The exact location of these areas is shown in Figure 7 of Part I of this report.

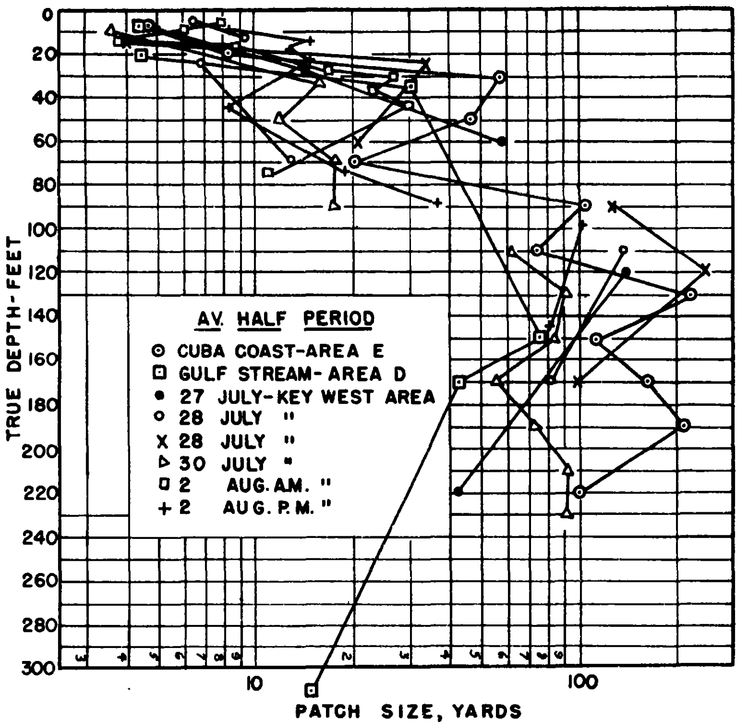

Fig. 2a - Patch size (average half period) vs depth

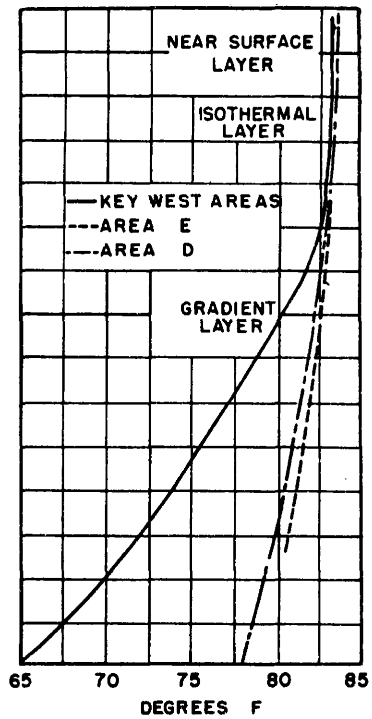

Fig. 2b - Typical bathythermograms

AUTOCORRELATION ANALYSIS

Mathematical Review

A convenient method of studying a time-variable process is by means of an autocorrelation analysis.† The autocorrelation function of a function y(t) is defined as the average of $[y(t) \cdot y(t+s)]$ where s is a time interval. It is a function therefore of s and of the function y, and will be denoted by R(s). By this definition,

$$R(s) \equiv \overline{y(t) \cdot y(t+s)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t) \cdot y(t+s)\, dt.$$

It is normally more convenient to work with the normalized autocorrelation function

$$\rho(s) = \frac{\overline{\left[y(t) - \bar{y}\right]\left[y(t+s) - \bar{y}\right]}}{(\overline{y^2}) - (\bar{y})^2},$$

where $\bar{y}$ is the average value of y. The normalization is such that $\rho(o) = 1$.

† James, H. M., Nichols, N. B. and Phillips, R. S., "Theory of Servomechanisms," M.I.T. Rad. Lab. Series vol. 25, Chap. 6, McGraw-Hill (1947)

The autocorrelation function is useful for determining the spectrum of a time-variant function, and therefore to reveal hidden periodicities in that function. If we denote by $A_T(f)$ the Fourier transform of y(t) in the interval -T to +T, that is,

$$A_T(f) = \int_{-T}^{T} y(t)e^{2\pi ift}\,dt,$$

then the power spectral density G(f) is defined by

$$G(f) = \lim_{T \to \infty} \frac{1}{2T}\left[A_T(f)\right]^2 .$$

If we imagine for a moment y(t) to represent the current flowing in a one-ohm resistor, the average power dissipation will be

$$\frac{1}{2T}\int_{-T}^{T} y^2(t)\,dt,$$

and it can be shown that

$$\lim_{T \to \infty} \frac{1}{2T}\int_{-T}^{T} y^2(t)\,dt = \int_{0}^{\infty} G(f)\,df, \; (\bar{y} = 0).$$

Thus G(f) is the power contained in a one-cycle band about the central frequency f. Since G(f) is a power spectrum, periodicities in y(t) will appear as peaks in G(f). Since the integral of G(f) over all f gives the average power in y(t), it is convenient to normalize it so as to give the normalized power spectrum,

$$S(f) = \frac{G(f)}{\int_0^{\infty} G(f)df},$$

which has the property that

$$\int_0^{\infty} S(f)df = 1 .$$

Both the autocorrelation function R(s) and the power spectrum G(f) depend on the product of y(t) by itself, and on the periodicities in y(t). It can be proved that R(s) and G(f) are Fourier transforms of each other, that is

$$R(s) = \int_0^{\infty} G(f)\cos 2\pi fs\,df; \quad G(f) = 4\int_0^{\infty} R(s)\cos 2\pi fs\,ds$$

with similar relations for their normalized equivalents $\rho(s)$ and S(f).

R(s) is much simpler in form than the function y(t) from which it was derived. This allows the computation of G(f) to be made with much less labor through the use of the second of the above equations, than by an ordinary Fourier analysis of the raw data y(t) itself. For example, most types of noise have the autocorrelation function $R(s) = e^{-\beta s}$ (s positive) and therefore the power spectrum,

$$G(f) = \frac{4\beta}{\beta^2 + (2\pi f)^2} .$$

R(s) can be obtained from the raw data by routine repeated multiplications for different values of s; should R(s) turn out to be of exponential (or other algebraic) form to which an algebraic expression may be fitted with permissible accuracy, the power spectrum can be computed by evaluation of its integral

$$4 \int_0^\infty R(s) \cos 2\pi fs \, ds .$$

Application to Microthermal Records

Autocorrelation functions have been found for several microthermal records in the isothermal layer and in the thermocline. Figure 3 shows normalized autocorrelation curves for three representative records of microthermal variation at near-surface depths. The abscissa is in yards rather than time, since the time scale of the original records can be converted into distance once the speed of the submarine is known. These curves (and the ones for greater depths shown later in Figure 5) were computed by the Mark 22 relay computer at NRL, and also by manual computation using a Marchant calculating machine.

In Figure 3 the autocorrelation function for record 206 is approximated by a curve of the form $e^{-as}\cos bs$, with a = .20 and b = .27. By evaluating the integral

$$4 \int_0^\infty e^{-as} \cos bs \cos 2\pi fs \, ds = \frac{2a}{a^2 + (b + 2\pi f)^2} + \frac{2a}{a^2 + (b - 2\pi f)^2} ,$$

we obtain the normalized power spectrum. The value of this function at abscissa f is the fraction of the total squared deviation (= 1 under normalization) from the mean of the original record which lies in a band one unit of f wide centered at f. Since in the present application s has the units of yards, f must have the units $(\text{yards})^{-1}$. It is perhaps more meaningful to replace f by $1/f \equiv T$ so that T has the units of yards, and then

$$\int_0^\infty G(f)df = \int_\infty^0 \frac{G(T)}{T^2} dt = 1.$$

Now the function $G(T)/T^2$ is the fraction of the total "power" which lies in a wavelength band one yard wide centered at wavelength T yards, and thus the function is a period power spectrum of the thermal inhomogeneities. For record 206, when approximated by the function given above, this spectrum is shown in Figure 4. No extension to very short periods is possible because of the finite response time of the thermopile and records with which the original records were obtained. This curve is probably a qualitatively

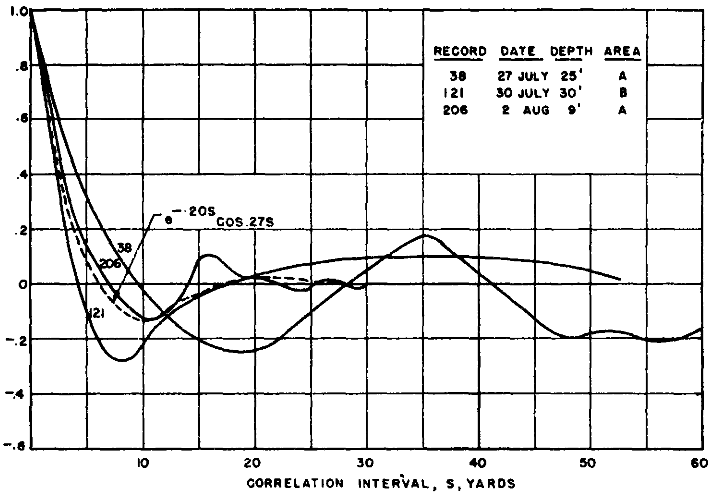

Fig. 3 - Normalized autocorrelation curves for three records at shallow depths

representative power spectrum of the upper ocean layers, in view of the rough similarity of the autocorrelation curves of typical near-surface records. There is evident a slight periodicity at a patch size of 8 yards.

No such similarity of autocorrelation function was found for typical records in the thermocline. A half dozen of these are shown in Figure 5. Although marked periodicity (period 120 yards) is evidenced by the curve for record 34, the remaining curves are not noticeably periodic. Because of the variety of autocorrelation curves, no power spectra have been computed for the thermal inhomogeneities in the thermocline.

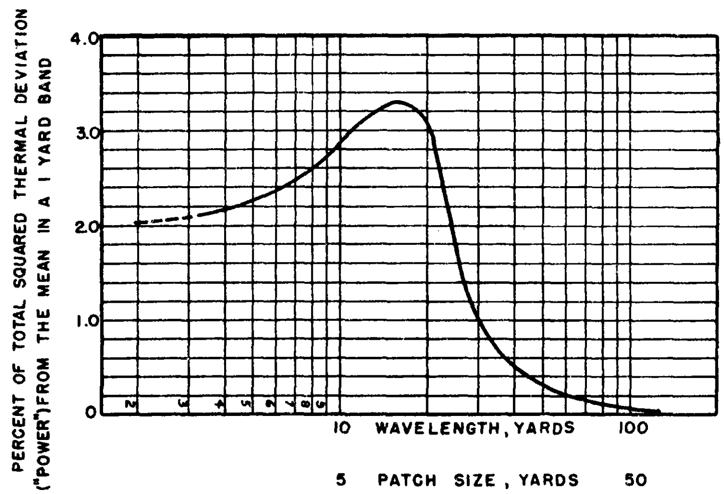

Fig. 4 - Wavelength power spectrum, shallow depths (based on $R(s) = e^{-.20s} \cos .27s$ with s in yards)

EFFECTS ON SOUND PROPAGATION

Ray Bending

The presence of thermal variations in the ocean indicates that the velocity of sound C and its refractive index $\eta = C/C_0$, (C_0 a constant) fluctuate slightly from point to point. An obvious result of such velocity variation is a normally irregular slight bending of sound ray paths about the average ray path predicted in the ordinary manner by the bathythermograph. The amount of such ray bending is of interest whenever accurate measurements of the direction of arrival of sound are important.

When the spatial size of the thermal variations is large compared to a wavelength, the methods of geometrical optics can be used to evaluate the ray bending in terms of the thermal variations. Let us consider a medium which, in the absence of thermal variations, is of uniform velocity, that is, one in which the ray paths are, on the average, straight lines. The analysis is not essentially different for the gradient layer case of a nearly uniform velocity gradient. We will also postulate that the velocity variations are such that the deviations of the ray paths from straight lines are small. Further, the medium is taken to be isotropic, that is, the velocity variations in all directions are statistically the same.

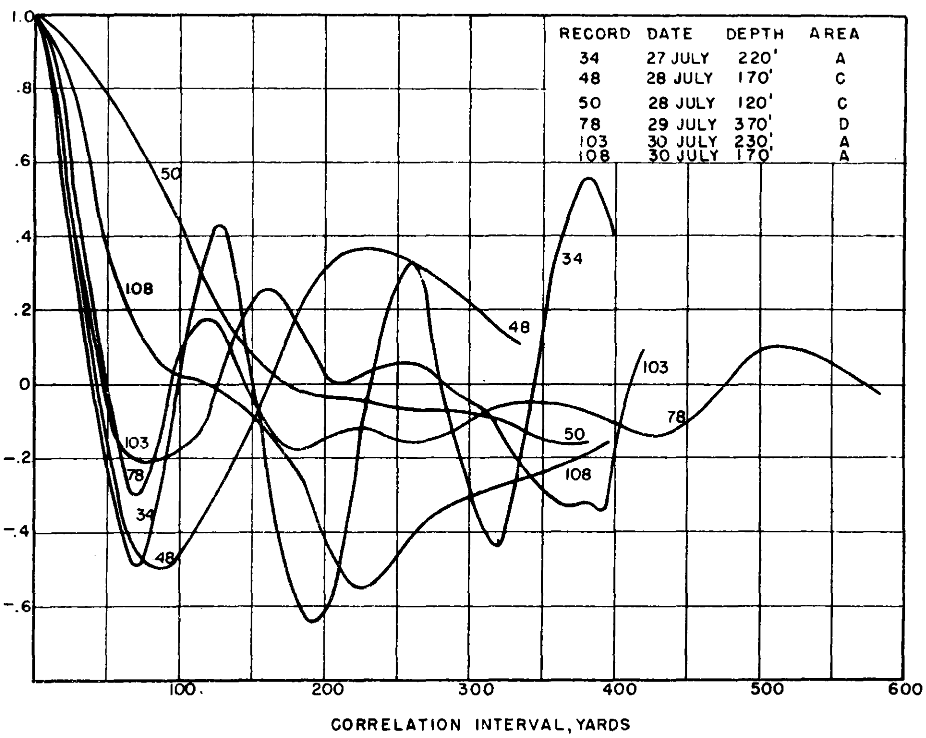

Fig. 5 - Normalized autocorrelation curves for records at depths in gradient layer

Consider a ray which travels between the points A and B and take the x-axis of coordinates to be the line A-B, with the y-axis at right angles. Let $C(x) = C_a(1 + \eta)$, where C_a is the velocity at A, so that $\eta = \Delta C/C_a$. Then by geometrical optics it can be shown that for $\eta \ll 1$ the ray satisfies $d^2y/dx^2 = d\eta/dy$.‡ Now by the assumption of isotropy, $d\eta/dy$ is statistically the same as $d\eta/dx$. We therefore assume $d^2y/dx^2 = d\eta/dx$ will give a statistically correct picture of the ray-bending. Integrating once between A and B, we have

$$\left.\frac{dy}{dx}\right|_B - \left.\frac{dy}{dx}\right|_A = \eta_B - \eta_A .$$

If the ray is directed by a projector A toward B, $dy/dx\,|_A = 0$; also, by the above definition of η, $\eta_A = 0$. Hence, calling the slope of the ray at the receiving point θ_B, we have $\theta_B = \eta_B$. $\eta_B = \Delta C/C$ can easily be obtained from the RMSM thermal deviation vs depth data of Figure 1. From tables of velocity against temperature for sea water we find that at a temperature of 85°F, C = 5060 ft/sec, and 1° F change of temperature is equivalent to 4 ft/sec change in velocity. Thus 1°C = 1.8°F $\approx$ 7.2/5060 units of $\Delta C/C$ = 0.0014 units of $\Delta C/C$. Using the RMSM values mentioned previously of 0.013°C and 0.45°C for the mixed and gradient water respectively, as obtained from Figure 1, we obtain for the mixed layer $\theta_B = 1.8 \times 10^{-5}$ radian = 3.7 seconds and for gradient water $\theta_B = 6.3 \times 10^{-4}$ = 2.1 minutes.

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‡ Synge, J. L., "Geometrical Optics - An Introduction to Hamilton's Method," Chap. V., Camb. Univ. Press (1937)

Reverberation

The scattering produced by random spherical small variations of velocity has been the subject of a short paper by Pekeris.§ It is there shown that the energy E scattered by a volume V into a solid angle $d\Omega$ at an angle θ_0 to the backward direction to the source is

$$E = E_0 \frac{k^4}{\pi} \cdot V \cdot d\Omega \cdot \overline{\left(\frac{\Delta C}{C}\right)^2} \int_0^{\infty} s^2 R(s) \frac{\sin bks}{bks} ds.$$

where $k = 2\pi/\lambda$, R(s) is the normalized autocorrelation function of the thermal variations, and $b = 2\cos(\theta_0/2)$. The reverberation encountered in echo ranging is sound scattered backward toward the source, so that $\theta_0 = 0$; $b = 2$. The volume-scattering coefficient, m, equal to the number of scatterers times the scattering cross section per scatterer, can be defined as 4π times the fraction of the incident energy scattered backward into a unit solid angle by a unit volume of ocean. Thus, the volume scattering coefficient for the thermal inhomogeneities becomes

$$m = \frac{4\pi}{V \cdot d\Omega} \cdot \frac{E}{E_0} = 4k^4 \overline{\left(\frac{\Delta C}{C}\right)^2} \int_0^{\infty} s^2 R(s) \frac{\sin bks}{ks} ds.$$

Pekeris finds that the validity of this expression holds only if $ak(\Delta C/C) \ll 1$, where a is the radius of a spherical scatterer. For the present data this restricts its applicability to the mixed layer where both the radius a and magnitude $\Delta C/C$ of the thermal variations are found to be small enough for this condition to be satisfied.

Let us attempt to obtain an estimate of m from some of the averaged data obtained previously. Let us take $R(s) = e^{-as} \cos bs$ where, as in Figure 3, a = 0.20 and b = 0.27 yards. Then the above integral becomes**

$$\int_0^{\infty} s^2 R(s) \frac{\sin 2ks}{ks} ds = \frac{1}{2} \frac{a(2k+b)}{[a^2 + (2k+b)^2]^2} + \frac{1}{2} \frac{a(2k-b)}{[a^2 + (2k-b)^2]^2} .$$

For sound at 24 kc, k = 90 reciprocal yards, so that $k \gg a$; $k \gg b$; and the above result becomes simply $a/8k^3$. Hence

$$m = \frac{a}{2} \overline{\left(\frac{\Delta C}{C}\right)^2}$$

the same as if the oscillatory term cos bs were omitted from R(s). For a RMSM temperature deviation of 0.013° C in the isothermal water, the corresponding value of $\overline{(\Delta C/C)^2}$ is 3.2×10^{-10}. Using a = 0.20, we obtain $m = 3.2 \times 10^{-11}$ yard^{-1}. Field observations, mostly near San Diego, show that measured values of m center about 10^{-6}, with a range of 5×10^{-6} to as small as 10^{-10}; no measurements are at hand for the Key West area in summer. Nevertheless, it seems probable that the contribution of the scattering by thermal inhomogeneities in "isothermal" water to volume reverberation will be found to be

§ Pekeris, C. L., Phys. Rev. 71, 268-269, February 1947

** Bierens de Haan, D., Nouvelles Tables D'Integrales Definies, Table 361, New York, Steckert (1939) negligible. This same conclusion has been reached†† by assuming rather extreme values for parameters which are here estimated more accurately. However, the effect of the thermal variations in gradient water cannot be estimated until a theory for the scattering by these spatially and thermally large inhomogeneities is developed.

Other Effects

Thermal variations are in theory possible causes of other effects on the propagation of sound in the ocean, such as fluctuation of signal level at a single receiving point. Theory also permits‡‡ the evaluation of the mean deviation of the acoustic path length along the ray to the geometrical path length, and the correlation of signal level between two receiving points. What is required for an estimate of these quantities is the autocorrelation function of the temperature gradient, and of the second space derivative of temperature. While the data of the present report could conceivably be analyzed so as to yield the temperature gradient, the validity of these gradients, in view of the time constants involved, was not felt to justify the labor involved. Such information could be obtained by field observations with two or four suitably connected thermopiles of the type used in this investigation.

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†† NDRC Summary Technical Report, Div. 6, vol. 8, 482 (Unclassified)

‡‡ Bergmann, P. G., Phys. Rev. 70, 486-492, October 1946