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TWO-DIMENSIONAL SIMULATION OF THE AUTOMATIC AIRCRAFT INTERCEPT CONTROL SYSTEM

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NAVAL RESEARCH LABORATORY

WASHINGTON, D.C.

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TWO-DIMENSIONAL SIMULATION OF THE AUTOMATIC AIRCRAFT INTERCEPT CONTROL SYSTEM

H. G. Paine

26 January 1950

Approved by:

Mr. E. F. Kulikowski, Head, Systems Utilization Branch
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NAVAL RESEARCH LABORATORY
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ABSTRACT

This report describes the results of the simulation studies of the Automatic Aircraft Intercept Control System (AAICS). The results indicate that a target can be tracked by Track-While-Scan equipment (T-W-S) if sufficient smoothing is provided. Stable operation of AAICS was shown to be possible. The importance of accuracies in the determination of bomber and fighter speeds was investigated. The interception accuracies obtained in the simulation were good. The AAICS loop operated satisfactorily at all data rates employed in the simulation. Greater smoothing was required for a maneuvering target than for a target on a straight-line course. A simple evasive maneuver started early in the bomber's course did not materially alter interception accuracies.

PROBLEM STATUS

This is an interim report; work is continuing on the problem.

AUTHORIZATION

NRL Problem RO7-25R.
NR 507-250

TWO-DIMENSIONAL SIMULATION OF THE AUTOMATIC AIRCRAFT INTERCEPT CONTROL SYSTEM

INTRODUCTION

The problem with which this report is concerned was established[1,2,3] at the Laboratory for the purpose of conducting a study to provide specific recommendations on the technical characteristics of an Automatic Aircraft Intercept Control System (AAICS). Several reports have been released[4,5,6] covering certain aspects of the problem. The present report, another of the series, covers the results of analysis and simulation of the proposed AAICS servo-loop.

For analysis purposes the AAICS can be divided into three general parts. The first covers human operations of detection, evaluation, briefing, and launching of aircraft which could not be mechanized for simulation, and hence was not embraced in this study. The second involves the automatic tracking, the automatic computation of intercept courses, and the relay of the necessary course commands to bring the interceptors to the point of acquisition of their targets. The third part is the aircraft duel. Only the second part is directly considered in this report.

The degree of automaticity possible in performing the functions of data gathering and computation of intercept course depends largely on the stability of these functions and on the complexity likely to be encountered in the operational equipments.

PRACTICAL AND THEORETICAL CONSIDERATIONS IN AAICS SIMULATION

The value of results obtained by simulating any problem is, of course, largely dependent upon the degree of realism attained in setting up the problem for simulation. For this reason attention needs to be given to the theoretical and practical considerations underlying the interceptions attempted, so that a fuller appreciation may be had of the significance of the results obtained.

To simplify the equations, only the two-dimensional phase of navigation was considered, i.e., the phase after the fighter had attained the bomber's altitude. To simplify the work further, all computers auxiliary to the main-course computer were omitted from the simulation loop. The derivation of the two-dimensional equations related to the navigational phase of AAICS is based on the general navigational triangle, Figure 1.

---

[1]CNO ltr. OP-413-C63/fic, FS-367 Ser. No. 1397P413 to ONR, BuAer, BuShips, BuOrd, dated 4 Sept., 1947, directing initiation of development plans.

[2]ONR ltr. to NRL Exos: ONR: N461: EOW, Ser. 52, dated 19 Jan. 1948. Request for establishment of project at NRL to conduct studies of Automatic Aircraft Intercept Control System as stated in correspondence from CNO, BuAer, and BuShips.

[3]NRL ltr. C-115-23/48 (04213) to ONR dated 4 Feb. 1948, confirming establishment of Problem R07-25R.

[4]Riccobono, S., "Automatic Aircraft Intercept Control System Study." NRL Report R-3342, August 30, 1948. (Secret)

[5]Alderson, W. S., Guarino, P.A., and Varela, A.A., "A Study of computers and radar for aircraft interception control." NRL Report R-3368, October 13, 1948. (Secret)

[6]Harrell, B. F., "An analysis of the attack approach to the aircraft duel." NRL Report R-3479, June 7, 1949. (Secret)

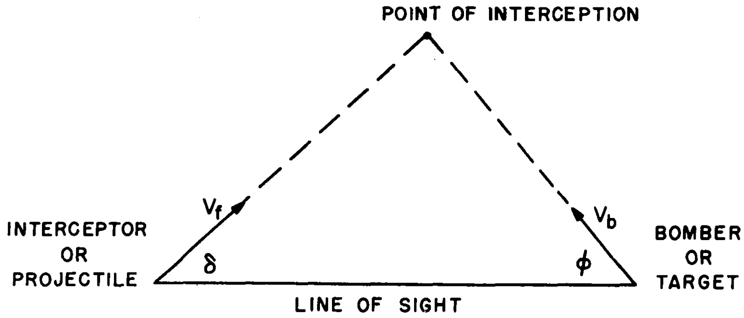

Figure 1 - Navigational triangle

In this triangle, the lead angle, δ, is given by the equation

$$\delta = \sin^{-1}(q \sin \phi) \tag{1}$$

in which

$$q = \frac{V_b}{V_f} \tag{2}$$

The solution δ, equation 1, would have no meaning to the pilot in AAICS except within sight of the target. Also the variables of equation 1 are not directly obtainable from the data-gathering equipment of AAICS. To obtain a useful relationship, the triangle is shown in Figure 2 oriented with respect to axes passing through the radar. It is required of the course computer that it accept coordinate data of the bomber and fighter as delivered by the radar plus T-W-S, process the data, and then deliver a command course, β, which is the fighter course that will result in interception. Using the coordinates and angles defined on Figure 2, the command course is

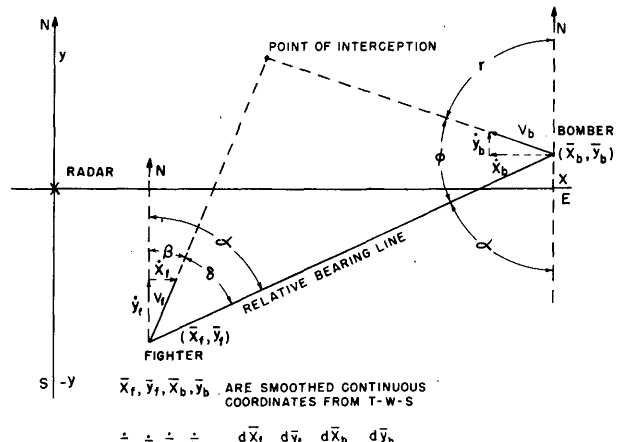

Figure 2 - Relationship of navigational triangle to radar-based axes

$$\beta = \tan^{-1}\left[\frac{\bar{X}_b - \bar{X}_f}{\bar{Y}_b - \bar{Y}_f}\right] - \sin^{-1}(q \sin \phi) \tag{3}$$

in which

$$\phi = 180° - \tan^{-1}\left[\frac{\bar{X}_b - \bar{X}_f}{\bar{Y}_b - \bar{Y}_f}\right] - \tan^{-1}\left[\frac{\dot{\bar{X}}_b}{\dot{\bar{Y}}_b}\right] \tag{4}$$

and

$$q = \left|\frac{\dot{\bar{X}}_b + j\,\dot{\bar{Y}}_b}{\dot{\bar{X}}_f + j\,\dot{\bar{Y}}_f}\right| = \frac{\sqrt{\dot{\bar{X}}_b^2 + \dot{\bar{Y}}_b^2}}{\sqrt{\dot{\bar{X}}_f^2 + \dot{\bar{Y}}_f^2}}\ . \tag{5}$$

These equations are general expressions of the relationship existing between coordinate data and the intercept course, β. Other mathematically equivalent statements of this relationship are, of course, possible, but no attempt was made at this point to choose the form or arrangement most advantageous to the design of a specific AAICS computer. The work attempted was a study of the stability of the AAICS loop, rather than a study of stability of individual units of the loop. It was assumed that, if stability occurred in the simulating computer, consisting of standard computer units arranged to solve these equations, the design of a stable special-purpose computer for AAICS was possible.

Stability

A course computer designed to solve the above equations, or their equivalent expressions, will of necessity contain many amplifier units and servo devices, which in turn will contain active elements and feedback. Such a system can be made to oscillate, if properly excited. An examination of the separate terms of these equations indicates the possibility of considerable variation in them from instant to instant under conditions of input data containing erratic and random errors. The coordinate data $\overline{X}_b$, $\overline{Y}_b$, $\overline{X}_f$, and $\overline{Y}_f$ delivered to the computer by the T-W-S contain position errors (which are herein designated by the letter E with the proper subscripts). The observed coordinates for either bomber or fighter can be expressed by the equations,

$$\overline{Y} = Y + E_y \tag{6}$$

and

$$\overline{X} = X + E_x, \tag{7}$$

in which X and Y are the exact position coordinates and E_y, E_x the position errors. Since the errors vary with time, the derivatives of the indicated position coordinates contain the rate of change of these errors. That is,

$$\dot{\overline{Y}} = \dot{Y} + \dot{E}_y \tag{8}$$

$$\dot{\overline{X}} = \dot{X} + \dot{E}_x \tag{9}$$

The values $\dot{X}$ and $\dot{Y}$ are the exact component velocities, and the values $\dot{E}_x$ and $\dot{E}_y$ are the errors in the indicated component velocities. Substitution of equations 6 through 9 into the various terms of equations 3, 4, and 5 gives an indication of the degree of variation to be expected in these terms with the assumption of reasonable values of E. The first term of the right side of equation 3 may be written as

$$\alpha = \tan^{-1}\left[\frac{(X_b - X_f) + (E_{xb} - E_{xf})}{(Y_b - Y_f) + (E_{yb} - E_{yf})}\right] \tag{10}$$

This term would not be likely to excite instabilities in the course computer except in the immediate vicinity of the interception point. This follows from the fact that everywhere except near the interception point the displacement between bomber and fighter will be very much larger than the position errors contained in the coordinates.

Instabilities resulting from variations occurring in this term are not of prime consideration in AAICS because tally-ho will occur well outside the region of collision. The last term of equation 3 contains the quantities ϕ and q. It is in these quantities that possibility of unstable operation lies. The quantities ϕ and q, expressed by equations 4 and 5, contain coordinate derivatives of the first and second power. These derivatives include the rates of change of position errors, equations 8 and 9. Only the last term of equation 4 needs to be considered. Substituting equations 8 and 9 into it, this term becomes

$$\gamma = \tan^{-1}\left[\frac{-\dot{X}_b - \dot{E}_{xb}}{\dot{Y}_b + \dot{E}_{yb}}\right] \tag{11}$$

The angle γ, which indicates the bomber's heading, is dependent on the rates of change of coordinate errors $\dot{E}_{xb}$ and $\dot{E}_{yb}$ as well as on the component velocities. The magnitudes of $\dot{E}_{xb}$ and $\dot{E}_{yb}$ that will be encountered in taking the derivative of the output of the finished T-W-S unit is not known, but on the basis of radar errors alone, it seems probable that they will be at least of the same order of magnitude as the component bomber velocities $\dot{X}_b$ and $\dot{Y}_b$. Assuming this to be the case, $\dot{E}_{yb}$ could at some instant equal $(-\dot{Y}_b)$, and $(-\dot{X}_b - \dot{E}_{xb})$ could be either positive or negative. The value of γ could then at one instant be $+90^\circ$ and a moment later be -90° under these conditions. Hence ϕ could momentarily vary about its steady value by a corresponding amount.

Similarly, considerable variation can occur in q. A substitution of equations 8 and 9 into equation 5 shows that, under the same assumed conditions as before, q could possibly have momentary values ranging from the vicinity of zero to values many times greater than unity. With a q greater than unity, there exist values of ϕ for which no solution of δ (equation 1) and hence β (equation 3) can be found. If the computer should fail through instability, or through inability to find a solution under conditions of erratic input data, the entire AAICS loop would become useless. It was primarily this consideration that prompted resort to simulation in the analysis of the problem.

Concept of Course Computer for Simulation

A firm concept of the exact requirements and specifications of a computer for AAICS awaits the results of other research programs. Therefore, simulation of the complete AAICS computer of the future, with all probable auxiliary functions included, was impossible at this stage. The primary function of the computer will of necessity be that of generating course direction to be transmitted to the fighter. Only this function was simulated for the two-dimensional problem.

An inspection of the course equations 3, 4, and 5 discloses two possible concepts of the course computer compatible with practical considerations. One is that the course computer be made fully automatic, i.e., solve all three equations, and the other that the computer be simplified by requiring the operator to determine q (equation 5) and insert it manually into the computer. Both possibilities were investigated in the simulation.

The fully automatic computer would have the advantage in practice of maintaining a minimum path course under all tactical situations, regardless of bomber course aspect or evasive maneuvers involving either speed changes or heading changes with respect to wind. It would further eliminate any delays resulting from the manual determination and insertion of q. Its sole disadvantage is the added complexity introduced in the computer and the resulting possible influence on stability of the computer.

With the simplified computer the operator would be dependent on data at hand and obtainable from equipment other than the computer for his determination of q. This might be accomplished by using bomber-speed estimates obtained during the evaluation period, supplemented by the operator's knowledge of enemy bomber types and the air speed command to the fighter. It is entirely possible that experience with past raids might prove a reliable guide to the operator in choosing a value of q. The manual determination of q in any case involves possible errors. Simulation of the simplified computer was accomplished by mechanizing equations 3 and 4 with the exact value of q set in. No errors in q were introduced in this simulation because a comparison of the stability and accuracy of the fully automatic computer with the simplified computer was desired.

Effect of Inaccuracies in q

The effect of inaccuracies in q on the accuracy of course command, β, can be analyzed readily by plotting equation 1, since errors in the lead angle δ appear directly in β. The ratio q was given an assumed value of 0.9, and equation 1 rearranged to

$$\delta = \sin^{-1} \left[(0.9 + \Delta q) \sin \phi\right] \tag{12}$$

The family of curves represented by equation 12 is shown in Figure 3. The angle ϕ, used as the abscissa, is the aspect of the bomber's course. The case, $\phi = 0°$, corresponds to head-on attack, and the case, $\phi = 180°$, corresponds to a tail chase. The solid curve is drawn for the condition of no error in q ($\Delta q = 0$) and is used as a reference. The dashed curves are identified by the assumed error in q with which they are labeled. The ordinate difference, $\Delta\delta$, between the reference curve and any other curve of the family, is the error in lead angle resulting from the error Δq indicated on the curve. It is a maximum at $\phi = 90°$ and reduces to zero at $\phi = 0°$ or 180°.

It can be seen in Figure 3 that, for attacks in which ϕ is 30° or less (approaching the condition of a head-on interception), an error of 0.1 in q results in a 3-degree or less error in the

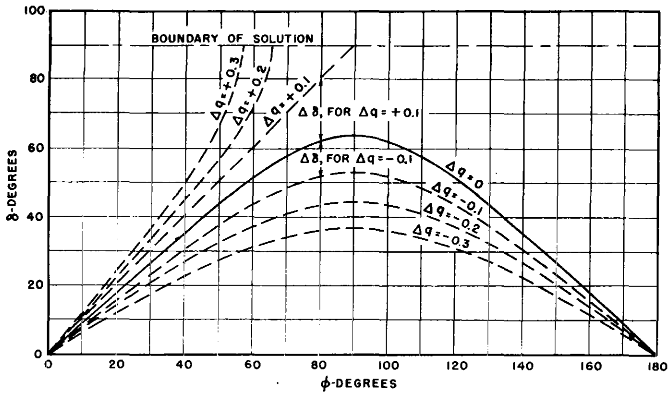

Figure 3 - Fffect of errors in q on lead angle

lead angle δ, and hence in the interceptor course β. It should be stated at this point, in considering the importance of accuracy in q, that an error in δ, and hence in β, does not necessarily result in an error in interception with a closed loop such as AAICS, but only in a longer curved path of the interceptor with a consequent deeper penetration of the bomber. The increased penetration of the bomber is a function of the error in q and the aspect of the bomber's course. This is illustrated for an assumed case in Figure 4.

Extreme accuracy in the determination of q therefore is not necessary for purposes of obtaining an interception. Accurate interceptions can be obtained over a considerable range of error in q with a closed loop system. The case illustrated in Figure 4 is somewhat extreme in that the initial bomber aspect angle of 90° produces the greatest error in fighter course for an error in q, and in that the values assumed for Δq are inordinately large. For a case in which ϕ is 30° or less and where a more reasonable error in q is assumed, the bomber's penetration beyond the minimum path interception point would be materially reduced and would not exceed that permitted by normal navigation errors of the interceptor.

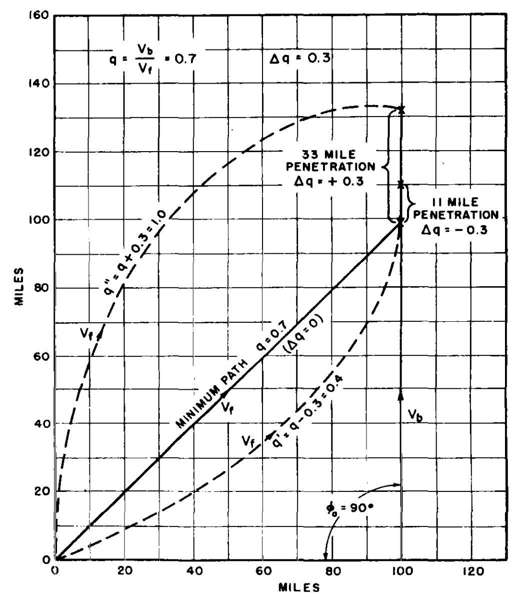

Figure 4 - Bomber penetration resulting from an error in q

Simulation Loop

The AAICS simulation loop covering the two-dimensional phase of navigation is shown in Figure 5. The bomber-course generator consisted of two integrators into which had been set rates and initial conditions. This generated exactly the straight-line bomber courses assumed. For the dog-leg bomber courses assumed, two sets of rates were used, with one set held back by a time delay unit until the point of turn had been reached. The two sets of rates were then connected thru a time-constant circuit so that a smooth transition from the initial rates to the final rates occurred, simulating quite accurately the same motion of a plane in flight.

The T-W-S simulator consisted of four error generators whose purpose was to introduce random errors in the exact position data. It was desired that the character of the random errors closely resemble that anticipated in a T-W-S unit. The arrangement of computer elements employed to accomplish the generation and insertion of errors is shown in Figure 6. The two relays at A, which opened and closed together, were actuated by the voltage cycle shown in Figure 7. The data period was adjusted for each run to correspond to the rate of antenna rotation assumed for that run. Likewise, the duration of closure was adjusted for each run to correspond to the time of radar illumination of the target occurring at that antenna rate with an approximate beamwidth of 2.5°. During the time that the relays (A in Figure 6) were closed, the computer arrangement shown acted as a second-order differential circuit expressed by the equation,

$$E = \frac{p^2}{p^2 + Rp + K} X . \tag{13}$$

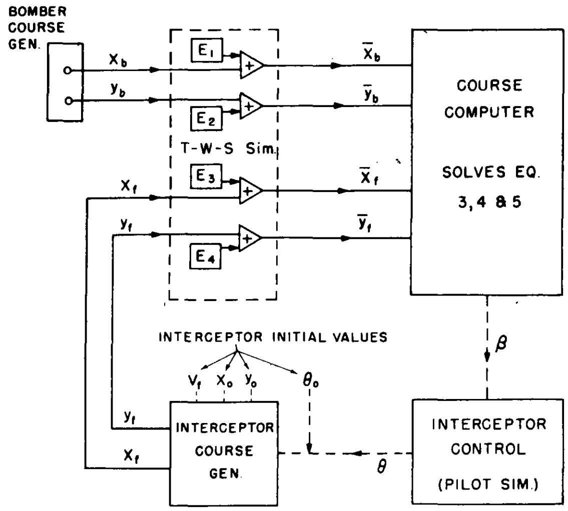

Figure 5 - AAICS simulation loop

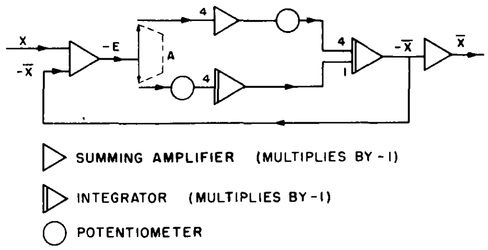

Figure 6 - T-W-S simulator

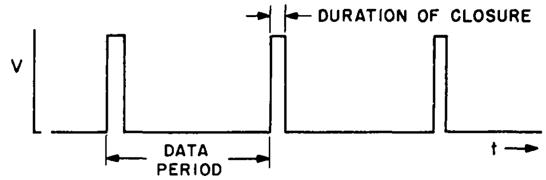

Figure 7 - Cycle of control voltage

The quantities R and K, which represent damping and angular velocity (during the time the relays are closed) were controlled by adjustment of the scaling potentiometers (Figure 6). The values of R and K used during simulation made equation 13 correspond to an overdamped circuit. When the relays closed, E would drop rapidly to zero or to a value close to zero before the relays could open. The first integrator integrated the error E during the time of closure. This integral of E was stored by the first integrator after the relays opened to provide a rate for the second integrator. The character of the output between sweeps ($-\overline{X}$) was then similar to a ramp function whose slope was controlled in sign and magnitude by E during the preceding period of closure.

The outputs from the four error generators were fed to the next block of the simulation loop of Figure 5, the course computer. Simulation of the AAICS course computer was accomplished by setting equations 3, 4, and 5 into standard REAC computer units. This simulation amounted to almost a direct substitution for an AAICS course computer of the electrical analogue type. As similar components are involved, similar accuracies and response characteristics might be expected.

The next block of Figure 5, marked "Interceptor Control," simulates the action of the pilot and plane in executing the course command, β. The output of this block, θ, is the actual course of the interceptor, which may or may not coincide exactly with β depending on limitations of both pilot and plane. The special mechanization necessary for interceptor control was based on the following assumptions, derived from conversations with fighter pilots:

a. The pilot will fly a prescribed course within ±3 degrees.

b. There will be a three-second average delay before a new course β is recognized by the pilot. The many demands on a fighter pilot's eyes preclude instant response to any one indicator. The pilot's reaction time and the plane's response time, however, both being much less than one second, were omitted.

c. Course corrections during the navigational phase will involve a turning rate of approximately 1.5 degrees per second.

The output of the interceptor control is fed to the last block in the loop, marked "Interceptor Course Generator," thus controlling the flight of the interceptor. The output of the generator is the exact position of the interceptor, i.e., coordinates X_f and Y_f, which are fed to the T-W-S simulator to obtain the indicated coordinates, $\overline{X}_f$ and $\overline{Y}_f$, thus completing the loop.

RESULTS OF SIMULATION TESTS CONDUCTED

The work of simulation was done at the Cyclone Laboratory at Reeves Instrument Corp., New York City.* The course equations 3, 4, and 5, plus bomber and fighter course parameters and the initial conditions pertaining to each interception, were presented to the Reeves Engineers for solution on the Cyclone simulator for seven representative cases so arranged as to facilitate the desired study. The seven cases, with the bomber and fighter course parameters, are presented in detail in the Appendix. The tactical situations assumed are shown in Figure 8. Cases I to V inclusive, which were repeats of the same bomber course, were used to obtain comparisons of changes in the course computer and to obtain the effect of insertion of T-W-S at 5-, 10-, and 15-second data rates. Cases VI and VII were bomber-course variations.

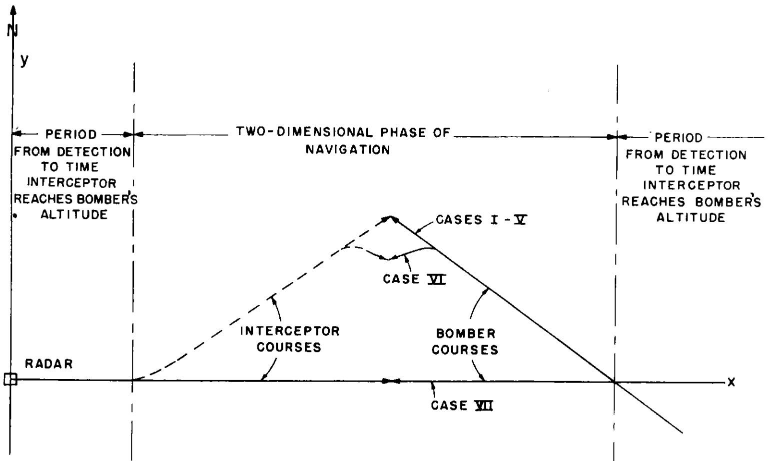

Figure 8 - Assumed bomber courses

It was further specified that, on completion of the seven cases outlined in the Appendix, they be re-run with an additional noise inserted in the position coordinates to simulate errors originating in equipment other than T-W-S. A total of seventeen runs were made to complete the seven cases. Twelve quantities were recorded for each run (Figures 9a and 9b). They were: fighter coordinates, $\overline{X}_f$ and $\overline{Y}_f$; bomber coordinates, $\overline{X}_b$ and $\overline{Y}_b$; fighter course, θ; course command, β; coordinate displacements, $(\overline{X}_b-\overline{X}_f)$ and $(\overline{Y}_b-\overline{Y}_f)$; and the errors in indicated position, E_{xb}, E_{yb}, E_{xf}, and E_{yf}. All but two of the runs resulted in successful interception. These two were successfully re-run after modification of the smoothing constants of the T-W-S.

The appraisal of loop stability was obtained by observing operation of the equipment and by studying the performance as indicated on the recordings. The course computer had minor transients and hunting of servos during most of the runs. These occurred in the parts of the computer associated with coordinate derivatives. The data input to the computer contained positional errors, E_{xf}, etc. (see Figure 10 for a sample). Consequently, high momentary values of coordinate derivatives could and did occur.

Smoothing of component velocities to eliminate these momentary peaks would have reduced or eliminated the minor transients occurring in the course computer. The output of the course computer, β, contained frequent but minor damped oscillations which were of greatest magnitude during the time the bomber was maneuvering. Variations of β and the execution of the mechanized pilot, θ, during a bomber maneuver are shown in Figure 11. The action of the mechanized pilot simulated quite well the integrating action of a pilot's eye in reading an indicator since only steady-state values of β were acted upon. Hence very good loop results were obtained even though some hunting or minor transients occurred during most of the runs.

The schedule of runs is given in Table I. Two runs, No. 11 and No. 14, were unstable. They were re-run satisfactorily after modification of the smoothing constant, K, and are designated as No. 12 and No. 15.

The errors shown in the last column of Table I are the Y displacements of the fighter from the bomber at the moment their X coordinates coincide. They are an indication of the over-all

*Made available thru courtesy of ONR and the Special Devices Center.

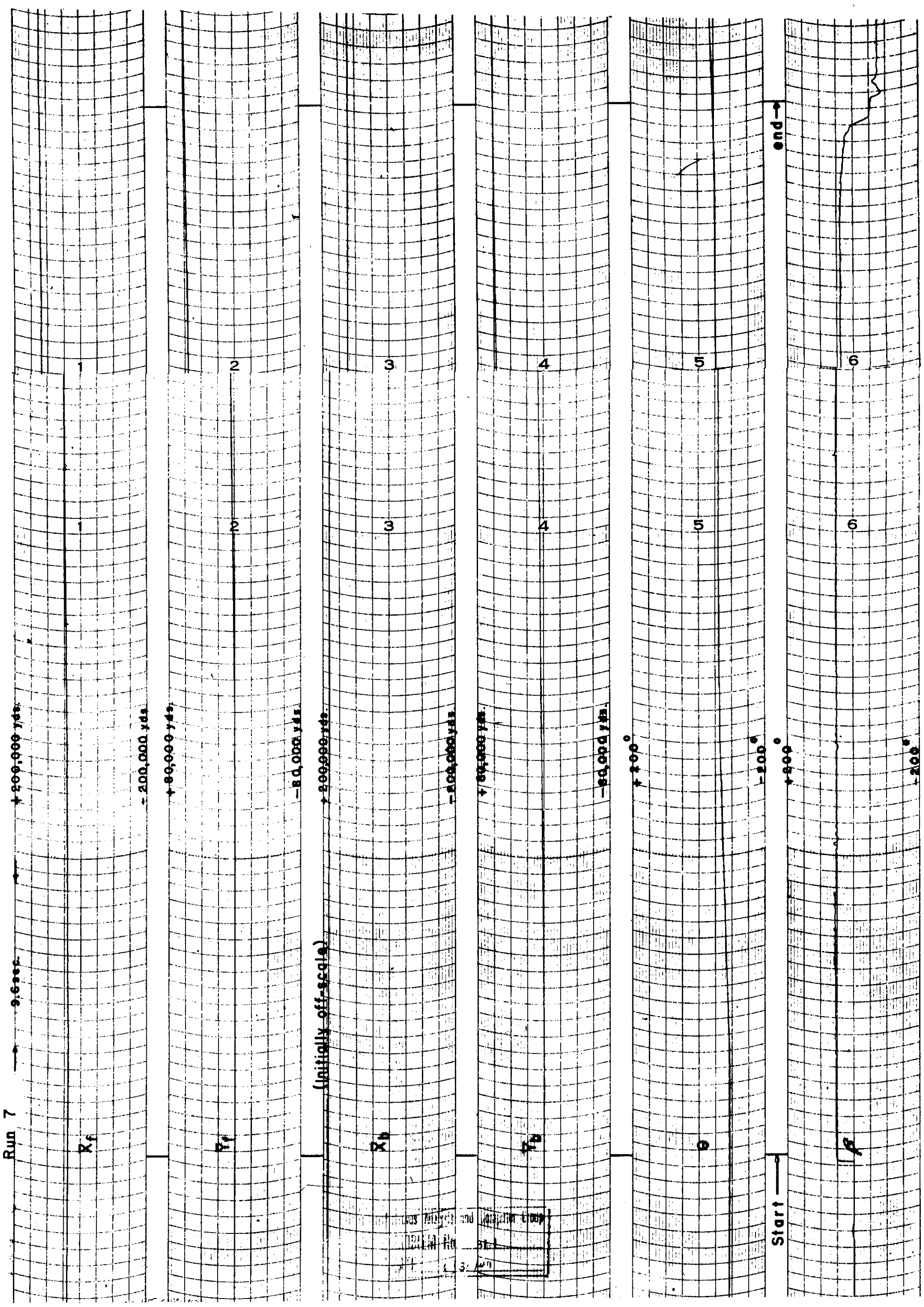

Figure 9a - Sample test data

Figure 9b - Sample test data

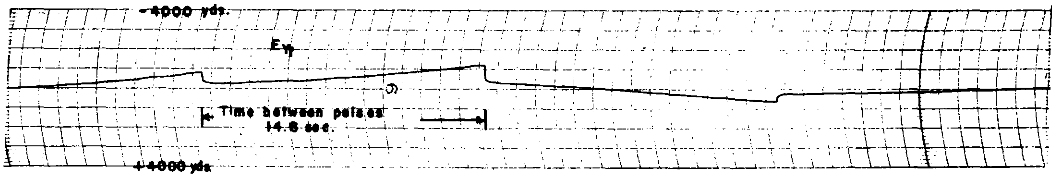

Figure 10 - Sample of error introduced in coordinate data to simulate T-W-S

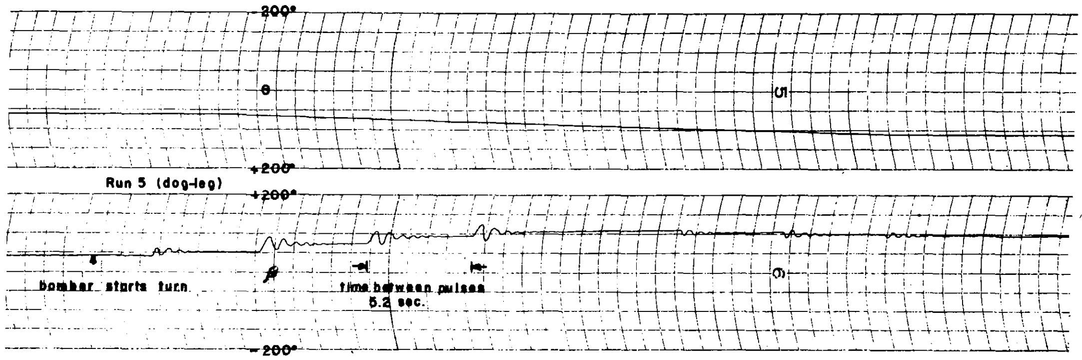

Figure 11 - Variation of β and θ during bomber manuever

performance of the system and involve errors related to the operation of T-W-S, to pilot errors assumed, and to errors of the computer. They are realistic of actual interception conditions to the extent that the assumption of pilot performance and T-W-S performance are realistic. The course computer, as set up in the Cyclone equipment, appeared to contribute very little to the errors shown in Table I. The pilot performance assumed is believed to be realistic in that it represents the average performance anticipated and is well within the realm of pilot attainment. It is felt that the simulation of T-W-S used in this problem was realistic in light of the similarity of the error E in Figure 10 to errors obtained from actual equipment under development. The maximum error amplitude during simulation may be in variance with actual values that will be obtained from the completed T-W-S, but the character of the curves is similar. The steepness of slope during closure, which is critical to parts of the computer associated with coordinate derivatives, was made as adverse in simulation as is expected in practice by making the duration of closure equal to the time of radar target illumination.

To simulate the possible existence of data errors originating in equipment extraneous to T-W-S, noise (Figure 12) was introduced in the coordinates during some of the runs shown in Table I. Each case in which noise was added was a re-run of a previous run. Only in No. 14 did the addition of this noise result in failure. The T-W-S, operating at a 15-second data period with the choice of smoothing constants for No. 14, was unstable with the presence of this noise in its input. Failure occurred shortly after the start of the run. It was satisfactorily re-run (No. 15) after changing the smoothing constant K.

A thorough study of the recordings of each run was made, including factors contributing to failure. The results are used as a basis for the conclusions.

CONCLUSIONS

Stability of Loop

The AAICS loop proved stable whenever individual components were stable. The two failures observed were related principally to the operation of T-W-S and to noise intentionally added to

TABLE I - TEST RESULTS

Run	Case	Input Data	Smoothing Constants* R	Smoothing Constants* K	Data Period (Sec)	Duration of Closure (Sec)	Computer	Results	Error
1	I	Exact position	--	--	--	--	Simplified (Eq. 2 & 3)	Stable	100 yds.
			(T-W-S inoperative)				q = 0.903 set-in		
2	II	T-W-S	12.8	3.2	5.20	0.04	Simplified (Eq. 2 & 3) q = 0.903 set-in	Stable	400 yds.
3	III	T-W-S	12.8	3.2	5.20	0.04	Complete (Eq. 2, 3 & 4)	Stable	200 yds.
4	III	T-W-S noise added†	12.8	3.2	5.20	0.04	Complete (Eq. 2, 3 & 4)	Stable	300 yds.
5	VI (dog-leg)	T-W-S	12.8	3.2	5.20	0.04	Complete (Eq. 2, 3 & 4)	Stable	500 yds.
6	VI (dog-leg)	T-W-S noise added†	12.8	3.2	5.20	0.04	Complete (Eq. 2, 3 & 4)	Stable	400 yds.
7	IV	T-W-S	12.8	3.2	10.05	0.07	Complete (Eq. 2, 3 & 4)	Stable	300 yds.
8	dog-leg	T-W-S	12.8	3.2	10.05	0.07	Complete (Eq. 2, 3 & 4)	Stable	400 yds.
9	IV	T-W-S noise added†	12.8	3.2	10.05	0.07	Complete (Eq. 2, 3 & 4)	Stable	0
10	dog-leg	T-W-S noise added†	12.8	3.2	10.05	0.07	Complete (Eq. 2, 3 & 4)	Stable	400 yds.
11	V	T-W-S	12.8	3.2	14.80	0.10	Complete (Eq. 2, 3 & 4)	Unstable due to poor smoothing constants. Incomplete run.	
12	V	T-W-S	12.8	2.4	14.80	0.10	Complete (Eq. 2, 3 & 4)	Stable	300 yds.
13	dog-leg	T-W-S	12.8	2.4	14.80	0.10	Complete (Eq. 2, 3 & 4)	Stable	200 yds.
14	dog-leg	T-W-S noise added†	12.8	2.4	14.80	0.10	Complete (Eq. 2, 3 & 4)	Unstable. Incomplete run.	
15	dog-leg	T-W-S noise added†	12.8	2.0	14.80	0.10	Complete (Eq. 2, 3 & 4)	Stable	1000 yds.
16	VII	T-W-S	12.8	3.2	5.10	0.04	Complete (Eq. 2, 3 & 4)	Stable	1000 yds.
17	VII	T-W-S noise added†	12.8	3.2	5.10	0.04	Complete (Eq. 2, 3 & 4)	Stable	700 yds.

* *R and K refer to constants in equation 13.*

† *For character of noise added to bomber and fighter coordinates, see Figure 12.*

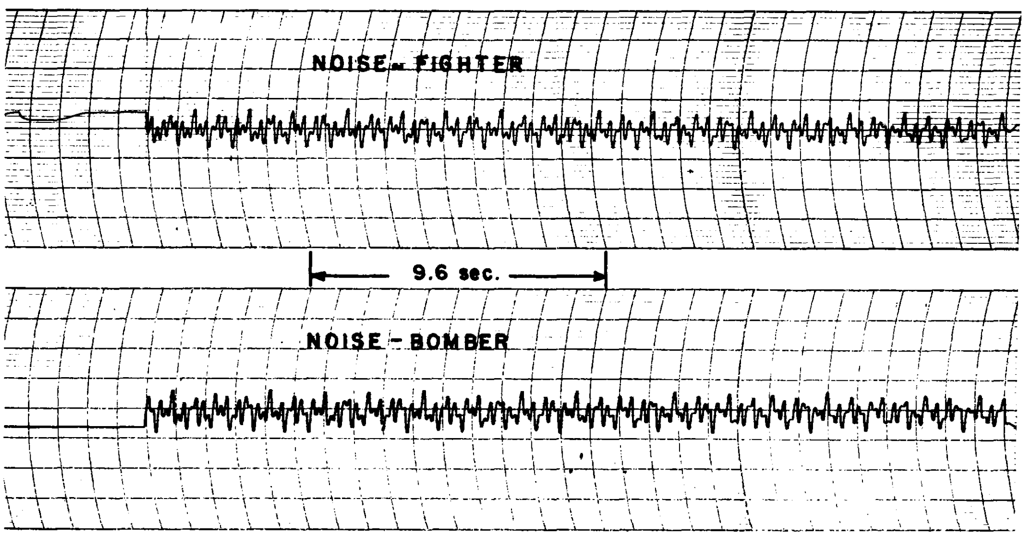

Figure 12 - Character of noise added to coordinates to simulate errors from radar and equipment other than T-W-S

the circuit. The computer and loop operated correctly at all data periods employed (i.e., 5, 10, and 15 seconds) provided that the tracking was stable. All harmful instabilities encountered originated in the T-W-S unit.

Stability of T-W-S

A Track-While-Scan system can be used for stable tracking provided suitable smoothing is used. The degree of smoothing required varies with the data period. The data period can be made very long, in the order of 15 to 20 seconds, and stable tracking obtained when the target flies a straight-line course. A maneuvering target can be tracked if the data period is not too long. It was found that a period of 10 seconds or less was satisfactory for the turns executed. For longer periods or sharper turns, the errors built up to large values which induced instability in the T-W-S. The stability of T-W-S is dependent upon the data rate, the duration of closure, and the constants of its smoothing circuit. Because of the critical factors involved, the design of practical T-W-S equipment may prove difficult.

Stability of Course Computer

The course computer operated with satisfactory stability on all data inputs provided by T-W-S up to the point where T-W-S failed to track and provide further data. Since the conditions of tests were made to reproduce actual interception conditions as closely as possible in the light of present knowledge, the design of a stable course computer is considered practicable.

Fully Automatic vs Simplified Computer

The automatic computation of q was easily accomplished with the Cyclone equipment. The influence of this operation on loop stability was found to be negligible. Considerable additional equipment was necessary for this operation, which might also be the case in the design of a special AAICS course computer. Therefore, the advantages of automatic computation of q should be weighed against the increased complexity of equipment before this feature is included in AAICS specifications.

Accuracy of Placement

The accuracy of interceptions was found to be unusually good, considering that average pilot performance for course navigation (i.e., a delay of 3 seconds, an error of ±3 degrees, and a moderate turning rate of 1.5° per second) was inserted in the loop right up to the time of interception. Placement errors ranged from one direct hit to a maximum of 1000 yds, with a mean error of 400 yds.

Evasive Maneuver

A simple dog-leg maneuver occurring early enough in the bomber's course was not effective in materially altering interception accuracies. However, a bomber turn occurring in the region of tally-ho could make interception impossible. A comprehensive determination of the point of breakdown in this maneuver could not be made at this stage. Any further simulation studies of evasive maneuvers should include the close-in phase; should be based on full fighter maneuverability; and should accurately simulate the actual data period, smoothing constants, and characteristics of the proposed Track-While-Scan equipment.

PROGRAM FOR THE NEXT PERIOD

The simulation herein reported assumed a simple two-dimensional problem in order to establish the soundness and validity of certain basic factors. Additional simulation tests are planned for the near future. More complex problems of AAICS will then be investigated for the purpose of establishing the merits of certain preliminary proposals to mechanize the tasks of tracking, smoothing, and computing, and of generating optimum intercept courses. A more thorough investigation of the operation of T-W-S while tracking a maneuvering target is desirable. It is proposed to investigate aided tracking as a possible interim substitute for T-W-S and to simulate interceptions on the basis of aided tracking.

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APPENDIX

SPECIFICATIONS OF TEST RUNS

Two-Dimensional AAICS Simulation

The following specifications describe the seven cases simulated:

Case I Simplified course computer without Track-While-Scan (exact position coordinates used). Equations 3 and 4 solved by course computer with the value, q=0.903, set into the computer.

Case II Simplified course computer same as Case I with Track-While-Scan operating at 5-second data rate and 0.04-second duration of closure.

Case III Complete computer (equations 3, 4, and 5) with Track-While-Scan at 5-second data rate and 0.04-second duration of closure.

Case IV Same as Case III, but at 10-second data rate and 0.07-second duration of closure.

Case V Same as Case III, but at 15-second data rate and 0.1-second duration of closure.

Case VI Dog-leg bomber course, otherwise same as Case III.

Case VII Head-on bomber course, otherwise same as Case III.

Interceptor Data	Cases Applicable
V_f = 310 yds/sec (550 knots)	All
X_0 = 40,000 yds	All
$Y_0 = 0$	All
$\theta_0 = 90°$	I, II, III, IV, V, VI
$\theta_0 = 0°$	VII
$Y_f = \int_0^t 310 \cos\theta \, dt$ (yds)	All
$X_f = 40{,}000 + \int_0^t 310 \sin\theta \, dt$ (yds)	All

Interceptor Control

Movement of θ was confined to the following restrictions in all seven cases:

1. When $\beta - \theta < \pm 3^\circ$, θ remained constant.
2. When $\beta - \theta \geq \pm 3^\circ$, θ remained constant for 3 seconds and then approached β at a rate:

$$\omega = \frac{d\theta}{dt} = 1.5^\circ/\text{sec}.$$

At the start and end of a θ change,

$$\left|\frac{d\omega}{dt}\right|_{ave} = \left|\frac{d^2\theta}{dt^2}\right|_{ave} \geq 4.5^\circ/\text{sec}^2.$$

Bomber Data	Cases Applicable
V_b = 280 yds/sec (500 knots)	All
$Y_b = 168t$ X_b = 200,000-224t (yds) t in sec.	I, II, III, IV, V
$Y_b = O$ Y_b = 200,000-280t	VII
Course of Figure 13	VI

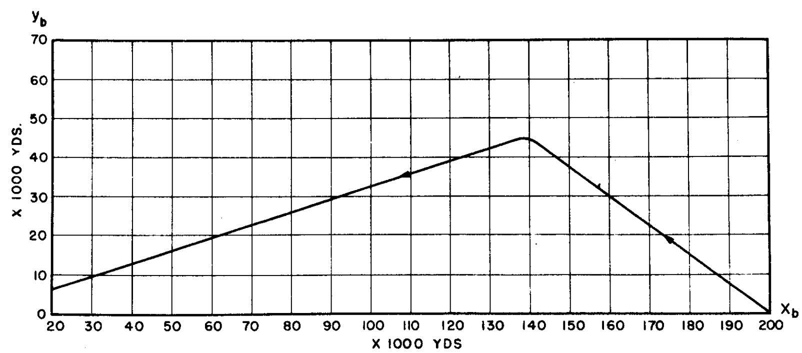

Figure 13 - Bomber course, case VI

NOMENCLATURE

θ heading of the interceptor

θ_0 initial heading of the interceptor

β directed course for the interceptor (computer output)

α true bearing of the bomber with respect to the interceptor.

δ relative bearing of the bomber with respect to the interceptor when on collision course (lead angle)

ϕ angle between the line of sight, bomber to interceptor, and the bomber's heading.

V_f interceptor speed.

V_b bomber speed.

q ratio of bomber speed to interceptor speed.

X_b, Y_b coordinates of the bomber.

X_f, Y_f coordinates of the interceptor.

X_0, Y_0 initial coordinates of the interceptor.

$\overline{X}_b$, $\overline{Y}_b$, $\overline{X}_f$, $\overline{Y}_f$ corresponding coordinates as observed by the track-while-scan unit.

* * *