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APPROXIMATE THEORY OF LOW REFLECTING FILMS

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ABSTRACT

For the purpose of facilitating the design of low reflecting transparent films on glass, the optical theory is outlined and the theoretical formulas are worked out to a second degree of approximation. Theoretical curves of the reflection are calculated for several cases of one and two films on glass for various thicknesses and refractive indices of the films and of the glass.

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CHAPTER 1

INTRODUCTION

Dr. Peter King of the Chemistry Division of this Laboratory has made low and high reflecting surfaces by depositing one and two films on glass or other transparent material, the thickness and the refractive index of each film being chosen and controlled to give a low or a high, or a desired, reflectivity. He wished to know how best to choose the optical constants of a film, so that by combination of one or more films any specified reflectivity may be obtained. Experimentally the number of variables is great. He suggested that a theoretical annalysis might direct experiment in the best direction.

Rayleigh[1] gave an exact solution of the reflection from a single glass plate, and from a pile of similar glass plates, immersed in air. The more complex case of one film on glass has not been worked out exactly. And, a fortiori, the much more complex cases of two films on glass, and of multiple films, has not been given exact annalysis. Cartwright[2], Blodgett[3] and Smakula[4] have outlined in varying degree similar approximate treatments of several cases of films on glass.

In this paper the cases of one and two transparent films on glass are worked out to a second degree of approximation and the theoretical equations are compared with certain of King's measurements. Curves are given which show the variation of the reflectivity with variation of the refractive indices and thicknesses of the films and the refractive index of the glass.

The approximate treatment of the following pages is useful for low reflecting films and yields results which give insight into the design of multiple films. However, the approximations that are made are not accurate for high reflecting films. A discussion of high reflecting films is left to the future.

The present survey of the problem indicates that an exact treatment of the single film case may not be very difficult and that an exact treatment of the two film case and the multiple film case may not be very simple.

CHAPTER 2

OBSERVATIONAL DATA

Certain experimental data of King, given in Plate 1, were selected for discussion. These are plotted in the dots of Plate 1; the curves of Plate 1 are theoretical and are described later. In plate 1 the ordinates are the reflectivities R of a single surface of glass coated with the film or films. The abscissas are the thicknesses of the film in wavelengths λ of light. The dots are the observed values of R for film thicknesses from zero to $\lambda/4$ and more.

Glass of refractive index 1.53 was used. Two film materials were used, one of relatively low and one of relatively high refractive index. Approximate measurements gave indices 1.47 and 1.75, respectively. Calculations from the reflectivity data, described later, gave the more exact values 1.45 and 1.71. These values are in Plate 1.

R was determined by observing with a Photox photoelectric cell the light from a tungsten lamp reflected by the coated glass surface. Actually the energy R_2 reflected from both surfaces of a glass plate equally coated on each side was measured. Then R for a single surface was calculated from the equation[5]

$$R = \frac{R_2}{2 - R_2} \qquad (1)$$

The reflectivity R_λ for monochromatic light was determined throughout the visible spectrum and was found to vary to some extent across the spectrum. These data are not reproduced here, but will be referred to later.

In expressing the thickness of the film in wavelengths of light in the abscissas of Plate 1, we have introduced a little theory. In the experiments the thickness of the film was increased by constant increments to a maximum or minimum value of R, but the thickness of an increment was not determined. However, the theory is so simple that one had no hesitancy in placing maximum or minimum R at the optical thickness $\lambda/4$, and laying off the other abscissas accordingly.

CHAPTER 3

GENERAL THEORY

Consideration is restricted to transparent media bounded by plane parallel surfaces and to rays incident perpendicularly on the surfaces. Therefore, no polarization effects are encountered. Inconsistently with the restriction the rays in Plates 2 and 5 are not drawn normal to the surfaces, but at an angle, merely in order that the course of the rays be seen clearly.

Let n be the refractive index of a material, with a subscript to denote the material. For a ray of unit energy incident on a boundary between two materials let a and b be the reflected and transmitted amplitudes, respectively, and r be the reflected energy. Then, for transparent materials with no absorption, the following relations follow from Fresnel's equations

$$a = \left| \frac{n_2 - n_1}{n_2 + n_1} \right| , \qquad (2)$$

$$b^2 = 1 - a^2 , \qquad (3)$$

$$r = a^2 . \qquad (4)$$

a is the absolute value given by (2), and is essentially positive. All quantities in (2), (3) and (4) refer to monochromatic light of wavelength λ.

Several coherent parallel rays of amplitudes a and phase differences Δ combine to give an amplitude A, where

$$A = \sum a \cos \Delta . \qquad (5)$$

If the several rays arise by reflection of an incident ray of unit energy from plane parallel surfaces, the reflectivity R of the surfaces is

$$R = A^2 = (\sum a \cos \Delta)^2 . \qquad (6)$$

Referring to the film, phase retardations of the rays occur due to reflection at the surfaces of the film and to traversing the film. Reflection in a rarer medium from a denser medium causes no phase change; therefore $\Delta = 0$, $\cos \Delta = 1$, and $a \cos \Delta$ is positive. Reflection in a denser medium from a rarer medium causes a phase change of π; therefore $\Delta = \pi$, $\cos \Delta = -1$, and $a \cos \Delta$ is negative.

If e cm is the thickness of the film and δ is the phase retardation of a ray that traverses the film twice

$$\delta = 4\pi ne/m \qquad (7)$$

where m is an integer. For example, in order to cause a reversal of phase, $\delta = \pi$, the optical thickness of the film is $ne = \lambda/4$, with $m = 1$ in (7).

In later equations in which the summation of (6) is written out it is convenient to use the sign of the term to take care of the phase changes upon reflection and to use Δ to give the path retardation.

CHAPTER 4

SINGLE FILM ON GLASS, SECOND APPROXIMATION

Low refractive index film

The foregoing relations are applied to the case of a single low refractive index film on glass. "Low" means that the refractive index of the film is less than that of the glass. The refractive indices of air, film and glass are n_o, n_1 and n_2, respectively, and $n_o < n_1 < n_2$. Referring to Plate 2A, a ray of unit amplitude and unit energy incident normally on the film-glass combination is reflected in a number of rays, of which four are shown. The signs marked on the rays indicate the changes in phase upon reflection. Rays 1 and 2 have each experienced one reflection, ray 3 three reflections, ray 4 five reflections, etc. The amplitude A and energy R of the reflected light are, from (2), (3), (5) and (6),

$$R = A^2 = (a_1 + a_2 b_1^2 \cos\delta - a_1 a_2^2 b_1^2 \cos 2\delta + a_1^2 a_2^3 b_1^2 \cos 3\delta - \text{---})^2, \quad (8)$$

where

$$a_1 = \left|\frac{n_1 - n_o}{n_1 + n_o}\right| , \quad a_2 = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| , \quad b_1^2 = 1 - a_1^2 . \quad (9)$$

Since we are concerned with low reflecting films and hence with values of a_1 and a_2 less than 0.3, three terms of (8) give values of R correct to less than 2 percent of R. The inclusion of the first three terms may be designated a "second" approximation, for the terms refer respectively to the two once reflected rays and the one thrice reflected ray. Then

$$R = A^2 \cong (a_1 + a_2 b_1^2 \cos\delta - a_1 a_2^2 b_1^2 \cos 2\delta) . \quad (10)$$

Differentiating (10) with respect to δ and equating to zero leads to $\delta = 0$ and π for maximum and minimum values of A and R. Then

$$\delta = 0$$
$$n_1 e = 0 \qquad R_{max} = (A_{max})^2 = (a_1 + a_2 b_1^2 - a_1 a_2^2 b_1^2)^2 , \quad (11)$$

$$\delta = \pi$$
$$n_1 e = \lambda/4 \qquad R_{min} = (A_{min})^2 = (a_1 - a_2 b_1^2 - a_1 a_2^2 b_1^2)^2 . \quad (12)$$

With $n_o = 1$ and $n_2 = 1.53$ the values of R_{max} and R_{min} were calculated from (11) and (12) for various values of n_1, and are plotted in the full line curves of Plate 3A. The observed values of R_{max} and R_{min}, from Plate 1A, were 0.0435 and 0.025, respectively; these are shown in the dotted lines of Plate 3A. It is seen that best agreement with the observed values occurs when $n_1 = 1.45$, in which case $R_{max} = 0.044$ and $R_{min} = 0.025$. The conclusion was that the reflection measurements gave for the low refractive index film

$$n_1 = 1.45$$

This is in satisfactory agreement with the approximate observed value 1.47, and is the value used in later calculations.

In passing it may be mentioned that the foregoing method of determining the refractive index of a film from interference maxima and minima is a well known optical method.

With $n_1 = 1.45$ the values of R for various film thicknesses were calculated from (10) and are plotted in the full line curve of Plate 1A. The curve agrees well with the observed points.

If we include only the two single reflected rays 1 and 2 of plate 2A, and neglect the reflection loss due to the passage of ray 2 through the air-film surface, (12) becomes

$$R_{min} = a_1 - a_2$$

$$= \frac{n_1 - 1}{n_1 + 1} - \frac{n_2 - n_1}{n_2 + n_1} \quad , \qquad (13)$$

Equating (13) to zero yields

$$n_1 = n_2^{1/2} \quad , \qquad (14)$$

which is a well known expression. It is thus seen that (14) is an approximation of a low order. In spite of this (14) is very accurate for the special case to which it applies. For example, if the refractive index of the $\lambda/4$ film is related to the refractive index of the glass by (14), one finds from (13) that $R_{min} = 0$, and from the more accurate expression (12) that $R_{min} = a_1^{10}$, which is a very small quantity for $a_1 < 0.3$.

High refractive index film

The case of a single high refractive index film on glass is treated in the same way as was done for the low refractive index film. "High" means that the refractive index of the film is greater than that of the glass. The reflected rays in this case are shown in Plate 2B, where the refractive indices of air film and glass are n_o, n_1 and n_2, respectively and $n_1 > n_2 > n_o$. Dealing only with three reflected rays, the amplitude A and energy R of the reflected light is

$$R = A^2 = (a_1 - a_2 b_1^2 \cos\delta - a_1 a_2^2 b_1^2 \cos 2\delta)^2 \qquad (15)$$

Maximum and minimum values of R are

$$\delta = 0$$

$$n_1 e = 0 \qquad R_{min} = (A_{min})^2 = (a_1 - a_2 b_1^2 - a_1 a_2^2 b_1^2)^2 \qquad (16)$$

$$\delta = \pi$$

$$n_1 e = \lambda/4 \qquad R_{max} = (A_{max})^2 = (a_1 + a_2 b_1^2 - a_1 a_2^2 b_1^2)^2 \qquad (17)$$

With $n_o = 1$ and $n_2 = 1.53$, R_{max} and R_{min} were calculated from (16) and (17) for various values of n_1. These were plotted in the full line curves of Plate 3B. Comparison with the observed values $R_{min} = 0.040$ and $R_{max} = 0.100$ of Plate 1B gave

$$n_1 = 1.71$$

This value is used in later calculations; it is in fair agreement with the approximately observed value 1.75. With $n_1 = 1.71$ the R, δ curve was calculated from (15) and is plotted in the full line curve of Plate 1B; the agreement with the observed points is tolerable.

It is of interest to compare the approximate equations (16) and (17) with exact expressions which Rayleigh derived for the special case of a plane parallel plate with the same medium on both sides of the plate, as a glass plate in air. Rayleigh's Eq. 7 (Ref. 1, page 493) reduces in our notation to

$$R_{min} = 0 \qquad (18)$$

and

$$R_{max} = \left(\frac{2a}{1+a^2}\right)^2 = (2a)^2 (1 - a^2 + a^4 - \text{—})^2 \qquad (19)$$

For this case $a_1 \simeq a_2 \simeq a$ and $b_1^2 \simeq 1 - a^2$

(16) reduces to

$$R_{min} \simeq \left[a - a(1 - a^2) - a^3 (1 - a^2)\right]^2$$
$$\simeq a^{10} \tag{20}$$

which is not greatly different from (18) for $a < 0.3$.

(17) reduces to

$$R_{max} \simeq \left[a + a (1 - a^2) - a^3 (1 - a^2)\right]^2$$
$$\simeq (2a)^2 \left[1 - a^2 + \tfrac{1}{2}a^4\right]^2 \tag{21}$$

and therefore differs from (19) by a quantity of order a^{10}, which is small.

CHAPTER 5

TWO FILMS ON GLASS, FIRST APPROXIMATION

The case of two films on glass is worked out to a first approximation by considering only the once reflected rays. It is shown in the next section that the changes occasioned by including the thrice reflected rays are not great for the cases of low reflection that are discussed here. Plate 2C is a diagram of two films on glass in the order air, low index film, high index film, and glass, of respective refractive indices n_o, n_1, n_2 and n_3, where $n_o < n_1 < n_3 < n_2$. The singly reflected rays are 1, 2 and 3 and the signs given them in Plate 2C indicate the changes in phase upon reflection. The amplitudes of reflection at, and of transmission through, the respective surfaces S_1, S_2 and S_3 of Plate 2C are a_1, a_2, a_3 and b_1, b_2 where

$$a_1 = \left|\frac{n_o - n_1}{n_o + n_1}\right| , \quad a_2 = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| , \quad a_3 = \left|\frac{n_2 - n_3}{n_2 + n_3}\right|$$

$$b_1^2 = 1 - a_1^2 , \quad b_2^2 = 1 - a_2^2 \tag{22}$$

The total reflected energy R and amplitude A of the three rays is

$$R = A^2 = \left[a_1 + a_2 b_1^2 \cos\delta_1 - a_3 b_1^2 b_2^2 \cos(\delta_1 + \delta_2)\right]^2 , \tag{23}$$

where δ_1 and δ_2 are the phase retardations due to a <u>double</u> passage through the low and high index films, respectively.

Eq. (23) is the approximate expression from which R is calculated for films of various thicknesses and refractive indices on glasses of various refractive indices. (23) is a complicated function of the optical quantities n_o, n_1, n_2, n_3, δ_1 and δ_2. It does not reduce to any simple relations that the quantities must satisfy in order to give a low or a zero value of R. However, it may be seen from (23),

(a) that within certain ranges of the quantities $R = 0$,

(b) That within these ranges there are an infinity of combinations that give $R = 0$,

(c) that outside of the ranges it is not possible to make $R = 0$ by any two-film combination.

The procedure followed here is to select certain specific materials and to work out from (23) the values of R to be expected from certain combinations. The materials and combinations selected are those of experimental interest. The various cases are presented as problems some of which are solved and some of which are suggested as exercises for the student.

If the thickness of the low index film is $\lambda/4$, then $n_1 e_1 = \lambda/4$, $\delta_1 = \pi$, $\cos \delta_1 = -\pi$ and (23) becomes

$$R = (a_1 - a_2 b_1^2 + a_3 b_1^2 b_2^2 \cos \delta_2)^2 \tag{24}$$

If the thicknesses of both the low and high index films are $\lambda/4$, (24) becomes

$$R = (a_1 - a_2 b_1^2 - a_3 b_1^2 b_2^2)^2 \tag{25}$$

<u>Problem 1.</u> <u>Variation of R with thickness of high index film.</u>

Given air $n_o = 1$
film, low, $n_1 = 1.45$, thickness $\lambda/4$
film, high, $n_2 = 1.71$, thickness varying
glass $n_3 = 1.53$

Eq. (24) becomes

$$\begin{aligned} R &= (0.1836 - 0.0802 + 0.0534 \cos \delta_2)^2 \\ &= (0.1034 + 0.0534 \cos \delta_2)^2 \end{aligned} \tag{26}$$

The maximum value of R occurs at $\delta_2 = 0$ (and $2m\pi$) corresponding to a film thickness zero; the minimum value of R at $\delta_2 = \pi$ and a film thickness $\lambda/4$. The minimum value of $R = (0.1034 - 0.0534)^2 = 0.0025$, which is not zero. Therefore with films and glass of the above indices there is no combination that gives $R = 0$.

R was calculated from (26) for various thicknesses of the high index film, and is plotted in the full line curve of Plate 1C. The curve does not agree very well with the observed points. At the point of minimum reflection the discrepancy between theory and experiment is of particular interest in connection with the production of very low reflecting surfaces. Here R from theory was 0.0025 and from experiment was about 0.006. There are two factors that may contribute to the discrepancy, namely, dispersion and scattering. With regard to dispersion the theoretical treatment dealt with monochromatic light on the assumption that values of refractive indices and reflectivities for unresolved light could be treated as monochromatic. However, since the optical properties of the films and the glass vary considerably across the spectrum the assumption may be faulty. Scattering, unless corrected for, increases the apparent observed value of R; and it is known that for very low values of R scattering of light is relatively important. Probably more complete attention to theory and experiment will lessen the discrepancy.

Problem 2. Variation of R with n_1

Given air $n_0 = 1$

film, low, n_1 varying, thickness $\lambda/4$

film, high, $n_2 = 1.71$, thickness $\lambda/4$

glass $n_3 = 1.53$

The values of R for various values of n_1 were calculated from (25) and are plotted in the curve Plate 4A. The observed value for $n_1 = 1.45$ is shown by the cross. It is seen that $R = 0$ for $n_1 = 1.38$.

Problem 3. Variation of R with n_2

Given air $n_0 = 1$

film, low, $n_1 = 1.45$, thickness $\lambda/4$

film, high, n_2 varying, thickness $\lambda/4$

glass $n_3 = 1.53$

Problem 4. Variation of R with n_3

Given air $n_o = 1$

film, low, $n_1 = 1.45$, thickness $\lambda/4$

film, high, $n_2 = 1.71$, thickness $\lambda/4$

glass n_3 varying

The values of R for various values of n_3 were calculated from (25) and are plotted in the curve Plate 4C. The observed value for $n_3 = 1.53$ is shown by the cross. It is seen that $R = 0$ for $n_3 = 1.37$.

Problem 5. Variation of R with λ

Assume that the refractive index changes with λ are not important and that the film thickness is the most important feature. Work out the variation of R with λ and show why a single-film or a two-film coating on glass, that gives $R = 0$ for green light viewed normally, appears purple when viewed normally. Show why the color changes from purple to reddish as the angle of view is inclined to the normal. Show why a two-film coating is likely to give a more saturated purple than a single-film coating.

Problem 6. Film-glass combination under water.

Show why a film-glass combination that is low reflecting in air becomes a moderately good reflector when placed under water.

Discuss what happens optically to a film-glass combination which is coated with a film of water. This amounts to discussing the three-film problem.

Problem 7. With films of indices 1.71 and 1.45 and glass of index 1.53 show that the lowest obtainable values of R are 0.0335 for a single film on the glass and 0.0025 for two films on the glass. (The second part of this problem has been done in Problem 1.)

Problem 8. Films on glass in a different order.

Given films in order, air, high index film, low index film, glass, of the same indices as Problem 1, thus;

air $n_o = 1$

film, high, $n_1 = 1.71$

film, low, $n_2 = 1.45$

glass $n_3 = 1.53$

Show that R in this case is given by (23) with the signs of the second and third terms changed. It will be found that the lowest value of R for this film combination is 0.026 and occurs when the thickness of the high index film is $\lambda/2$ and of the low index film $\lambda/4$. The value is about 10 times the value $R = 0.0025$ that is obtainable with the films of the same index but in opposite order, as in Problem 1.

CHAPTER 6

TWO FILMS ON GLASS, SECOND APPROXIMATION

In the case of two films on glass a second approximation is obtained by considering, in addition to the once reflected rays, the rays that have been reflected three times. For films in the order air, low index film, high index film, glass, the rays are drawn in Plate 5 in which A shows the once reflected rays 1, 2, 3 and the beginnings F, G, H of the other rays. The continuance of F, G, H and their emergence in the thrice reflected rays 4, 5, 6, 7, 8 are shown in B, C, D of Plate 5. The complete courses of the rays 4,5,6,7,8 are drawn in E, Plate 5. The signs on the rays indicate the changes in phase upon reflection.

As in the preceding section the respective reflected amplitudes at, and transmitted amplitudes through, the surfaces S_1, S_2 and S_3, are a_1, a_2, a_3 and b_1, b_2. The phase differences due to double transmission through the low and high index films are δ_1 and δ_2, respectively. $\underline{a}$ and $\underline{b}$ with appropriate subscripts are given by (2) and (3). The reflectivity R given by the eight rays is, from (6),

$$
\begin{aligned}
R^{\frac{1}{2}} = {} & a_1 + a_2 b_1^2 \cos \delta_1 - a_3 b_1^2 b_2^2 \cos(\delta_1 + \delta_2) \\
& - a_1 a_2^2 b_1^2 \cos 2\delta_1 + a_1 a_2 a_3 b_1^2 b_2^2 \cos(2\delta_1 + \delta_2) \\
& + a_1 a_2 a_3 b_1^2 b_2^2 \cos(2\delta_1 + \delta_2) - a_1 a_3^2 b_1^2 b_2^4 \cos(2\delta_1 + 2\delta_2) \\
& - a_2 a_3^2 b_1^2 b_2^2 \cos(\delta_1 + 2\delta_2) \qquad (27)
\end{aligned}
$$

The eight terms of (27) refer in order to the rays 1 to 8. The first three terms of (27) are of course the same as (23).

Two of the problems of the preceding section are now worked out from (27) and the results are compared with those from the first approximation formulas.

Problem 1, second approximation. Variation of R with thickness of high index film.

Given

Air $n_0 = 1$

film, low, $n_1 = 1.45$, thickness $\lambda/4$

film, high, $n_2 = 1.71$, thickness varying

glass, $n_3 = 1.53$

R was calculated from (27) and is plotted in the dotted curve of Plate 6A. The full line curve gives R from (24) or (26) and is the same as the curve of Plate 1C.

Problem 3, second approximation. Variation of R with n_2.

Given

air $n_0 = 1$

film, low, $n_1 = 1.45$, thickness $\lambda/4$

film, high, n_2 varying, thickness $\lambda/4$

glass, $n_3 = 1.53$

R was calculated from (27) and is plotted in the dotted curve of Plate 6B. The full line curve gives R from (25) and is the same as the curve of Plate 4B.

It is seen from Plate 6 that conclusions derived from the first approximation formulas concerning changes in R with the optical constants of the films are not greatly different from conclusions derived from the second approximation formulas. For example, both curves of Plate 6B agree in a value well below 0.001 for the minimum value of R_{min} at about $n_2 = 1.81$. The actual values, however, that the first and second approximation formulas give in this case differ considerably, being respectively 0.000009 and 0.000019.

If higher order approximations were made one may infer that the resulting curves would probably lie between the dotted and full line curves of Plate 6.

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4. A. Smakula, Phys, Zeit. 43, 217-222, 1943.
5. Eq. (1) is derived in reports by King.

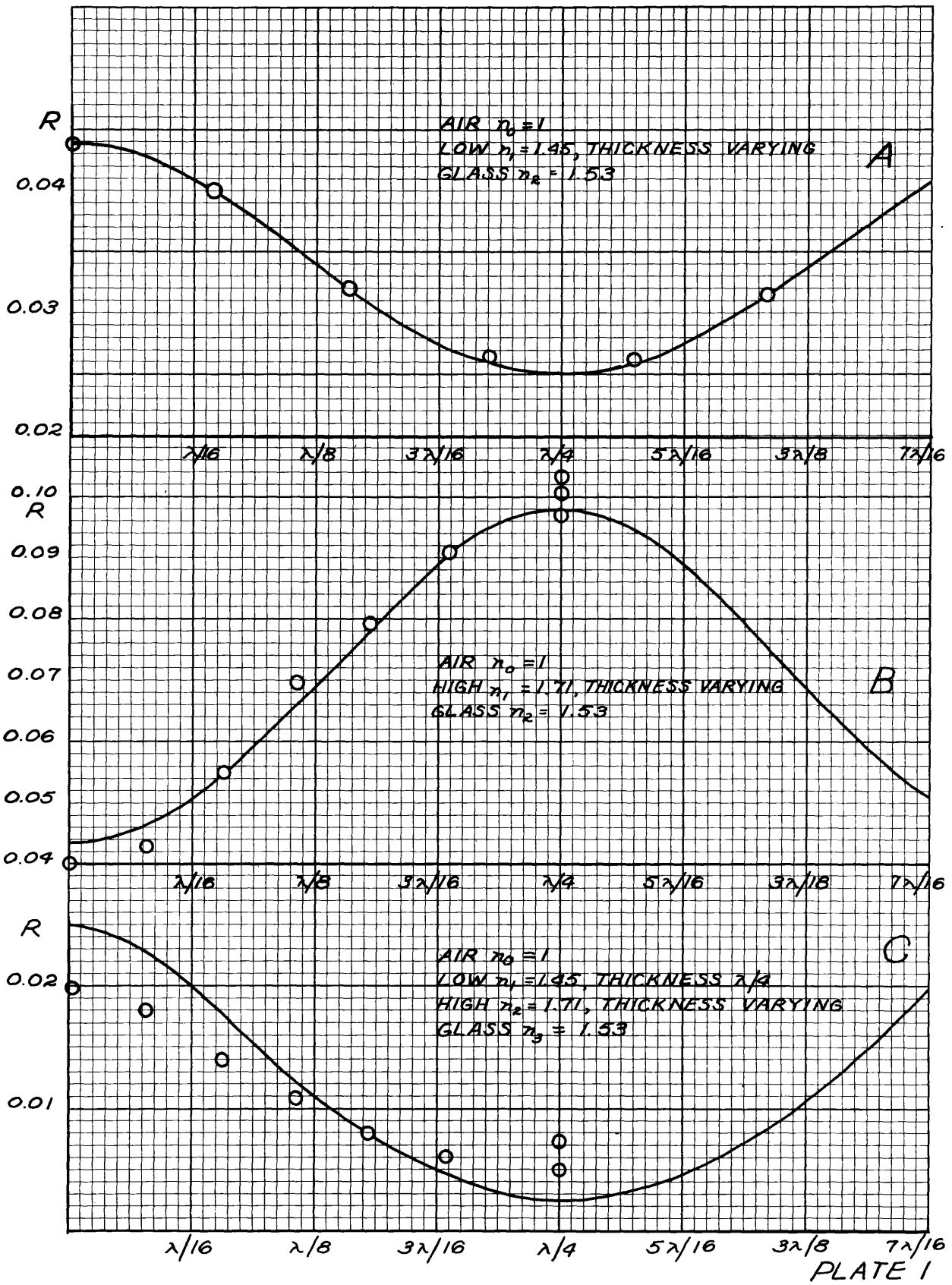
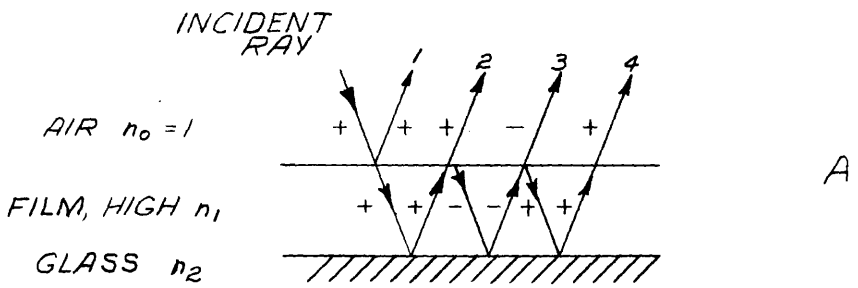
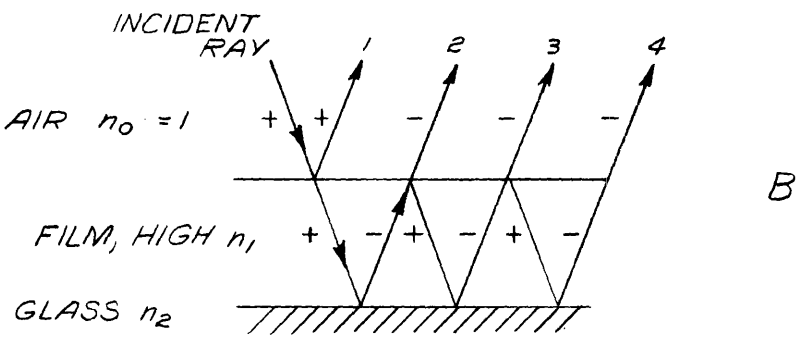

R
0.04
0.03
0.02
AIR $n_0 = 1$
LOW $n_1 = 1.45$, THICKNESS VARYING
GLASS $n_2 = 1.53$
A
λ/16
λ/8
3λ/16
λ/4
5λ/16
3λ/8
7λ/16
0.10
R
0.09
0.08
0.07
0.06
0.05
0.04
AIR $n_0 = 1$
HIGH $n_1 = 1.71$, THICKNESS VARYING
GLASS $n_2 = 1.53$
B
λ/16
λ/8
3λ/16
λ/4
5λ/16
3λ/8
7λ/16
R
0.02
0.01
AIR $n_0 = 1$
LOW $n_1 = 1.45$, THICKNESS λ/4
HIGH $n_2 = 1.71$, THICKNESS VARYING
GLASS $n_3 = 1.53$
C
λ/16
λ/8
3λ/16
λ/4
5λ/16
3λ/8
7λ/16

PLATE I


INCIDENT RAY
1
2
3
4
AIR $n_0 = 1$
FILM, HIGH n_1
GLASS n_2
A


INCIDENT RAY
1
2
3
4
AIR $n_0 = 1$
FILM, HIGH n_1
GLASS n_2
B

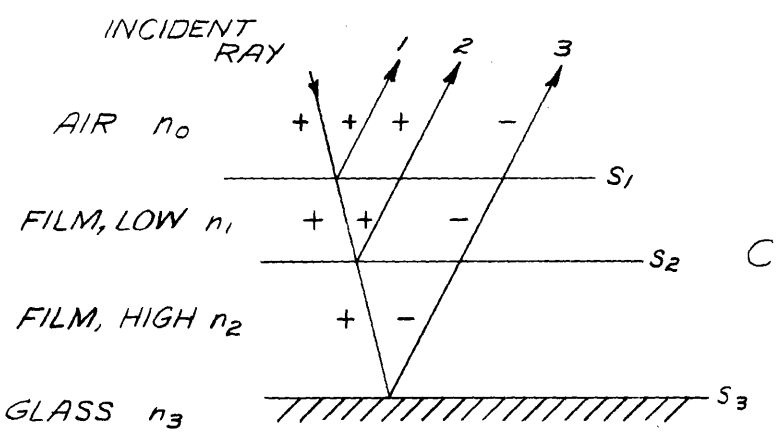

INCIDENT RAY
1
2
3
AIR n_0
S_1
FILM, LOW n_1
S_2
FILM, HIGH n_2
GLASS n_3
S_3
C

PLATE 2

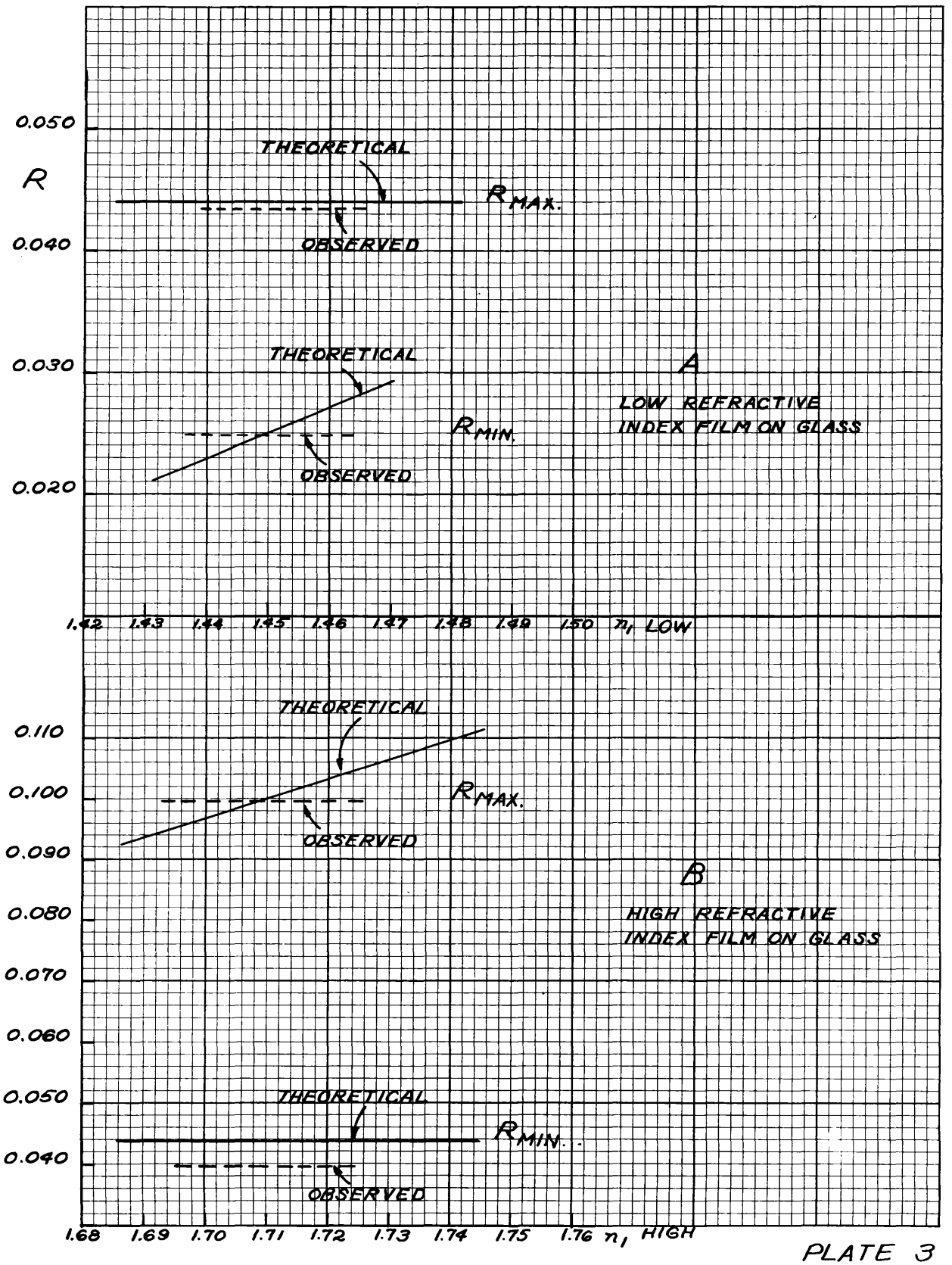

0.050
R
0.040
0.030
0.020
THEORETICAL
$R_{MAX.}$
OBSERVED
THEORETICAL
$R_{MIN.}$
OBSERVED
A
LOW REFRACTIVE
INDEX FILM ON GLASS
1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49 1.50 n_1 LOW
0.110
0.100
0.090
0.080
0.070
0.060
0.050
0.040
THEORETICAL
$R_{MAX.}$
OBSERVED
B
HIGH REFRACTIVE
INDEX FILM ON GLASS
THEORETICAL
$R_{MIN.}$
OBSERVED
1.68 1.69 1.70 1.71 1.72 1.73 1.74 1.75 1.76 n_1 HIGH

PLATE 3

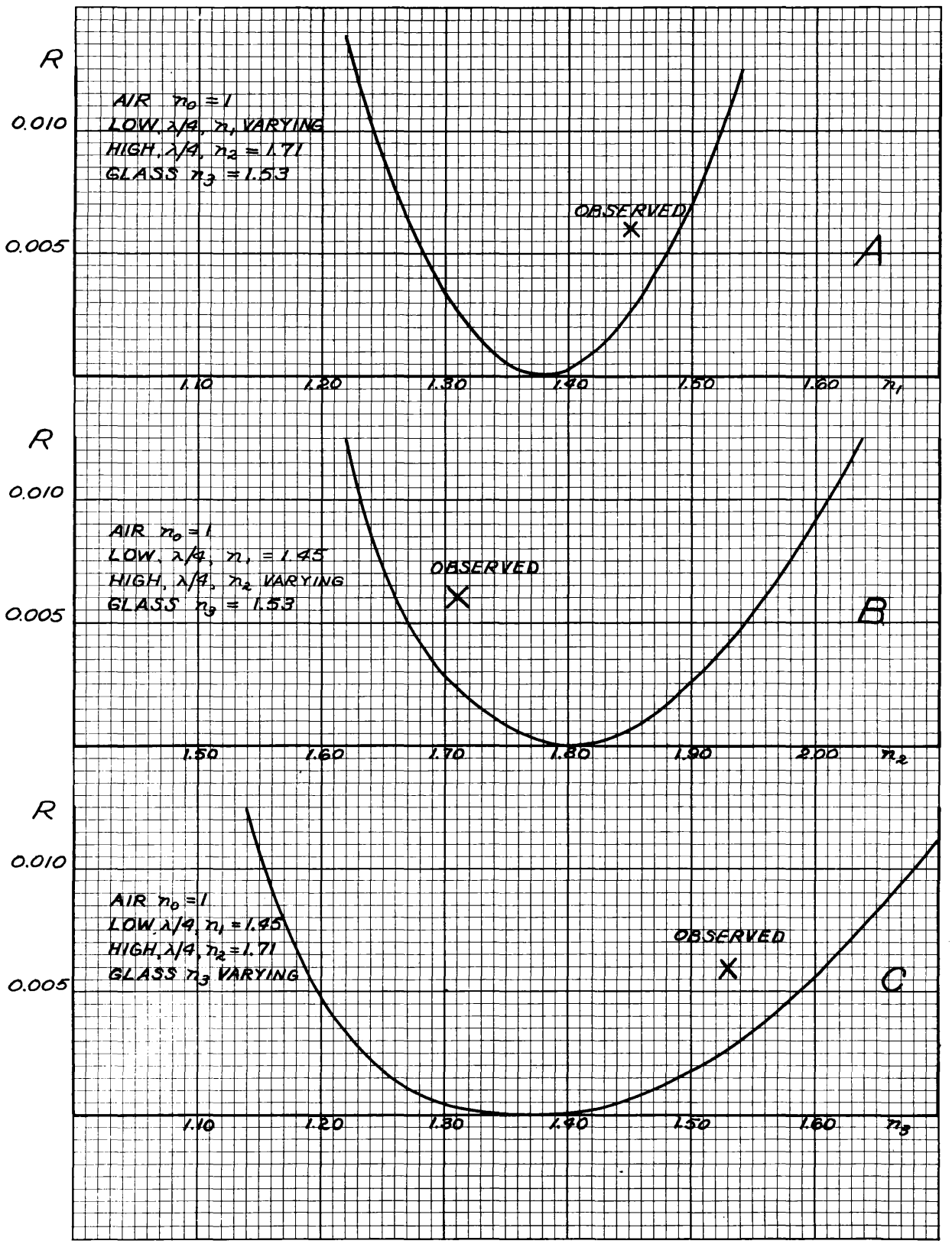
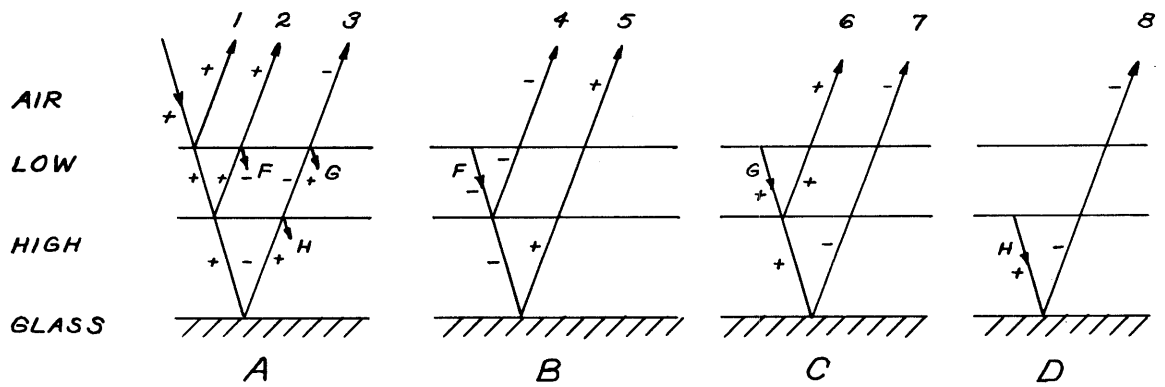

R
0.010
0.005
AIR $n_0 = 1$
LOW, $\lambda/4$, n_1 VARYING
HIGH, $\lambda/4$, $n_2 = 1.71$
GLASS $n_3 = 1.53$
OBSERVED
A
1.10
1.20
1.30
1.40
1.50
1.60
n_1
R
0.010
0.005
AIR $n_0 = 1$
LOW, $\lambda/4$, $n_1 = 1.45$
HIGH, $\lambda/4$, n_2 VARYING
GLASS $n_3 = 1.53$
OBSERVED
B
1.50
1.60
1.70
1.80
1.90
2.00
n_2
R
0.010
0.005
AIR $n_0 = 1$
LOW, $\lambda/4$, $n_1 = 1.45$
HIGH, $\lambda/4$, $n_2 = 1.71$
GLASS n_3 VARYING
OBSERVED
C
1.10
1.20
1.30
1.40
1.50
1.60
n_3

PLATE 4


1 2 3
4 5
6 7
8
AIR
LOW
HIGH
GLASS
F
G
H
A
B
C
D

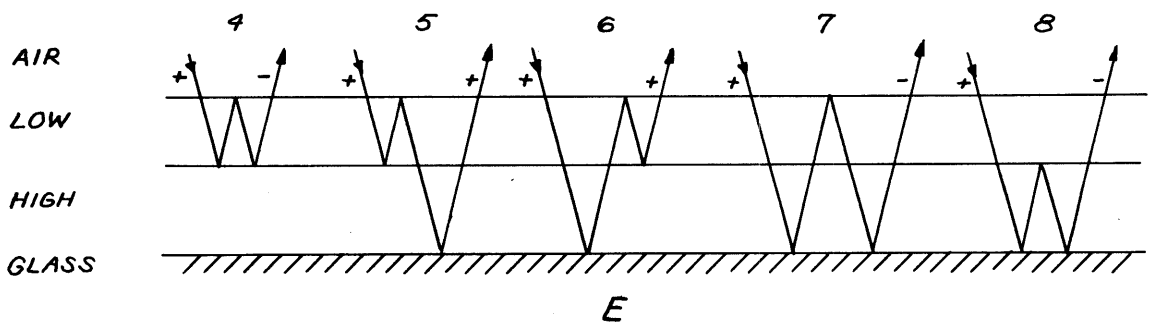

4
5
6
7
8
AIR
LOW
HIGH
GLASS
E

PLATE 5

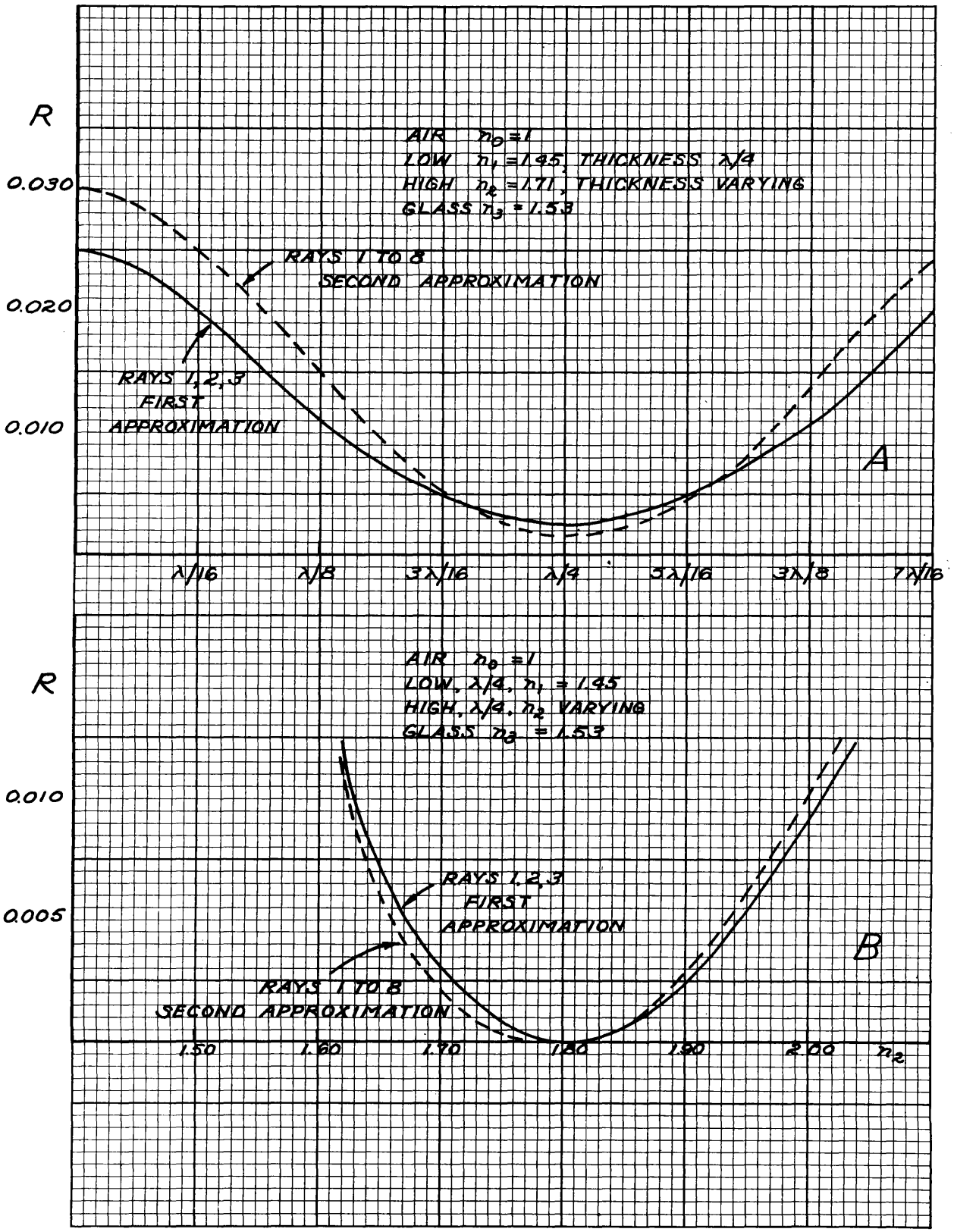

R
0.030
0.020
0.010
AIR $n_0 = 1$
LOW $n_1 = 1.45$, THICKNESS $\lambda/4$
HIGH $n_2 = 1.71$, THICKNESS VARYING
GLASS $n_3 = 1.53$
RAYS 1 TO 8
SECOND APPROXIMATION
RAYS 1,2,3
FIRST
APPROXIMATION
A
λ/16
λ/8
3λ/16
λ/4
5λ/16
3λ/8
7λ/16
R
0.010
0.005
AIR $n_0 = 1$
LOW, $\lambda/4$, $n_1 = 1.45$
HIGH, $\lambda/4$, n_2 VARYING
GLASS $n_3 = 1.53$
RAYS 1,2,3
FIRST
APPROXIMATION
RAYS 1 TO 8
SECOND APPROXIMATION
B
1.50
1.60
1.70
1.80
1.90
2.00
n_2

PLATE 6